

Elastic demand at the monopoly price

Before investigating the relationship between marginal revenue and elasticity of demand, we will need to digress a moment and recall the elasticity coefficient, E_d . By definition, E_d is the (absolute value of the) percentage change in quantity demanded divided by the percentage change in price: $E_d = \left| \frac{dQ/Q}{dP/P} \right|$.

A simple rearrangement of this formula shows that $E_d = \left| \frac{dQ}{dP} \cdot \frac{P}{Q} \right|$.

With this in mind, suppose a monopolist's demand curve is given by $P = f(Q)$, revenue is $R(Q) = QP = Qf(Q)$ and marginal revenue is $MR = R'(Q) = f(Q) + Qf'(Q)$. Suppose we now both divide and multiply the right-hand-side of MR by $P = f(Q)$: $MR = P \left(\frac{f(Q)}{P} + \frac{Q}{P} f'(Q) \right) = P \left(1 + \frac{Q}{P} f'(Q) \right)$. At this point we make use of the facts that demand is monotonically decreasing and $f'(Q) = dP/dQ$ to assert that $\frac{dP}{dQ} = 1/\frac{dQ}{dP}$ so that $E_d = \left| \frac{dQ}{dP} \cdot \frac{P}{Q} \right| = -1/\left(\frac{Q}{P} f'(Q) \right)$. Inverting both sides, we see that $\frac{Q}{P} f'(Q) = -1/E_d$.

Substituting this into our equation for MR we obtain the following: $MR = P(1 - 1/E_d)$.

We now have an expression that relates MR to the elasticity of demand: $MR = P(1 - 1/E_d)$. If demand is inelastic, $E_d < 1$ and $MR < 0$. Alternatively, $MR > 0$ if and only if $E_d > 1$. As profit maximization requires that $MR = MC$ and MC is always positive, we see that a monopolist must always price in the elastic portion of the demand curve.