# CHAPTER 2

## Solved Problems

**P.2.12** An investor wishes to choose the better of the two equally costly cashflow streams, namely, Annuity $X$ ($AX$) and Annuity $Y$ ($AY$). While $AX$ is an annuity due (i.e. cash flows occur at the beginning of the year) with a cash inflow of Rs 90,000 for each of 6 years, AY is an ordinary annuity (i.e. cash flows occur at the end of the year) with a cash inflow of Rs 1,00,000 for each of 6 years. Assuming 15 per cent return on investment (a) find the future value at the end of year 6 ($FVA_6$) for both $AX$ and $AY$ and (b) which annuity is more attractive?

### Solution

**(a)** Annuity $X$, $FAV_6$ = Rs 90,000 × FVIA (15.6 × 0.15)

$\qquad$ = Rs 90,000 × 8.754 × 1.15 = Rs 9,06,039

$\quad$ Annuity $Y$, $FVA_6$ = Rs 1,00,000 × $FVIFA_{(15,6)}$

$\qquad$ = Rs 1,00,000 × 8.754 = Rs 8,75,400

**(b)** $AX$ is more attractive as its $FVA_6$ is larger than that of $AY$. This is so because the benefit of receiving the cash flows of $AX$ at the beginning of the year more than offsets the fact that its flows are Rs 10,000 less than those of $AY$, which has year-end cash flows. The cash flows of $AX$ earn 15 per cent for an extra year, thereby enhancing its future value.

**P.2.13** You have a choice of accepting either of two 5-year cashflow streams or lump-sum amounts given below.

| End of year | Cash flow stream | |
| --- | --- | --- |
| | Alternative I | Alternative II |
| 1 | Rs 7,000 | Rs 11,000 |
| 2 | 7,000 | 9,000 |
| 3 | 7,000 | 7,000 |
| 4 | 7,000 | 5,000 |
| 5 | 7,000 | 3,000 |
| | Lump-sum amount | |
| At time zero ($t = 0$) | 28,250 | 28,000 |

Assuming 10 per cent required rate of return, which alternative (I or II) and in which form (Cash flow or lump-sum) would you prefer and why?

### Solution

**Alternative I:**

Cash flow stream (annuity):

$\qquad PVA_5$ = Rs 7,000 × $PVIFA_{(10,5)}$

$\qquad\qquad$ = Rs 7,000 × 3.791 = Rs 26,537

$\quad$ Lump sum = Rs 28,250

**Alternative II:**

**Cash flow stream (mixed stream)**

| Year ($n$) | Cash flow | $PVIF_{(10,n)}$ | Present value |
| --- | --- | --- | --- |
| | (1) | (2) | [(1) × (2)] (3) |
| 1 | Rs 11,000 | 0.909 | Rs 9,999 |
| 2 | 9,000 | 0.826 | 7,434 |
| 3 | 7,000 | 0.751 | 5,257 |
| 4 | 5,000 | 0.683 | 3,415 |
| 5 | 3,000 | 0.621 | 1,863 |
| | | | 27,968 |

Lump sum = Rs 28,000

**Conclusion:** Alternative I in the form of lump-sum payment is preferable as it has the largest present value.

**P.2.14** You wish to accumulate Rs 80,00,000 by the end of 5 years by making equal annual year-end deposits over the next 5 years. Assuming 7 per cent rate of return, how much should you deposit at the end of each year to accumulate Rs 8,00,000?

***Solution***   $FVA_n = X[FVIFA_{(r,n)}]$

$FVA_5 = \text{Rs } 80{,}000; \; FVIFA_{(7,5)} = 5.751$

$\text{Rs } 8{,}00{,}000 = 5.751 \; X$

$X = \text{Rs } 8{,}00{,}000 \div 5.751 = \text{Rs } 1{,}39{,}106.$

**P.2.15** Compute the future values of (1) an initial Rs 100 compounded annually for 10 years at 10 per cent and (2) an annuity of Rs 100 for 10 years at 10 per cent.

### Solution

(1) The future value of an investment compounded annually $= F_n = P(1 + i)^n = P \times FIVF_{i,n} = F_{10} = \text{Rs } 100$
$(1 + 0.10)^{10} = \text{Rs } 100 \, (2.5937) = \text{Rs } 259.4$

(2) The future value of an annuity $= S_n = A \times FVIFA_{i,n} = \text{Rs } 100 \times 15.937 = \text{Rs } 1{,}593.7.$

**P.2.16** An investor has two options to choose from: (a) Rs 6,000 after 1 year; (b) Rs 9,000 after 4 years. Assuming a discount rate of (i) 10 per cent and (ii) 20 per cent, which alternative should he opt for?

