CHAPTER 3

Solved Problems

P.3.15 The risk-free rate is 6 per cent and the expected rate of return on the market portfolio is 16 per cent, with a standard deviation of 8 per cent. An aggressive investor is keen to earn 20 per cent return. Is it possible for a rational investor to achieve the target return? How? Explain the nature of risk-return trade-off for him and verify results.

Solution The capital market line (CML) shows that returns in excess of market portfolio can be obtained by creating a margined or leveraged portfolio, that is borrowing at the risk-free rate and investing the whole amount in the market portfolio. With borrowings, the weight of the market portfolio (risky asset) is taken as *w*, which is greater than one. Since the sum of the weights should be zero, the weight of risk-free asset (T-bills or any other asset like savings deposit) is one minus w (1 – *w*).

$$E(r_p) = w_a r_a + w_b r_b = w\,(16\%) + (1 - w)\,(6\%)$$
$$= 20 = 16w - 6w + 6$$
$$10w = 20 - 6 = 14$$
$$w = 14/10 = 1.4$$
$$1 - w = 1.0 - 1.4 = -0.4$$

Thus, the investor should borrow a sum equivalent to 40% of his owned funds at risk-free rate and invest the owned plus borrowed funds in the market portfolio. His risk return trade-off is implicit in the CML, which constitutes an efficient frontier. With risk-free lending (borrowing), the portfolio risk is simply the weight of risky asset times the standard deviation of the market portfolio. Thus, portfolio risk (σ_p) = $w\,\sigma_m$ = 1.4 × 8% = 11.2 per cent.

Verification: Using the CML, the portfolio return may be obtained using the formula

$$E(r_p) = r_f + [(r_m - r_f)/\sigma_m]\,\sigma_p$$
$$= 6 + [(16\text{–}6)/8] \times 11.2 = 6 + 14 = 20 \text{ per cent}$$

Alternatively,

Return on the market portfolio = $w \times r_m$ = 1.4 × 16% = 22.4 per cent

Interest cost of borrowings = $(1 - w) \times r_f$ = – 0.4 × 6% = 2.4 per cent

Net return from investment = 22.4% – 2.4% = 20 per cent

P.3.16 If the simple CAPM holds good, comment on the following situations.

(a)

Portfolio	*E(r)*	σ
Aries (%)	30	25
Taurus (%)	40	15

(b)

Portfolio	*E(r)*	σ
Risk-free asset (%)	10	0
Market	18	24
Libra	18.8	27

Solution

(a) Positive incremental return 10 per cent (40% – 30%) is available with lower risk (standard deviation). This violates the basic assumption of the CAPM. Between Aries and Taurus. Aries is dominated by Taurus.

(b) The equation of the capital market line (CML) as per the CAPM is:

$$E(r_r) = r_f + (\sigma\rho/\sigma_m)\,[E(r_m) - r_f]$$
$$= 10\% + (27/24)\,(18 - 10) = 10\% + 27/3\% = 19\%$$

The expected return on Libra is not commensurate with the total variability in returns (standard deviation). It is an inefficient portfolio and lies below the CML.

P.3.17 Risk-return features of two securities X and Y are:

Portfolio	*E(r)*	σ
X (%)	20	16
Y (%)	25	20

If the correlation coefficient between X and Y is 0.6, determine:

(a) Weights of X and Y, which would produce minimum portfolio risk (standard deviation), calculate expected return for these weights

(b) Portfolio risk and return, if weights are equal

(c) Portfolio risk and return, if weights are 3:1

(d) Portfolio risk and return, if weights are 1:3

Solution

(a) Weights that produce minimum variance in a 2 security portfolio may be obtained as:

$W^*_x = (\sigma^2_Y - COV_{xy})/(\sigma^2_x + \sigma^2_y - 2COV_{xy})$

where $COV_{xy} = \rho\ \sigma_x\ \sigma_y$

$COV_{xy} = 0.6 \times 16 \times 20 = 192$

$W^*_x = [(20)^2 - 192]/[(16)^2 + (20)^2 - 2.192] = (400 - 192)/(256 + 400 - 384)$

$= 208/272 = 0.765 = 76.5$ per cent

$W_y = 1 - W^*_x = 1–0.765 = 0.235 = 23.5$ per cent

$\sigma^2_p = (W_x\ \sigma_x)^2 \times (W_y\ \sigma_y)^2 + 2\ \sigma_x \sigma_y\ W_x\ W_y\ \rho$

