Contents

2.1

Preface	xiii
Acknowledgements	xvii

PART-I

PRELIMINARIES

- 1. Vector Algebra, Theory of Equations, and **Complex Numbers** 1.1
 - 1.1 Vector Algebra 1.1
 - 1.2 Theory of Equations 1.13
 - 1.3 Cardano's Method 1.14
 - 1.4 Ferrari's Method 1.17
 - 1.5 Complex Numbers 1.19
 - Roots of Complex Numbers 1.22 1.6

PART-II

DIFFERENTIAL AND INTEGRAL CALCULUS

2. Differential Calculus

- 2.1 Derivation of n^{th} Derivative of Some Elementary Functions 2.1
- 2.2 Leibnitz's Theorem (Rule or Formula) 2.7
- 2.3 Angle Between the Radius Vector and the Tangent 2.10
- 2.4 Rolle's Theorem 2.14
- 2.5 Lagrange's Mean Value Theorem 2.17
- 2.6 Cauchy's Mean Value Theorem 2.21
- 2.7 Generalized Mean Value Theorem 2.23
- 2.8 Taylor's Series and Maclaurin's Series Expansions 2.26
- 2.9 Indeterminate Forms 2.33
- 2.10 Derivatives of Arcs 2.40
- 2.11 Curvature 2.44
- 2.12 Evolute 2.61
- 2.13 Envelopes 2.65

3. Partial Differentiation 3.1

- 3.1 Functions of Several Variables: Limit and Continuity 3.1
- 3.2 Partial Differentiation 3.5

- Variable Treated as Constant 3.9 3.3
- 3.4 Total Derivative 3.11
- 3.5 Partial Differentiation of Composite Functions: Change of Variables 3.13
- 3.6 Differentiation of an Implicit Function 3.16
- 3.7 Euler's Theorem 3.18
- 3.8 Jacobian 3.22
- 3.9 Functional Dependence 3.27
- Errors and Approximations 3.29 3.10
- 3.11 Differentiation Under Integral Sign: Leibnitz's Rule 3.31

4. Maxima and Minima

- Taylor's Theorem for Function of Two 41 Variables 4.1
- Maxima and Minima of Functions of Two 4.2 Variables: With and Without Constraints 4.5
- 4.3 Lagrange's Method of Undetermined Multipliers 4.10

5. Curve Tracing

- 5.1 Curve Tracing: Curves in Cartesian Form 5.1
- 5.2 Curve Tracing: Standard Curves in Cartesian Form 5.8
- Curve Tracing: Polar Curves 5.15 5.3
- 5.4 Curve Tracing: Standard Polar Curves 5.20
- 5.5 Curve Tracing: Parametric Curves 5.24
- Curve Tracing: Standard Parametric 5.6 Curves 5.26

6. Integral Calculus

- 6.1 Reduction Formulae 6.1
- 6.2 Area of a Plane Region: Quadrature 6.2
- 6.3 Length of Plane Curve: Rectification 6.20
- 6.4 Volume of Solid of Revolution 6.26 Area of the Surface of a Solid of 6.5
- Revolution 6.38 6.6
- Improper Integrals 6.45

4.1

5.1

viii Contents

7. Multiple Integrals

- 7.1 Double Integral 7.1
- 7.2 Application of Double Integral 7.4
- 7.3 Change of Order of Integration: Double Integral 7.12

7.1

- 7.4 General Change of Variables in Double Integral 7.14
- 7.5 Triple Integrals 7.21
- 7.6 General Change of Variables in a Triple Integral 7.24
- 7.7 Dirichlet's Integral 7.31

