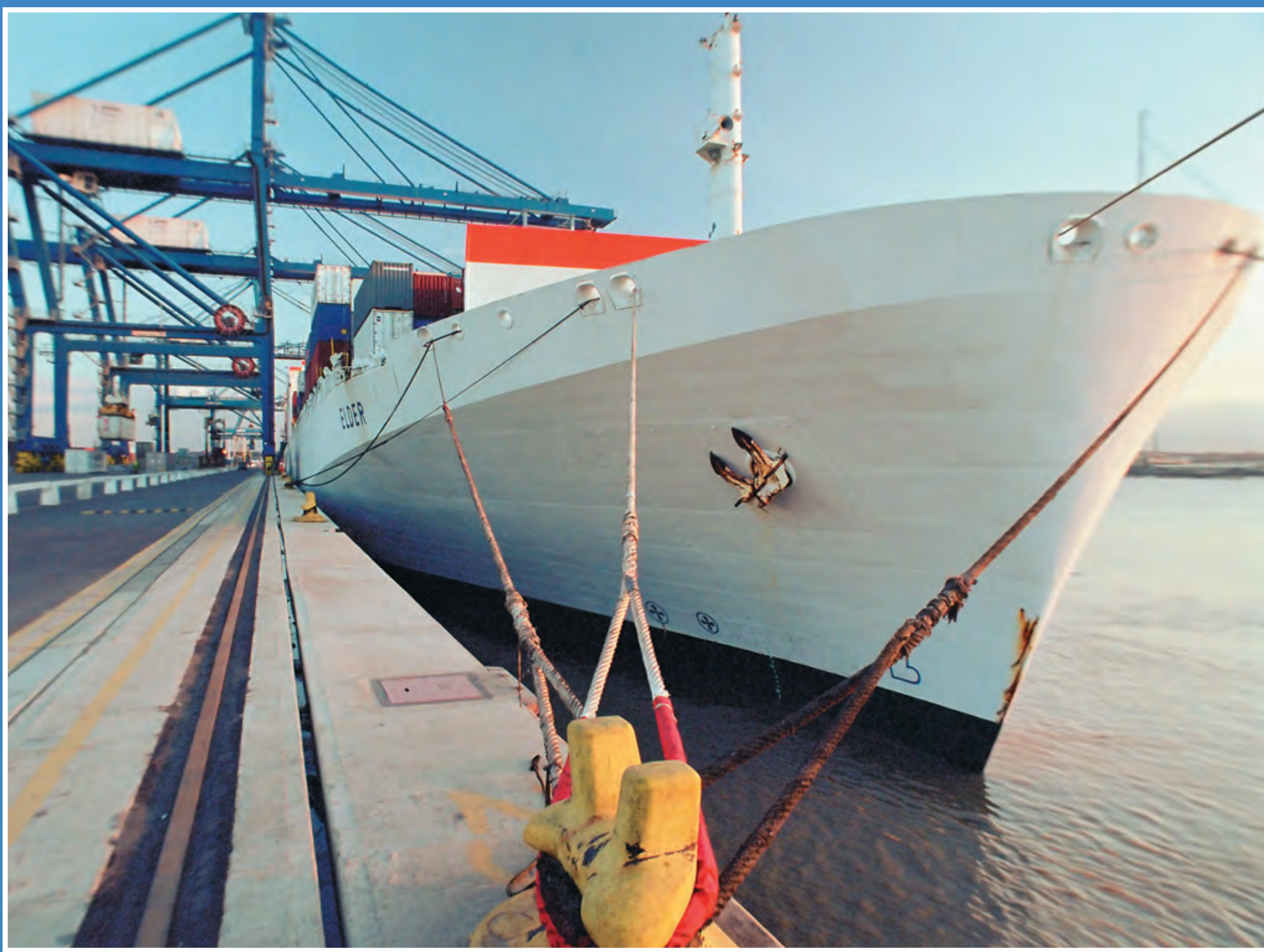
CHAPTER 2

Statics of Particles



Many engineering problems can be solved by considering the *equilibrium* of a *particle.* In this chapter you will learn that by treating the bollard as a particle, the relation among the tensions in the ropes can be obtained.

STATICS OF PARTICLES

2.1. INTRODUCTION

In this chapter you will study the effect of forces acting on particles. First you will learn how to replace two or more forces acting on a given particle by a single force having the same effect as the original forces. This single equivalent force is the *resultant* of the original forces acting on the particle. Later the relations which exist among the various forces acting on a particle in a state of *equilibrium* will be derived and used to determine some of the forces acting on the particle.

The use of the word *particle* does not imply that our study will be limited to that of small corpuscles. What it means is that the size and shape of the bodies under consideration will not significantly affect the solution of the problems treated in this chapter and that all the forces acting on a given body will be assumed to be applied at the same point. Since such an assumption is verified in many practical applications, you will be able to solve a number of engineering problems in this chapter.

The first part of the chapter is devoted to the study of forces contained in a single plane, and the second part to the analysis of forces in three-dimensional space.

FORCES IN A PLANE

2.2. FORCE ON A PARTICLE. RESULTANT OF TWO FORCES

A force represents the action of one body on another and is generally characterized by its *point of application,* its *magnitude,* and its *direction.* Forces acting on a given particle, however, have the same point of application. Each force considered in this chapter will thus be completely defined by its magnitude and direction.

The magnitude of a force is characterized by a certain number of units. As indicated in Chap. 1, the SI units used by engineers to measure the magnitude of a force are the newton (N) and its multiple the kilonewton (kN), equal to 1000 N, while the U.S. customary units used for the same purpose are the pound (lb) and its multiple the kilopound (kip), equal to 1000 lb. The direction of a force is defined by the *line of action* and the *sense* of the force. The line of action is the infinite straight line along which the force acts; it is characterized by the angle it forms with some fixed axis (Fig. 2.1).

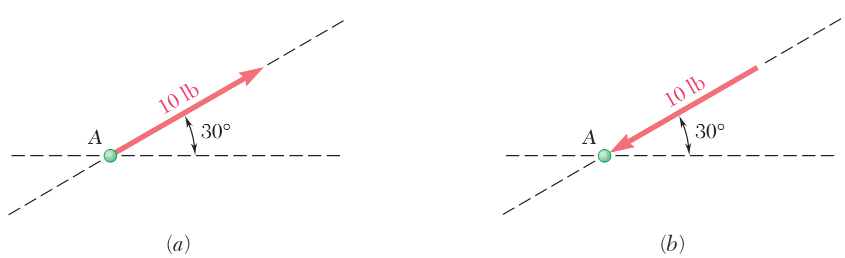


Fig. 2.1

bee76985_ch02_15-72 5/9/07 12:06 PM Page 17

The force itself is represented by a segment of that line; through the use of an appropriate scale, the length of this segment may be chosen to represent the magnitude of the force. Finally, the sense of the force should be indicated by an arrowhead. It is important in defining a force to indicate its sense. Two forces having the same magnitude and the same line of action but different sense, such as the forces shown in Fig. 2.1*a* and *b*, will have directly opposite effects on a particle.

Experimental evidence shows that two forces **P** and **Q** acting on a particle *A* (Fig. 2.2*a*) can be replaced by a single force **R** which has the same effect on the particle (Fig. 2.2*c*). This force is called the *resultant* of the forces **P** and **Q** and can be obtained, as shown in Fig. 2.2*b*, by constructing a parallelogram, using **P** and **Q** as two adjacent sides of the parallelogram. *The diagonal that passes through A represents the resultant.* This method for finding the resultant is known as the *parallelogram law* for the addition of two forces. This law is based on experimental evidence; it cannot be proved or derived mathematically.

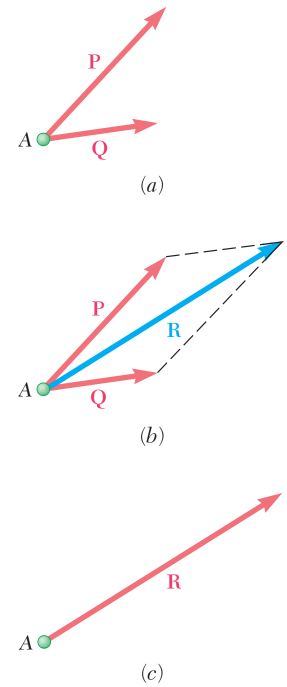


Fig. 2.2

2.3. VECTORS

It appears from the above that forces do not obey the rules of addition defined in ordinary arithmetic or algebra. For example, two forces acting at a right angle to each other, one of 4 lb and the other of 3 lb, add up to a force of 5 lb, *not* to a force of 7 lb. Forces are not the only quantities which follow the parallelogram law of addition. As you will see later, *displacements, velocities, accelerations,* and *momenta* are other examples of physical quantities possessing magnitude and direction that are added according to the parallelogram law. All these quantities can be represented mathematically by *vectors,* while those physical quantities which have magnitude but not direction, such as *volume, mass,* or *energy,* are represented by plain numbers or *scalars.*

Vectors are defined as *mathematical expressions possessing magnitude and direction, which add according to the parallelogram law.* Vectors are represented by arrows in the illustrations and will be distinguished from scalar quantities in this text through the use of boldface type (**P**). In longhand writing, a vector may be denoted by drawing a short arrow above the letter used to represent it ($\vec{P}$) or by underlining the letter ($\underline{P}$). The magnitude of a vector defines the length of the arrow used to represent the vector. In this text, italic type will be used to denote the magnitude of a vector. Thus, the magnitude of the vector **P** will be denoted by *P.*

A vector used to represent a force acting on a given particle has a well-defined point of application, namely, the particle itself. Such a vector is said to be a *fixed,* or *bound,* vector and cannot be moved without modifying the conditions of the problem. Other physical quantities, however, such as couples (see Chap. 3), are represented by vectors which may be freely moved in space; these vectors are called *free* vectors. Still other physical quantities, such as forces

bee76985_ch02_15-72 5/9/07 12:06 PM Page 18

acting on a rigid body (see Chap. 3), are represented by vectors which can be moved, or slid, along their lines of action; they are known as *sliding* vectors.†

Two vectors which have the same magnitude and the same direction are said to be *equal*, whether or not they also have the same point of application (Fig. 2.4); equal vectors may be denoted by the same letter.

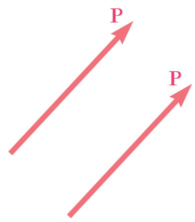


Fig. 2.4

The *negative vector* of a given vector **P** is defined as a vector having the same magnitude as **P** and a direction opposite to that of **P** (Fig. 2.5); the negative of the vector **P** is denoted by $-\mathbf{P}$. The vectors **P** and $-\mathbf{P}$ are commonly referred to as *equal and opposite* vectors. Clearly, we have

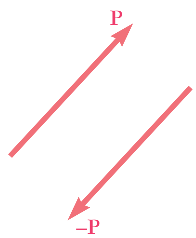


Fig. 2.5

$$\mathbf{P} + (-\mathbf{P}) = 0$$

2.4. ADDITION OF VECTORS

We saw in the preceding section that, by definition, vectors add according to the parallelogram law. Thus, the sum of two vectors **P** and **Q** is obtained by attaching the two vectors to the same point A and constructing a parallelogram, using **P** and **Q** as two sides of the parallelogram (Fig. 2.6). The diagonal that passes through A represents the sum of the vectors **P** and **Q**, and this sum is denoted by $\mathbf{P} + \mathbf{Q}$. The fact that the sign + is used to denote both vector and scalar addition should not cause any confusion if vector and scalar quantities are always carefully distinguished. Thus, we should note that the magnitude of the vector $\mathbf{P} + \mathbf{Q}$ is *not*, in general, equal to the sum $P + Q$ of the magnitudes of the vectors **P** and **Q**.

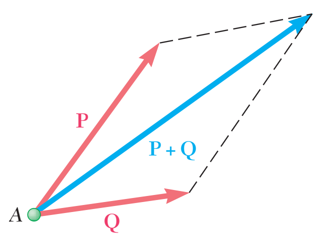


Fig. 2.6

Since the parallelogram constructed on the vectors **P** and **Q** does not depend upon the order in which **P** and **Q** are selected, we conclude that the addition of two vectors is *commutative*, and we write

$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P} \tag{2.1}$$

†Some expressions have magnitude and direction, but do not add according to the parallelogram law. While these expressions may be represented by arrows, they *cannot* be considered as vectors.

A group of such expressions is the finite rotations of a rigid body. Place a closed book on a table in front of you, so that it lies in the usual fashion, with its front cover up and its binding to the left. Now rotate it through 180° about an axis parallel to the binding (Fig. 2.3*a*); this rotation may be represented by an arrow of length equal to 180 units and oriented as shown. Picking up the book as it lies in its new position, rotate it now through

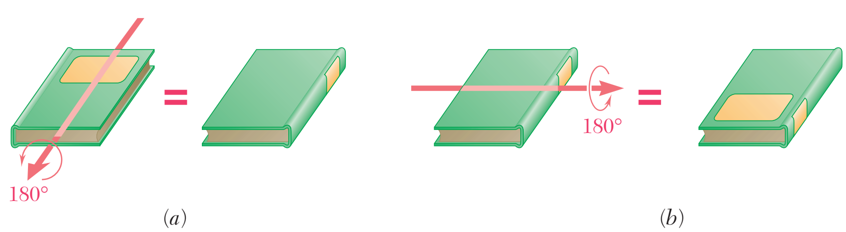


Fig. 2.3 Finite rotations of rigid body

bee76985_ch02_15-72 5/9/07 12:06 PM Page 19

From the parallelogram law, we can derive an alternative method for determining the sum of two vectors. This method, known as the *triangle rule,* is derived as follows. Consider Fig. 2.6, where the sum of the vectors **P** and **Q** has been determined by the parallelogram law. Since the side of the parallelogram opposite **Q** is equal to **Q** in magnitude and direction, we could draw only half of the parallelogram (Fig. 2.7*a*). The sum of the two vectors can thus be found by *arranging* **P** *and* **Q** *in tip-to-tail fashion and then connecting the tail of* **P** *with the tip of* **Q**. In Fig. 2.7*b*, the other half of the parallelogram is considered, and the same result is obtained. This confirms the fact that vector addition is commutative.

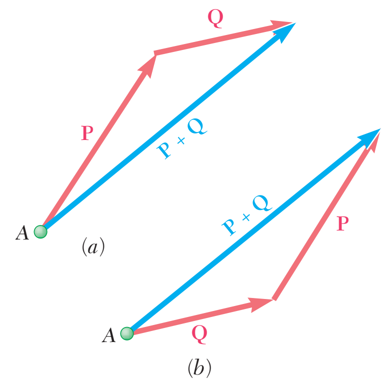


Fig. 2.7

The *subtraction* of a vector is defined as the addition of the corresponding negative vector. Thus, the vector $\mathbf{P} - \mathbf{Q}$ representing the difference between the vectors **P** and **Q** is obtained by adding to **P** the negative vector $-\mathbf{Q}$ (Fig. 2.8). We write

$$\mathbf{P} - \mathbf{Q} = \mathbf{P} + (-\mathbf{Q}) \tag{2.2}$$

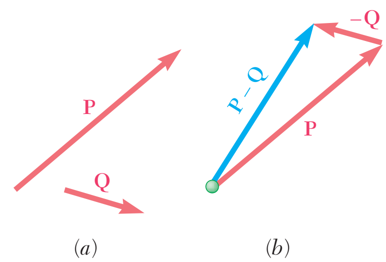


Fig. 2.8

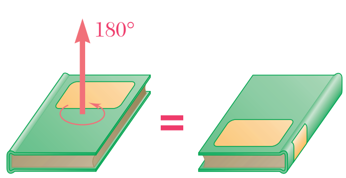
Here again we should observe that, while the same sign is used to denote both vector and scalar subtraction, confusion will be avoided if care is taken to distinguish between vector and scalar quantities.

We will now consider the *sum of three or more vectors.* The sum of three vectors **P**, **Q**, and **S** will, *by definition,* be obtained by first adding the vectors **P** and **Q** and then adding the vector **S** to the vector $\mathbf{P} + \mathbf{Q}$. We thus write

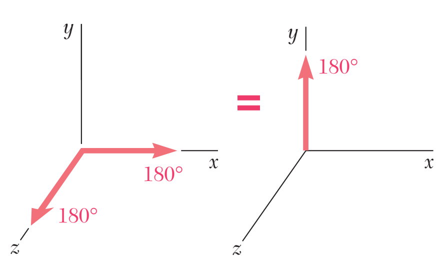
$$\mathbf{P} + \mathbf{Q} + \mathbf{S} = (\mathbf{P} + \mathbf{Q}) + \mathbf{S} \tag{2.3}$$

Similarly, the sum of four vectors will be obtained by adding the fourth vector to the sum of the first three. It follows that the sum of any number of vectors can be obtained by applying repeatedly the parallelogram law to successive pairs of vectors until all the given vectors are replaced by a single vector.

180° about a horizontal axis perpendicular to the binding (Fig. 2.3*b*); this second rotation may be represented by an arrow 180 units long and oriented as shown. But the book could have been placed in this final position through a single 180° rotation about a vertical axis (Fig. 2.3*c*). We conclude that the sum of the two 180° rotations represented by arrows directed respectively along the z and x axes is a 180° rotation represented by an arrow directed along the y axis (Fig. 2.3*d*). Clearly, the finite rotations of a rigid body *do not* obey the parallelogram law of addition; therefore, they *cannot* be represented by vectors.



(*c*)



(*d*)

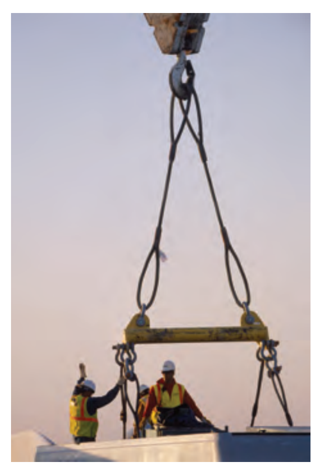


Photo 2.1 As we have shown, either the parallelogram law or the triangle rule can be used to determine the resultant force exerted by the two long cables on the hook.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 20

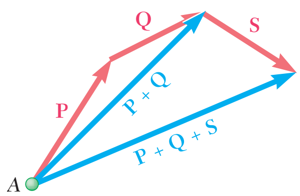


Fig. 2.9

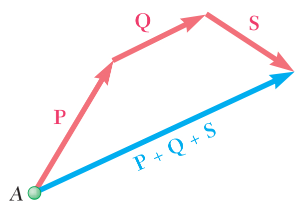


Fig. 2.10

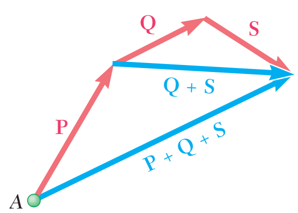


Fig. 2.11

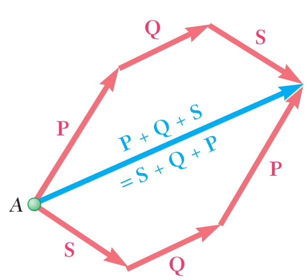


Fig. 2.12

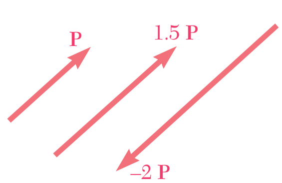


Fig. 2.13

If the given vectors are *coplanar,* that is, if they are contained in the same plane, their sum can be easily obtained graphically. For this case, the repeated application of the triangle rule is preferred to the application of the parallelogram law. In Fig. 2.9 the sum of three vectors $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{S}$ was obtained in that manner. The triangle rule was first applied to obtain the sum $\mathbf{P} + \mathbf{Q}$ of the vectors $\mathbf{P}$ and $\mathbf{Q}$; it was applied again to obtain the sum of the vectors $\mathbf{P} + \mathbf{Q}$ and $\mathbf{S}$. The determination of the vector $\mathbf{P} + \mathbf{Q}$, however, could have been omitted and the sum of the three vectors could have been obtained directly, as shown in Fig. 2.10, by *arranging the given vectors in tip-to-tail fashion and connecting the tail of the first vector with the tip of the last one.* This is known as the *polygon rule* for the addition of vectors.

