PART 03

VALUATION OF FUTURE CASH FLOWS

CHAPTER 4

INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY

AFTER STUDYING THIS CHAPTER, YOU SHOULD BE ABLE TO:

- **LO 1** **Determine the future value of an investment made today.**
- **LO 2** **Determine the present value of cash to be received at a future date.**
- **LO 3** **Calculate the return on an investment**
- **LO 4** **Predict how long it takes for an investment to reach a desired value.**

In January 2009, the National Bank for Agriculture and Rural Development (NABARD) in India announced the issue of Bhavishya Nirman Bonds (BNB), which are essentially ten-year zero-coupon bonds.[1] By March 2009, NABARD raised just over Rs 27.6 billion (about US$600 million). NABARD uses the resources generated under the scheme for the development of agriculture and rural sectors in India. The bonds will be issued at a discount and repaid at their face value upon maturity. The face value per bond will be Rs 20,000.

The deal was to produce an effective yield of 8.62%. In simple terms, if you made a nominal investment of Rs 8750, the NABARD promised to repay you Rs 20 000 at the end of ten years, with investors receiving no payment in the intervening period. Investors gave up Rs 8750 in January 2009 in return for a promise of a Rs 20 000 payment ten years later. Such a security, for which you pay some amount today in exchange for a promised lump sum to be received at a specified future date, is about the simplest possible type of investment.

Is giving up Rs 8750 today in exchange for Rs 20 000 in ten years a good deal? On the plus side, you get back 2.29 times every rupee you put up. That probably sounds good, but, on the downside, you have to wait ten years to get it. What you need to know is how to analyse this trade-off, and this chapter gives you the tools you need.

Specifically our goal here is to introduce you to one of the most important principles in finance, the time value of money. What you will learn is how to determine the value today of some cash flow to be received later. This is a very basic business skill, and it underlies the analysis of many different types of investments and financing arrangements. In fact, almost all business activities, whether they originate in marketing, management, operations or strategy, involve comparing outlays made today with benefits projected for the future. Everyone needs to understand how to make this comparison; this chapter gets you started.

1 A zero-coupon bond pays no interest; rather, the interest component is bundled up and included in the final repayment upon maturity.

One of the basic problems faced by the financial manager is how to determine the value today of cash flows expected in the future. For example, in 2009 the New South Wales 'Win for Life' $5.00 instant lottery ticket (scratchie) had a $1 125 000 first prize. Does this mean the winning ticket was worth $1 125 000? The answer is no, because the jackpot was actually going to be paid out over a fifteen-year period at a rate of $75 000 per year. How much was the ticket worth then? The answer depends on the time value of money, the subject of this chapter.

In the most general sense, the phrase *time value of money* refers to the fact that a dollar in the hand today is worth more than a dollar promised at some time in the future. On a practical level, one reason for this is that you could earn interest while you waited; so, a dollar today would grow to more than a dollar later. The trade-off between money now and money later thus depends on, among other things, the rate you can earn by investing. Our goal in this chapter is to explicitly evaluate this trade-off between dollars today and dollars at some future time.

A thorough understanding of the material in this chapter is critical to understanding material in subsequent chapters, so you should study it with particular care. We will present a number of examples in this chapter. In many problems, your answer may differ slightly from ours. This can happen because of rounding and it is not a cause for concern.

FUTURE VALUE AND COMPOUNDING

4.1

The first thing we will study is future value. **Future value (FV)** refers to the amount of money an investment will grow to over some period of time at some given interest rate. Put another way, future value is the cash value of an investment at some time in the future. We start out by considering the simplest case, a single-period investment.

future value (FV)
The amount an investment is worth after one or more periods.

INVESTING FOR A SINGLE PERIOD

Suppose you were to invest $100 in a savings account that pays 10% interest per year. How much would you have in one year? You would have $110. This $110 is equal to your original *principal* of $100 plus the $10 in interest that you earn. We say that $110 is the future value of $100 invested for one year at 10%, and we simply mean that $100 today is worth $110 in one year, given that 10% is the interest rate.

In general, if you invest for one period at an interest rate of r, your investment will grow to $(1 + r)$ per dollar invested. In our example, r is 10%, so your investment grows to $1 + 0.10 = 1.10$ dollars per dollar invested. You invested $100 in this case, so you ended up with $\$100 \times 1.10 = \$110$.

compounding
The process of accumulating interest in an investment over time to earn more interest.

INVESTING FOR MORE THAN ONE PERIOD

Going back to our $100 investment, what will you have after two years, assuming the interest rate does not change? If you leave the entire $110 in the bank, you will earn $\$110 \times 0.10 = \$11$ in interest during the second year, so you will have a total of $\$110 + 11 = \$121$. This $121 is the future value of $100 in two years at 10%. Another way of looking at it is that, one year from now, you are effectively investing $110 at 10% for a year. This is a single-period problem, so you will end up with $1.10 for every dollar invested, or $\$110 \times 1.1 = \$121$ total.

This $121 has four parts. The first part is the $100 original principal. The second part is the $10 in interest you earn in the first year, and the third part is another $10 you earn in the second year, for a total of $120. The last $1 you end up with (the fourth part) is interest you earn in the second year on the interest paid in the first year: $\$10 \times 0.10 = \$1$.

This process of leaving your money and any accumulated interest in an investment for more than one period, thereby reinvesting the interest, is called **compounding**. Compounding the interest means earning **interest on interest**, so we call the result **compound interest**. With **simple interest**, the interest is not reinvested, so interest is earned each period only on the original principal.

interest on interest
Interest earned on the reinvestment of previous interest payments.

compound interest
Interest earned on both the initial principal and the interest reinvested from prior periods.

simple interest
Interest earned only on the original principal amount invested.

EXAMPLE 4.1

INTEREST ON INTEREST

Suppose you find a two-year investment that pays 14% per year. If you invest $325, how much will you have at the end of the two years? How much of this is simple interest? How much is compound interest?

At the end of the first year, you will have $325 × (1 + 0.14) = $370.50. If you reinvest this entire amount, and thereby compound the interest, you will have $370.50 × 1.14 = $422.37 at the end of the second year. The total interest you earn is thus $422.37 – 325 = $97.37. Your $325 original principal earns $325 × 0.14 = $45.50 in interest each year, for a two-year total of $91 in simple interest. The remaining $97.37 – 91 = $6.37 results from compounding. You can check this by noting that the interest earned in the first year is $45.50. The interest on interest earned in the second year thus amounts to $45.50 × 0.14 = $6.37, as we calculated.

We now take a closer look at how we calculated the $121 future value. We multiplied $110 by 1.1 to get $121. The $110, however, was $100 also multiplied by 1.1. In other words:

$$\begin{aligned}\$121 &= \$110 \times 1.1 \\ &= (\$100 \times 1.1) \times 1.1 \\ &= \$100 \times (1.1 \times 1.1) \\ &= \$100 \times (1.1)^2 \\ &= \$100 \times 1.21\end{aligned}$$

At the risk of labouring the obvious, let us ask: how much would our $100 grow to after three years? In two years, we will be investing $121 for one period at 10%. We will end up with $1.10 for every dollar we invest, or $121 × 1.10 = $133.10 in total. This $133.10 is thus:

$$\begin{aligned}\$133.10 &= \$121 \times 1.1 \\ &= (\$110 \times 1.1) \times 1.1 \\ &= (\$100 \times 1.1) \times 1.1 \times 1.1 \\ &= \$100 \times (1.1 \times 1.1 \times 1.1) \\ &= \$100 \times (1.1)^3 \\ &= \$100 \times 1.331\end{aligned}$$

You are probably noticing a pattern to these calculations, so we can now go ahead and state the general result. As our examples suggest, the future value of $1 invested for t periods at a rate of r per period is:

$$\text{Future value} = \$1 \times (1 + r)^t \quad [4.1]$$

The expression $(1 + r)^t$ is sometimes called the *future value interest factor* (or just *future value factor*) for $1 invested at r% for t periods and can be abbreviated as FVIF(r, t).