### Solution

(i) (a) Rs 6,000 after 1 year at 10 per cent discount $= P = \text{Rs } 6{,}000(0.9091) = \text{Rs } 5{,}454.6.$

(i) (b) Rs 9,000 after 4 years at 10 per cent discount $= P = \text{Rs } 9{,}000(0.6830) = \text{Rs } 6{,}147.$
   At 10 per cent required rate, the investor should choose Rs 9,000 after 4 years.

(ii) (a) Rs 6,000 after 1 year at 20 per cent discount $= P = \text{Rs } 6{,}000(0.8333) = \text{Rs } 4{,}999.8.$

(ii) (b) Rs 9,000 after 4 years at 20 per cent discount $= P = \text{Rs } 9{,}000(0.4823) = \text{Rs } 4{,}340.7.$
   At 20 per cent required rate, the investor should choose Rs 6,000 after 1 year.

**P.2.17** An investor is 50 years of age today. He will retire at the age of 60. In order to receive Rs 2,00,000 annually for 10 years after retirement, how much amount should he have at the time of retirement? Assume the required rate of return is 10 per cent.

### Solution $P_n$(present value of annuity) $= A \times PVIFA_{i,n} = P_{10} = \text{Rs } 2{,}00{,}000 \, (6.1446) = \text{Rs } 12{,}28{,}920.$

**P.2.18** A person would need Rs 100, 5 years from now. How much amount should he deposit each year in his bank account, if the yearly interest rate is 10 per cent?

### Solution $S_n = A \times FVIFA_{i,n}$ or $A = S_n/FVIFA_{i,n} = \text{Rs } 100/6.1051 = \text{Rs } 16.38$

**P.2.19** *X* has taken a 20-month car loan of Rs 6,00,000. The rate of interest is 12 per cent per annum. What will be the amount of monthly loan amortisation?

### Solution $A = \text{Rs } 6{,}00{,}000/PVIFA_{1,20} = \text{Rs } 6{,}00{,}000/18.0456 = \text{Rs } 33{,}249.1.$ Monthly interest $= 12$ per cent/12

$= 1$ per cent.

**P.2.20** ABC Ltd has borrowed Rs 1,000 to be repaid in equal instalments at the end of each of the next 3 years. The interest rate is 15 per cent. Prepare a amortisation schedule.

### Solution Amount of equal instalment, $A = P_n/PVIFA_{i,n} = \text{Rs } 1{,}000/2.2832 = \text{Rs } 437.98$

Amortisation schedule

| Year | Payment | Interest* | Repayment of principal | Balance outstanding |
|---|---|---|---|---|
| 1 | Rs 437.98 | Rs 150.00 | Rs 287.98 | Rs 712.02 |
| 2 | 437.98 | 106.80 | 331.18 | 380.84 |
| 3 | 437.98 | 57.13 | 380.85 | |

*= Loan balance at the beginning of the year × interest rate, e.g., year 1 = (Rs 1,000 × 0.15) = Rs 150.

**P.2.21** ABC Ltd has borrowed Rs 1,000 to be repaid in 12 monthly instalments of Rs 94.56. Compute the annual interest.

### Solution

$P_n = A \times PVIFA_{i,n}$

$PVIFA_{i,n} = P_n/A = \text{Rs } 1{,}000/\text{Rs } 94.56 = 10.5753$

According to Table A-4 (Appendix), a PVIFA of 10.5753 for 12 periods at interest (i) $= 2$ per cent. The annual interest rate is therefore $0.02 \times 12 = 24$ per cent.

## Review Questions

**2.23** Using the following information, determine the present value of the mixed stream of cash flows, assuming 5 per cent discount rate:

| Year | Cash flows |
|------|-----------|
| 1 | Rs 8,000 |
| 2 | 9,000 |
| 3 | 10,000 |
| 4 | 15,000 |
| 5 | 20,000 |

How much would you be willing to pay for an opportunity to buy this stream, assuming that you can at best earn (a) 5 per cent (b) 7 per cent on your investments?

**2.24** An insurance agent offers you an immediate retirement annuity which, for a lump-sum fee paid today, would provide you with Rs 2,00,000 per year for the next 20 years. You currently earn 8 per cent on low-risk investment compared to the retirement annuity. Ignoring taxes, what is the maximum amount you would pay for the annuity?

**2.25** Assuming two alternative options (a) Rs 40,000 at the end of each year of the next 20 years and (b) Rs 4,00,000 paid immediately, which alternative would you prefer, if you expect to earn 5 per cent annually ignoring taxes and other considerations? At approximately what earnings rate would you be indifferent in choosing between the two plans?