We observe that the result obtained would have been unchanged if, as shown in Fig. 2.11, the vectors $\mathbf{Q}$ and $\mathbf{S}$ had been replaced by their sum $\mathbf{Q} + \mathbf{S}$. We may thus write

$$\mathbf{P} + \mathbf{Q} + \mathbf{S} = (\mathbf{P} + \mathbf{Q}) + \mathbf{S} = \mathbf{P} + (\mathbf{Q} + \mathbf{S}) \tag{2.4}$$

which expresses the fact that vector addition is *associative.* Recalling that vector addition has also been shown, in the case of two vectors, to be commutative, we write

$$\begin{aligned} \mathbf{P} + \mathbf{Q} + \mathbf{S} &= (\mathbf{P} + \mathbf{Q}) + \mathbf{S} = \mathbf{S} + (\mathbf{P} + \mathbf{Q}) \\ &= \mathbf{S} + (\mathbf{Q} + \mathbf{P}) = \mathbf{S} + \mathbf{Q} + \mathbf{P} \end{aligned} \tag{2.5}$$

This expression, as well as others which may be obtained in the same way, shows that the order in which several vectors are added together is immaterial (Fig. 2.12).

Product of a Scalar and a Vector. Since it is convenient to denote the sum $\mathbf{P} + \mathbf{P}$ by $2\mathbf{P}$, the sum $\mathbf{P} + \mathbf{P} + \mathbf{P}$ by $3\mathbf{P}$, and, in general, the sum of n equal vectors $\mathbf{P}$ by the product $n\mathbf{P}$, we will define the product $n\mathbf{P}$ of a positive integer n and a vector $\mathbf{P}$ as a vector having the same direction as $\mathbf{P}$ and the magnitude nP. Extending this definition to include all scalars, and recalling the definition of a negative vector given in Sec. 2.3, we define the product $k\mathbf{P}$ of a scalar k and a vector $\mathbf{P}$ as a vector having the same direction as $\mathbf{P}$ (if k is positive), or a direction opposite to that of $\mathbf{P}$ (if k is negative), and a magnitude equal to the product of P and of the absolute value of k (Fig. 2.13).

2.5. RESULTANT OF SEVERAL CONCURRENT FORCES

Consider a particle A acted upon by several coplanar forces, that is, by several forces contained in the same plane (Fig. 2.14*a*). Since the forces considered here all pass through A, they are also said to be *concurrent.* The vectors representing the forces acting on A may be added by the polygon rule (Fig. 2.14*b*). Since the use of the polygon rule is equivalent to the repeated application of the parallelogram law, the vector $\mathbf{R}$ thus obtained represents the resultant of the given concurrent forces, that is, the single force which has the same effect on the particle A as the given forces. As indicated above, the order in which the vectors $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{S}$ representing the given forces are added together is immaterial.

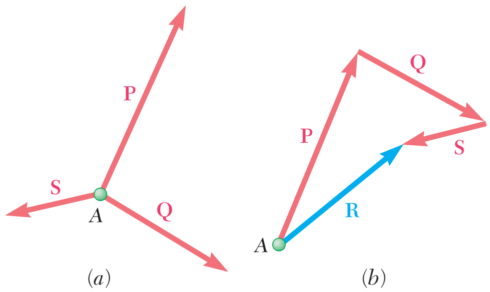


Fig. 2.14

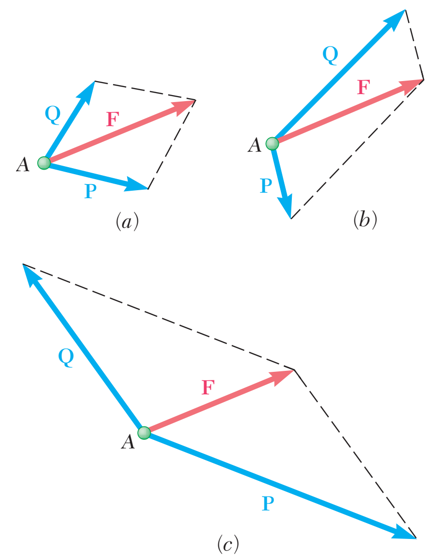


Fig. 2.15

2.6. RESOLUTION OF A FORCE INTO COMPONENTS

We have seen that two or more forces acting on a particle may be replaced by a single force which has the same effect on the particle. Conversely, a single force **F** acting on a particle may be replaced by two or more forces which, together, have the same effect on the particle. These forces are called the *components* of the original force **F**, and the process of substituting them for **F** is called *resolving the force* **F** *into components.*

Clearly, for each force **F** there exist an infinite number of possible sets of components. Sets of *two components* **P** *and* **Q** are the most important as far as practical applications are concerned. But, even then, the number of ways in which a given force **F** may be resolved into two components is unlimited (Fig. 2.15). Two cases are of particular interest:

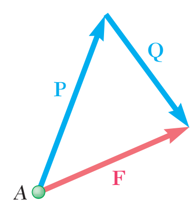


Fig. 2.16

1. *One of the Two Components,* **P**, *Is Known.* The second component, **Q**, is obtained by applying the triangle rule and joining the tip of **P** to the tip of **F** (Fig. 2.16); the magnitude and direction of **Q** are determined graphically or by trigonometry. Once **Q** has been determined, both components **P** and **Q** should be applied at *A*.
2. *The Line of Action of Each Component Is Known.* The magnitude and sense of the components are obtained by applying the parallelogram law and drawing lines, through the tip of **F**, parallel to the given lines of action (Fig. 2.17). This process leads to two well-defined components, **P** and **Q**, which can be determined graphically or computed trigonometrically by applying the law of sines.

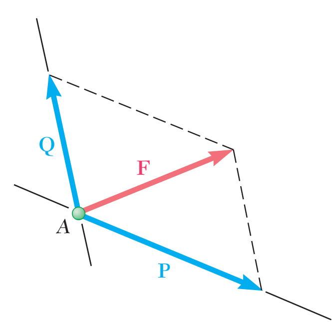
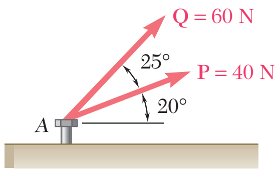


Fig. 2.17

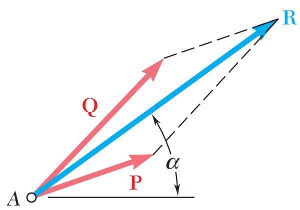
Many other cases can be encountered; for example, the direction of one component may be known, while the magnitude of the other component is to be as small as possible (see Sample Prob. 2.2). In all cases the appropriate triangle or parallelogram which satisfies the given conditions is drawn.



SAMPLE PROBLEM 2.1

The two forces **P** and **Q** act on a bolt *A*. Determine their resultant.

SOLUTION

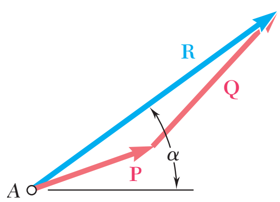


Graphical Solution. A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant are measured and found to be

$$R = 98\text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98\text{ N} \measuredangle 35° \quad ◀$$

The triangle rule may also be used. Forces **P** and **Q** are drawn in tip-to-tail fashion. Again the magnitude and direction of the resultant are measured.

$$R = 98\text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98\text{ N} \measuredangle 35° \quad ◀$$



Trigonometric Solution. The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.

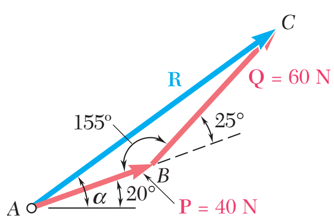
$$R^2 = P^2 + Q^2 - 2PQ \cos B$$
$$R^2 = (40\text{ N})^2 + (60\text{ N})^2 - 2(40\text{ N})(60\text{ N}) \cos 155°$$
$$R = 97.73\text{ N}$$

Now, applying the law of sines, we write

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60\text{ N}} = \frac{\sin 155°}{97.73\text{ N}} \tag{1}$$

Solving Eq. (1) for sin *A*, we have

$$\sin A = \frac{(60\text{ N}) \sin 155°}{97.73\text{ N}}$$

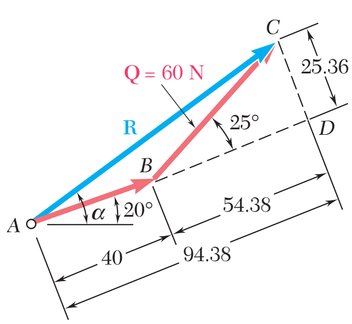


Using a calculator, we first compute the quotient, then its arc sine, and obtain

$$A = 15.04° \qquad \alpha = 20° + A = 35.04°$$

We use 3 significant figures to record the answer (see Sec. 1.6):

$$\mathbf{R} = 97.7\text{ N} \measuredangle 35.0° \quad ◀$$



Alternative Trigonometric Solution. We construct the right triangle *BCD* and compute

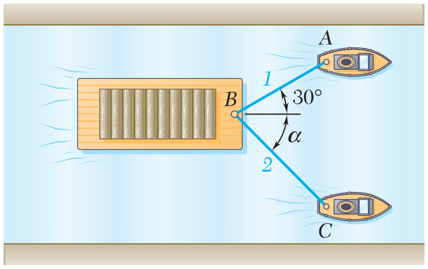
$$CD = (60\text{ N}) \sin 25° = 25.36\text{ N}$$
$$BD = (60\text{ N}) \cos 25° = 54.38\text{ N}$$

Then, using triangle *ACD*, we obtain

$$\tan A = \frac{25.36\text{ N}}{94.38\text{ N}} \qquad A = 15.04°$$
$$R = \frac{25.36}{\sin A} \qquad R = 97.73\text{ N}$$

Again, $\qquad \alpha = 20° + A = 35.04° \qquad \mathbf{R} = 97.7\text{ N} \measuredangle 35.0° \quad ◀$

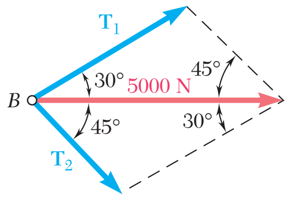
bee76985_ch02_15-72 5/9/07 12:06 PM Page 23



SAMPLE PROBLEM 2.2

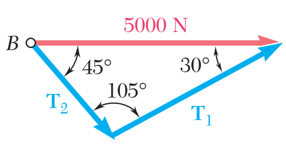
A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-N force directed along the axis of the barge, determine (*a*) the tension in each of the ropes knowing that $\alpha = 45°$, (*b*) the value of α for which the tension in rope 2 is minimum.

SOLUTION



a. Tension for $\alpha = 45°$. ***Graphical Solution.*** The parallelogram law is used; the diagonal (resultant) is known to be equal to 5000 lb and to be directed to the right. The sides are drawn parallel to the ropes. If the drawing is done to scale, we measure

$$T_1 = 3700 \text{ N} \qquad T_2 = 2600 \text{ N} \quad ◀$$

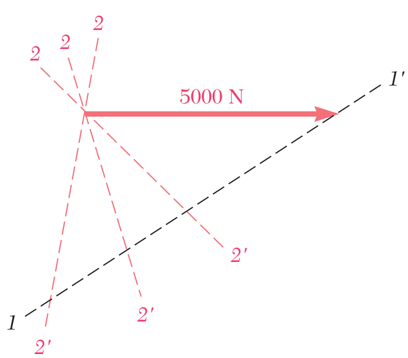


Trigonometric Solution. The triangle rule can be used. We note that the triangle shown represents half of the parallelogram shown above. Using the law of sines. we write

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000 \text{ N}}{\sin 105°}$$

With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by sin 45° and sin 30°, we obtain

$$T_1 = 3660 \text{ N} \qquad T_2 = 2590 \text{ N} \quad ◀$$



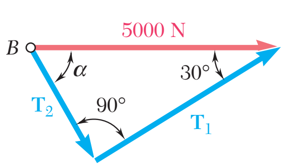
b. Value of α for Minimum T_2. To determine the value of α for which the tension in rope 2 is minimum, the triangle rule is again used. In the sketch shown, line *1-1′* is the known direction of $\mathbf{T}_1$. Several possible directions of $\mathbf{T}_2$ are shown by the lines *2-2′*. We note that the minimum value of T_2 occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular. The minimum value of T_2 is

$$T_2 = (5000 \text{ N}) \sin 30° = 2500 \text{ N}$$

Corresponding values of T_1 and α are

$$T_1 = (5000 \text{ N}) \cos 30° = 4330 \text{ N}$$

$$\alpha = 90° - 30° \qquad \alpha = 60° \quad ◀$$



SOLVING PROBLEMS ON YOUR OWN

The preceding sections were devoted to introducing and applying the *parallelogram law* for the addition of vectors.

You will now be asked to solve problems on your own. Some may resemble one of the sample problems; others may not. What all problems and sample problems in this section have in common is that they can be solved by the direct application of the parallelogram law.

Your solution of a given problem should consist of the following steps:

1. Identify which of the forces are the applied forces and which is the resultant. It is often helpful to write the vector equation which shows how the forces are related. For example, in Sample Prob. 2.1 we would have

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

You should keep that relation in mind as you formulate the next part of your solution.

2. Draw a parallelogram with the applied forces as two adjacent sides and the resultant as the included diagonal (Fig. 2.2). Alternatively, you can *use the triangle rule,* with the applied forces drawn in tip-to-tail fashion and the resultant extending from the tail of the first vector to the tip of the second (Fig. 2.7).

3. Indicate all dimensions. Using one of the triangles of the parallelogram, or the triangle constructed according to the triangle rule, indicate all dimensions—whether sides or angles—and determine the unknown dimensions either graphically or by trigonometry. If you use trigonometry, remember that the law of cosines should be applied first if two sides and the included angle are known [Sample Prob. 2.1], and the law of sines should be applied first if one side and all angles are known [Sample Prob. 2.2].

As is evident from the figures of Sec. 2.6, the two components of a force need not be perpendicular. Thus, when asked to resolve a force into two components, it is essential that you align the two adjacent sides of your parallelogram with the specified lines of action of the components.

If you have had prior exposure to mechanics, you might be tempted to ignore the solution techniques of this lesson in favor of resolving the forces into rectangular components. While this latter method is important and will be considered in the next section, use of the parallelogram law simplifies the solution of many problems and should be mastered at this time.

Problems†

2.1 Two forces **P** and **Q** are applied as shown at point A of a hook support. Knowing that $P = 15$ N and $Q = 25$ N, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

2.2 Two forces **P** and **Q** are applied as shown at point A of a hook support. Knowing that $P = 45$ N and $Q = 15$ N, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

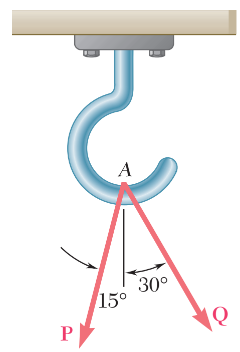


Fig. P2.1 and P2.2

2.3 Two forces are applied to an eye bolt fastened to a beam. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

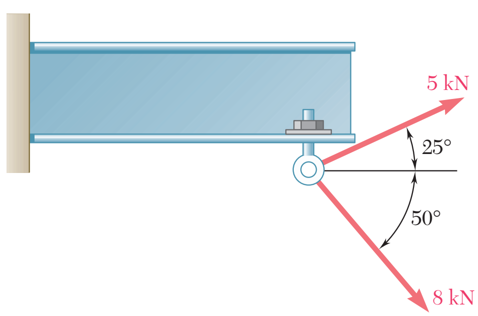


Fig. *P2.3*

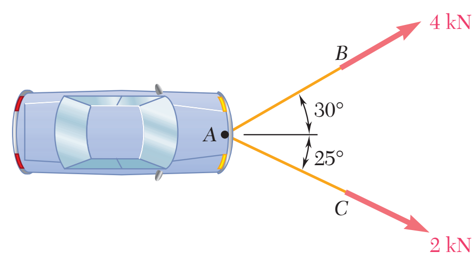


Fig. P2.4

2.4 A disabled automobile is pulled by means of ropes subjected to the two forces as shown. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

2.5 The 200-N force is to be resolved into components along lines a-a' and b-b'. (*a*) Determine the angle α using trigonometry knowing that the component along a-a' is to be 150 N. (*b*) What is the corresponding value of the component along b-b'?

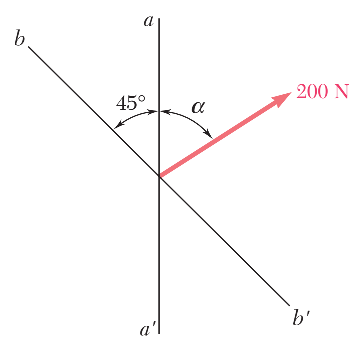


Fig. P2.5 and P2.6

2.6 The 200-N force is to be resolved into components along lines a-a' and b-b'. (*a*) Determine the angle α using trigonometry knowing that the component along b-b' is to be 120 N. (*b*) What is the corresponding value of the component along a-a'?

2.7 Two forces are applied as shown to a hook support. Using trigonometry and knowing that the magnitude of **P** is 600 N, determine (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be vertical, (*b*) the corresponding magnitude of **R**.

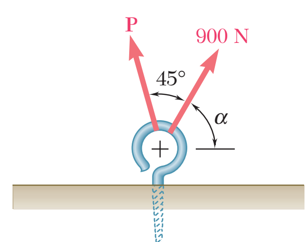


Fig. P2.7

†Answers to all problems set in straight type (such as **2.1**) are given at the end of the book. Answers to problems with a number set in italic type (such as **2.3**) are not given.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 26

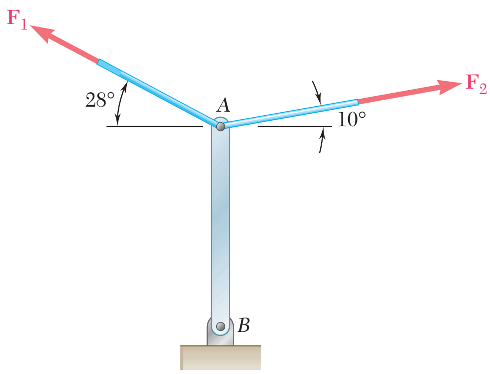


Fig. P2.8 and *P2.9*

2.8 Two control rods are attached at A to lever AB. Using trigonometry and knowing that the force in the left-hand rod is $F_1 = 30$ N, determine (a) the required force F_2 in the right-hand rod if the resultant $\mathbf{R}$ of the forces exerted by the rods on the lever is to be vertical, (b) the corresponding magnitude of $\mathbf{R}$.

2.9 Two control rods are attached at A to lever AB. Using trigonometry and knowing that the force in the right-hand rod is $F_2 = 20$ N, determine (a) the required force F_1 in the left-hand rod if the resultant $\mathbf{R}$ of the forces exerted by the rods on the lever is to be vertical, (b) the corresponding magnitude of $\mathbf{R}$.

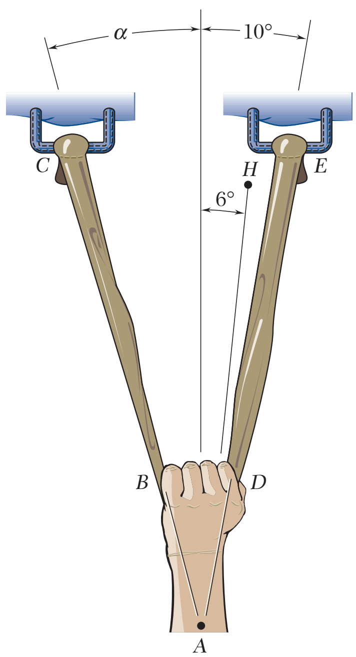


Fig. P2.10

2.10 An elastic exercise band is grasped and then is stretched as shown. Knowing that the tensions in portions BC and DE of the band are 80 N and 60 N, respectively, determine, using trigonometry, (a) the required angle α if the resultant $\mathbf{R}$ of the two forces exerted on the hand at A is to be vertical, (b) the corresponding magnitude of $\mathbf{R}$.