In our example, what would your $100 be worth after five years? We can first compute the relevant future value factor as:

$$(1 + r)^t = (1 + 0.10)^5 = 1.1^5 = 1.6105$$

Your $100 will thus grow to:

$$\$100 \times 1.6105 = \$161.05$$

The growth of your $100 each year is illustrated in Table 4.1. As shown, the interest earned in each year is equal to the beginning amount multiplied by the interest rate of 10%.

Year	Beginning Amount	Interest Earned	Ending Amount
1	$100.00	$10.00	$110.00
2	110.00	11.00	121.00
3	121.00	12.10	133.10
4	133.10	13.31	146.41
5	146.41	14.64	161.05
		Total interest $61.05	

Table 4.1

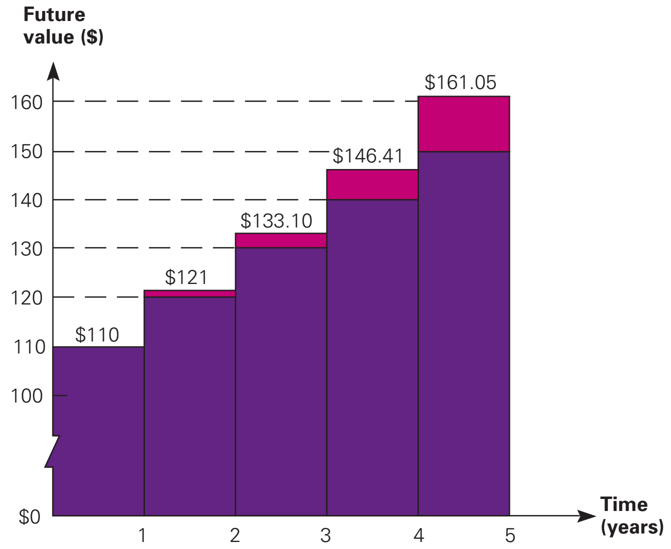
Future value of $100 at 10%

In Table 4.1, notice that the total interest you earn is $61.05. Over the five-year span of this investment, the simple interest is $\$100 \times 0.10 = \$10$ per year, so you accumulate $50 this way. The other $11.05 is from compounding.

Figure 4.1 illustrates the growth of the compound interest in Table 4.1. Notice how the simple interest is constant each year, but the compound interest you earn gets bigger every year. The size of the compound interest keeps increasing because more and more interest builds up and there is thus more to compound.

Future values depend critically on the assumed interest rate, particularly for long-lived investments. Figure 4.2 illustrates this relationship by plotting the growth of $1 for different rates and lengths of time. Notice that the future value of $1 after 10 years is about $6.20 at a 20% rate, but it is only about $2.60 at 10%. In this case, doubling the interest rate more than doubles the future value.

To solve future value problems, we need to come up with the relevant future value factors. There are several different ways of doing this. In our example, we could have multiplied 1.1 by itself five times. This would work just fine, but it would become very tedious for, say, a thirty-year investment.



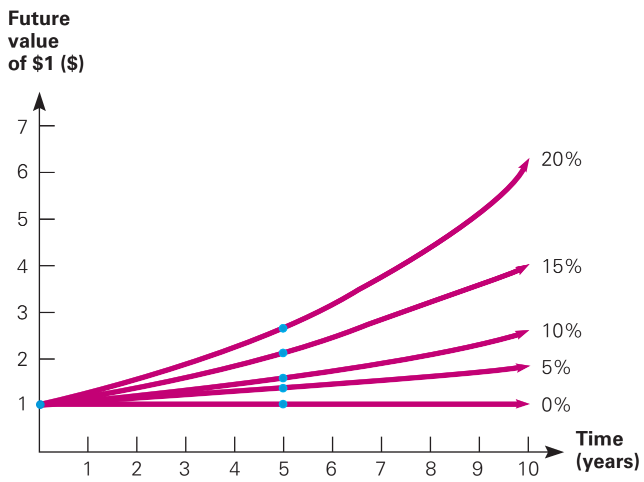
Growth of $100 original amount at 10% per year. Red shaded area represents the portion of the total that results from compounding of interest.

Figure 4.1

Future value, simple interest and compound interest

Figure 4.2

Future value of $1 for different periods and rates



Fortunately there are several easier ways to get future value factors. Most calculators have a key labelled 'y^x'. You can usually just enter 1.1, press this key, enter 5, and press the = key to get the answer. This is an easy way to calculate future value factors because it is quick and accurate.

Alternatively, you can use a table that contains future value factors for some common interest rates and time periods. Table 4.2 contains some of these factors. Table A.1 in Appendix A at the end of the book contains a much larger set. To use the table, find the column that corresponds to 10%. Then look down the rows until you come to five periods. You should find the factor that we calculated, 1.6105.

Table 4.2

Future value interest factors

Number of Periods	Interest Rates			
	5%	**10%**	**15%**	**20%**
1	1.0500	1.1000	1.1500	1.2000
2	1.1025	1.2100	1.3225	1.4400
3	1.1576	1.3310	1.5209	1.7280
4	1.2155	1.4641	1.7490	2.0736
5	1.2763	1.6105	2.0114	2.4883

Tables such as Table 4.2 are not as common as they once were because they predate inexpensive calculators and are only available for a relatively small number of rates. Interest rates are often quoted to three or four decimal places, so the tables needed to deal with these accurately would be quite large. As a result, the 'real world' has moved away from using them. We will emphasise the use of a calculator in this chapter.

These tables still serve a useful purpose. To make sure you are doing the calculations correctly, pick a factor from the table and then calculate it yourself to see that you get the same answer. There are plenty of numbers to choose from.

COMPOUND INTEREST

EXAMPLE 4.2

You have found an investment that pays 12%. That rate sounds good to you, so you invest \$400. How much will you have in three years? How much will you have in seven years? At the end of seven years, how much interest have you earned? How much of that interest results from compounding?

Based on our discussion, we can calculate the future value factor for 12% and three years as:

$$(1 + r)^t = (1.12)^3 = 1.4049$$

Your \$400 thus grows to:

$$\$400 \times 1.4049 = \$561.97$$

After seven years, you will have:

$$\$400 \times (1.12)^7 = \$400 \times 2.2107 = \$884.27$$

Thus, you will more than double your money over seven years.

Since you invested \$400, the interest in the \$884.27 future value is \$884.27 – 400 = \$484.27. At 12%, your \$400 investment earns \$400 × 0.12 = \$48 in simple interest every year. Over seven years, the simple interest thus totals 7 × \$48 = \$336. The other \$484.27 – 336 = \$148.27 is due to compounding.

The effect of compounding is not great over short time periods, but it really starts to add up as the horizon grows. To take an extreme case, suppose one of your more frugal ancestors had invested \$5 for you at a 6% interest rate 200 years ago. How much would you have today? The future value factor is a substantial $(1.06)^{200} = 115\,125.90$ (you will not find this one in a table), so you would have \$5 × 115 125.90 = \$575 629.50 today. Notice that the simple interest is just \$5 × 0.06 = \$0.30 per year. After 200 years, this amounts to \$60. The rest is from reinvesting. Such is the power of compound interest!

How much do you need at retirement? Check out the 'Personal Centre/ Your Retirement/ Sobering Retirement Facts' link at <www.cba.com.au>.