PART—III ORDINARY DIFFERENTIAL EQUATIONS

8. Ordinary Differential Equations: First Order and First Degree 8.1

- 8.1 Introduction to Mathematical Modeling 8.1
- 8.2 Basic Definitions 8.2
- 8.3 First Order First Degree Differential Equations 8.4
- 8.4 Variables Separable or Separable Equation *8.4*
- 8.5 Homogeneous Equation–Reduction to Separable Form 8.6
- 8.6 Non-homogeneous Equations Reducible to Homogeneous Form 8.8
- 8.7 Exact Differential Equations 8.10
- 8.8 Reduction of Non-exact Differential Equations: Using Integrating Factors 8.12
- 8.9 Linear Differential Equation: First Order 8.19
- 8.10 Bernoulli Equation 8.22
- 8.11 First Order Nonlinear Differential Equations 8.25
- 8.12 Clairaut's Equation 8.29
- 8.13 Lagrange's Equation 8.32
- 8.14 Formation of Ordinary Differential Equations by Elimination of Arbitrary Constants 8.33
- 8.15 Geometrical Applications 8.36
- 8.16 Orthogonal. Trajectories of Curves 8.39
- 8.17 Law of Natural Growth 8.44

- 8.18 Law of Natural Decay 8.45
- 8.19 Newton's Law of Cooling 8.46
- 8.20 Velocity of Escape from Earth 8.48
- 8.21 Simple Electric Circuits 8.50
- 9. Linear Differential Equations of Second Order and Higher Order 9.1
 - 9.1 Linear Independence and Dependence 9.1
 - 9.2 Linear Differential Equations of Second Order with Variable Coefficients 9.2
 - 9.3 Second Order Differential Equations with Constant Coefficients; Homogeneous 9.2
 - 9.4 Higher Order Linear Homogeneous Differential Equations 9.5
 - 9.5 Non-homogeneous Equations 9.9
 - 9.6 Differential Equations with Variable Coefficients: Reducible to Equations with Constant Coefficients 9.25
 - 9.7 Method of Variation of Parameters 9.29
 - 9.8 The Method of Undetermined Coefficients 9.32
 - 9.9 System of Simultaneous Linear D.E. with Constant Coefficients 9.38
 - 9.10 Method of Reduction of Order 9.41
 - 9.11 Higher Order Linear Equations with Variable Coefficients 9.43
 - 9.12 Simple Harmonic Motion 9.48
 - 9.13 Mass-spring Mechanical System 9.50

- 9.14 RLC-Circuit 9.59
- 9.15 Simple Pendulum 9.63
- 10. Series Solutions
 - 10.1 Classification of Singularities 10.1
 - 10.2 Power Series Solution 10.2
 - 10.3 Frobenius Method 10.7
 - 10.4 Orthogonality of Functions 10.17
 - 10.5 Sturm-Liouville Problems 10.21
 - 10.6 Gram-Schmidt Orthogonalization Process 10.28
- 11. Special Functions—Gamma, Beta, Bessel and Legendre 11.1
 - 11.1 Gamma Function 11.1
 - 11.2 Beta Function 11.2
 - 11.3 Bessel's Functions 11.9
 - 11.4 Differential Equations Reducible to Bessel's Equation 11.19

- 11.5 Legendre Functions 11.21
- 11.6 Fourier-Legendre and Fourier-Bessel Series 11.30
- 11.7 Chebyshev Polynomials 11.35

12. Laplace Transform

12.1

- 12.1 Laplace Transform 12.1
 12.2 Applications, Advantages and Sufficient Conditions for Existence of Laplace Transform 12.3
- 12.3 General Properties of Laplace Transform 12.3
- 12.4 Laplace Transform of Periodic Function 12.18
- 12.5 Inverse Laplace Transform 12.20
- 12.6 General Properties of Inverse Laplace Transform 12.21
- 12.7 Use of Partial Fractions to Find Inverse L.T. 12.30
- 12.8 Convolution 12.33
- 12.9 Application of Laplace Transform to Differential Equations with Constant Coefficients 12.35
- 12.10 Application of Laplace Transform to System of Simultaneous Differential Equations 12.38
- 12.11 Table of General Properties of Laplace Transform 12.41
- 12.12 Table of Some Laplace Transforms 12.41

PART-IV

LINEAR ALGEBRA AND VECTOR CALCULUS

13. Matrices

- 13.1 Inverse of a Matrix 13.2
- 13.2 Rank of a Matrix *13.3*
- 13.3 Normal Form 13.8
- 13.4 System of Linear Non-homogeneous Equations 13.10
- 13.5 System of Homogeneous Equations 13.15
- 13.6 Gaussian Elimination Method 13.17
- 13.7 LU-decompositions 13.20
- 13.8 LU-decomposition from Gaussian Elimination *13.21*
- 13.9 Solution to Tridiagonal Systems 13.25