2.11 To steady a sign as it is being lowered, two cables are attached to the sign at A. Using trigonometry and knowing that $\alpha = 25°$, determine (a) the required magnitude of the force $\mathbf{P}$ if the resultant $\mathbf{R}$ of the two forces applied at A is to be vertical, (b) the corresponding magnitude of $\mathbf{R}$.

2.12 To steady a sign as it is being lowered, two cables are attached to the sign at A. Using trigonometry and knowing that the magnitude of $\mathbf{P}$ is 70 N, determine (a) the required angle α if the resultant $\mathbf{R}$ of the two forces applied at A is to be vertical, (b) the corresponding magnitude of $\mathbf{R}$.

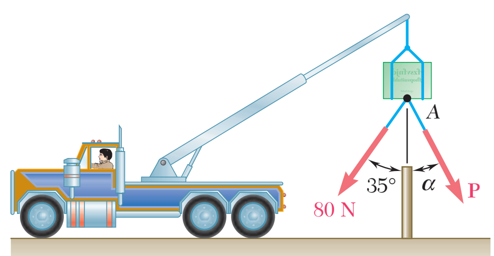


Fig. P2.11 and *P2.12*

2.13 As shown in Fig. P2.11, two cables are attached to a sign at A to steady the sign as it is being lowered. Using trigonometry, determine (a) the magnitude and direction of the smallest force $\mathbf{P}$ for which the resultant $\mathbf{R}$ of the two forces applied at A is vertical, (b) the corresponding magnitude of $\mathbf{R}$.

2.14 As shown in Fig. P2.10, an elastic exercise band is grasped and then is stretched. Knowing that the tension in portion DE of the band is 70 N, determine, using trigonometry, (a) the magnitude and direction of the smallest force in portion BC of the band for which the resultant $\mathbf{R}$ of the two forces exerted on the hand at A is directed along a line joining points A and H, (b) the corresponding magnitude of $\mathbf{R}$.

2.15 Solve Prob. 2.1 using trigonometry.

2.16 Solve Prob. 2.2 using trigonometry.

2.17 Solve Prob. 2.3 using trigonometry.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 27

2.18 For the hook support of Prob. 2.7, determine, using trigonometry, the magnitude and direction of the resultant of the two forces applied to the support knowing that $P = 500$ N and $\alpha = 60°$.

2.19 Two structural members A and B are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 30 kN in member A and 20 kN in member B, determine, using trigonometry, the magnitude and direction of the resultant of the forces applied to the bracket by members A and B.

2.20 Two structural members A and B are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 20 kN in member A and 30 kN in member B, determine, using trigonometry, the magnitude and direction of the resultant of the forces applied to the bracket by members A and B.

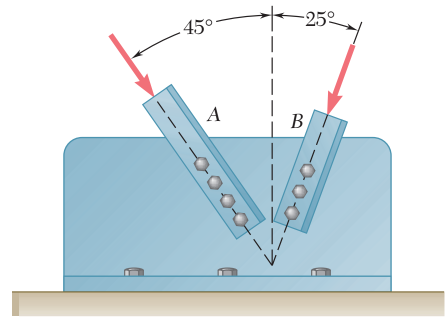


Fig. P2.19 and *P2.20*

2.7. RECTANGULAR COMPONENTS OF A FORCE. UNIT VECTORS†

In many problems it will be found desirable to resolve a force into two components which are perpendicular to each other. In Fig. 2.18, the force **F** has been resolved into a component $\mathbf{F}_x$ along the x axis and a component $\mathbf{F}_y$ along the y axis. The parallelogram drawn to obtain the two components is a *rectangle,* and $\mathbf{F}_x$ and $\mathbf{F}_y$ are called *rectangular components.*

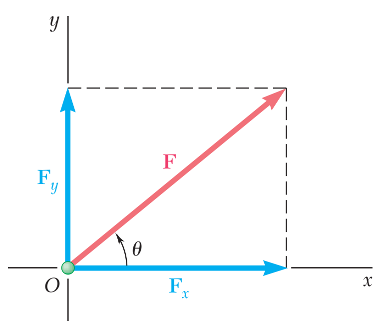


Fig. 2.18

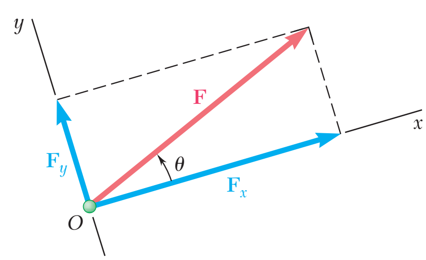


Fig. 2.19

The x and y axes are usually chosen horizontal and vertical, respectively, as in Fig. 2.18; they may, however, be chosen in any two perpendicular directions, as shown in Fig. 2.19. In determining the rectangular components of a force, the student should think of the construction lines shown in Figs. 2.18 and 2.19 as being *parallel* to the x and y axes, rather than *perpendicular* to these axes. This practice will help avoid mistakes in determining *oblique* components as in Sec. 2.6.

†The properties established in Secs. 2.7 and 2.8 may be readily extended to the rectangular components of any vector quantity.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 28

Two vectors of unit magnitude, directed respectively along the positive x and y axes, will be introduced at this point. These vectors are called *unit vectors* and are denoted by $\mathbf{i}$ and $\mathbf{j}$, respectively (Fig. 2.20). Recalling the definition of the product of a scalar and a vector given in Sec. 2.4, we note that the rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$ of a force $\mathbf{F}$ may be obtained by multiplying respectively the unit vectors $\mathbf{i}$ and $\mathbf{j}$ by appropriate scalars (Fig. 2.21). We write

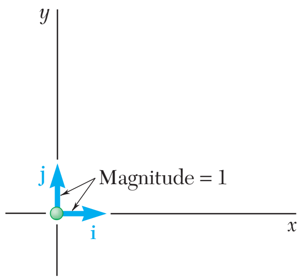


Fig. 2.20

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j} \tag{2.6}$$

and

$$\boxed{\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}} \tag{2.7}$$

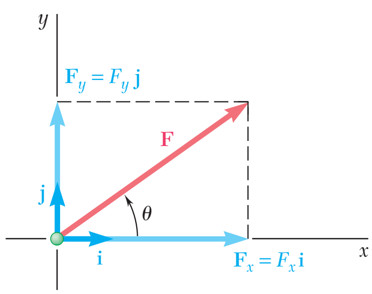


Fig. 2.21

While the scalars F_x and F_y may be positive or negative, depending upon the sense of $\mathbf{F}_x$ and of $\mathbf{F}_y$, their absolute values are respectively equal to the magnitudes of the component forces $\mathbf{F}_x$ and $\mathbf{F}_y$. The scalars F_x and F_y are called the *scalar components* of the force $\mathbf{F}$, while the actual component forces $\mathbf{F}_x$ and $\mathbf{F}_y$ should be referred to as the *vector components* of $\mathbf{F}$. However, when there exists no possibility of confusion, the vector as well as the scalar components of $\mathbf{F}$ may be referred to simply as the *components* of $\mathbf{F}$. We note that the scalar component F_x is positive when the vector component $\mathbf{F}_x$ has the same sense as the unit vector $\mathbf{i}$ (that is, the same sense as the positive x axis) and is negative when $\mathbf{F}_x$ has the opposite sense. A similar conclusion may be drawn regarding the sign of the scalar component F_y.

Denoting by F the magnitude of the force $\mathbf{F}$ and by θ the angle between $\mathbf{F}$ and the x axis, measured counterclockwise from the positive x axis (Fig. 2.21), we may express the scalar components of $\mathbf{F}$ as follows:

$$\boxed{F_x = F\cos\theta \qquad F_y = F\sin\theta} \tag{2.8}$$

We note that the relations obtained hold for any value of the angle θ from 0° to 360° and that they define the signs as well as the absolute values of the scalar components F_x and F_y.

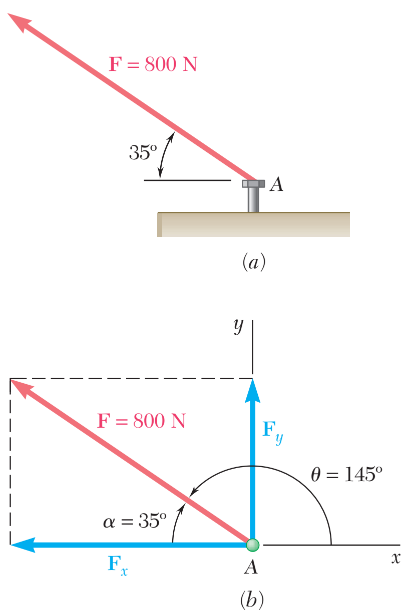


Fig. 2.22

Example 1. A force of 800 N is exerted on a bolt A as shown in Fig. 2.22*a*. Determine the horizontal and vertical components of the force.

In order to obtain the correct sign for the scalar components F_x and F_y, the value 180° − 35° = 145° should be substituted for θ in Eqs. (2.8). However, it will be found more practical to determine by inspection the signs of F_x and F_y (Fig. 2.22*b*) and to use the trigonometric functions of the angle $\alpha = 35°$. We write, therefore,

$$F_x = -F\cos\alpha = -(800\text{ N})\cos 35° = -655\text{ N}$$
$$F_y = +F\sin\alpha = +(800\text{ N})\sin 35° = +459\text{ N}$$

The vector components of $\mathbf{F}$ are thus

$$\mathbf{F}_x = -(655\text{ N})\mathbf{i} \qquad \mathbf{F}_y = +(459\text{ N})\mathbf{j}$$

and we may write $\mathbf{F}$ in the form

$$\mathbf{F} = -(655\text{ N})\mathbf{i} + (459\text{ N})\mathbf{j}$$

bee76985_ch02_15-72 5/9/07 12:06 PM Page 29

Example 2. A man pulls with a force of 300 N on a rope attached to a building, as shown in Fig. 2.23*a*. What are the horizontal and vertical components of the force exerted by the rope at point *A*?

It is seen from Fig. 2.23*b* that

$$F_x = +(300 \text{ N}) \cos \alpha \qquad F_y = -(300 \text{ N}) \sin \alpha$$

Observing that $AB = 10$ m, we find from Fig. 2.23*a*

$$\cos \alpha = \frac{8 \text{ m}}{AB} = \frac{8 \text{ m}}{10 \text{ m}} = \frac{4}{5} \qquad \sin \alpha = \frac{6 \text{ m}}{AB} = \frac{6 \text{ m}}{10 \text{ m}} = \frac{3}{5}$$

We thus obtain

$$F_x = +(300 \text{ N})\tfrac{4}{5} = +240 \text{ N} \qquad F_y = -(300 \text{ N})\tfrac{3}{5} = -180 \text{ N}$$

and write

$$\mathbf{F} = (240 \text{ N})\mathbf{i} - (180 \text{ N})\mathbf{j}$$

When a force **F** is defined by its rectangular components F_x and F_y (see Fig. 2.21), the angle θ defining its direction can be obtained by writing

$$\tan \theta = \frac{F_y}{F_x} \qquad (2.9)$$

The magnitude F of the force can be obtained by applying the Pythagorean theorem and writing

$$F = \sqrt{F_x^2 + F_y^2} \qquad (2.10)$$

or by solving for F one of the Eqs. (2.8).

Example 3. A force $\mathbf{F} = (700 \text{ N})\mathbf{i} + (1500 \text{ N})\mathbf{j}$ is applied to a bolt *A*. Determine the magnitude of the force and the angle θ it forms with the horizontal.

First we draw a diagram showing the two rectangular components of the force and the angle θ (Fig. 2.24). From Eq. (2.9), we write

$$\tan \theta = \frac{F_y}{F_x} = \frac{1500 \text{ N}}{700 \text{ N}}$$

Using a calculator,† we enter 1500 N and divide by 700 N; computing the arc tangent of the quotient, we obtain $\theta = 65.0°$. Solving the second of Eqs. (2.8) for F, we have

$$F = \frac{F_y}{\sin \theta} = \frac{1500 \text{ N}}{\sin 65.0°} = 1655 \text{ N}$$

The last calculation is facilitated if the value of F_y is stored when originally entered; it may then be recalled to be divided by $\sin \theta$.

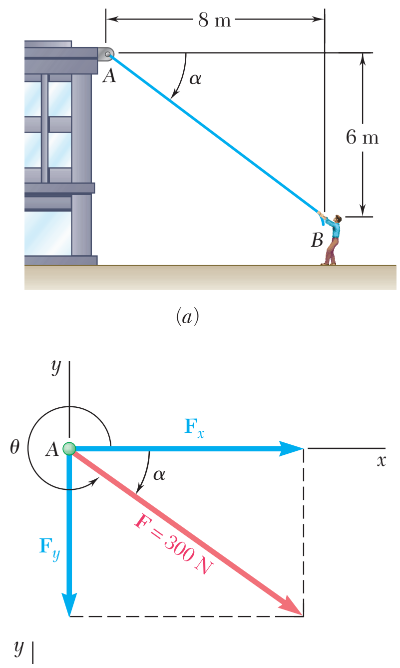


Fig. 2.23

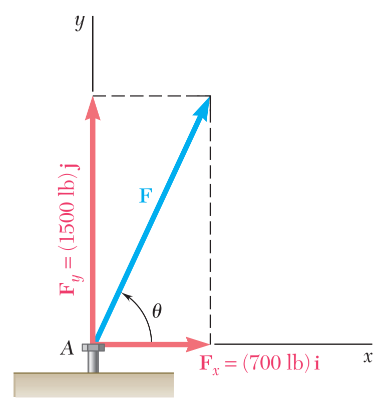


Fig. 2.24

†It is assumed that the calculator used has keys for the computation of trigonometric and inverse trigonometric functions. Some calculators also have keys for the direct conversion of rectangular coordinates into polar coordinates, and vice versa. Such calculators eliminate the need for the computation of trigonometric functions in Examples 1, 2, and 3 and in problems of the same type.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 30

2.8. ADDITION OF FORCES BY SUMMING *X* AND *Y* COMPONENTS

It was seen in Sec. 2.2 that forces should be added according to the parallelogram law. From this law, two other methods, more readily applicable to the *graphical* solution of problems, were derived in Secs. 2.4 and 2.5: the triangle rule for the addition of two forces and the polygon rule for the addition of three or more forces. It was also seen that the force triangle used to define the resultant of two forces could be used to obtain a *trigonometric* solution.

When three or more forces are to be added, no practical trigonometric solution can be obtained from the force polygon which defines the resultant of the forces. In this case, an *analytic* solution of the problem can be obtained by resolving each force into two rectangular components. Consider, for instance, three forces **P**, **Q**, and **S** acting on a particle A (Fig. 2.25*a*). Their resultant **R** is defined by the relation

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{S} \tag{2.11}$$

Resolving each force into its rectangular components, we write

$$\begin{aligned} R_x\mathbf{i} + R_y\mathbf{j} &= P_x\mathbf{i} + P_y\mathbf{j} + Q_x\mathbf{i} + Q_y\mathbf{j} + S_x\mathbf{i} + S_y\mathbf{j} \\ &= (P_x + Q_x + S_x)\mathbf{i} + (P_y + Q_y + S_y)\mathbf{j} \end{aligned}$$

from which it follows that

$$R_x = P_x + Q_x + S_x \qquad R_y = P_y + Q_y + S_y \tag{2.12}$$

or, for short,

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \tag{2.13}$$

We thus conclude that *the scalar components R_x and R_y of the resultant* **R** *of several forces acting on a particle are obtained by adding algebraically the corresponding scalar components of the given forces.*†

In practice, the determination of the resultant **R** is carried out in three steps as illustrated in Fig. 2.25. First the given forces shown in Fig. 2.25*a* are resolved into their x and y components (Fig. 2.25*b*). Adding these components, we obtain the x and y components of **R** (Fig. 2.25*c*). Finally, the resultant $\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$ is determined by applying the parallelogram law (Fig. 2.25*d*). The procedure just described will be carried out most efficiently if the computations are arranged in a table. While it is the only practical analytic method for adding three or more forces, it is also often preferred to the trigonometric solution in the case of the addition of two forces.

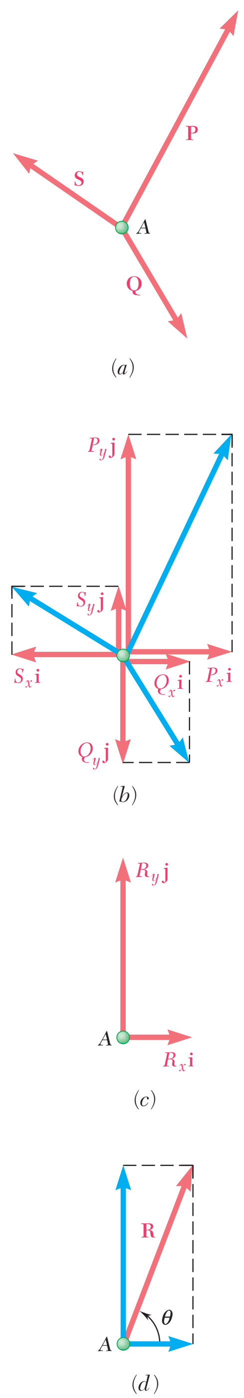
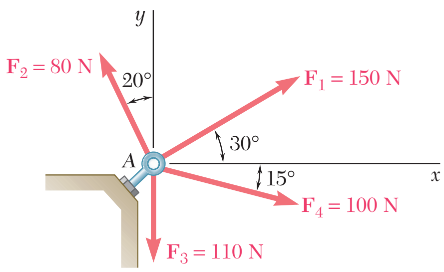


Fig. 2.25

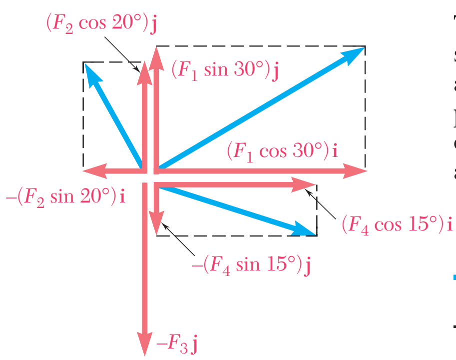
†Clearly, this result also applies to the addition of other vector quantities, such as velocities, accelerations, or momenta.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 31



SAMPLE PROBLEM 2.3

Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.



SOLUTION

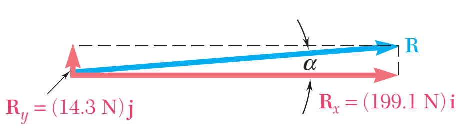
The x and y components of each force are determined by trigonometry as shown and are entered in the table below. According to the convention adopted in Sec. 2.7, the scalar number representing a force component is positive if the force component has the same sense as the corresponding coordinate axis. Thus, x components acting to the right and y components acting upward are represented by positive numbers.

Force	Magnitude, N	x Component, N	y Component, N
$\mathbf{F}_1$	150	$+129.9$	$+75.0$
$\mathbf{F}_2$	80	-27.4	$+75.2$
$\mathbf{F}_3$	110	0	-110.0
$\mathbf{F}_4$	100	$+96.6$	-25.9
		$R_x = +199.1$	$R_y = +14.3$

Thus, the resultant $\mathbf{R}$ of the four forces is

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (199.1\text{ N})\mathbf{i} + (14.3\text{ N})\mathbf{j} \quad ◀$$

The magnitude and direction of the resultant may now be determined. From the triangle shown, we have



$$\tan \alpha = \frac{R_y}{R_x} = \frac{14.3\text{ N}}{199.1\text{ N}} \qquad \alpha = 4.1°$$

$$R = \frac{14.3\text{ N}}{\sin \alpha} = 199.6\text{ N} \qquad \mathbf{R} = 199.6\text{ N} \measuredangle 4.1° \quad ◀$$

With a calculator, the last computation may be facilitated if the value of R_y is stored when originally entered; it may then be recalled to be divided by sin α. (Also see the footnote on p. 29.)