HOW MUCH FOR THAT ISLAND?

EXAMPLE 4.3

To further illustrate the effect of compounding for long horizons, consider this famous American case of Peter Minuit and the Indians. In 1626 Minuit bought all of Manhattan Island, New York, for about \$24 in goods and trinkets. This sounds cheap, but the Indians may have got the better end of the deal. To see why, suppose the Indians had sold the goods and invested the \$24 at 10%. How much would it be worth today?

Roughly 380 years have passed since the transaction. At 10%, \$24 will grow by quite a bit over that time. How much? The future value factor is approximately:

$$(1 + r)^t = 1.1^{380} = 5\,000\,000\,000\,000\,000$$

That is, 5 followed by 15 zeroes. The future value is thus in the order of \$24 × 5 quadrillion, or about \$120 *quadrillion* (give or take a few hundreds of trillions).

Well \$120 quadrillion is a lot of money. How much? If you had it, you could buy the United States. All of it. Cash. With money left over to buy Canada, Mexico and the rest of the world, for that matter.

This example is something of an exaggeration, of course. In 1626 it would not have been easy to locate an investment that would pay 10% every year without fail for the next 380 years.

CALCULATOR HINTS

USING A FINANCIAL CALCULATOR

Although you can calculate future values in the various ways we have described so far, many of you will decide that a financial calculator is the way to go. If you are planning on using one, you should read this extended hint; otherwise, skip it.

A financial calculator is simply an ordinary calculator with a few extra features. In particular, it knows some of the most commonly used financial formulas, so it can directly compute things like future values.

Financial calculators have the advantage that they handle a lot of the computation, but that is really all. In other words, you still have to understand the problem; the calculator just does some of the arithmetic. In fact, there is an old joke (somewhat modified) that goes like this: anyone can make a mistake on a time value of money problem, but to really screw one up takes a financial calculator! We therefore have two goals for this section. First, we will discuss how to compute future values. After that, we will show you how to avoid the most common mistakes people make when they start using financial calculators.

HOW TO CALCULATE FUTURE VALUES WITH A FINANCIAL CALCULATOR

Examining a typical financial calculator, you will find five keys of particular interest. They usually look like this:

N **I/Y** **PMT** **PV** **FV**

For now, we need to focus on four of these. The keys labelled **PV** and **FV** are just what you would guess: present value and future value. The key labelled **N** refers to the *n*umber of periods, which is what we have been calling *t*. Finally, **I/Y** stands for the *i*nterest rate, which we have called *r*.[2]

If we have the financial calculator set up correctly (see our next section), then calculating a future value is very simple. Take a look back at our question involving the future value of $100 at 10% for five years. We have seen that the answer is $161.05. The exact keystrokes will differ, depending on what type of calculator you use, but here is basically all you do:

1. Enter –100. Press the **PV** key. (The negative sign is explained below.)
2. Enter 10. Press the **I/Y** key. (Notice that we entered 10, not 0.10; see below.)
3. Enter 5. Press the **N** key.

Now we have entered all of the relevant information. To solve for the future value, we need to ask the calculator what the FV is. Depending on your calculator, you press the button labelled either 'CPT' (for compute) or 'CAL' and then press **FV**, or else you just press **FV**. Either way, you should get 161.05. If you do not (and you probably will not if this is the first time you have used a financial calculator!), we will offer some help in our next section.

Before we explain the kinds of problems that you are likely to run into, we want to establish a standard format for showing you how to use a financial calculator. Using the example we just looked at, in the future, we will illustrate such problems like this:

Enter	5	10		–100	
	N	**I/Y**	**PMT**	**PV**	**FV**
Solve for					161.05

Here is an important tip: the textbook website contains some more detailed instructions for the most common types of financial calculators. See if yours is included, and, if it is, follow the instructions there if you need help. Of course, if all else fails, you can read the manual that came with the calculator.

HOW TO GET THE WRONG ANSWER USING A FINANCIAL CALCULATOR

A couple of common (and frustrating) problems cause a lot of trouble with financial calculators. In this section, we provide some important *dos* and *don'ts*. If you just cannot seem to get a problem to work out, you should come back to this section.

There are two categories we examine: three things you need to do only once and three things you need to do every time you work a problem. The things you need to do just once deal with the following calculator settings:

1. *Make sure your calculator is set to display a large number of decimal places.* Most financial calculators display only two decimal places; this

2 The reason financial calculators use **N** and **I/Y** is that the most common use for these calculators is determining loan payments. In this context, **N** is the number of payments and **I/Y** is the interest rate on the loan. But, as we will see, there are many other uses for financial calculators that do not involve loan payments and interest rates.

causes problems because we frequently work with numbers—like interest rates—that are very small.

2. *Make sure your calculator is set to assume only one payment per period or per year.* Most financial calculators assume monthly payments (12 per year) unless you say otherwise (the payments-per-year function).
3. *Make sure your calculator is in 'end' mode.* This is usually the default, but you can accidentally change to 'begin' mode.

If you do not know how to set these three things, see the textbook website or your calculator's operating manual.

There are also three things you need to do *every time you work a problem*:

1. *Before you start, completely clear out the calculator.* This is very important. Failure to do this is the number one reason for wrong answers; you simply must get into the habit of clearing the calculator every time you start a problem. How you do this depends on the calculator (see the textbook website), but you must do more than just clear the display. For example, on a Texas Instruments BA II Plus you must press **2nd** then **CLR TVM** for *clear time value of money*. There is a similar command on your calculator. Learn it!

   Note that turning the calculator off and back on will not do it. Most financial calculators remember everything you enter, even after you turn them off. In other words, they remember all your mistakes unless you explicitly clear them out. Also, if you are in the middle of a problem and make a mistake *clear it out and start over.* Better to be safe than sorry.
2. *Put a negative sign on cash outflows.* Most financial calculators require you to put a negative sign on cash outflows and a positive sign on cash inflows.3 As a practical matter this usually just means that you should enter the present value amount with a negative sign (because normally the present value represents the amount you give up today in exchange for cash inflows later). You enter a negative value on the BA II Plus by first entering a number and then pressing the **+/–** key. By the same token, when you solve for a present value, you should not be surprised to see a negative sign.
3. *Enter the rate correctly.* Financial calculators assume that rates are quoted in percentages, so if the rate is 0.08 (or 8%), you should enter 8, not 0.08.

If you follow these guidelines (especially the one about clearing out the calculator), you should have no problem using a financial calculator to work almost all of the problems in this and the next few chapters. We will provide some additional examples and guidance where appropriate.

3 The Texas Instruments BA-35 is an exception; it does not require nagative signs to be entered.

CONCEPT QUESTIONS

4.1a What do we mean by the future value of an investment?

4.1b What does it mean to compound interest? How does compound interest differ from simple interest?

4.1c In general, what is the future value of $1 invested at *r* per period for *t* periods?

PRESENT VALUE AND DISCOUNTING 4.2

When we discuss future value, we are thinking of questions such as the following: what will my $2000 investment grow to if it earns a 6.5% return every year for the next six years? The answer to this question is what we call the future value of $2000 invested at 6.5% for six years (verify that the answer is about $2918).

There is another type of question that comes up even more often in financial management that is obviously related to future value. Suppose you need to have $10 000 in ten years, and you can earn 6.5% on your money. How much do you have to invest today to reach your goal? You can verify that the answer is $5327.26. How do we know this? Read on.