13.10 Crout Reduction for Tridiagonal Linear Systems 13.25

14. Eigen Values and Eigen Vectors 14.1

- 14.1 Linear Transformation 14.1
- 14.2 Eigen Values and Eigen Vectors *14.1*14.3 Properties of Eigen Values and Eigen
- Vectors 14.2
- 14.4 Cayley-Hamilton Theorem 14.9
- 14.5 Diagonalization Powers of a Matrix 14.13
- 14.6 Real Matrices: Symmetric, Skewsymmetric, Orthogonal Quadratic Form 14.18
- 14.7 Canonical Form: Or Sum of the Squares Form 14.21
- 14.8 Transformation (Reduction) of Quadratic Form to Canonical Form 14.23
- 14.9 Complex Matrices: Hermitian, Skew-Hermitian, Unitary Matrices 14.27
 14.10 Sylvester's Law of Inertia 14.32
- 14.10 Sylvester's Law of mertia 14.52

15. Vector Differential Calculus: Gradient,
Divergence and Curl25.1

15.1 Vector Differentiation 15.1

- 15.2 Directional Derivative, Gradient of a Scalar Function and Conservative Field 15.5
- 15.3 Divergence 15.10
- 15.4 Curl 15.11
- 15.5 Related Properties of Gradient, Divergence and Curl of Sums 15.14
- 15.6 Second-order Differential Operator 15.16
- 15.7 Curvilinear Coordinates: Cylindrical and Spherical Coordinates 15.20

16. Vector Integral Calculus

16.1 Vector Integration: Integration of a Vector Function of a Scalar Argument *16.1*

16.1

- 16.2 Line Integrals: Work Done, Potential, Conservative Field and Area 16.4
- 16.3 Surface Integrals: Surface Area and Flux *16.11*
- 16.4 Volume Integrals 16.16
- 16.5 Green's Theorem in Plane: Transformation Between Line Integral and Double Integral Area in Cartesian and Polar Coordinates 16.19

x Contents

- 16.6 Stokes' Theorem *16.24*
- 16.7 Gauss Divergence Theorem 16.29

PART-V

FOURIER ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS

17. Fourier Series17.1

- 17.1 Euler's (Fourier–Euler) Formulae *17.2*17.2 Fourier Series for Even and Odd
- Functions 17.7
- 17.3 Fourier Series for Functions Having Period 2L 17.10
- 17.4 Half Range Expansions: Fourier Cosine and Sine Series 17.14

18.1

19.1

17.5 Practical Harmonic Analysis 17.20

18. Partial Differential Equations

- 18.1 Partial Differential Equations 18.1
- 18.2 Partial Differential Equations of First Order 18.7
- 18.3 Linear Partial Differential Equations of First Order 18.7
- 18.4 Non-linear Partial Differential Equations of First Order 18.11
- 18.5 Charpit's Method 18.16
- 18.6 Homogeneous Linear Partial Differential Equations with Constant Coefficients 18.18
- 18.7 Non-homogeneous Linear Partial Differential Equations with Constant Coefficients 18.22
- 18.8 Cauchy Type Differential Equation 18.29
- 18.9 Non-linear Partial Differential Equations of Second Order: Monge's Method 18.30
- 18.10 Solution of Second Order P.D.E.: Miscellaneous 18.36

19. Application of Partial Differential Equations

- 19.1 Method of Separation of Variables 19.1
- 19.2 Classification of Partial Differential Equations of Second Order 19.2
- 19.3 Derivation of One-dimensional Heat Equation 19.3

- 19.4 Solution of One-dimensional Heat Equation *19.3*
- 19.5 Derivation of One-dimensional Wave Equation 19.12
- 19.6 Solution of One-dimensional Wave Equation by Separation of Variables 19.13
- 19.7 Laplace's Equation or Potential Equation or Two-dimensional Steady-state Heat Flow 19.17
- 19.8 Laplace Equation in Polar Coordinates 19.25
- 19.9 Derivation of Two-dimensional Heat Equation 19.30
- 19.10 Derivation of Two-dimensional Wave Equation 19.37
- 19.11 Vibrations of Circular Membrane 19.42
- 19.12 Transmission Line Equations 19.45