SOLVING PROBLEMS ON YOUR OWN

You saw in the preceding lesson that the resultant of two forces can be determined either graphically or from the trigonometry of an oblique triangle.

A. When three or more forces are involved, the determination of their resultant **R** is best carried out by first resolving each force into *rectangular components.* Two cases may be encountered, depending upon the way in which each of the given forces is defined:

Case 1. The force* F *is defined by its magnitude F and the angle α it forms with the x axis. The x and y components of the force can be obtained by multiplying F by cos α and sin α, respectively [Example 1].

Case 2. The force* F *is defined by its magnitude F and the coordinates of two points A and B on its line of action (Fig. 2.23). The angle α that **F** forms with the x axis may first be determined by trigonometry. However, the components of **F** may also be obtained directly from proportions among the various dimensions involved, without actually determining α [Example 2].

B. Rectangular components of the resultant. The components R_x and R_y of the resultant can be obtained by adding algebraically the corresponding components of the given forces [Sample Prob. 2.3].

You can express the resultant in *vectorial form* using the unit vectors **i** and **j**, which are directed along the x and y axes, respectively:

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

Alternatively, you can determine the *magnitude and direction* of the resultant by solving the right triangle of sides R_x and R_y for R and for the angle that **R** forms with the x axis.

In the examples and sample problem of this lesson, the x and y axes were horizontal and vertical, respectively. You should remember, however, that for some problems it will be more efficient to rotate the axes to align them with one or more of the applied forces.

Problems

2.21 Determine the x and y components of each of the forces shown.

2.22 Determine the x and y components of each of the forces shown.

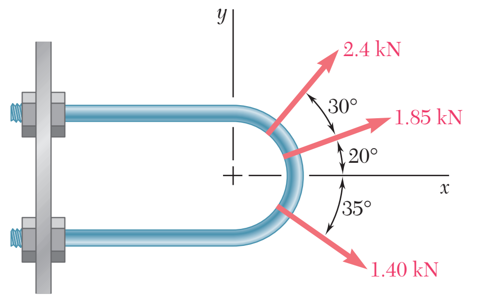


Fig. P2.21

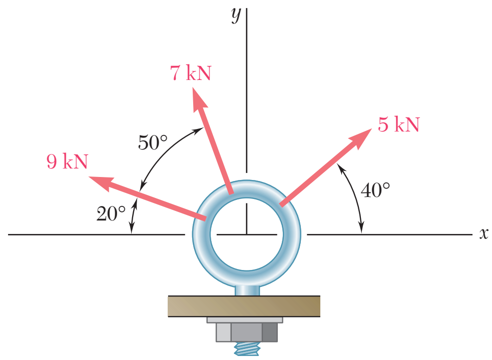


Fig. P2.22

***2.23* and 2.24** Determine the x and y components of each of the forces shown.

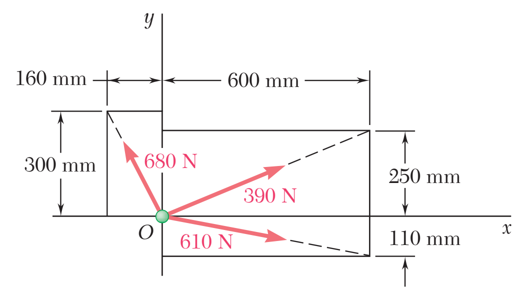


Fig. *P2.23*

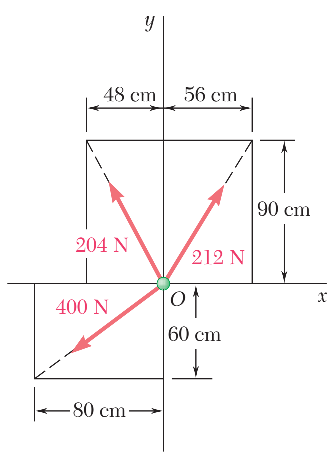


Fig. P2.24

2.25 Member BD exerts on member ABC a force $\mathbf{P}$ directed along line BD. Knowing that $\mathbf{P}$ must have a 960-N vertical component, determine (a) the magnitude of the force $\mathbf{P}$, (b) its horizontal component.

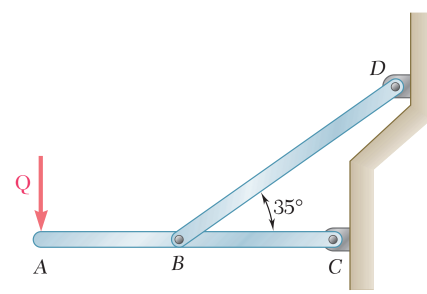


Fig. P2.25

bee76985_ch02_15-72 5/9/07 12:06 PM Page 34

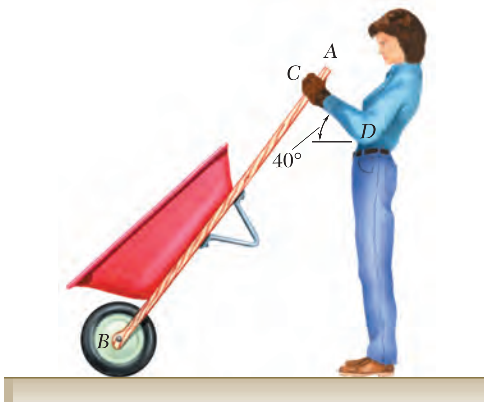


Fig. P2.26

2.26 While emptying a wheelbarrow, a gardener exerts on each handle *AB* a force **P** directed along line *CD*. Knowing that **P** must have a 30-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

2.27 Activator rod *AB* exerts on crank *BCD* a force **P** directed along line *AB*. Knowing that **P** must have a 100-N component perpendicular to arm *BC* of the crank, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *BC*.

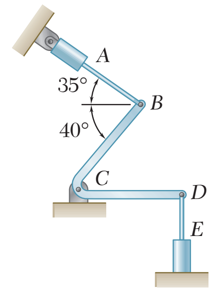


Fig. *P2.27*

2.28 Member *CB* of the vise shown exerts on block *B* a force **P** directed along line *CB*. Knowing that **P** must have a 260-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

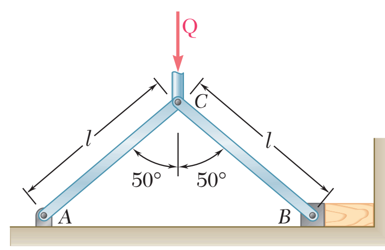


Fig. P2.28

Fig. P2.29 and *P2.30*

2.29 A window pole is used to open a window as shown. Knowing that the pole exerts on the window a force **P** directed along the pole and that the magnitude of the vertical component of **P** is 45 N, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

2.30 A window pole is used to open a window as shown. Knowing that the pole exerts on the window a force **P** directed along the pole and that the magnitude of the horizontal component of **P** is 18 N, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

2.31 Determine the resultant of the three forces of Prob. 2.21.

2.32 Determine the resultant of the three forces of Prob. 2.22.

2.33 Determine the resultant of the three forces of Prob. 2.24.

2.34 Determine the resultant of the three forces of Prob. 2.23.

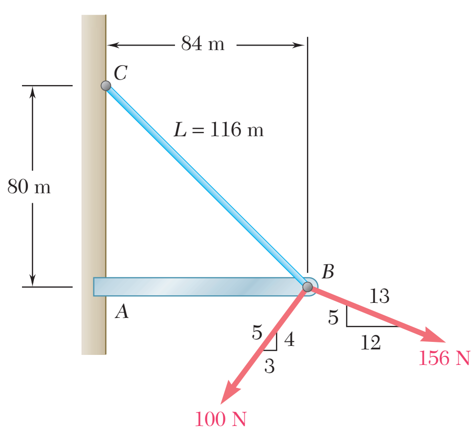


Fig. P2.35

2.35 Knowing that the tension in cable *BC* is 145 N, determine the resultant of the three forces exerted at point *B* of beam *AB*.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 35

2.36 A collar that can slide on a vertical rod is subjected to the three forces shown. Determine (*a*) the value of the angle α for which the resultant of the three forces is horizontal, (*b*) the corresponding magnitude of the resultant.

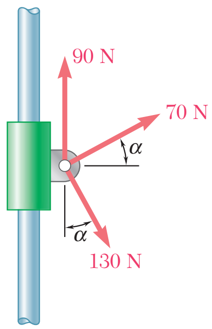


Fig. *P2.36*

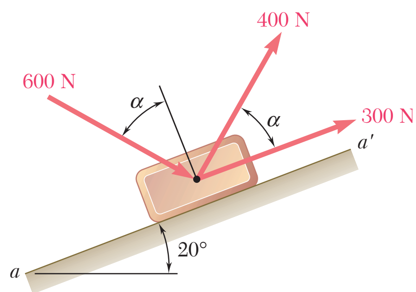


Fig. P2.37

2.37 Knowing that $\alpha = 65°$, determine the resultant of the three forces shown.

2.38 Knowing that $\alpha = 50°$, determine the resultant of the three forces shown.

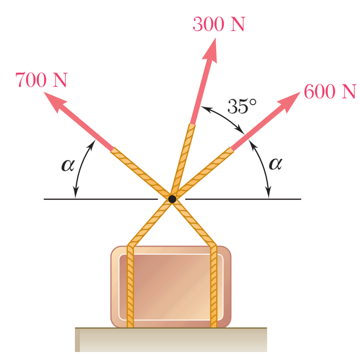


Fig. P2.38

2.39 For the beam of Prob. 2.35, determine (*a*) the required tension in cable *BC* if the resultant of the three forces exerted at point *B* is to be vertical, (*b*) the corresponding magnitude of the resultant.

2.40 For the three forces of Prob. 2.38, determine (*a*) the required value of α if the resultant is to be vertical, (*b*) the corresponding magnitude of the resultant.

2.41 For the block of Prob. 2.37, determine (*a*) the required value of α if the resultant of the three forces shown is to be parallel to the incline, (*b*) the corresponding magnitude of the resultant.

2.42 Boom *AB* is held in the position shown by three cables. Knowing that the tensions in cables *AC* and *AD* are 900 N and 1200 N, respectively, determine (*a*) the tension in cable *AE* if the resultant of the tensions exerted at point *A* of the boom must be directed along *AB*, (*b*) the corresponding magnitude of the resultant.

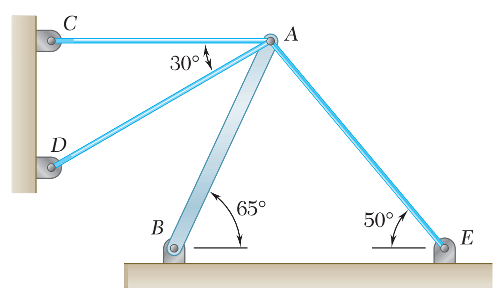


Fig. *P2.42*

2.9. EQUILIBRIUM OF A PARTICLE

In the preceding sections, we discussed the methods for determining the resultant of several forces acting on a particle. Although it has not occurred in any of the problems considered so far, it is quite possible for the resultant to be zero. In such a case, the net effect of the given forces is zero, and the particle is said to be in equilibrium. We thus have the following definition: *When the resultant of all the forces acting on a particle is zero, the particle is in equilibrium.*

A particle which is acted upon by two forces will be in equilibrium if the two forces have the same magnitude and the same line of action but opposite sense. The resultant of the two forces is then zero. Such a case is shown in Fig. 2.26.

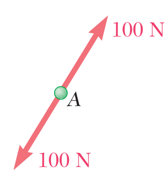


Fig. 2.26

bee76985_ch02_15-72 5/9/07 12:06 PM Page 36

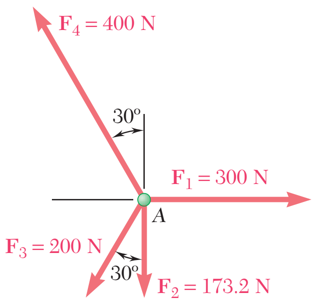


Fig. 2.27

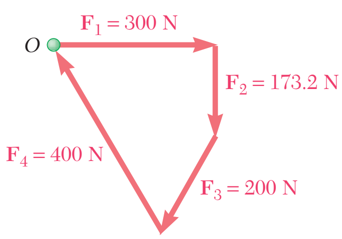


Fig. 2.28

Another case of equilibrium of a particle is represented in Fig. 2.27, where four forces are shown acting on A. In Fig. 2.28, the resultant of the given forces is determined by the polygon rule. Starting from point O with $\mathbf{F}_1$ and arranging the forces in tip-to-tail fashion, we find that the tip of $\mathbf{F}_4$ coincides with the starting point O. Thus the resultant $\mathbf{R}$ of the given system of forces is zero, and the particle is in equilibrium.

The closed polygon drawn in Fig. 2.28 provides a *graphical* expression of the equilibrium of A. To express *algebraically* the conditions for the equilibrium of a particle, we write

$$\mathbf{R} = \Sigma\mathbf{F} = 0 \tag{2.14}$$

Resolving each force $\mathbf{F}$ into rectangular components, we have

$$\Sigma(F_x\mathbf{i} + F_y\mathbf{j}) = 0 \qquad \text{or} \qquad (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} = 0$$

We conclude that the necessary and sufficient conditions for the equilibrium of a particle are

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \tag{2.15}$$

Returning to the particle shown in Fig. 2.27, we check that the equilibrium conditions are satisfied. We write

$$\begin{aligned}
\Sigma F_x &= 300 \text{ N} - (200 \text{ N}) \sin 30° - (400 \text{ N}) \sin 30° \\
&= 300 \text{ N} - 100 \text{ N} - 200 \text{ N} = 0 \\
\Sigma F_y &= -173.2 \text{ N} - (200 \text{ N}) \cos 30° + (400 \text{ N}) \cos 30° \\
&= -173.2 \text{ N} - 173.2 \text{ N} + 346.4 \text{ N} = 0
\end{aligned}$$

2.10. NEWTON'S FIRST LAW OF MOTION

In the latter part of the seventeenth century, Sir Isaac Newton formulated three fundamental laws upon which the science of mechanics is based. The first of these laws can be stated as follows:

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

From this law and from the definition of equilibrium given in Sec. 2.9, it is seen that a particle in equilibrium either is at rest or is moving in a straight line with constant speed. In the following section, various problems concerning the equilibrium of a particle will be considered.

2.11. PROBLEMS INVOLVING THE EQUILIBRIUM OF A PARTICLE. FREE-BODY DIAGRAMS

In practice, a problem in engineering mechanics is derived from an actual physical situation. A sketch showing the physical conditions of the problem is known as a *space diagram.*

The methods of analysis discussed in the preceding sections apply to a system of forces acting on a particle. A large number of problems involving actual structures, however, can be reduced to problems concerning the equilibrium of a particle. This is done by choosing a significant particle and drawing a separate diagram showing this particle and all the forces acting on it. Such a diagram is called a *free-body diagram.*

As an example, consider the 75-kg crate shown in the space diagram of Fig. 2.29*a.* This crate was lying between two buildings, and it is now being lifted onto a truck, which will remove it. The crate is supported by a vertical cable, which is joined at A to two ropes which pass over pulleys attached to the buildings at B and C. It is desired to determine the tension in each of the ropes AB and AC.

In order to solve this problem, a free-body diagram showing a particle in equilibrium must be drawn. Since we are interested in the rope tensions, the free-body diagram should include at least one of these tensions or, if possible, both tensions. Point A is seen to be a good free body for this problem. The free-body diagram of point A is shown in Fig. 2.29*b.* It shows point A and the forces exerted on A by the vertical cable and the two ropes. The force exerted by the cable is directed downward, and its magnitude is equal to the weight W of the crate. Recalling Eq. (1.4), we write

$$W = mg = (75 \text{ kg})(9.81 \text{ m/s}^2) = 736 \text{ N}$$

and indicate this value in the free-body diagram. The forces exerted by the two ropes are not known. Since they are respectively equal in magnitude to the tensions in rope AB and rope AC, we denote them by $\mathbf{T}_{AB}$ and $\mathbf{T}_{AC}$ and draw them away from A in the directions shown in the space diagram. No other detail is included in the free-body diagram.

Since point A is in equilibrium, the three forces acting on it must form a closed triangle when drawn in tip-to-tail fashion. This *force triangle* has been drawn in Fig. 2.29*c.* The values T_{AB} and T_{AC} of the tension in the ropes may be found graphically if the triangle is drawn to scale, or they may be found by trigonometry. If the latter method of solution is chosen, we use the law of sines and write

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{AC}}{\sin 40°} = \frac{736 \text{ N}}{\sin 80°}$$

$$T_{AB} = 647 \text{ N} \qquad T_{AC} = 480 \text{ N}$$

When a particle is in *equilibrium under three forces,* the problem can be solved by drawing a force triangle. When a particle is in *equilibrium under more than three forces,* the problem can be solved graphically by drawing a force polygon. If an analytic solution is desired, the *equations of equilibrium* given in Sec. 2.9 should be solved:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \tag{2.15}$$

These equations can be solved for no more than *two unknowns;* similarly, the force triangle used in the case of equilibrium under three forces can be solved for two unknowns.

The more common types of problems are those in which the two unknowns represent (1) the two components (or the magnitude and direction) of a single force, (2) the magnitudes of two forces, each of known direction. Problems involving the determination of the maximum or minimum value of the magnitude of a force are also encountered (for example, see Probs. 2.59 through 2.63).

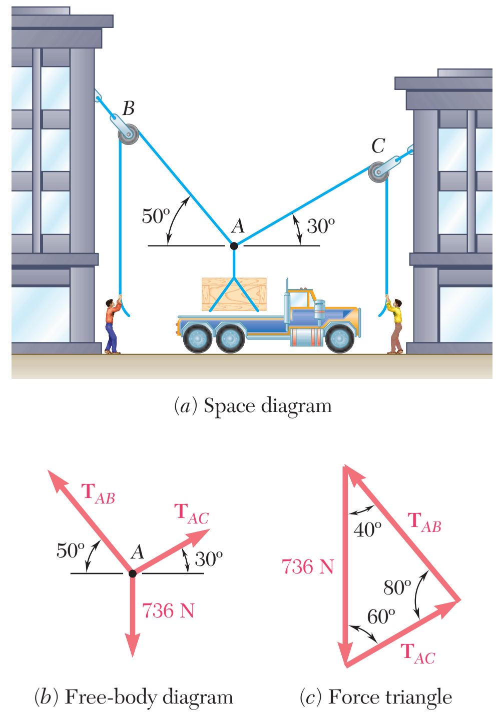
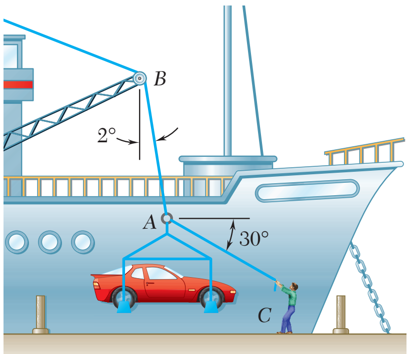


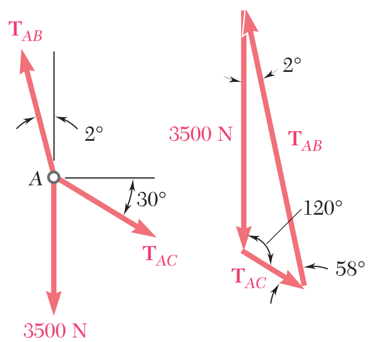
Fig. 2.29

Photo 2.2 As illustrated in the above example, it is possible to determine the tensions in the cables supporting the shaft shown by treating the hook as a particle and then applying the equations of equilibrium to the forces acting on the hook.