THE SINGLE-PERIOD CASE

We have seen that the future value of $1 invested for one year at 10% is $1.10. We now ask a slightly different question: how much do we have to invest today at 10% to get $1 in one year? In other words, we know the future value here is $1, but what is the **present value (PV)**? The answer is not too hard to work out. Whatever we invest today will be 1.1 times bigger at the end of the year. Since we need $1 at the end of the year:

present value (PV) The current value of future cash flows discounted at the appropriate discount rate.

$$\text{Present value} \times 1.1 = \$1$$

Or, solving for the present value:

$$\text{Present value} = \$1/1.1 = \$0.909$$

In this case, the present value is the answer to the following question: what amount, invested today, will grow to $1 in one year if the interest rate is 10%? Present value is thus just the reverse of future value. Instead of compounding the money forward into the future, we **discount** it back to the present.

discount Calculate the present value of some future amount.

From our examples, the present value of $1 to be received in one period is generally given as:

$$PV = \$1 \times [1/(1 + r)] = \$1/(1 + r)$$

EXAMPLE 4.4

SINGLE-PERIOD PV

Suppose you need $400 to buy textbooks next year. You can earn 7% on your money. How much do you have to put up today?

We need to know the PV of $400 in one year at 7%. Proceeding as above:

$$\text{Present value} \times 1.07 = \$400$$

We can now solve for the present value:

$$\text{Present value} = \$400 \times (1/1.07) = \$373.83$$

Thus, $373.83 is the present value. Again, this just means that investing this amount for one year at 7% will result in a future value of $400.

We next examine how to calculate the present value of an amount to be paid in two or more periods into the future.

PRESENT VALUES FOR MULTIPLE PERIODS

Suppose you need to have $1000 in two years. If you can earn 7%, how much do you have to invest today to ensure that you have the $1000 when you need it? In other words, what is the present value of $1000 in two years if the relevant interest rate is 7%?

Based on your knowledge of future values, you know that the amount invested must grow to $1000 over the two years. In other words, it must be the case that:

$$\begin{aligned}\$1000 &= PV \times 1.07 \times 1.07\\ &= PV \times (1.07)^2\\ &= PV \times 1.1449\end{aligned}$$

Given this, we can solve for the present value:

$$\text{Present value} = \$1000/1.1449 = \$873.44$$

Therefore, you must invest $873.44 in order to achieve your goal.

SAVING UP

EXAMPLE 4.5

You would like to buy a new car. You have \$50 000, but the car costs \$68 500. If you can earn 9%, how much do you have to invest today to buy the car in two years? Do you have enough? Assume the price will stay the same.

We need to know the present value of \$68 500 to be paid in two years, assuming a 9% rate. Based on our discussion, this is:

$$PV = \$68\,500/(1.09)^2 = \$68\,500/1.1881 = \$57\,655.08$$

You are still about \$7655 short, even if you are willing to wait two years.

As you have probably recognised by now, calculating present values is quite similar to calculating future values, and the general result looks much the same. The present value of \$1 to be received t periods into the future at a discount rate of r is:

$$PV = \$1 \times [1/(1 + r)] = \$1/(1 + r)^t \quad [4.2]$$

The quantity in brackets, $1/(1 + r)^t$, goes by several different names. Since it is used to discount a future cash flow, it is often called a *discount factor*. With this name, it is not surprising that the rate used in the calculation is often called the **discount rate**. We will generally use this term in talking about present values. The quantity in square brackets is also called the *present value interest factor* (or just *present value factor*) for \$1 at r% for t periods and is sometimes abbreviated as PVIF(r, t). Finally, calculating the present value of a future cash flow to determine its worth today is commonly called **discounted cash flow (DCF) valuation**.

discount rate
The rate used to calculate the present value of future cash flows.

discounted cash flow (DCF) valuation
Valuation calculating the present value of a future cash flow to determine its value today.

To illustrate, suppose you need \$1000 in three years. You can earn 15% on your money. How much do you have to invest today? To find out, we have to determine the present value of \$1000 in three years at 15%. We do this by discounting \$1000 back three periods at 15%. With these numbers, the discount factor is:

$$1/(1 + 0.15)^3 = 1/1.5209 = 0.6575$$

The amount you must invest is thus:

$$\$1000 \times 0.6575 = \$657.50$$

We say that \$657.50 is the present, or discounted, value of \$1000 to be received in three years at 15%.

There are tables for present value factors just as there are tables for future value factors, and you use them in the same way (if you use them at all). Table 4.3 contains a small set of these factors. A much larger set can be found in Table A.2 in Appendix A.

In Table 4.3, the discount factor we just calculated, 0.6575, can be found by looking down the column labelled '15%' until you come to the third row. Of course, you could use a financial calculator, as we illustrate next.

CALCULATOR HINTS

You solve present value problems on a financial calculator just like you do future value problems. For the example we just examined (the present value of \$1000 to be received in three years at 15%), you would do the following:

Enter	3	15			1000
	N	**I/Y**	**PMT**	**PV**	**FV**
Solve for				-657.50	

Notice that the answer has a negative sign; as we discussed above, that's because it represents an outflow today in exchange for the \$1000 inflow later.

Table 4.3

Present value interest factors

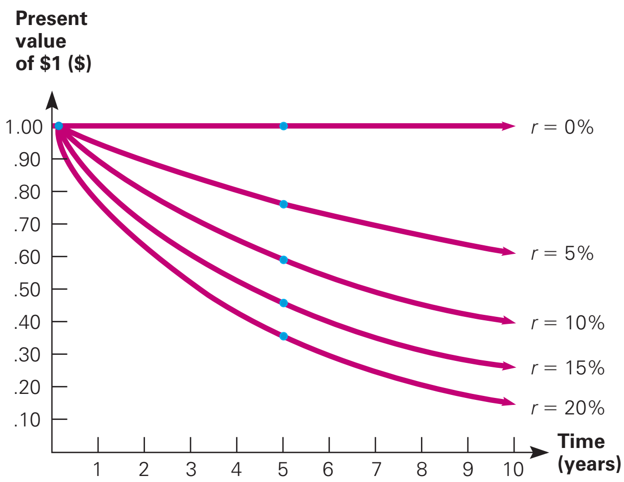
Number of Periods	**Interest Rates**			
	5%	**10%**	**15%**	**20%**
1	0.9524	0.9091	0.8696	0.8333
2	0.9070	0.8264	0.7561	0.6944
3	0.8638	0.7513	0.6575	0.5787
4	0.8227	0.6830	0.5718	0.4823
5	0.7832	0.6209	0.4972	0.4019

As the length of time until payment grows, present values decline. As Example 4.6 illustrates, present values tend to become small as the time horizon grows. If you look out far enough, they will always get close to zero. Also, for a given length of time, the higher the discount rate is, the lower the present value. Put another way, present values and discount rates are inversely related. Increasing the discount rate decreases the PV and vice versa.

The relationship between time, discount rates and present values is illustrated in Figure 4.3. Notice that by the time we get to ten years, the present values are all substantially smaller than the future amounts.

Figure 4.3

Present value of $1 for different periods and rates



CONCEPT QUESTIONS

4.2a What do we mean by the present value of an investment?

4.2b The process of discounting a future amount back to the present is the opposite of doing what?

4.2c What do we mean by discounted cash flow, or DCF, valuation?

4.2d In general, what is the present value of $1 to be received in *t* periods, assuming a discount rate of *r* per period?

MORE ON PRESENT AND FUTURE VALUES

4.3

If you look back at the expressions we came up with for present and future values, you will see there is a very simple relationship between the two. We explore this relationship and some related issues in this section.

PRESENT VERSUS FUTURE VALUE

What we called the present value factor is just the reciprocal of (i.e. 1 divided by) the future value factor:

Future value factor = $(1 + r)^t$

Present value factor = $1/(1 + r)^t$

For a downloadable, Windows-based financial calculator, go to <www.calculator.org>.