20. Fourier Integral, Fourier Transforms and Integral Transforms 20.1

- 20.1 Fourier Integral Theorem 20.1
- 20.2 Fourier Transform 20.3
- 20.3 Convolution 20.4
- 20.4 Finite Fourier Sine and Cosine Transforms 20.12
- 20.5 Parseval's Identity for Fourier Transforms 20.17

21. Linear Difference Equations and Z-Transforms

21.1

22.1

- 21.1 Linear Difference Equations 21.1
- 21.2 Z-transforms 21.12
- 21.3 Standard Z-transforms 21.20

PART—VI

COMPLEX ANALYSIS

22. Complex Function Theory

- 22.1 Complex Function 22.1
- 22.2 Continuity 22.2
- 22.3 Differentiability 22.2
- 22.4 Analyticity 22.3
- 22.5 Cauchy-Riemann (C-R) Equations: In Cartesian Coordinates 22.3
- 22.6 Harmonic and Conjugate Harmonic Functions 22.4
- 22.7 Cauchy-Riemann Equations: In Polar Coordinates 22.5

	22.8	Elementary Functions 22.14	
23.	Com	plex Integration 23.1	
	23.1	Line Integral in Complex Plane 23.1	
	23.2	Cauchy's Integral Theorem 23.6	
	23.3	Cauchy's Integral Formula 23.12	
	23.4	Derivative of Analytic Functions 23.14	
	23.5	Complex Sequence, Series and Power	
		Series 23.16	
	23.6	Taylor's Series (Theorem) 23.17	
	23.7	Laurent Series 23.21	
	23.8	Zeros and Poles 23.27	
24.	Theo	rv of Residues 24.1	
	24.1	Residue 24.1	
	24.2	Residue Theorem 24.2	
	24.3	Evaluation of Real Integrals 24.7	
	24.4	Argument Principle 24.17	
	24.5	Rouche's Theorem 24.19	
	24.6	Fundamental Theorem of Algebra 24.21	
	24.7	Liouville Theorem 24.21	
25.	Conf	ormal Mapping 25.1	
	25.1	Mapping (or Transformation or	
		Operator) 25.1	
	25.2	Conformal Mapping 25.1	
	25.3	Conformal Mapping by Elementary	
		Functions 25.2	
	25.4	Transformation: $w = z^n - 25.5$	

26. Probability

- 26.1 Review of Set Theory 26.1
- 26.2 Review of Counting 26.4
- 26.3 Introduction to Probability 26.6
- 26.4 Theorem of Total Probability (or the Rule of Elimination) 26.18
- 26.5 Bayes' Theorem (or Bayes' Rule) 26.20

27. Probability Distributions 27.1 27.1 Probability Distributions 27.1 27.2 Chebyshev's Theorem 27.3 27.3 Discrete Uniform Distribution 27.9 27.4 Binomial Distribution 27.11 27.5 Hypergeometric Distribution 27.15 27.6 Poisson Distribution 27.19 27.7 Poisson Process 27.24 27.8 Continuous Uniform Distribution 27.26 27.9 Normal Distribution 27.28 27.10 Normal Approximation to Binomial Distribution 27.39 27.11 Error Function 27.41 27.12 The Exponential Distribution 27.44 27.13 The Gamma Distribution 27.48 27.14 The Weibull Distribution 27.50 28. Sampling Distribution 28.1 28.1 Population and Sample 28.1 28.2 Sampling Distribution 28.2 28.3 Sampling Distribution of Means: (σ Known) 28.3 28.4 Sampling Distribution of Proportions 28.9 Sampling Distribution of Differences and 28.5 Sums 28.9 28.6 Sampling Distribution of Mean (σ Unknown): *t*-distribution 28.12 28.7 Chi-squared Distribution 28.15 28.8 Sampling Distribution of Variance s^2 28.16 28.9 F-Distribution 28.16 29. Estimation and Test of Hypothesis 29.1 Point Estimation 29.1 29.2 Interval Estimation 29.3 29.3 Bayesian Estimation 29.5 29.4 Test of Hypothesis 29.7 29.5 Test of Hypothesis Concerning Single Population Mean µ: (With Known Variance σ^2 : Large Sample) 29.9