**2.26** To supplement your planned retirement after 30 years, you estimate that you need to accumulate Rs 22,00,000 by the end of 30 years from today. You plan to make equal annual end-of- year deposits into an account paying 6 per cent annual interest.
(a) What should be the annual deposit? (b) If you can afford to deposit Rs 60,000 per year, how much will you accumulate at the end of 30 years?

**2.27** *X* borrowed Rs 1,50,000 at 12 per cent annual rate of interest to be repaid over 3 years. The loan is amortised into three equal annual end-of year payments.
(a) Calculate the annual end-of-year loan payment.
(b) Prepare a loan amortisation schedule showing the interest and principal of each of the three loan repayments.

**2.28** You are given the series of cash flows shown below.

| Year | A | B | C |
|------|-----|-----|-----|
| | | Cash flows | |
| 1 | Rs 5,000 | Rs 15,000 | Rs 25,000 |
| 2 | 5,600 | 15,500 | 26,000 |
| 3 | 6,400 | 16,100 | 26,500 |
| 4 | 7,200 | 16,800 | 26,500 |
| 5 | 8,200 | 17,600 | 28,000 |
| 6 | — | 18,500 | 28,500 |
| 7 | — | 19,500 | 29,000 |
| 8 | — | 20,600 | — |
| 9 | — | 21,700 | — |
| 10 | — | 22,800 | — |

(a) Compute the compound annual growth rate associated with each cash flow stream.
(b) If year–1 values represents initial deposits in a savings account paying annual interest, what is the annual rate of interest earned on each account?
(c) Compare and discuss the growth rate found in (a) and interest rate found in (b).

**2.29** You have Rs 15,000 to invest. An investment that pays no stated interest but would return Rs 20,000 at the end of 3 years. What annual rate of return would you earn on your investments? Another investment, of equal risk, earns an annual return of 8 per cent. Which investment should you take? Why?

**2.30** What is the rate of return on an investment of Rs 10,60,000 if the company expects to receive Rs 2,00,000 each year for the next years 10 years?

**2.31** The information relating to the three loan alternatives to finance the purchase of a used car is summarised below.

| Loan | Principal | Annual payment | Term (year) |
|------|-----------|----------------|-------------|
| A | Rs 5,00,000 | Rs 1,35,281 | 5 |

| | | | |
|---|---|---|---|
| B | 5,00,000 | 1,54,321 | 4 |
| C | 5,00,000 | 2,01,045 | 3 |

Determine the interest associated with each of the loans. Which loan should be taken?

**2.32** Given the data below, determine for each of the perpetuities (a) the appropriate present value interest factor (PVIF) and (b) the present value.

| Perpetuity | Annual amount | Discount rate (%) |
|---|---|---|
| A | Rs 2,00,000 | 7 |
| B | 5,00,000 | 9 |
| C | 30,000 | 5 |
| D | 3,00,000 | 4 |

**2.33** Mr *X* will retire at the end of 10 years. Upon retirement he is entitled to receive an annual end-of-year payment of Rs 1,20,000 for 15 years. If he dies prior to the end of the 15-year period, his heirs would be entitled to the contractual payment.

His employers, Avon Industries Ltd, has to accumulate funds to provide a retirement annuity by making an equal annual year-end deposits into an account earning 7 per cent interest. When the 15-year payment period begins, the company would transfer the accumulated fund into an account earning a guaranteed 9 per cent. At the end of the payment period, the account balance would be zero.

Required:

1. How much would the company accumulate by the end of the year?
2. What should be the company's equal annual end-of-year deposit into the account be over the 10-year period to fund fully the retirement annuity?
3. How much would the company have to deposit annually during the 10-year accumulation period, if it could earn 8 per cent?
4. How much would the company have to deposit annually during the 10-year period of accumulation if the retirement annuity was a perpetuity?

**2.34** You have Rs 15,000 to invest today at 7 per cent compounded annually:

(a) How much would you have accumulated in the account at the end of 3 years, 6 years and 9 years?
(b) Using your findings in (a), calculate the amount of interest earned in the (1) first 3 years (years 1-3), (2) second 3 years (years 3-6) and (3) third 3 years (years 6-9).
(c) Explain why the interest earned in (b) increases in each succeeding 3 years.

**2.35** Using annual, semi-annual and quarterly compounding periods for each of the following (1) compute the future value if Rs 1,000 is initially deposited and (2) determine the effective rate of interest.

(a) At 10 per cent annual interest for 5 years
(b) At 8 per cent annual interest for 6 years
(c) At 6 per cent annual interest for 10 years

**2.36** You have just won a lottery that promises to pay you Rs 50 lakh exactly 5 years from today. You can sell the claim today for an immediate lump-sum cash payment. What is the minimum amount you would sell your claim for if you could earn the following rates on similar risk investments during the 5-year period: (1) 6 per cent (2) 9 per cent and (3) 12 per cent?