In fact, the easy way to calculate a present value factor on many calculators is to calculate the future value factor first and then press the **1/X** key to flip it over.

If we let FV_t stand for the future value after t periods, then the relationship between future value and present value can be written very simply as one of the following:

$$PV \times (1 + r)^t = FV_t \quad [4.3]$$

$$PV = FV_t/(1 + r)^t = FV_t \times [1/(1 + r)^t]$$

This last result we will call the *basic present value equation*, and we will use it throughout the text. There are a number of variations that come up, but this simple equation underlies many of the most important ideas in finance.

EVALUATING INVESTMENTS

EXAMPLE 4.6

To give you an idea of how we will be using present and future values, consider the following simple investment. Your company proposes to buy an asset for \$335. This investment is very safe. You will sell the asset in three years for \$400. You know you could invest the \$335 elsewhere at 10% with very little risk. What do you think of the proposed investment?

This is not a good investment. Why not? Because if you can invest the \$335 elsewhere at 10% after three years it will grow to \$445.89:

$$\begin{aligned}\$335(1 + r)^t &= \$335 \times 1.1^3 \\ &= \$335 \times 1.331 \\ &= \$445.89\end{aligned}$$

Since the proposed investment only pays out \$400, it is not as good as other alternatives we have. Another way of saying the same thing is to notice that the present value of \$400 in three years at 10% is:

$$400 \times [1/(1 + r)^t] = \$400/1.1^3 = \$400/1.331 = \$300.53$$

This tells us that we only have to invest about \$300 to get \$400 in three years, not \$335. We will return to this type of analysis later on.

DETERMINING THE DISCOUNT RATE

We frequently need to determine what discount rate is implicit in an investment. We can do this by looking at the basic present value equation:

$$PV = FV_t/(1 + r)^t$$

EXAMPLE 4.7

FINDING r FOR A SINGLE-PERIOD INVESTMENT

You are considering a one-year investment. If you put up \$1250, you will get back \$1350. What rate is this investment paying?

First, in this single-period case, the answer is fairly obvious. You are getting a total of \$100 in addition to your \$1250. The implicit rate on this investment is thus 8% (\$100/1250).

More formally, from the basic present value equation, the present value (the amount you must put up today) is \$1250. The future value (what the present value grows to) is \$1350. The time involved is one period, so we have:

$$\$1250 = \$1350/(1 + r)^1$$
$$1 + r = \$1350/1250$$
$$= 1.08$$
$$r = 8\%$$

In this simple case, of course, there was no need to go through this calculation, but, as we describe below, it gets a little harder when there is more than one period.

There are only four parts to this equation: the present value (PV), the future value (FV_t), the discount rate (r) and the life of the investment (t). Given that we know any three of these, we can always find the fourth.

To illustrate what happens with multiple periods, let us say that we are offered an investment that costs us \$100 and will double our money in eight years. To compare this with other investments, we would like to know what discount rate is implicit in these numbers. This discount rate is called the *rate of return*, or sometimes just *return*, on the investment. In this case, we have a present value of \$100, a future value of \$200 (double our money), and an eight-year life. To calculate the return, we can write the basic present value equation as:

$$PV = FV_t/(1 + r)^t$$

$$\$100 = \$200/(1 + r)^8$$

It could also be written as:

$$(1 + r)^8 = \$200/100 = 2$$

We now need to solve for r. There are three ways we could do it:

1. Use a financial calculator (see below).
2. Solve the equation for $1 + r$ by taking the eighth root of both sides. Since this is the same thing as raising both sides to the power of $\frac{1}{8}$, or 0.125, this is actually easy to do with the y^x key on a calculator. Just enter 2, then press y^x, enter 0.125, and press the = key. The eighth root should be about 1.09, which implies that r is 9%.
3. Use a future value table. The future value factor for eight years is equal to 2. If you look across the row corresponding to eight periods in Table A.1, you will see that a future value factor of 2 corresponds with the 9% column, again implying that the return here is 9%.

Why does the Rule of 72 work? See **<www.moneychimp.com>.**

Actually, in this particular example, there is a useful 'back of the envelope' means of solving for r—the Rule of 72. For reasonable rates of return, the time it takes to double your money is given approximately by $72/r\%$. In our example, this means that $72/r\% = 8$ years, implying that r is 9%, as we calculated. This rule is fairly accurate for discount rates in the 5% to 20% range.

The nearby *Reality Bytes* box provides some examples of rates of return on collectibles. See if you can verify the numbers reported there.

DOUBLE YOUR FUN

EXAMPLE 4.8

You have been offered an investment that promises to double your money every ten years. What is the approximate rate of return on the investment?

From the Rule of 72, the rate of return is given approximately by $72/r\% = 10$, so the rate is approximately $72/10 = 7.2\%$. Verify that the exact answer is 7.177%.

REALITY BYTES

A QUACK INVESTMENT?

It used to be that trading in collectibles such as American baseball cards, art and old toys occurred mostly at auctions, swap meets and collectible shops, all of which were limited to regional traffic. However, with the growing popularity of on-line auctions such as eBay, trading in collectibles has expanded to an international arena. The most visible form of collectible is probably the American baseball card, but more recently Furbies, Beanie Babies and Pokémon cards have been extremely hot. However, it is not just fad items that spark the collector's interest; virtually anything of sentimental value from days gone by is considered collectible, and, more and more, collectibles are being viewed as investments.

Collectibles typically provide no cash flows, except when sold, and condition and buyer sentiment are the major determinants of value. The rates of return have been staggering at times, but care is needed in interpreting them. For example, Picasso's *Boy with a Pipe* painting sold in 1950 for $30 000 and again in 2004 for $104 million. This represented an actual return on investment of about 16% per year. However, for most investors, the long-run return in such an opaque market is no better than that on traditional investments like the stock market. The Mei/Moses index, calculated by two economists from New York University in 2001 and using data from 1954 onwards, shows a long-term annual return of 8.2% in art market investment, compared with 8.9% for the Dow Jones share index.

Comic books have always been a popular collectible. Back in the early 1900s most comic books were purchased for values between 10 and 15 cents. Many of these comic books have been sold for hundreds of thousands of dollars today and are said to be worth more. It is doubtful that any superhero is stronger than Superman, who first appeared in June 1938 in a 10-cent comic book called *Action Comics #1*. Now, *Action Comics #1* is the world's most valuable comic book. Its present value is estimated to be $350 000. While this looks like an extraordinary return to the untrained eye, check for yourself that the actual return on the investment was about 23.8% per year. Plus, you had the problem of storing the comic book in mint condition.

Barbie dolls have historically been a hot collectible, but Barbie's tale also shows the downside of investing in collectibles. An original Barbie sold for about $3 when it was introduced in March 1959. An original in mint condition (and never removed from its package) might have been worth $7000 in 2000, which represents a whopping return of 20.8% per year. However, at an auction in 2002, the top bid for a vintage 1959, in-the-box Barbie, was $4400. If you had purchased Barbie in 2000 and sold it in 2002 you would have had a return of –20.72% per year (notice the negative sign). The return on your original Barbie purchased in 1959 would have dropped to 18.48% per year.

Stamp collecting (or philately) is a popular activity. The first stamp issued in Australia by the state of New South Wales in 1850 was a one penny stamp and showed the seal of the New South Wales colony as it was then. Today, if you have one in 'mint' condition, it is worth about $5000. Again, to the untrained eye it appears to be a huge gain, but the return is only 8.84% per year. On the other hand, the first stamp issued in England, called the 'Penny Black', was printed in 1840. Unfortunately, one of these stamps in mint condition would sell today for approximately only $1000, because there were 68 million of them printed.