- 29.6 Test of Hypothesis Concerning Two Means 29.13
- 29.7 Test for One Mean (Small Sample: T-distribution) 29.16
- 29.8 Small-sample Test Concerning Difference between Two Means 29.18
- 29.9 Paired-sample t-test 29.21

29.1

26.1

- Transformation: w = z25.5
- 25.5 Mapping $w = z^2$ 25.6
- 25.6 Transformation $w = e^z$ 25.9
- 25.7 Transformation $w = \sin z$ 25.11
- 25.8 Joukvowski's (Zhukovsky's) Transformation 25.15
- 25.9 Bilinear Transformation 25.15
- 25.10 Schwarz-Christoffel
 - Transformation 25.20

PART-VII

PROBABILITY AND STATISTICS

- 29.10 Test of Hypothesis: One Proportion: Small Samples 29.23
- 29.11 Test of Hypothesis: One Proportion: Large Sample 29.24
- 29.12 Test of Hypothesis: Two Proportions 29.26
- 29.13 Test of Hypothesis for Several Proportions 29.29
- 29.14 Analysis of $r \times c$ Tables (Contingency Tables) 29.31
- 29.15 Goodness of Fit Test 29.34
- 29.16 Estimation of Proportions 29.37

30. Curve Fitting, Regression and Correlation Analysis 30.1

- 30.1 Curve Fitting 30.1
- 30.2 Regression Analysis 30.4
- 30.3 Inferences Based on the Least Squares Estimation 30.5
- 30.4 Curvilinear (or Nonlinear) Regression 30.8
- 30.5 Curve Fitting by a Sum of Exponentials 30.11
- 30.6 Linear Weighted Least Squares Approximation 30.16
- 30.7 Non-linear Weighted Least Squares Approximation 30.19
- 30.8 Multiple Regression 30.22
- 30.9 Correlation Analysis 30.25
- 30.10 Rank Correlation or Spearman's Correlation 30.32
- 30.11 Correlation for Bivariate Frequency Distribution 30.34

31. Joint Probability Distribution and Markov 31.1

- Chains
- 31.1 Joint Probability Distribution 31.1
- 31.2 Markov Chains 31.7

PART-VIII NUMERICAL ANALYSIS

32. Numerical Analysis

- 32.1 Roots of Transcendental Equations 32.1
- 32.2 Finite Differences 32.7
- 32.3 Interpolation 32.12

- 32.4 Newton-Gregory Forward Interpolation Formula 32.13
- Central Differences 32.18 32.5
- 32.6 Stirling and Bessel's Interpolation Formulae 32.19
- 32.7 Lagrange's Interpolation 32.22
- 32.8 Inverse Interpolation Using Lagrange's Interpolation Formula 32.23
- 32.9 Divided Differences 32.26
- 32.10 Newton's Divided Differences Formula 32.27
- 32.11 Errors in Polynomial Interpolation 32.29
- 32.12 Symbolic Relations and Separation of Symbols 32.32
- 32.13 Numerical Differentiation 32.36
- 32.14 Numerical Integration 32.39
- 32.15 Spline Interpolation 32.47
- 32.16 Numerical Methods in Linear Algebra: Gauss-Seidel Method 32.53
- 32.17 Largest Eigen Value and the Corresponding Eigen Vector: By Power Method 32.55

33. Numerical Solutions of ODE and PDE 33.1

- 33.1 Numerical Solutions of First Order Ordinary Differential Equations 33.1
- 33.2 Picard's Method of Successive Approximation 33.7
- 33.3 Adams-Bashforth-Moulton Method (ABM Method) 33.9
- 33.4 Numerical Solutions to Partial Differential Equations 33.11
- Numerical Solution to One Dimensional 33.5 Heat Equation 33.12
- 33.6 Numerical Solution to One Dimensional Wave Equation 33.15
- 33.7 Numerical Solution to Two Dimensional Laplace Equation 33.17
- Appendix A: Statistical Tables A.1-A.28
- Appendix B: Basic Results B.1-B.5

Bibliography C.1-C.2

Index 1.1-1.18