SAMPLE PROBLEM 2.4

In a ship-unloading operation, a 3500-N automobile is supported by a cable. A rope is tied to the cable at A and pulled in order to center the automobile over its intended position. The angle between the cable and the vertical is 2°, while the angle between the rope and the horizontal is 30°. What is the tension in the rope?



SOLUTION

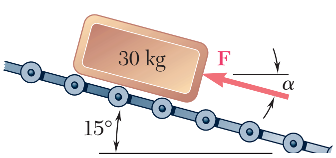
Free-Body Diagram. Point A is chosen as a free body, and the complete free-body diagram is drawn. T_{AB} is the tension in the cable AB, and T_{AC} is the tension in the rope.

Equilibrium Condition. Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Using the law of sines, we write

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{3500\text{ N}}{\sin 58°}$$

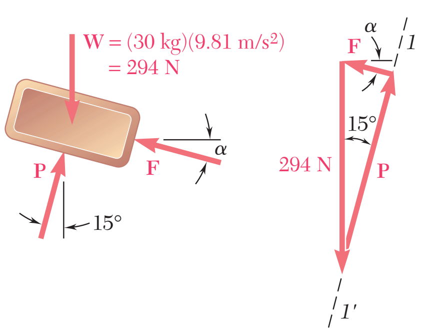
With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by sin 120° and sin 2°, we obtain

$$T_{AB} = 3570\text{ N} \qquad T_{AC} = 144\text{ N} \blacktriangleleft$$



SAMPLE PROBLEM 2.5

Determine the magnitude and direction of the smallest force **F** which will maintain the package shown in equilibrium. Note that the force exerted by the rollers on the package is perpendicular to the incline.



SOLUTION

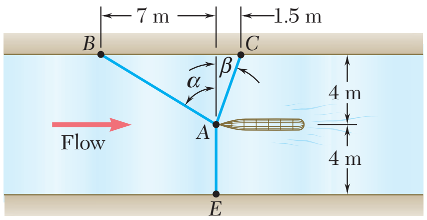
Free-Body Diagram. We choose the package as a free body, assuming that it can be treated as a particle. We draw the corresponding free-body diagram.

Equilibrium Condition. Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Line *1-1′* represents the known direction of **P**. In order to obtain the minimum value of the force **F**, we choose the direction of **F** perpendicular to that of **P**. From the geometry of the triangle obtained, we find

$$F = (294\text{ N}) \sin 15° = 76.1\text{ N} \qquad \alpha = 15°$$

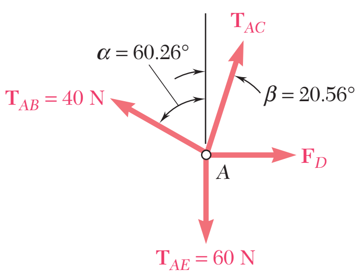
$$\mathbf{F} = 76.1\text{ N} \measuredangle 15° \blacktriangleleft$$

bee76985_ch02_15-72 5/9/07 12:06 PM Page 39



SAMPLE PROBLEM 2.6

As part of the design of a new sailboat, it is desired to determine the drag force which may be expected at a given speed. To do so, a model of the proposed hull is placed in a test channel and three cables are used to keep its bow on the centerline of the channel. Dynamometer readings indicate that for a given speed, the tension is 40 N in cable AB and 60 N in cable AE. Determine the drag force exerted on the hull and the tension in cable AC.



SOLUTION

Determination of the Angles. First, the angles α and β defining the direction of cables AB and AC are determined. We write

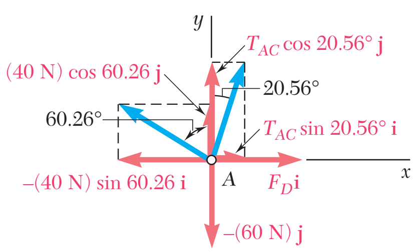
$$\tan\alpha = \frac{7\text{ m}}{4\text{ m}} = 1.75 \qquad \tan\beta = \frac{1.5\text{ m}}{4\text{ m}} = 0.375$$

$$\alpha = 60.26° \qquad \beta = 20.56°$$

Free-Body Diagram. Choosing the hull as a free body, we draw the free-body diagram shown. It includes the forces exerted by the three cables on the hull, as well as the drag force $\mathbf{F}_D$ exerted by the flow.

Equilibrium Condition. We express that the hull is in equilibrium by writing that the resultant of all forces is zero:

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AE} + \mathbf{F}_D = 0 \tag{1}$$



Since more than three forces are involved, we resolve the forces into x and y components:

$$\begin{aligned}
\mathbf{T}_{AB} &= -(40\text{ N})\sin 60.26°\mathbf{i} + (40\text{ N})\cos 60.26°\mathbf{j} \\
&= -(34.73\text{ N})\mathbf{i} + (19.84\text{ N})\mathbf{j} \\
\mathbf{T}_{AC} &= T_{AC}\sin 20.56°\mathbf{i} + T_{AC}\cos 20.56°\mathbf{j} \\
&= 0.3512T_{AC}\mathbf{i} + 0.9363T_{AC}\mathbf{j} \\
\mathbf{T}_{AE} &= -(60\text{ N})\mathbf{j} \\
\mathbf{F}_D &= F_D\mathbf{i}
\end{aligned}$$

Substituting the expressions obtained into Eq. (1) and factoring the unit vectors $\mathbf{i}$ and $\mathbf{j}$, we have

$$(-34.73\text{ N} + 0.3512T_{AC} + F_D)\mathbf{i} + (19.84\text{ N} + 0.9363T_{AC} - 60\text{ N})\mathbf{j} = 0$$

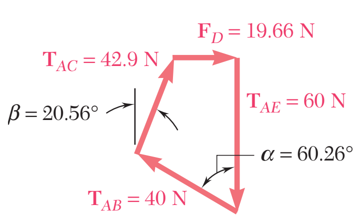
This equation will be satisfied if, and only if, the coefficients of $\mathbf{i}$ and $\mathbf{j}$ are equal to zero. We thus obtain the following two equilibrium equations, which express, respectively, that the sum of the x components and the sum of the y components of the given forces must be zero.

$$\Sigma F_x = 0: \qquad -34.73\text{ N} + 0.3512T_{AC} + F_D = 0 \tag{2}$$

$$\Sigma F_y = 0: \qquad 19.84\text{ N} + 0.9363T_{AC} - 60\text{ N} = 0 \tag{3}$$

From Eq. (3) we find $\qquad T_{AC} = +42.9\text{ lb}$ ◀

and, substituting this value into Eq. (2), $\qquad F_D = +19.66\text{ lb}$ ◀



In drawing the free-body diagram, we assumed a sense for each unknown force. A positive sign in the answer indicates that the assumed sense is correct. The complete force polygon may be drawn to check the results.

SOLVING PROBLEMS ON YOUR OWN

When a particle is in *equilibrium*, the resultant of the forces acting on the particle must be zero. Expressing this fact in the case of a particle under *coplanar forces* will provide you with two relations among these forces. As you saw in the preceding sample problems, these relations can be used to determine two unknowns—such as the magnitude and direction of one force or the magnitudes of two forces.

Drawing a free-body diagram is the first step in the solution of a problem involving the equilibrium of a particle. This diagram shows the particle and all the forces acting on it. Indicate on your free-body diagram the magnitudes of known forces as well as any angle or dimensions that define the direction of a force. Any unknown magnitude or angle should be denoted by an appropriate symbol. Nothing else should be included on the free-body diagram.

Drawing a clear and accurate free-body diagram is a must in the solution of any equilibrium problem. Skipping this step might save you pencil and paper, but is very likely to lead you to a wrong solution.

Case 1. If only three forces are involved in the free-body diagram, the rest of the solution is best carried out by drawing these forces in tip-to-tail fashion to form a *force triangle.* This triangle can be solved graphically or using trigonometry for no more than two unknowns [Sample Probs. 2.4 and 2.5].

Case 2. If more than three forces are involved, it is to your advantage to use an *analytic solution.* Begin by selecting appropriate x and y axes and resolve each of the forces shown in the free-body diagram into x and y components. Expressing that the sum of the x components and the sum of the y components of all the forces are both zero, you will obtain two equations which you can solve for no more than two unknowns [Sample Prob. 2.6].

It is strongly recommended that when using an analytic solution the equations of equilibrium be written in the same form as Eqs. (2) and (3) of Sample Prob. 2.6. The practice adopted by some students of initially placing the unknowns on the left side of the equation and the known quantities on the right side may lead to confusion in assigning the appropriate sign to each term.

We have noted that regardless of the method used to solve a two-dimensional equilibrium problem we can determine at most two unknowns. If a two-dimensional problem involves more than two unknowns, one or more additional relations must be obtained from the information contained in the statement of the problem.

Some of the following problems contain small pulleys. We will assume that the pulleys are frictionless, so the tension in the rope or cable passing over a pulley is the same on each side of the pulley. In Chap. 4 we will discuss why the tension is the same. Lastly, as we will discuss in Chap. 10, the magnitude of the force **F** exerted on a body by a stretched or compressed spring is given by $F = kx$, where k is the spring constant and x is the amount the spring is stretched or compressed from its undeformed length.

Problems

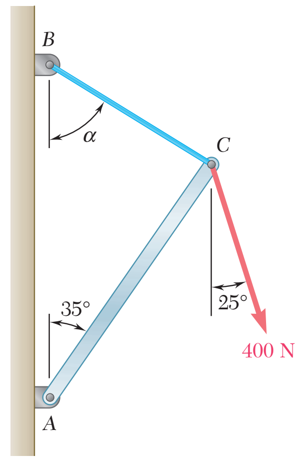


Fig. P2.43

2.43 Knowing that $\alpha = 50°$ and that boom AC exerts on pin C a force directed along line AC, determine (*a*) the magnitude of that force, (*b*) the tension in cable BC.

2.44 Two cables are tied together at C and are loaded as shown. Determine the tension (*a*) in cable AC, (*b*) in cable BC.

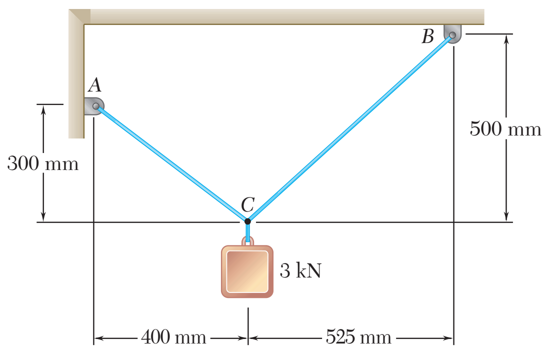


Fig. P2.44

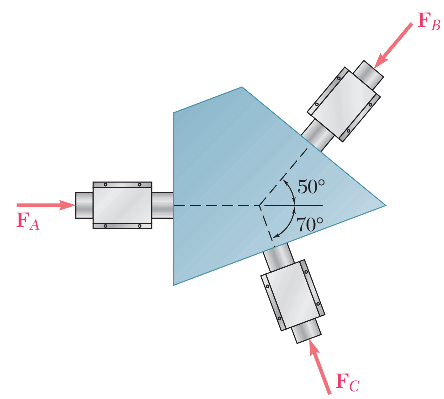


Fig. *P2.45*

2.45 An irregularly shaped machine component is held in the position shown by three clamps. Knowing that $F_A = 940$ N, determine the magnitudes of the forces F_B and F_C exerted by the other two clamps.

2.46 Ropes AB and AC are thrown to a boater whose canoe had capsized. Knowing that $\alpha = 25°$ and that the magnitude of the force $\mathbf{F}_R$ exerted by the river on the boater is 70 N, determine the tension (*a*) in rope AB, (*b*) in rope AC.

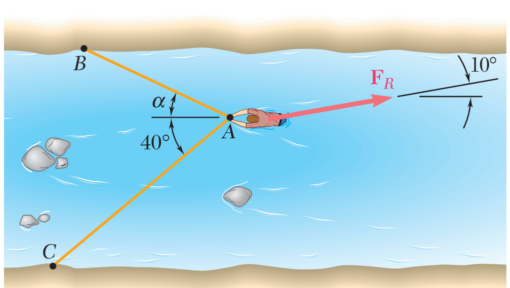


Fig. P2.46

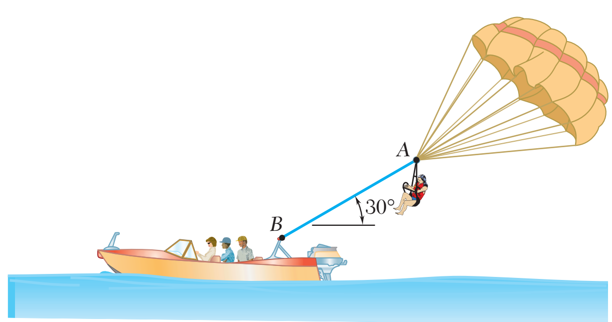


Fig. P2.47

2.47 A boat is pulling a parasail and rider at a constant speed. Knowing that the rider weighs 550 N and that the resultant force $\mathbf{R}$ exerted by the parasail on the towing yoke A forms an angle of 65° with the horizontal, determine (*a*) the tension in the tow rope AB, (*b*) the magnitude of $\mathbf{R}$.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 42

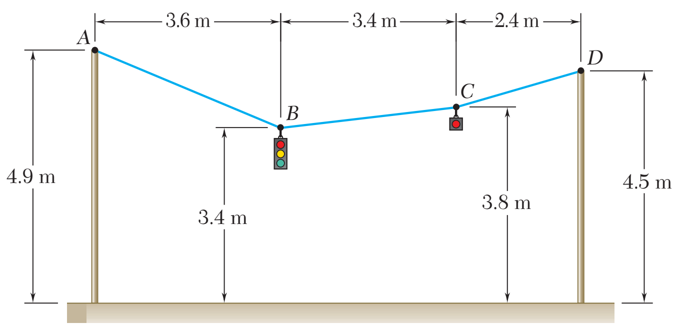


Fig. *P2.48*

2.48 Two traffic signals are temporarily suspended from a cable as shown. Knowing that the signal at B weighs 300 N, determine the weight of the signal at C.

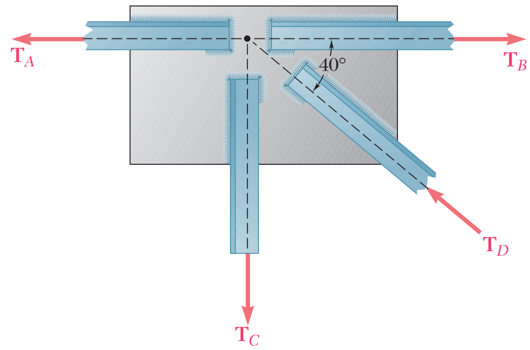


Fig. P2.49 and *P2.50*

2.49 Two forces of magnitude $T_A = 8$ kN and $T_B = 15$ kN are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces T_C and T_D.

2.50 Two forces of magnitude $T_A = 6$ kN and $T_C = 9$ kN are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces T_B and T_D.

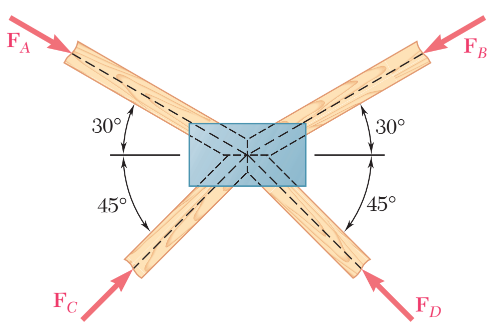


Fig. P2.51 and P2.52

2.51 Four wooden members are joined with metal plate connectors and are in equilibrium under the action of the four forces shown. Knowing that $F_A = 2.3$ kN and $F_B = 2.1$ kN, determine the magnitudes of the other two forces.

2.52 Four wooden members are joined with metal plate connectors and are in equilibrium under the action of the four forces shown. Knowing that $F_A = 1.9$ kN and $F_C = 2.4$ kN, determine the magnitudes of the other two forces.

2.53 In a circus act, an aerialist performs a handstand on a wheel while being pulled across a high wire ABC of length 8 m by another performer as shown. Knowing that the tension in rope DE is 35 N when the aerialist is held in equilibrium at $a = 2.5$ m, determine (a) the weight of the aerialist, (b) the tension in the wire.

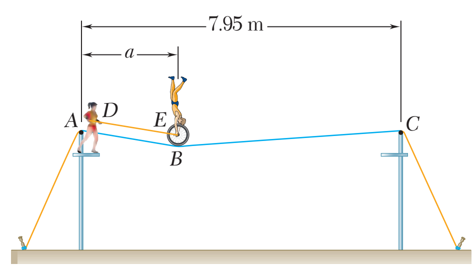


Fig. P2.53 and P2.54

2.54 In a circus act, an aerialist performs a handstand on a wheel while being pulled across a high wire ABC of length 8 m by another performer as shown. Knowing that the aerialist weighs 720 N and is being held in equilibrium at $a = 3$ m, determine (a) the tension in the wire, (b) the tension in rope DE.

2.55 Two cables tied together at C are loaded as shown. Knowing that $W = 190$ N, determine the tension (a) in cable AC, (b) in cable BC.

2.56 Two cables tied together at C are loaded as shown. Determine the range of values of W for which the tension will not exceed 240 N in either cable.

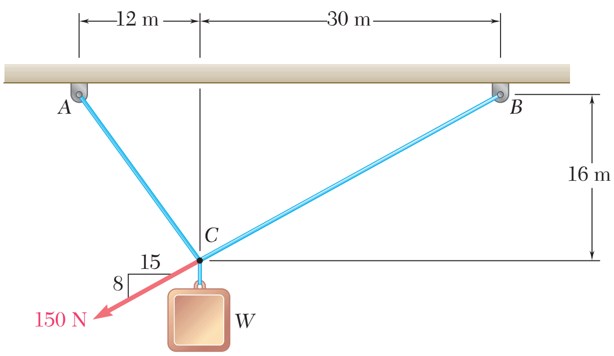


Fig. P2.55 and *P2.56*

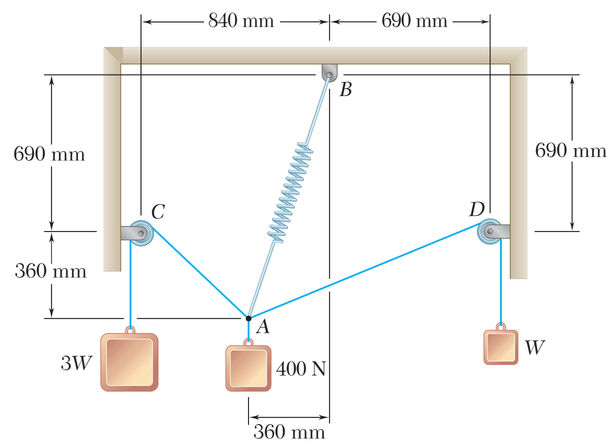


Fig. P2.57

2.57 A load of weight 400 N is suspended from a spring and two cords that are attached to blocks of weights $3W$ and W as shown. Knowing that the constant of the spring is 800 N/m, determine (a) the value of W, (b) the unstretched length of the spring.

2.58 A block of weight W is suspended from a 25-cm-long cord and two springs of which the unstretched lengths are 22.5 cm. Knowing that the constants of the springs are $k_{AB} = 9$ N/cm and $k_{AD} = 3$ N/cm, determine (a) the tension in the cord, (b) the weight of the block.