Looking back at these investments, the comic book had the highest return recently. The problem is that to earn this return, you had to purchase the comic book when it was new and store it for all those years. Looking ahead, the corresponding problem is predicting what

Continued

the future value of the collectible will be. You can earn a positive return only if the market value of your asset rises above the purchase price at some point in the future. That, of course, is rarely assured. For example, most collectors say the Barbies that are mass-marketed at discount stores today will probably have little or no value as collectibles at any time in the future, so we don't recommend them for your retirement investing.

A slightly more extreme example involves money bequeathed by Benjamin Franklin, who died on 17 April 1790. In his will he gave 1000 pounds sterling to Massachusetts and the City of Boston. He gave the same amount to Pennsylvania and the City of Philadelphia. The money was paid to Franklin when he held political office, but he believed that politicians should not be paid for their service (it appears that this view is not widely shared by modern politicians).

Franklin originally specified that the money should be paid out 100 years after his death and used to train young people. Later, however, after some legal wrangling, it was agreed that the money would be paid out in 1990, 200 years after Franklin's death. By that time, the Pennsylvania bequest had grown to about US$2 million; the Massachusetts bequest had grown to US$4.5 million. The money was used to fund the Franklin Institutes in Boston and Philadelphia. Assuming that 1000 pounds sterling was equivalent to US$1000, what rate of return did the two states earn (the dollar did not become the official US currency until 1792)?

For Pennsylvania, the future value is US$2 million and the present value is US$1000. There are 200 years involved, so we need to solve for r in the following:

$$\$1000 = \$2 \text{ million}/(1 + r)^{200}$$

$$(1 + r)^{200} = 2000$$

Solving for r, we see that the Pennsylvania money grew at about 3.87% per year. The Massachusetts money did better; verify that the rate of return in this case was 4.3%. Small differences can add up!

CALCULATOR HINTS

We can illustrate how to calculate unknown rates with a financial calculator using these numbers. For Pennsylvania, you would do the following:

	N	I/Y	PMT	PV	FV
Enter	200			-1000	-2000000
Solve for		3.87			

As in our previous examples, notice the minus sign on the present value, representing Franklin's outlay made many years ago. What do you change to work the problem for Massachusetts?

EXAMPLE 4.9

SAVING FOR UNIVERSITY

You estimate that you will need about $80 000 to send your child to university in eight years. You have about $35 000 now. If you can earn 20% per year, will you make it? At what rate will you just reach your goal?

If you can earn 20%, the future value of your $35 000 in eight years will be:

$$FV = \$35\,000 \times 1.20^8 = \$35\,000 \times 4.2998 = \$150\,493.59$$

So, you will make it easily. The minimum rate is the unknown r in the following:

$$FV = \$35\,000 \times (1 + r)^8 = \$80\,000$$

$$(1 + r)^8 = \$80\,000/35\,000 = 2.2857$$

Therefore, the future value factor is 2.2857. Looking at the row in Table A.1 that corresponds to eight periods, we see that our future value factor is roughly halfway between the ones shown for 10% (2.1436) and 12% (2.4760), so you will just reach your goal if you earn approximately 11%. To get the exact answer, we could use a financial calculator or we could solve for r:

$$(1 + r)^8 = \$80\,000/35\,000 = 2.2857$$

$$1 + r = (2.2857)^{(1/8)} = (2.2857)^{0.125}$$

$$1 + r = 1.1089$$

$$r = 1.1089 - 1 = 0.1089$$

$$= 10.89\%$$

ONLY 18 262.5 DAYS TO RETIREMENT

EXAMPLE 4.10

You would like to retire in fifty years as a millionaire. If you have $10 000 today, what rate of return do you need to earn to achieve your goal?

The future value is $1 000 000. The present value is $10 000, and there are fifty years until retirement. We need to calculate the unknown discount rate in the following:

$$\$10\,000 = \$1\,000\,000/(1 + r)^{50}$$

$$(1 + r)^{50} = 100$$

The future value factor is thus 100. You can verify that the implicit rate is about 9.65%.

FINDING THE NUMBER OF PERIODS

Suppose we were interested in purchasing an asset that costs $50 000. We currently have $25 000. If we can earn 12% on this $25 000, how long until we have the $50 000? Finding the answer involves solving for the last variable in the basic present value equation, the number of periods. You already know how to get an approximate answer to this particular problem. Notice that we need to double our money. From the Rule of 72, this will take about 72/12 = 6 years at 12%.

To come up with the exact answer, we can again manipulate the basic present value equation. The present value is $25 000, and the future value is $50 000. With a 12% discount rate, the basic equation takes one of the following forms:

$$\$25\,000 = \$50\,000/(1.12)^t$$

$$(1.12)^t = \$50\,000/25\,000 = 2$$

We thus have a future value factor of 2 for a 12% rate. We now need to solve for t. If you look down the column in Table A.1 that corresponds to 12%, you will see that a future value factor of 1.9738 occurs at six periods. It will thus take about six years, as we calculated. To get the exact answer, we have to explicitly solve for t (or use a financial calculator). If you do this, you will find that the answer is 6.1163 years, so our approximation was quite close in this case.

CALCULATOR HINTS

If you do use a financial calculator, here are the relevant entries:

	N	I/Y	PMT	PV	FV
Enter		12		-25 000	50 000
Solve for	6.1163				

EXAMPLE 4.11

WAITING FOR NIPPER

You have been saving up to buy the Nipper Company. The total cost will be $10 million. You currently have about $2.3 million. If you can earn 5% on your money, how long will you have to wait? At 16%, how long must you wait?

At 5%, you will have to wait a long time. From the basic present value equation:

$$\$2.3 = \$10/(1.05)^t$$
$$(1.05)^t = 4.35$$
$$t = 30 \text{ years}$$

At 16%, things are a little better. Verify for yourself that it will take about ten years.

Learn more about using Excel for time value and other calculations at <www.studyfinance.com>.

This example finishes our introduction to basic time value of money concepts. Table 4.4 on page 110 summarises present value and future value calculations for future reference. As our nearby *Work the Web* box shows, online calculators are widely available to handle these calculations, but it is still important to know what is going on.

SPREADSHEET STRATEGIES

USING A SPREADSHEET FOR TIME VALUE OF MONEY CALCULATIONS

More and more, business people from many different areas (and not just finance and accounting) rely on spreadsheets to do all the different types of calculations that come up in the real world. For that reason, in this section we will show you how to use a spreadsheet to handle the various time value of money problems we presented in this chapter. We will use Microsoft Excel™, but the commands are similar for other types of software. We assume you are already familiar with basic spreadsheet operations.

As we have seen, you can solve for any one of the following four potential unknowns: future value, present value, the discount rate or the number of periods. With a spreadsheet, there is a separate formula for each. In Excel, these are as follows:

TO FIND	ENTER THIS FORMULA
Future value	= FV (rate, nper, pmt, pv)
Present value	= PV (rate, nper, pmt, fv)
Discount rate	= RATE (nper, pmt, pv, fv)
Number of periods	= NPER (rate, pmt, pv, fv)

In these formulas, pv and fv are present and future value, nper is the number of periods, and rate is the discount, or interest, rate.

There are two things that are a little tricky here. First, unlike a financial calculator, the spreadsheet requires that the rate be entered as a decimal. Second, as with most financial calculators, you have to put a negative sign on either the present value or the future value to solve for the rate or the number of periods. For the same reason, if you solve for a present value, the answer will have a negative sign unless you input a negative future value. The same is true when you compute a future value.

