Marginal and average cost

Total cost is the sum of fixed cost and variable cost. Dividing each term by total output, we derive a similar relationship between averages: Average total cost equals average fixed cost plus average variable cost. In symbols, we could write $ATC(Q) = TC(Q)/Q = FC(Q)/Q + VC(Q)/Q$. (Although fixed cost is written as a function of output, recall that its level is independent of output, while total cost and variable cost vary positively with output.)

To determine the relationship of marginal cost to AVC and ATC, we begin by finding the slope of the average total cost function. Using the quotient rule, $\frac{\mathrm{d}ATC(Q)}{\mathrm{d}Q} = \frac{Q\frac{\mathrm{d}FC(Q)}{\mathrm{d}Q} - FC(Q)}{Q^2} + \frac{Q\frac{\mathrm{d}VC(Q)}{\mathrm{d}Q} - VC(Q)}{Q^2}$. We can simplify this by noting that $\mathrm{d}FC(Q)/\mathrm{d}Q = 0$ and dividing both numerator and denominator by Q to obtain $\frac{\mathrm{d}ATC(Q)}{\mathrm{d}Q} = \frac{\frac{\mathrm{d}VC(Q)}{\mathrm{d}Q} - \left(\frac{FC(Q)+VC(Q)}{Q}\right)}{Q}$. Consider the first term in the numerator, $\frac{\mathrm{d}VC(Q)}{\mathrm{d}Q}$. This is simply marginal cost. The term in parenthesis is recognizable as the sum of average fixed cost and average variable cost, which equals average total cost. Hence, the slope of the average total cost function can be written as $\frac{\mathrm{d}ATC(Q)}{\mathrm{d}Q} = \frac{MC - ATC}{Q}$. Consequently, we see that average total cost is decreasing if MC is less than ATC, ATC is increasing if MC exceeds ATC, and ATC achieves its minimum if $MC = ATC$. (The first-order condition for a minimum of ATC is that the slope of ATC equals zero.)

Likewise, the slope of AVC is easily found to be $\frac{\mathrm{d}AVC(Q)}{\mathrm{d}Q} = \frac{MC - AVC}{Q}$, leading to similar conclusions regarding the relationship between marginal cost and average variable cost. The latter reaches its minimum at the output at which marginal cost equals average variable cost; average variable cost will be falling (rising) if marginal cost is less than (greater than) average variable cost.