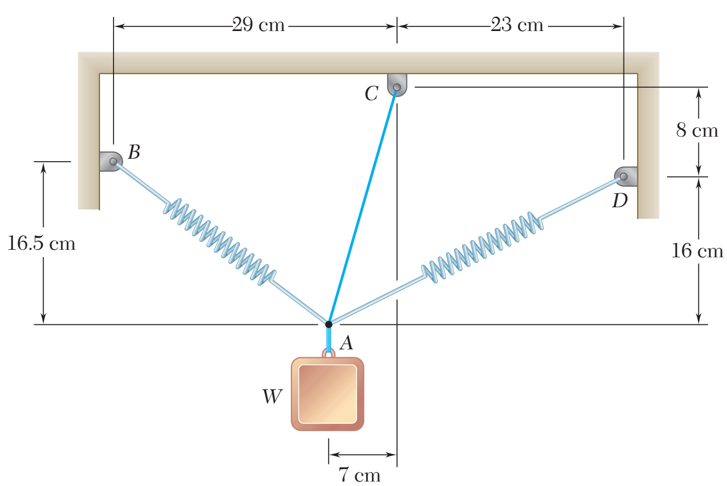


Fig. P2.58

2.59 For the ropes and force of the river of Prob. 2.46, determine (a) the value of α for which the tension in rope AB is as small as possible, (b) the corresponding value of the tension.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 44

2.60 Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension in each cable is 900 N, determine (a) the magnitude of the largest force $\mathbf{P}$ which may be applied at C, (b) the corresponding value of α.

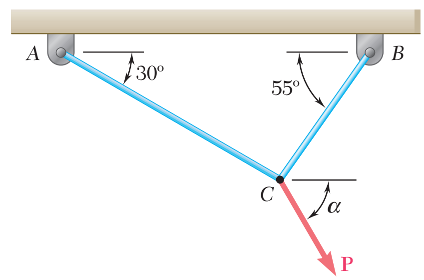


Fig. P2.60 and P2.61

2.61 Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension is 1400 N in cable AC and 700 N in cable BC, determine (a) the magnitude of the largest force $\mathbf{P}$ which may be applied at C, (b) the corresponding value of α.

2.62 Knowing that portions AC and BC of cable ACB must be equal, determine the shortest length of cable that can be used to support the load shown if the tension in the cable is not to exceed 870 N.

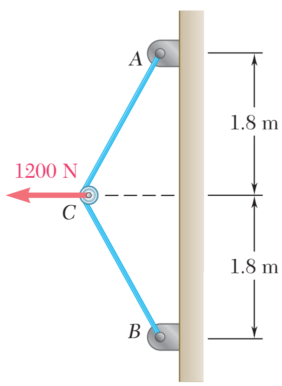


Fig. P2.62

2.63 For the structure and loading of Prob. 2.43, determine (a) the value of α for which the tension in cable BC is as small as possible, (b) the corresponding value of the tension.

2.64 Collar A can slide on a frictionless vertical rod and is attached as shown to a spring. The constant of the spring is 4 N/cm, and the spring is unstretched when $h = 12$ cm. Knowing that the system is in equilibrium when $h = 16$ cm, determine the weight of the collar.

2.65 The 9-N collar A can slide on a frictionless vertical rod and is attached as shown to a spring. The spring is unstretched when $h = 12$ cm. Knowing that the constant of the spring is 3 N/cm, determine the value of h for which the system is in equilibrium.

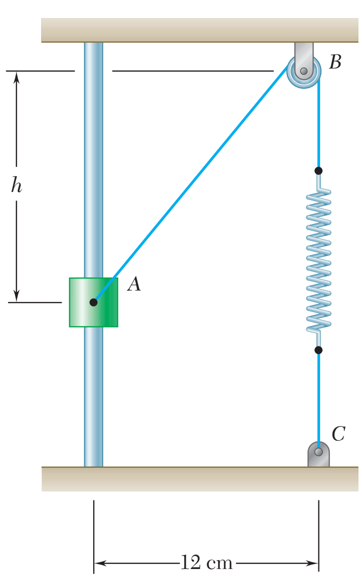


Fig. *P2.64* and P2.65

2.66 Boom AB is supported by cable BC and a hinge at A. Knowing that the boom exerts on pin B a force directed along the boom and that the tension in rope BD is 310 N, determine (a) the value of α for which the tension in cable BC is as small as possible, (b) the corresponding value of the tension.

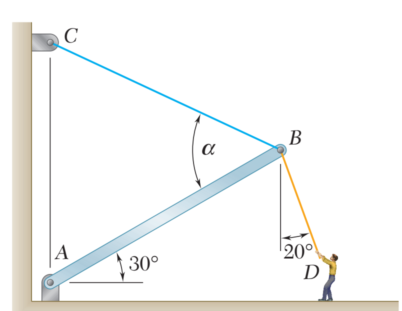


Fig. P2.66

bee76985_ch02_15-72 5/9/07 12:06 PM Page 45

2.67 The force **P** is applied to a small wheel that rolls on the cable *ACB*. Knowing that the tension in both parts of the cable is 140 N, determine the magnitude and direction of **P**.

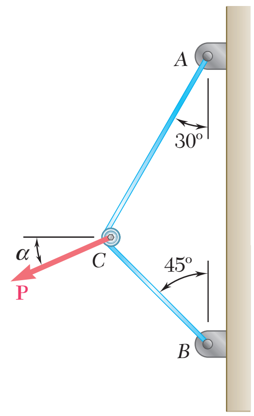


Fig. P2.67

2.68 A 280-kg crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chap. 4.)

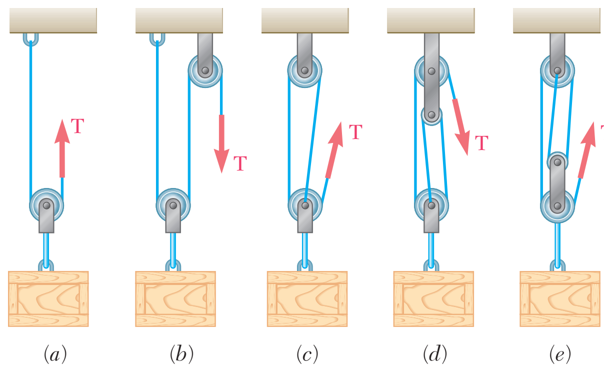


Fig. *P2.68*

2.69 Solve parts *b* and *d* of Prob. 2.68 assuming that the free end of the rope is attached to the crate.

2.70 A load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Knowing that $P = 800$ N, determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **Q**.

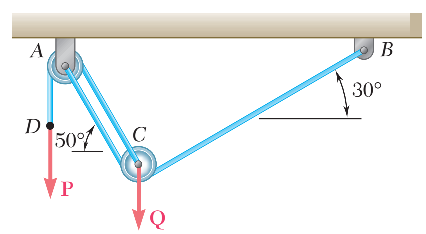


Fig. P2.70 and *P2.71*

2.71 A 2000-N load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **P**.

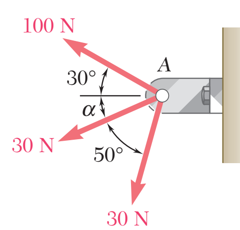


Fig. P2.72

2.72 Three forces are applied to a bracket. The directions of the two 30-N forces can vary, but the angle between these forces is always 50°. Determine the range of values of α for which the magnitude of the resultant of the forces applied to the bracket is less than 120 N.

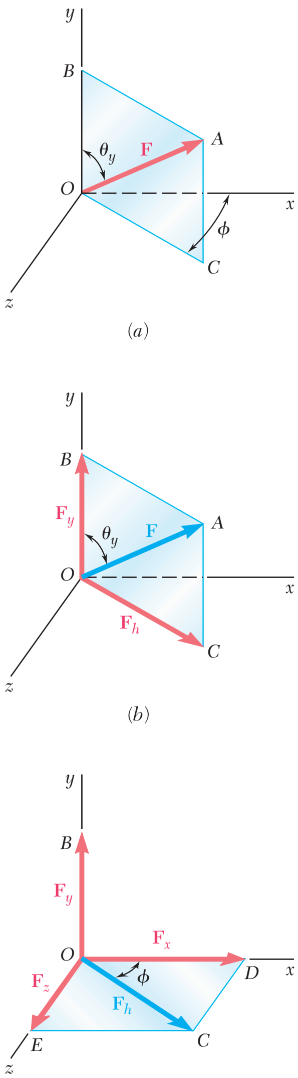
FORCES IN SPACE

2.12. RECTANGULAR COMPONENTS OF A FORCE IN SPACE

The problems considered in the first part of this chapter involved only two dimensions; they could be formulated and solved in a single plane. In this section and in the remaining sections of the chapter, we will discuss problems involving the three dimensions of space.

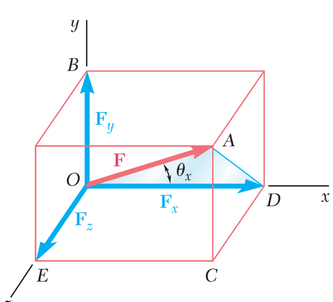
Consider a force **F** acting at the origin *O* of the system of rectangular coordinates *x*, *y*, *z*. To define the direction of **F**, we draw the

bee76985_ch02_15-72 5/9/07 12:06 PM Page 46



(a) (b) (c)

Fig. 2.30



(a)

Fig. 2.31

vertical plane *OBAC* containing **F** (Fig. 2.30*a*). This plane passes through the vertical y axis; its orientation is defined by the angle ϕ it forms with the xy plane. The direction of **F** within the plane is defined by the angle θ_y that **F** forms with the y axis. The force **F** may be resolved into a vertical component $\mathbf{F}_y$ and a horizontal component $\mathbf{F}_h$; this operation, shown in Fig. 2.30*b*, is carried out in plane *OBAC* according to the rules developed in the first part of the chapter. The corresponding scalar components are

$$F_y = F \cos \theta_y \qquad F_h = F \sin \theta_y \tag{2.16}$$

But $\mathbf{F}_h$ may be resolved into two rectangular components $\mathbf{F}_x$ and $\mathbf{F}_z$ along the x and z axes, respectively. This operation, shown in Fig. 2.30*c*, is carried out in the xz plane. We obtain the following expressions for the corresponding scalar components:

$$\begin{aligned} F_x &= F_h \cos \phi = F \sin \theta_y \cos \phi \\ F_z &= F_h \sin \phi = F \sin \theta_y \sin \phi \end{aligned} \tag{2.17}$$

The given force **F** has thus been resolved into three rectangular vector components $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$, which are directed along the three coordinate axes.

Applying the Pythagorean theorem to the triangles *OAB* and *OCD* of Fig. 2.30, we write

$$\begin{aligned} F^2 &= (OA)^2 = (OB)^2 + (BA)^2 = F_y^2 + F_h^2 \\ F_h^2 &= (OC)^2 = (OD)^2 + (DC)^2 = F_x^2 + F_z^2 \end{aligned}$$

Eliminating F_h^2 from these two equations and solving for F, we obtain the following relation between the magnitude of **F** and its rectangular scalar components:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \tag{2.18}$$

The relationship existing between the force **F** and its three components $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$ is more easily visualized if a "box" having $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$ for edges is drawn as shown in Fig. 2.31. The force **F** is then represented by the diagonal *OA* of this box. Figure 2.31*b* shows the right triangle *OAB* used to derive the first of the formulas (2.16): $F_y = F \cos \theta_y$. In Fig. 2.31*a* and *c*, two other right triangles have also been drawn: *OAD* and *OAE*. These triangles are seen to occupy in the box positions comparable with that of triangle *OAB*. Denoting by θ_x and θ_z, respectively, the angles that **F** forms with the x and z axes, we can derive two formulas similar to $F_y = F \cos \theta_y$. We thus write

$$F_x = F \cos \theta_x \qquad F_y = F \cos \theta_y \qquad F_z = F \cos \theta_z \tag{2.19}$$

The three angles θ_x, θ_y, and θ_z define the direction of the force **F**; they are more commonly used for this purpose than the angles θ_y and ϕ introduced at the beginning of this section. The cosines of θ_x, θ_y, and θ_z are known as the *direction cosines* of the force **F**.

Introducing the unit vectors **i**, **j**, and **k**, directed respectively along the x, y, and z axes (Fig. 2.32), we can express **F** in the form

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \tag{2.20}$$

where the scalar components F_x, F_y, F_z are defined by the relations (2.19).

Example 1. A force of 500 N forms angles of 60°, 45°, and 120°, respectively, with the x, y, and z axes. Find the components F_x, F_y, and F_z of the force.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 47

Substituting $F = 500$ N, $\theta_x = 60°$, $\theta_y = 45°$, $\theta_z = 120°$ into formulas (2.19), we write

$$F_x = (500\text{ N}) \cos 60° = +250\text{ N}$$
$$F_y = (500\text{ N}) \cos 45° = +354\text{ N}$$
$$F_z = (500\text{ N}) \cos 120° = -250\text{ N}$$

Carrying into Eq. (2.20) the values obtained for the scalar components of **F**, we have

$$\mathbf{F} = (250\text{ N})\mathbf{i} + (354\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k}$$

As in the case of two-dimensional problems, a plus sign indicates that the component has the same sense as the corresponding axis, and a minus sign indicates that it has the opposite sense.

The angle a force **F** forms with an axis should be measured from the positive side of the axis and will always be between 0 and 180°. An angle θ_x smaller than 90° (acute) indicates that **F** (assumed attached to O) is on the same side of the yz plane as the positive x axis; $\cos \theta_x$ and F_x will then be positive. An angle θ_x larger than 90° (obtuse) indicates that **F** is on the other side of the yz plane; $\cos \theta_x$ and F_x will then be negative. In Example 1 the angles θ_x and θ_y are acute, while θ_z is obtuse; consequently, F_x and F_y are positive, while F_z is negative.

Substituting into (2.20) the expressions obtained for F_x, F_y, and F_z in (2.19), we write

$$\mathbf{F} = F(\cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k}) \qquad (2.21)$$

which shows that the force **F** can be expressed as the product of the scalar F and the vector

$$\boldsymbol{\lambda} = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k} \qquad (2.22)$$

Clearly, the vector $\boldsymbol{\lambda}$ is a vector whose magnitude is equal to 1 and whose direction is the same as that of **F** (Fig. 2.33). The vector $\boldsymbol{\lambda}$ is referred to as the *unit vector* along the line of action of **F**. It follows from (2.22) that the components of the unit vector $\boldsymbol{\lambda}$ are respectively equal to the direction cosines of the line of action of **F**:

$$\lambda_x = \cos \theta_x \qquad \lambda_y = \cos \theta_y \qquad \lambda_z = \cos \theta_z \qquad (2.23)$$

We should observe that the values of the three angles θ_x, θ_y, and θ_z are not independent. Recalling that the sum of the squares of the components of a vector is equal to the square of its magnitude, we write

$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1$$

or, substituting for λ_x, λ_y, and λ_z from (2.23),

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1 \qquad (2.24)$$

In Example 1, for instance, once the values $\theta_x = 60°$ and $\theta_y = 45°$ have been selected, the value of θ_z *must* be equal to 60° or 120° in order to satisfy identity (2.24).

When the components F_x, F_y, and F_z of a force **F** are given, the magnitude F of the force is obtained from (2.18). The relations (2.19) can then be solved for the direction cosines,

$$\cos \theta_x = \frac{F_x}{F} \qquad \cos \theta_y = \frac{F_y}{F} \qquad \cos \theta_z = \frac{F_z}{F} \qquad (2.25)$$

and the angles θ_x, θ_y, θ_z characterizing the direction of **F** can be found.

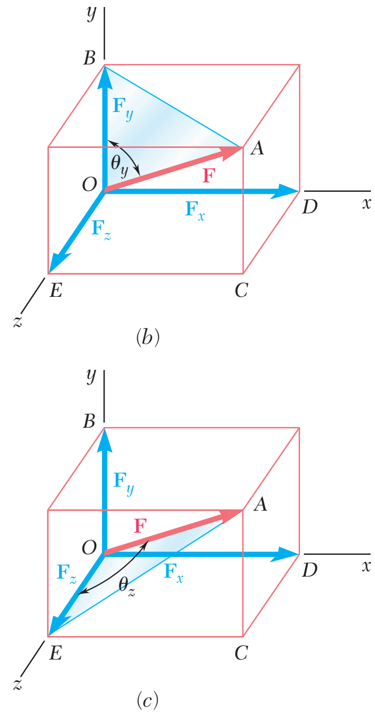


Fig. 2.31

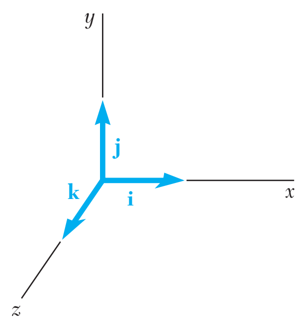


Fig. 2.32

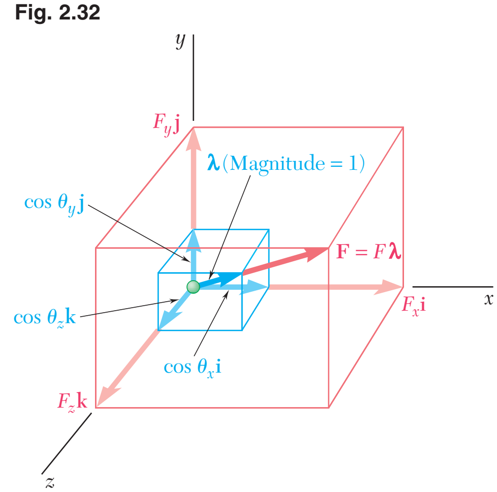


Fig. 2.33

bee76985_ch02_15-72 5/9/07 12:06 PM Page 48

Example 2. A force **F** has the components $F_x = 20$ N, $F_y = -30$ N, $F_z = 60$ N. Determine its magnitude F and the angles θ_x, θ_y, and θ_z it forms with the coordinate axes.

From formula (2.18) we obtain†

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2 + F_z^2} \\ &= \sqrt{(20\text{ N})^2 + (-30\text{ N})^2 + (60\text{ N})^2} \\ &= \sqrt{4900}\text{ N} = 70\text{ N} \end{aligned}$$

Substituting the values of the components and magnitude of **F** into Eqs. (2.25), we write

$$\cos\theta_x = \frac{F_x}{F} = \frac{20\text{ N}}{70\text{ N}} \qquad \cos\theta_y = \frac{F_y}{F} = \frac{-30\text{ N}}{70\text{ N}} \qquad \cos\theta_z = \frac{F_z}{F} = \frac{60\text{ N}}{70\text{ N}}$$

Calculating successively each quotient and its arc cosine, we obtain

$$\theta_x = 73.4° \qquad \theta_y = 115.4° \qquad \theta_z = 31.0°$$

These computations can be carried out easily with a calculator.

2.13. FORCE DEFINED BY ITS MAGNITUDE AND TWO POINTS ON ITS LINE OF ACTION

In many applications, the direction of a force **F** is defined by the coordinates of two points, $M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$, located on its line of action (Fig. 2.34). Consider the vector $\overrightarrow{MN}$ joining M and N

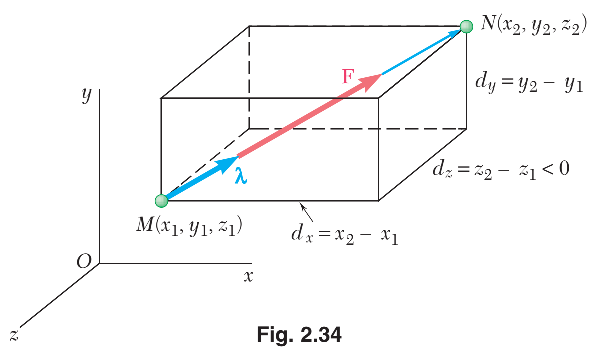


Fig. 2.34

and of the same sense as **F**. Denoting its scalar components by d_x, d_y, and d_z, respectively, we write

$$\overrightarrow{MN} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k} \tag{2.26}$$

The unit vector $\boldsymbol{\lambda}$ along the line of action of **F** (i.e., along the line MN) can be obtained by dividing the vector $\overrightarrow{MN}$ by its magnitude MN. Substituting for $\overrightarrow{MN}$ from (2.26) and observing that MN is equal to the distance d from M to N, we write

$$\boldsymbol{\lambda} = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \tag{2.27}$$

†With a calculator programmed to convert rectangular coordinates into polar coordinates, the following procedure will be found more expeditious for computing F: First determine F_h from its two rectangular components F_x and F_z (Fig. 2.30*c*), then determine F from its two rectangular components F_h and F_y (Fig. 2.30*b*). The actual order in which the three components F_x, F_y, and F_z are entered is immaterial.