To illustrate how you might use these formulas, we will go back to an example in the chapter. If you invest $25 000 at 12% per year, how long will it be until you have $50 000? You might set up a spreadsheet like this:

	A	B	C	D	E	F	G	H
1								
2	**Using a spreadsheet for time value of money calculations**							
3								
4	If we invest $25 000 at 12%, how long until we have $50 000? We need to solve for the							
5	unknown number of periods, so we use the formula NPER (rate, pmt, pv, fv)							
6								
7	Present value (PV):	$25 000						
8	Future value (FV):	$50 000						
9	Rate (rate):	0.12						
10								
11	Periods:	**6.116255**						
12								
13	The formula entered in cell B11 is = NPER (B9, 0, –B7, B8); notice that pmt is zero and that pv has a negative sign on it.							
14	Also notice that the rate is entered as a decimal, not a percentage							

WORK THE WEB

How important is the time value of money? A recent search on one internet engine returned over 3.2 million hits! It is important to understand the calculations behind the time value of money, but the advent of financial calculators and spreadsheets has eliminated the need for tedious calculations. In fact many websites offer time value of money calculators. The following is an example from Moneychimp's website <**www.moneychimp.com/calculator_compoundinterest_calculator**>. You have $40 000 today and want to know how much this will be worth in fifteen years if you can invest your money at 9.8%. How much do you need to deposit today? With the Moneychimp calculator, you simply enter the values and hit calculate:

The results look like this:

Inputs

Current Principal	$	40,000.00
Annual Addition	$	0
Years to grow		15
Interest Rate		9.8 %

Compound interest [1] time(s) annually

Make additions at ○ start ⊙ end of each compounding period

[Calculate]

Results

Future Value	$	162,590.47

Who said time value of money calculations are hard?

Table 4.4

Summary of time value of money calculations

I. Symbols

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future value of *C* invested at *r*% per period for *t* periods

$FV_t = C \times (1 + r)^t$

The term $(1 + r)^t$ is called the *future value factor*.

III. Present value of *C* to be received in *t* periods at *r*% per period

$PV = C / (1 + r)^t$

The term $1/(1 + r)^t$ is called the *present value factor*.

IV. The basic present value equation giving the relationship between present and future value is:

$PV = FV_t/(1 + r)^t$

CONCEPT QUESTIONS

4.3a What is the basic present value equation?

4.3b What is the Rule of 72?

SUMMARY AND CONCLUSIONS

This chapter has introduced you to the basic principles of present value and discounted cash flow valuation. In it, we explained a number of things about the time value of money, including:

1. For a given rate of return, the value at some point in the future of an investment made today can be determined by calculating the future value of that investment.
2. The current worth of a future cash flow can be determined for a given rate of return by calculating the present value of the cash flow involved.
3. The relationship between present value and future value for a given rate *r* and time *t* is given by the basic present value equation:

   $PV = FV_t /(1 + r)^t$

As we have shown, it is possible to find any one of the four components (PV, FV_t, *r*, or *t*) given the other three.

The principles developed in this chapter will figure prominently in the chapters to come. The reason for this is that most investments, whether they involve real assets or financial assets, can be analysed using the discounted cash flow, or DCF, approach. As a result, the DCF approach is broadly applicable and widely used in practice. Before going on, therefore, you might want to do some of the following problems.

CHAPTER REVIEW AND SELF-TEST PROBLEMS

4.1 Calculating Future Values. Assume you deposit $1000 today in an account that pays 8% interest. How much will you have in four years?

4.2 Calculating Present Values. Suppose you have just celebrated your nineteenth birthday. A rich uncle set up a trust fund for you that will pay you $100 000 when you turn 25. If the relevant discount rate is 11%, how much is this fund worth today?

4.3 Calculating Rates of Return. You have been offered an investment that will double your money in twelve years. What rate of return are you being offered? Check your answer using the Rule of 72.

4.4 Calculating the Number of Periods. You have been offered an investment that will pay you 7% per year. If you invest $10 000, how long until you have $20 000? How long until you have $30 000?

ANSWERS TO CHAPTER REVIEW AND SELF-TEST PROBLEMS

4.1 We need to calculate the future value of $1000 at 8% for four years. The future value factor is:

$(1.08)^4 = 1.3605$

The future value is thus $1000 × 1.3605 = $1360.50.

4.2 We need the present value of $100 000 to be paid in six years at 11%. The discount factor is:

$1/(1.11)^6 = 1/1.8704 = 0.5346$

The present value is thus about $53 460.

4.3 Suppose you invest, say, $100. You will have $200 in twelve years with this investment. So, $100 is the amount you have today, the present value, and $200 is the amount you will have in twelve years, the future value. From the basic present value equation, we have:

$$\$200 = \$100 \times (1 \times r)^{12}$$
$$2 = (1 \times r)^{12}$$

From here, we need to solve for r, the unknown rate. As shown in the chapter, there are several different ways to do this. We will take the 12th root of 2 (by raising 2 to the power of $\frac{1}{12}$):

$$2^{(1/12)} = 1 + r$$
$$1.0595 = 1 + r$$
$$r = 5.95\%$$

Using the Rule of 72, we have $72/t = r\,\%$, or 72/12 = 6%, so our answer looks good (remember that the Rule of 72 is only an approximation).

4.4 The basic equation is:

$$\$20\,000 = \$10\,000 \times (1 + 0.07)^t$$
$$2 = (1.07)^t$$

If we solve for t, we get $t = 10.24$ years. Using the Rule of 72, we get 72/7 = 10.29 years, so, once again, our answer looks good. To get $30 000, verify for yourself that you will have to wait 16.24 years.

CRITICAL THINKING AND CONCEPTS REVIEW

4.1 Present Value. The basic present value equation has four parts. What are they? **LO 1**

4.2 Compounding. What is compounding? What is discounting? **LO 1**

4.3 Compounding and Periods. As you increase the length of time involved, what happens to future values? What happens to present values? **LO 1**

4.4 Compounding and Interest Rates. What happens to a future value if you increase the rate r? What happens to a present value? **LO 1**

Continued

4.5 Future Values. Suppose you deposit a large sum of money in an account that earns a low interest rate and simultaneously deposit a small sum in an account with a high interest rate. Which account will have the larger value future value? **LO 1**

To answer the next four questions, refer to the NABARD zero coupon bond issue we discussed to open the chapter.

4.6 Time Value of Money. Why would NABARD be willing to accept such a small amount today (Rs 8750) in exchange for a promise to repay 2.2 times that amount (Rs 20 000) in the future? **LO 3**

4.7 Call Provisions. NABARD has the right to buy back the securities any time it wishes by paying Rs 20 000 (this is one of the terms of this particular deal). What impact does this feature have on the desirability of this security as an investment? **LO 3**

4.8 Time Value of Money. Would you be willing to pay $8750 today in exchange for $20 000 in ten years? What would be the key considerations in answering yes or no? Would your answer depend on who is making the promise to repay? **LO 3**

4.9 Investment Comparison. Suppose that when NABARD offered the security, the Australian government had offered an essentially identical security. Do you think it would have had a higher or lower price? Why? **LO 3**

QUESTIONS AND PROBLEMS

BASIC QUESTIONS (1–15)

1. **Simple Interest versus Compound Interest.** First Bronte Bank pays 8% simple interest on its savings account balances, whereas First Wellington Bank pays 8% interest compounded annually. If you made a $6000 deposit in each bank, how much more money would you earn from your First Wellington Bank account at the end of ten years? **LO 1**

2. **Calculating Future Values.** For each of the following, compute the future value:

PRESENT VALUE	YEARS	INTEREST RATE	FUTURE VALUE
$ 3150	5	18%	
8453	10	6	
89 305	17	11	
227 382	20	5	

LO 1

3. **Calculating Present Values.** For each of the following, compute the present value:

PRESENT VALUE	YEARS	INTEREST RATE	FUTURE VALUE
	12	4%	$ 15 451
	4	9	51 557
	16	17	886 073
	21	20	901 450

LO 2

4. **Calculating Interest Rates.** Solve for the unknown interest rate in each of the following:

PRESENT VALUE	YEARS	INTEREST RATE	FUTURE VALUE
$ 221	6		$ 307
425	7		905
25 000	18		143 625
40 200	21		255 810

LO 3

5. **Calculating the Number of Periods.** Solve for the unknown number of years in each of the following:

PRESENT VALUE	YEARS	INTEREST RATE	FUTURE VALUE
$ 250		8%	$ 1 105
1 941		5	3 700
32 805		14	387 120
32 500		24	198 212

LO 4

6. **Calculating Interest Rates.** Assume the total cost of a university education will be $250 000 when your child enters university in eighteen years. You presently have $35 000 to invest. What annual rate of interest must you earn on your investment to cover the cost of your child's university education? LO 3
7. **Calculating the Number of Periods.** At 9% interest, how long does it take to double your money? To quadruple it? LO 4
8. **Calculating Rates of Return**. In 2009 a 50-cent bicentennial coin issued in 1988 sold for $1300. What is the annual rate of return on this investment? LO 3
9. **Calculating the Number of Periods.** You are trying to save to buy a new $160 000 Ferrari. You have $30 000 today that can be invested at your bank. The bank pays 4.7% annual interest on its accounts. How long will it be before you have enough to buy the car? LO 4
10. **Calculating Present Values.** Kiwi Fruit Ltd has an unfunded superannuation liability of $800 million that must be paid in twenty years. To assess the value of the firm's shares, financial analysts want to discount this liability back to the present. If the relevant discount rate is 8%, what is the present value of this liability? LO 2
11. **Calculating Present Values.** You have just received notification that you have won the $2 million first prize in the state lottery. However, the prize will be awarded on your one-hundredth birthday (assuming you are around to collect), eighty years from now. What is the present value of your windfall if the appropriate discount rate is 11%? LO 2
12. **Calculating Future Values.** Your coin collection contains fifty 1952 US silver dollars. If your American grandparents purchased them for their face value when they were new, how much will your collection be worth when you retire in 2060, assuming they appreciate at a 5.7% annual rate? LO 2
13. **Calculating Interest Rates and Future Values.** In 1904 the first Australian Open Golf Championship was held. The winner's prize money was a modest $100. In 2009 the winner's payment was $1 500 000. What was the annual percentage increase in the winner's payment over this period? If the winner's prize increases at the same rate, what will it be in 2040? LO 1 & LO 2
14. **Calculating Present Values.** In 2009 a painting by Joseph William Turner, *Temple of Jupiter Panellenius*, sold for £9.1 millon. The seller bought the painting in 1982 for £648 000. What was the annual return for this investor? LO 3
15. **Calculating Rates of Return.** Although appealing to more refined tastes, art as a collectible has not always performed so profitably. During 2003 Sotheby's sold the Edgar Degas bronze sculpture *Petite danseuse de quatorze ans* at auction for a price of $10 311 500. Unfortunately for the previous owner, who had purchased the sculpture in 1999 for $12 377 500. What was his annual rate of return on this sculpture? LO 3

INTERMEDIATE QUESTIONS (16–25)

16. **Calculating Rates of Return.** Referring to the NABARD security we discussed at the very beginning of the chapter:
    - **a.** Based upon the Rs 8750 price, what rate was NABARD paying to borrow money?
    - **b.** Suppose that on 1 January 2011 this security's price was Rs 13 000. If an investor had

Continued

purchased it for Rs 8750 at the offering and sold it on this day, what annual rate of return would she have earned?

c. If an investor had purchased the security at market on 1 January 2011, and held it until it matured, what annual rate of return would she have earned? **LO 3**

17. **Calculating Present Values.** Suppose you are still committed to owning a $160 000 Ferrari (see Question 9). If you believe your investment fund can achieve a 10.75% annual rate of return, and you want to buy the car in ten years time on the day you turn 30, how much must you invest today? **LO 2**

18. **Calculating Future Values.** You have just made your first $5000 contribution to your individual superannuation account. Assuming you earn a 10% rate of return and make no additional contributions, what will your account be worth when you retire in thirty years? What if you wait ten years before contributing? (Does this suggest an investment strategy?) **LO 1**

19. **Calculating Future Values.** You are scheduled to receive $15 000 in two years. When you receive it, you will invest it for six more years at 9% per year. How much will you have in eight years? **LO 1**

20. **Calculating the Number of Periods.** You expect to receive $30 000 at graduation in two years. You plan on investing it at 9% until you have $160 000. How long will you wait from now? (Better than the situation in Question 9, but still no Ferrari.) **LO 4**

21. **Calculating Future Values.** You have $7000 to deposit. Roten Bank offers 12% per year compounded monthly (1% per month), while Brook Bank offers 12% per year, but will only compound annually. How much will your investment be worth in twenty years at each bank? **LO 1**

22. **Calculating Interest Rates.** An investment offers to triple your money in twenty-four months (do not believe it). What rate per three months are you being offered? **LO 3**

23. **Calculating the Number of Periods.** You can earn 0.45% per month at your bank. If you deposit $1500, how long must you wait until your account has grown to $3600? **LO 4**

24. **Calculating Present Values.** You need $75 000 in ten years. If you can earn 0.55% per month, how much will you have to deposit today? **LO 2**

25. **Calculating Present Values.** You have decided that you want to be a millionaire when you retire in forty years. If you can earn a 12% annual return, how much do you have to invest today? What if you can earn 6%? **LO 2**

CHALLENGE QUESTIONS (26–27)

26. **Calculating Future Values**. You have $10 000 you want to invest for the next forty years. You are offered an investment plan that will pay you 8% per year for the next twenty years and 12% for the last twenty years. How much will you have earned at the end of forty years? Does it matter if the investment plans to pay you 12% for the first twenty years and 8% for the next twenty years? Why or why not? **LO 1**

27. **Calculating Present Values.** You have been offered the chance to invest in a quick no-frills promotion scheme. The entry fee is $500 000 and the venture is to capitalise on the fame of the film *Star Trek* and will produce *Star Trek* costumes, given the great success of the movie. The promoter offers investors $200 000 back in four years and $800 000 in seven years. If investors think they require a 10% return on this type of investment, is this a good investment? **LO 2**

WHAT'S ON THE WEB?

4.1 Calculating Future Values. Go to <**www.webwombat.com.au**> and follow the 'Compound Interest' link. If you currently have $10 000 and invest this money at 9%, how much will you have in thirty years? Assume you will not make any additional contributions. How much will you have if you can earn 11%?

4.2 Calculating the Number of Periods. Go to the Bank of Queensland website at <**www.boq.com.au**> and follow the 'Personal Banking/Savings and Investment/Calculators' link. You want to be a millionaire. You can earn 11.5% per year. Using your current age, at what age will you become a millionaire if you have $25 000 to invest, assuming you make no other deposits (ignore inflation)?

4.3 Calculating the Number of Periods. Cigna has a financial calculator available at <**www.cigna.com**>. To get to the calculator, follow the 'Calculator and Tools' link, then the 'Present/Future Value Calculator' link. You want to buy a Lamborghini Murciélago. The current market price of the car is $330 000 and you have $33 000. If you can earn an 11% return, how many years until you can buy this car (assuming the price stays the same)?

4.4 Calculating Rates of Return. Use the Cigna financial calculator to solve the following problem. You still want to buy the Lamborghini Murciélago, but you have $50 000 to deposit and want to buy the car in fifteen years. What interest rate do you have to earn to accomplish this (assuming the price stays the same)?