$= (0.765 \times 16)^2 + (0.235 \times 20)^2 + 2\ (0.765)\ (0.235)\ 16 \times 20 \times 0.6)$

$= 149.82 + 22.09 + 69.03 = 240.94$ per cent

$\sigma\rho = 15.52$ per cent

$E\ (r_p) = w_x.r_x + w_y\ r_y = 0.765 \times 20\% + 0.235 \times 25\% = 15.3\% + 5.88\% = 21.18$ per cent

(b) $\sigma^2_p = (0.5 \times 16)^2 + (0.5 \times 20)^2 + 2\ (0.6 \times 16 \times 20)\ (0.5)\ (0.5) = 64 + 100 + 96 = 260$ per cent

$\sigma_p = 16.12$ per cent

$E\ (r_p) = 0.5 \times 20\% + 0.5 \times 25\% = 10\% + 12.5\% = 22.5$ per cent

(c) $\sigma^2_p = (0.75 \times 16)^2 + (0.25 \times 20)^2 + 2\ (0.6 \times 16 \times 20)\ (\ 0.75 \times 0.25)$

$= 144 + 25 + 72 = 241$ per cent

$\sigma_p = 15.52$ per cent

$E\ (r_p) = 0.75 \times 20\% \times 0.25 \times 25\% = 15\% + 6.25\% = 21.25$ per cent

(d) $\sigma^2_p = (0.25 \times 16)^2 + (0.75 \times 20)^2 + 2\ (0.6 \times 16 \times 20)\ (0.75 \times 0.25)$

$= 16 + 225 + 72 = 313$ per cent

$\sigma_p = 17.69$ per cent

$E\ (r_p) = 0.25 \times 20\% + 0.75 \times 25\% = 5\% + 18.75\% = 23.75$ per cent

Review Questions

3.18 The probability distribution of expected future returns are as follows:

Probability	*Return on shares (percentage)*	
	X	*Y*
0.1	(16)	(18)
0.2	2	12
0.4	8	18
0.2	12	32
0.1	20	40

Compute the (a) standard deviation of expected returns of each share, (b) coefficient of variation. Which share is more risky? Why?

3.19 The expected return (r) and standard deviation (s) of shares of X Ltd and Y Ltd are:

	r	σ
X Ltd	0.14	0.20
Y Ltd	0.09	0.30

Required: If the expected correlation between the two shares (ρ_{xy}) is (a) 0.1, (b)–1, compute the return and risk for each of the following portfolios:
(i) *X*, 100 per cent, (ii) *Y*, 100 per cent, (iii) *X*, 50 per cent–*Y*, 50 per cent.

3.20 The rate on T-bills (risk-free return, r_f) is currently 7.75 per cent, while the expected market return (r_m) is 14.25 per cent. Compute the required rate of return of each security listed below:

Security	*Beta*
X_1	1.5
X_2	1.2
X_3	1.0
X_4	0.9

3.21 Assume the following facts:
Risk-free return, r_f, 7.75 per cent
Beta, 2
Expected return of investors, *r*, 16 per cent
Applying CAPM, compute the expected market return (r_m).

3.22 The following facts are available:
r_m = 0.14
r_f = 0.0825
r = 0.18
Compute the beta coefficient (β).

3.23 Given the following information about the market r_m = 18 per cent, r_f = 6 per cent, σ_m = 30 per cent.
(a) Calculate the slope of the capital market line (CML).
(b) Calculate the expected return for three mutual funds, namely, Aries (σ = 15 per cent), Taurus (β = 18 per cent), Gemini (σ = 24 per cent).
State the assumptions necessary for the application of the model.

Answers

3.18 (a) Standard deviation 8.94 per cent (X), 14.98 per cent (Y).
(b) Coefficient of variation 1.4 (X), 0.82 (Y); Share X is more risky as its C.V. is higher

3.19 (a) (i) Return 14 per cent, Risk 20 per cent (ii) Return 9 per cent, Risk 30 per cent (iii) Return 11.5 per cent, Risk 18.84 per cent.
(b) (i) and (ii) are the same as in (a) (i) and (ii), (iii) 5 per cent.

3.20 Required rate of return 17.5 per cent (X1), 15.55 per cent (X2), 14.25 per cent (X3) and 13.60 per cent (X4).

3.21 Expected market return (rm)=11.87 per cent.

3.22 *Beta=1.7*

3.23 (a) Slope of the CML =0.4.
(b) Expected return of Aries (12 per cent), Taurus (13.2 per cent) and Gemini 15.6 per cent.