Recalling that **F** is equal to the product of F and λ, we have

$$\mathbf{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \tag{2.28}$$

from which it follows that the scalar components of **F** are, respectively,

$$F_x = \frac{Fd_x}{d} \qquad F_y = \frac{Fd_y}{d} \qquad F_z = \frac{Fd_z}{d} \tag{2.29}$$

The relations (2.29) considerably simplify the determination of the components of a force **F** of given magnitude F when the line of action of **F** is defined by two points M and N. Subtracting the coordinates of M from those of N, we first determine the components of the vector $\overrightarrow{MN}$ and the distance d from M to N:

$$d_x = x_2 - x_1 \qquad d_y = y_2 - y_1 \qquad d_z = z_2 - z_1$$
$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

Substituting for F and for d_x, d_y, d_z, and d into the relations (2.29), we obtain the components F_x, F_y, and F_z of the force.

The angles θ_x, θ_y, and θ_z that **F** forms with the coordinate axes can then be obtained from Eqs. (2.25). Comparing Eqs. (2.22) and (2.27), we can also write

$$\cos\theta_x = \frac{d_x}{d} \qquad \cos\theta_y = \frac{d_y}{d} \qquad \cos\theta_z = \frac{d_z}{d} \tag{2.30}$$

and determine the angles θ_x, θ_y, and θ_z directly from the components and magnitude of the vector $\overrightarrow{MN}$.

2.14. ADDITION OF CONCURRENT FORCES IN SPACE

The resultant **R** of two or more forces in space will be determined by summing their rectangular components. Graphical or trigonometric methods are generally not practical in the case of forces in space.

The method followed here is similar to that used in Sec. 2.8 with coplanar forces. Setting

$$\mathbf{R} = \Sigma\mathbf{F}$$

we resolve each force into its rectangular components and write

$$\begin{aligned} R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} &= \Sigma(F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}) \\ &= (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} + (\Sigma F_z)\mathbf{k} \end{aligned}$$

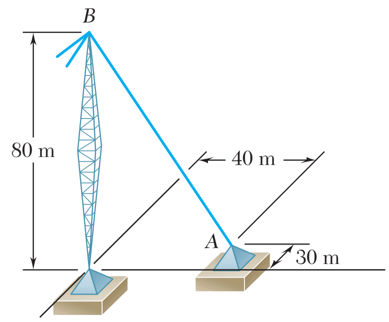
from which it follows that

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z \tag{2.31}$$

The magnitude of the resultant and the angles θ_x, θ_y, and θ_z that the resultant forms with the coordinate axes are obtained using the method discussed in Sec. 2.12. We write

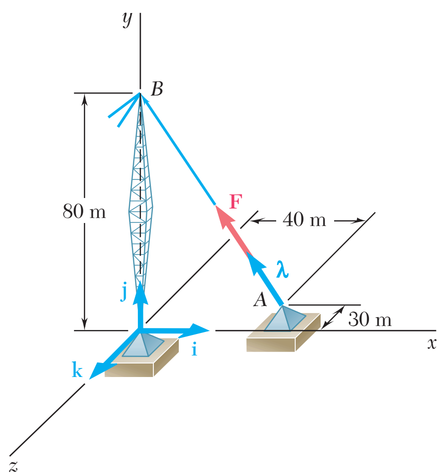
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \tag{2.32}$$

$$\cos\theta_x = \frac{R_x}{R} \qquad \cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R} \tag{2.33}$$



SAMPLE PROBLEM 2.7

A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 2500 N. Determine (a) the components F_x, F_y, and F_z of the force acting on the bolt, (b) the angles θ_x, θ_y, and θ_z defining the direction of the force.



SOLUTION

***a.* Components of the Force.** The line of action of the force acting on the bolt passes through A and B, and the force is directed from A to B. The components of the vector $\overrightarrow{AB}$, which has the same direction as the force, are

$$d_x = -40 \text{ m} \qquad d_y = +80 \text{ m} \qquad d_z = +30 \text{ m}$$

The total distance from A to B is

$$AB = d = \sqrt{d_x^2 + d_y^2 + d_z^2} = 94.3 \text{ m}$$

Denoting by $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ the unit vectors along the coordinate axes, we have

$$\overrightarrow{AB} = -(40 \text{ m})\mathbf{i} + (80 \text{ m})\mathbf{j} + (30 \text{ m})\mathbf{k}$$

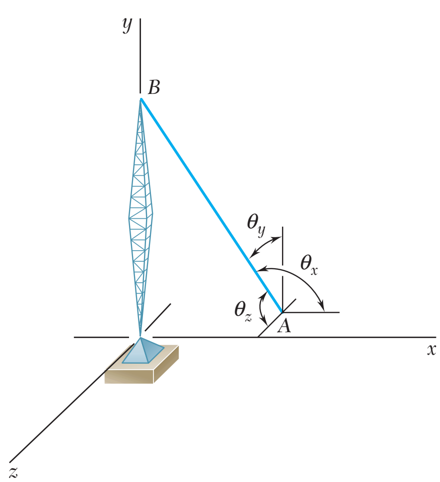
Introducing the unit vector $\boldsymbol{\lambda} = \overrightarrow{AB}/AB$, we write

$$\mathbf{F} = F\boldsymbol{\lambda} = F\frac{\overrightarrow{AB}}{AB} = \frac{2500 \text{ N}}{94.3 \text{ m}}\overrightarrow{AB}$$

Substituting the expression found for $\overrightarrow{AB}$, we obtain

$$\mathbf{F} = \frac{2500 \text{ N}}{94.3 \text{ m}}[-(40 \text{ m})\mathbf{i} + (80 \text{ m})\mathbf{j} + (30 \text{ m})\mathbf{k}]$$

$$\mathbf{F} = -(1060 \text{ N})\mathbf{i} + (2120 \text{ N})\mathbf{j} + (795 \text{ N})\mathbf{k}$$



The components of $\mathbf{F}$, therefore, are

$$F_x = -1060 \text{ N} \qquad F_y = +2120 \text{ N} \qquad F_z = +795 \text{ N} \quad ◀$$

***b.* Direction of the Force.** Using Eqs. (2.25), we write

$$\cos\theta_x = \frac{F_x}{F} = \frac{-1060 \text{ N}}{2500 \text{ N}} \qquad \cos\theta_y = \frac{F_y}{F} = \frac{+2120 \text{ N}}{2500 \text{ N}}$$

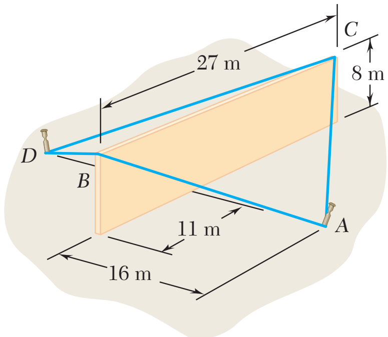
$$\cos\theta_z = \frac{F_z}{F} = \frac{+795 \text{ N}}{2500 \text{ N}}$$

Calculating successively each quotient and its arc cosine, we obtain

$$\theta_x = 115.1° \qquad \theta_y = 32.0° \qquad \theta_z = 71.5° \quad ◀$$

(*Note:* This result could have been obtained by using the components and magnitude of the vector $\overrightarrow{AB}$ rather than those of the force $\mathbf{F}$.)

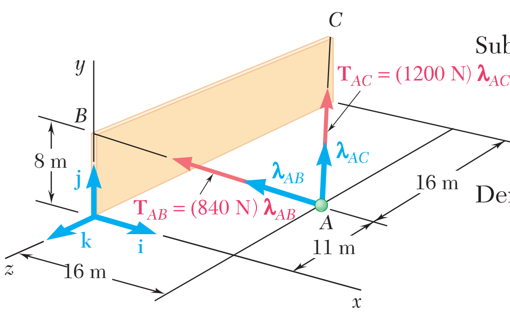
bee76985_ch02_15-72 5/9/07 12:06 PM Page 51



SAMPLE PROBLEM 2.8

A wall section of precast concrete is temporarily held by the cables shown. Knowing that the tension is 840 N in cable *AB* and 1200 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted by cables *AB* and *AC* on stake *A*.

SOLUTION



Components of the Forces. The force exerted by each cable on stake *A* will be resolved into x, y, and z components. We first determine the components and magnitude of the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$, measuring them from *A* toward the wall section. Denoting by **i**, **j**, and **k** the unit vectors along the coordinate axes, we write

$$\overrightarrow{AB} = -(16\text{ m})\mathbf{i} + (8\text{ m})\mathbf{j} + (11\text{ m})\mathbf{k} \qquad AB = 21\text{ m}$$
$$\overrightarrow{AC} = -(16\text{ m})\mathbf{i} + (8\text{ m})\mathbf{j} - (16\text{ m})\mathbf{k} \qquad AC = 24\text{ m}$$

Denoting by $\boldsymbol{\lambda}_{AB}$ the unit vector along *AB*, we have

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{840\text{ N}}{21\text{ m}}\overrightarrow{AB}$$

Substituting the expression found for $\overrightarrow{AB}$, we obtain

$$\mathbf{T}_{AB} = \frac{840\text{ N}}{21\text{ m}}[-(16\text{ m})\mathbf{i} + (8\text{ m})\mathbf{j} + (11\text{ m})\mathbf{k}]$$
$$\mathbf{T}_{AB} = -(640\text{ N})\mathbf{i} + (320\text{ N})\mathbf{j} + (440\text{ N})\mathbf{k}$$

Denoting by $\boldsymbol{\lambda}_{AC}$ the unit vector along *AC*, we obtain in a similar way

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{1200\text{ N}}{24\text{ m}}\overrightarrow{AC}$$
$$\mathbf{T}_{AC} = -(800\text{ N})\mathbf{i} + (400\text{ N})\mathbf{j} - (800\text{ N})\mathbf{k}$$

Resultant of the Forces. The resultant **R** of the forces exerted by the two cables is

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = -(1440\text{ N})\mathbf{i} + (720\text{ N})\mathbf{j} - (360\text{ N})\mathbf{k}$$

The magnitude and direction of the resultant are now determined:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(-1440)^2 + (720)^2 + (-360)^2}$$

$$R = 1650\text{ N} \quad \blacktriangleleft$$

From Eqs. (2.33) we obtain

$$\cos\theta_x = \frac{R_x}{R} = \frac{-1440\text{ N}}{1650\text{ N}} \qquad \cos\theta_y = \frac{R_y}{R} = \frac{+720\text{ N}}{1650\text{ N}}$$
$$\cos\theta_z = \frac{R_z}{R} = \frac{-360\text{ N}}{1650\text{ N}}$$

Calculating successively each quotient and its arc cosine, we have

$$\theta_x = 150.8° \qquad \theta_y = 64.1° \qquad \theta_z = 102.6° \quad \blacktriangleleft$$

SOLVING PROBLEMS ON YOUR OWN

In this lesson we saw that a *force in space* can be defined by its magnitude and direction or by its three rectangular components F_x, F_y, and F_z.

A. When a force is defined by its magnitude and direction, its rectangular components F_x, F_y, and F_z can be found as follows:

Case 1. If the direction of the force **F** is defined by the angles θ_y and ϕ shown in Fig. 2.30, projections of **F** through these angles or their complements will yield the components of **F** [Eqs. (2.17)]. Note that the x and z components of **F** are found by first projecting **F** onto the horizontal plane; the projection $\mathbf{F}_h$ obtained in this way is then resolved into the components $\mathbf{F}_x$ and $\mathbf{F}_z$ (Fig. 2.30*c*).

When solving problems of this type, we strongly encourage you first to sketch the force **F** and then its projection $\mathbf{F}_h$ and components F_x, F_y, and F_z before beginning the mathematical part of the solution.

Case 2. If the direction of the force **F** is defined by the angles θ_x, θ_y, and θ_z that **F** forms with the coordinate axes, each component can be obtained by multiplying the magnitude F of the force by the cosine of the corresponding angle [Example 1]:

$$F_x = F \cos \theta_x \qquad F_y = F \cos \theta_y \qquad F_z = F \cos \theta_z$$

Case 3. If the direction of the force **F** is defined by two points M and N located on its line of action (Fig. 2.34), you will first express the vector $\overrightarrow{MN}$ drawn from M to N in terms of its components d_x, d_y, and d_z and the unit vectors **i**, **j**, and **k**:

$$\overrightarrow{MN} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}$$

Next, you will determine the unit vector $\boldsymbol{\lambda}$ along the line of action of **F** by dividing the vector $\overrightarrow{MN}$ by its magnitude MN. Multiplying $\boldsymbol{\lambda}$ by the magnitude of **F**, you will obtain the desired expression for **F** in terms of its rectangular components [Sample Prob. 2.7]:

$$\mathbf{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k})$$

It is advantageous to use a consistent and meaningful system of notation when determining the rectangular components of a force. The method used in this text is illustrated in Sample Prob. 2.8 where, for example, the force $\mathbf{T}_{AB}$ acts from stake A toward point B. Note that the subscripts have been ordered to agree with the direction of the force. It is recommended that you adopt the same notation, as it will help you identify point 1 (the first subscript) and point 2 (the second subscript).

When forming the vector defining the line of action of a force, you may think of its scalar components as the number of steps you must take in each coordinate direction to go from point 1 to point 2. It is essential that you always remember to assign the correct sign to each of the components.

B. When a force is defined by its rectangular components F_x, F_y, and F_z, you can obtain its magnitude F by writing

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

You can determine the direction cosines of the line of action of **F** by dividing the components of the force by F:

$$\cos \theta_x = \frac{F_x}{F} \qquad \cos \theta_y = \frac{F_y}{F} \qquad \cos \theta_z = \frac{F_z}{F}$$

From the direction cosines you can obtain the angles θ_x, θ_y, and θ_z that **F** forms with the coordinate axes [Example 2].

C. To determine the resultant* R *of two or more forces in three-dimensional space, first determine the rectangular components of each force by one of the procedures described above. Adding these components will yield the components R_x, R_y, and R_z of the resultant. The magnitude and direction of the resultant can then be obtained as indicated above for the force **F** [Sample Prob. 2.8].

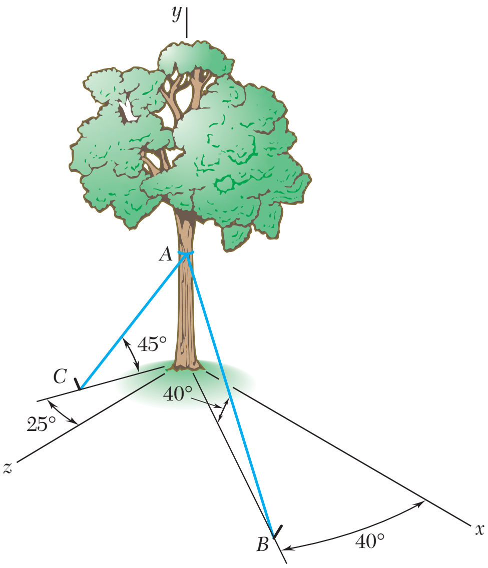


Fig. P2.73 and *P2.74*

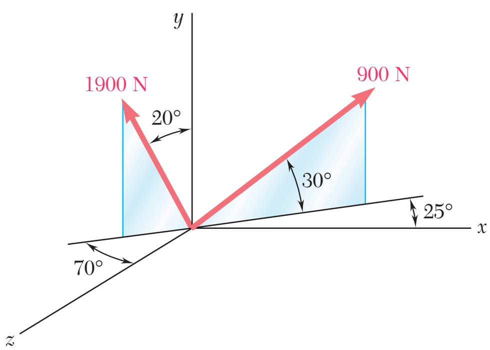


Fig. P2.75 and P2.76

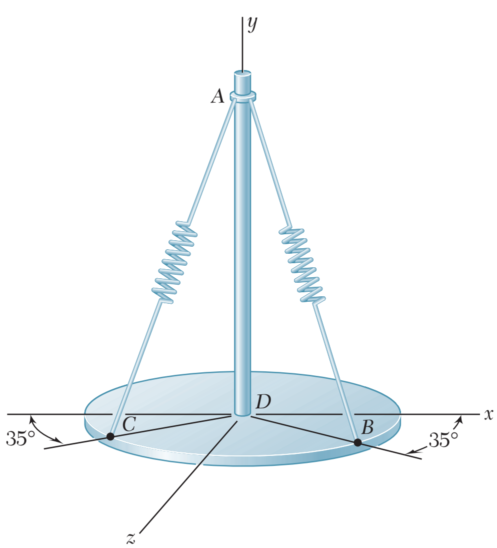


Fig. P2.79 and P2.80

Problems

2.73 To stabilize a tree partially uprooted in a storm, cables *AB* and *AC* are attached to the upper trunk of the tree and then are fastened to steel rods anchored in the ground. Knowing that the tension in cable *AB* is 950 N, determine (*a*) the components of the force exerted by this cable on the tree, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with axes at *A* which are parallel to the coordinate axes.

2.74 To stabilize a tree partially uprooted in a storm, cables *AB* and *AC* are attached to the upper trunk of the tree and then are fastened to steel rods anchored in the ground. Knowing that the tension in cable *AC* is 810 N, determine (*a*) the components of the force exerted by this cable on the tree, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with axes at *A* which are parallel to the coordinate axes.

2.75 Determine (*a*) the *x*, *y*, and *z* components of the 900-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

2.76 Determine (*a*) the *x*, *y*, and *z* components of the 1900-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

2.77 A gun is aimed at a point *A* located 20° west of north. Knowing that the barrel of the gun forms an angle of 35° with the horizontal and that the maximum recoil force is 180 N, determine (*a*) the *x*, *y*, and *z* components of that force, (*b*) the values of the angles θ_x, θ_y, and θ_z that define the direction of the recoil force. (Assume that the *x*, *y*, and *z* axes are directed, respectively, east, up, and south.)

2.78 Solve Prob. 2.77 assuming that point *A* is located 25° north of west and that the barrel of the gun forms an angle of 30° with the horizontal.

2.79 The angle between the spring *AB* and the post *DA* is 30°. Knowing that the tension in the spring is 220 N, determine (*a*) the *x*, *y*, and *z* components of the force exerted by this spring on the plate, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

2.80 The angle between the spring *AC* and the post *DA* is 30°. Knowing that the *x* component of the force exerted by spring *AC* on the plate is 180 N, determine (*a*) the tension in spring *AC*, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at *C* forms with the coordinate axes.

2.81 Determine the magnitude and direction of the force $\mathbf{F} = (65\text{ N})\mathbf{i} - (80\text{ N})\mathbf{j} - (200\text{ N})\mathbf{k}$.

2.82 Determine the magnitude and direction of the force $\mathbf{F} = (450\text{ N})\mathbf{i} + (600\text{ N})\mathbf{j} - (1800\text{ N})\mathbf{k}$.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 55

2.83 A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 43.2°$ and $\theta_z = 83.8°$. Knowing that the y component of the force is -50 N, determine (*a*) the angle θ_y, (*b*) the other components and the magnitude of the force.

2.84 A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 113.2°$ and $\theta_y = 78.4°$. Knowing that the z component of the force is -35 N, determine (*a*) the angle θ_z, (*b*) the other components and the magnitude of the force.

2.85 A force **F** of magnitude 250 N acts at the origin of a coordinate system. Knowing that $F_x = 80$ N, $\theta_y = 72.4°$, and $F_z > 0$, determine (*a*) the components F_y and F_z, (*b*) the angles θ_x and θ_z.

2.86 A force **F** of magnitude 320 N acts at the origin of a coordinate system. Knowing that $\theta_x = 104.5°$, $F_z = -120$ N, and $F_y < 0$, determine (*a*) the components F_x and F_y, (*b*) the angles θ_y and θ_z.

2.87 A steel rod is bent into a semicircular ring of radius 36 cm and is supported in part by cables *BD* and *BE* which are attached to the ring at *B*. Knowing that the tension in cable *BD* is 55 N, determine the components of the force exerted by the cable on the support at *D*.

2.88 A steel rod is bent into a semicircular ring of radius 36 cm and is supported in part by cables *BD* and *BE* which are attached to the ring at *B*. Knowing that the tension in cable *BE* is 60 N, determine the components of the force exerted by the cable on the support at *E*.

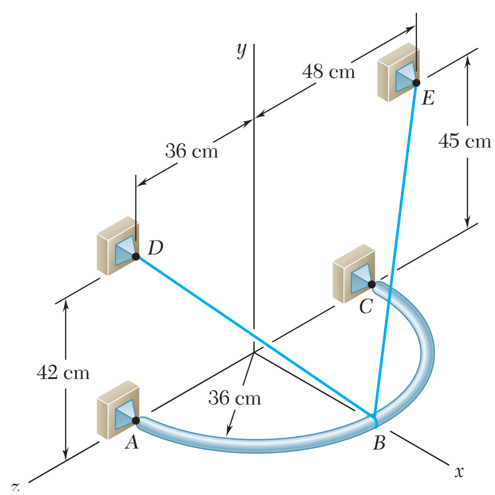


Fig. P2.87 and P2.88

2.89 A transmission tower is held by three guy wires anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AB* is 2100 N, determine the components of the force exerted by the wire on the bolt at *B*.

2.90 A transmission tower is held by three guy wires anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AD* is 1260 N, determine the components of the force exerted by the wire on the bolt at *D*.

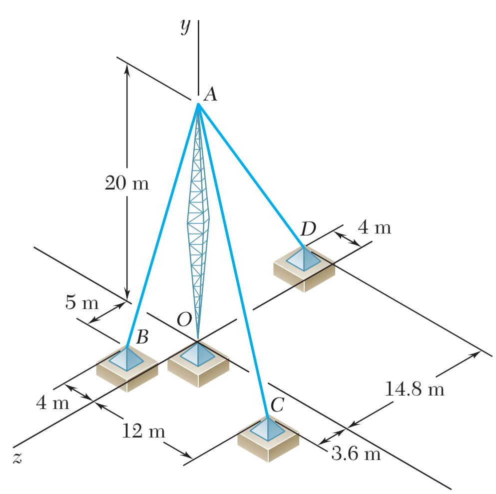


Fig. *P2.89* and P2.90

2.91 Two cables BG and BH are attached to the frame ACD as shown. Knowing that the tension in cable BG is 450 N, determine the components of the force exerted by cable BG on the frame at B.

2.92 Two cables BG and BH are attached to the frame ACD as shown. Knowing that the tension in cable BH is 600 N, determine the components of the force exerted by cable BH on the frame at B.

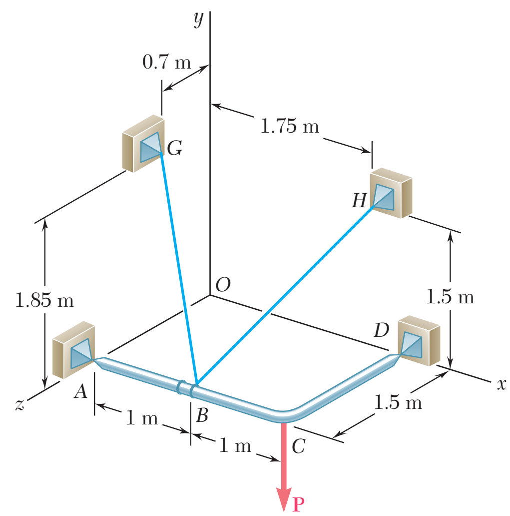


Fig. P2.91 and *P2.92*

2.93 Determine the magnitude and direction of the resultant of the two forces shown knowing that $P = 4$ kN and $Q = 8$ kN.

2.94 Determine the magnitude and direction of the resultant of the two forces shown knowing that $P = 6$ kN and $Q = 7$ kN.

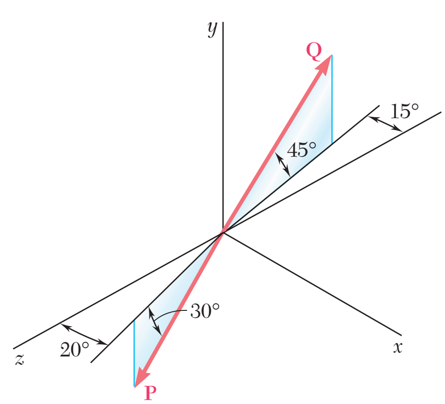


Fig. P2.93 and *P2.94*

2.95 The boom *OA* carries a load **P** and is supported by two cables as shown. Knowing that the tension is 510 N in cable *AB* and 765 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

2.96 Assuming that in Prob. 2.95 the tension is 765 N in cable *AB* and 510 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

2.97 For the tree of Prob. 2.73, knowing that the tension in cable *AB* is 760 N and that the resultant of the forces exerted at *A* by cables *AB* and *AC* lies in the *yz* plane, determine (*a*) the tension in *AC*, (*b*) the magnitude and direction of the resultant of the two forces.

2.98 For the tree of Prob. 2.73, knowing that the tension in cable *AC* is 980 N and that the resultant of the forces exerted at *A* by cables *AB* and *AC* lies in the *yz* plane, determine (*a*) the tension in *AB*, (*b*) the magnitude and direction of the resultant of the two forces.

2.99 For the boom of Prob. 2.95, knowing that $\alpha = 0°$, the tension in cable *AB* is 600 N, and the resultant of the load **P** and the forces exerted at *A* by the two cables is directed along *OA*, determine (*a*) the tension in cable *AC*, (*b*) the magnitude of the load **P**.

2.100 For the transmission tower of Prob. 2.89, determine the tensions in cables *AB* and *AD* knowing that the tension in cable *AC* is 1770 N and that the resultant of the forces exerted by the three cables at *A* must be vertical.

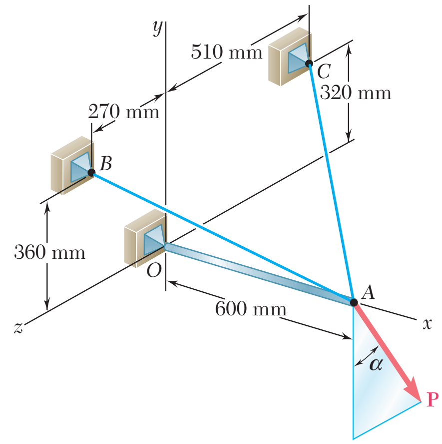


Fig. P2.95

2.15. EQUILIBRIUM OF A PARTICLE IN SPACE

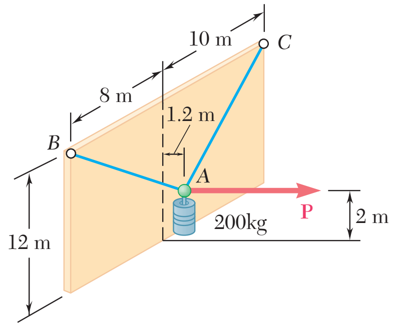
According to the definition given in Sec. 2.9, a particle *A* is in equilibrium if the resultant of all the forces acting on *A* is zero. The components R_x, R_y, and R_z of the resultant are given by the relations (2.31); expressing that the components of the resultant are zero, we write

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{2.34}$$

Equations (2.34) represent the necessary and sufficient conditions for the equilibrium of a particle in space. They can be used to solve problems dealing with the equilibrium of a particle involving no more than three unknowns.

To solve such problems, you first should draw a free-body diagram showing the particle in equilibrium and *all* the forces acting on it. You can then write the equations of equilibrium (2.34) and solve them for three unknowns. In the more common types of problems, these unknowns will represent (1) the three components of a single force or (2) the magnitude of three forces, each of known direction.

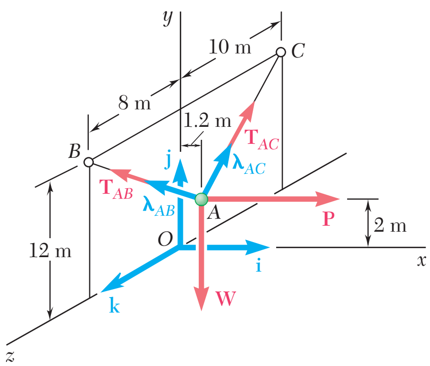
Photo 2.3 While the tension in the *four* cables supporting the shipping container cannot be found using the *three* equations of (2.34), a relation between the tensions can be obtained by considering the equilibrium of the hook.



SAMPLE PROBLEM 2.9

A 200-kg cylinder is hung by means of two cables AB and AC, which are attached to the top of a vertical wall. A horizontal force $\mathbf{P}$ perpendicular to the wall holds the cylinder in the position shown. Determine the magnitude of $\mathbf{P}$ and the tension in each cable.

SOLUTION



Free-body Diagram. Point A is chosen as a free body; this point is subjected to four forces, three of which are of unknown magnitude.

Introducing the unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, we resolve each force into rectangular components.

$$\mathbf{P} = P\mathbf{i}$$
$$\mathbf{W} = -mg\mathbf{j} = -(200 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(1962 \text{ N})\mathbf{j} \qquad (1)$$

In the case of $\mathbf{T}_{AB}$ and $\mathbf{T}_{AC}$, it is necessary first to determine the components and magnitudes of the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Denoting by $\boldsymbol{\lambda}_{AB}$ the unit vector along AB, we write

$$\overrightarrow{AB} = -(1.2 \text{ m})\mathbf{i} + (10 \text{ m})\mathbf{j} + (8 \text{ m})\mathbf{k} \qquad AB = 12.862 \text{ m}$$
$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{12.862 \text{ m}} = -0.09330\mathbf{i} + 0.7775\mathbf{j} + 0.6220\mathbf{k}$$
$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = -0.09330T_{AB}\mathbf{i} + 0.7775T_{AB}\mathbf{j} + 0.6220T_{AB}\mathbf{k} \qquad (2)$$

Denoting by $\boldsymbol{\lambda}_{AC}$ the unit vector along AC, we write in a similar way

$$\overrightarrow{AC} = -(1.2 \text{ m})\mathbf{i} + (10 \text{ m})\mathbf{j} - (10 \text{ m})\mathbf{k} \qquad AC = 14.193 \text{ m}$$
$$\boldsymbol{\lambda}_{AC} = \frac{\overrightarrow{AC}}{14.193 \text{ m}} = -0.08455\mathbf{i} + 0.7046\mathbf{j} - 0.7046\mathbf{k}$$
$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = -0.08455T_{AC}\mathbf{i} + 0.7046T_{AC}\mathbf{j} - 0.7046T_{AC}\mathbf{k} \qquad (3)$$

Equilibrium Condition. Since A is in equilibrium, we must have

$$\Sigma\mathbf{F} = 0: \qquad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{W} = 0$$

or, substituting from (1), (2), and (3) for the forces and factoring $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$,

$$(-0.09330T_{AB} - 0.08455T_{AC} + P)\mathbf{i} + (0.7775T_{AB} + 0.7046T_{AC} - 1962 \text{ N})\mathbf{j} + (0.6220T_{AB} - 0.7046T_{AC})\mathbf{k} = 0$$

Setting the coefficients of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ equal to zero, we write three scalar equations, which express that the sums of the x, y, and z components of the forces are respectively equal to zero.

$$\Sigma F_x = 0: \qquad -0.09330T_{AB} - 0.08455T_{AC} + P = 0$$
$$\Sigma F_y = 0: \qquad +0.7775T_{AB} + 0.7046T_{AC} - 1962 \text{ N} = 0$$
$$\Sigma F_z = 0: \qquad +0.6220T_{AB} - 0.7046T_{AC} = 0$$

Solving these equations, we obtain

$$P = 235 \text{ N} \qquad T_{AB} = 1402 \text{ N} \qquad T_{AC} = 1238 \text{ N} \quad ◀$$

SOLVING PROBLEMS ON YOUR OWN

We saw earlier that when a particle is in *equilibrium*, the resultant of the forces acting on the particle must be zero. Expressing this fact in the case of the equilibrium of a *particle in three-dimensional space* will provide you with three relations among the forces acting on the particle. These relations can be used to determine three unknowns—usually the magnitudes of three forces.

Your solution will consist of the following steps:

1. *Draw a free-body diagram of the particle.* This diagram shows the particle and all the forces acting on it. Indicate on the diagram the magnitudes of known forces, as well as any angles or dimensions that define the direction of a force. Any unknown magnitude or angle should be denoted by an appropriate symbol. Nothing else should be included on your free-body diagram.

2. *Resolve each of the forces into rectangular components.* Following the method used in the preceding lesson, you will determine for each force $\mathbf{F}$ the unit vector $\boldsymbol{\lambda}$ defining the direction of that force and express $\mathbf{F}$ as the product of its magnitude F and the unit vector $\boldsymbol{\lambda}$. When two points on the line of action of $\mathbf{F}$ are known, you will obtain an expression of the form

$$\mathbf{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k})$$

where d, d_x, d_y, and d_z are dimensions obtained from the free-body diagram of the particle. We also showed the direction of $\mathbf{F}$ can be defined in terms of the angles θ_y and ϕ. If a force is known in magnitude as well as in direction, then F is known and the expression obtained for $\mathbf{F}$ is completely defined; otherwise F is one of the three unknowns that should be determined.

3. *Set the resultant, or sum, of the forces exerted on the particle equal to zero.* You will obtain a vectorial equation consisting of terms containing the unit vectors $\mathbf{i}$, $\mathbf{j}$, or $\mathbf{k}$. You will group the terms containing the same unit vector and factor that vector. For the vectorial equation to be satisfied, the coefficient of each of the unit vectors must be equal to zero. Thus, setting each coefficient equal to zero will yield three scalar equations that you can solve for no more than three unknowns [Sample Prob. 2.9].

bee76985_ch02_15-72 5/9/07 12:06 PM Page 60

Problems

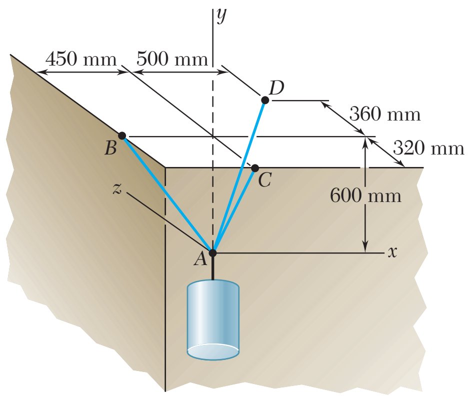


Fig. ***P2.101*** **and P2.102**

2.101 A container is supported by three cables that are attached to a ceiling as shown. Determine the weight W of the container knowing that the tension in cable AB is 6 kN.

2.102 A container is supported by three cables that are attached to a ceiling as shown. Determine the weight W of the container knowing that the tension in cable AD is 4.3 kN.

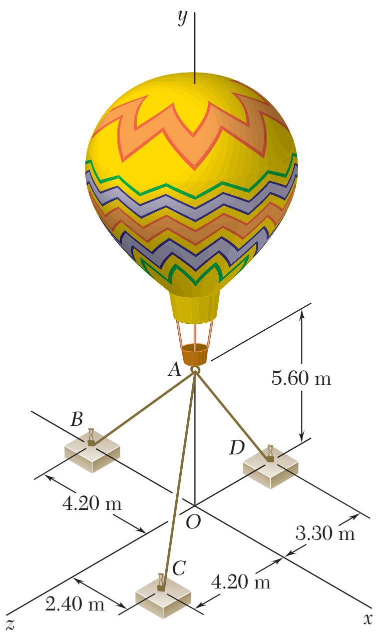


Fig. P2.103 and ***P2.104***

2.103 Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AB is 259 N.

2.104 Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AC is 444 N.

2.105 The support assembly shown is bolted in place at B, C, and D and supports a downward force **P** at A. Knowing that the forces in members AB, AC, and AD are directed along the respective members and that the force in member AB is 29.2 N, determine the magnitude of **P**.

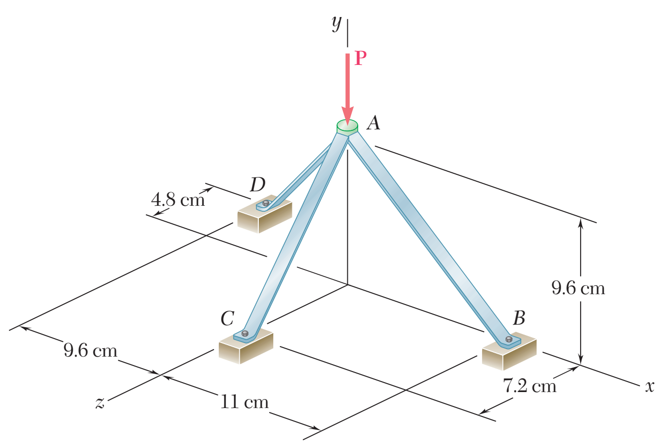


Fig. P2.105 and P2.106

2.106 The support assembly shown is bolted in place at B, C, and D and supports a downward force **P** at A. Knowing that the forces in members AB, AC, and AD are directed along the respective members and that $P = 45$ N, determine the forces in the members.

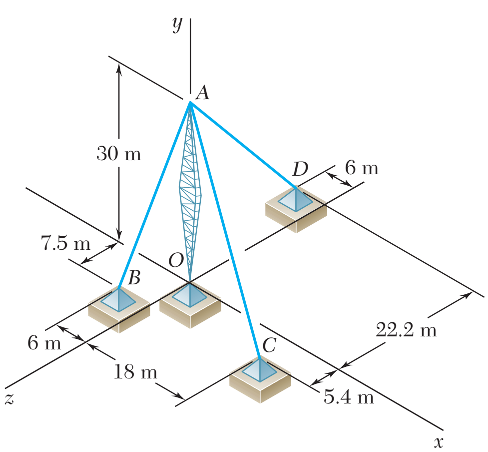


Fig. P2.107 and P2.108

2.107 A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AB is 3.6 kN, determine the vertical force **P** exerted by the tower on the pin at A.

2.108 A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AC is 2.6 kN, determine the vertical force **P** exerted by the tower on the pin at A.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 61

2.109 A 320 N load of lumber is lifted using a triple leg sling. Knowing that at the instant shown the lumber is at rest, determine the tension in each leg of the sling.

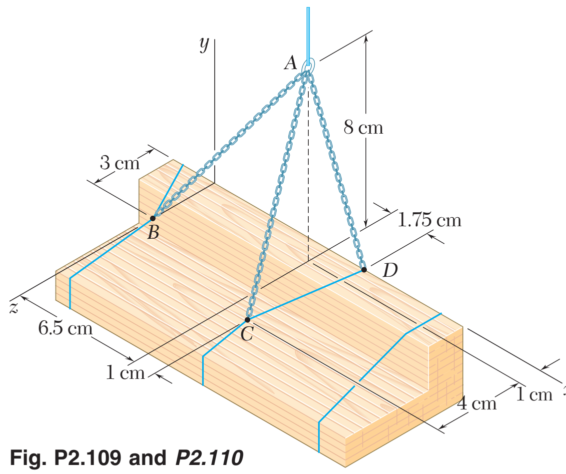


Fig. P2.109 and *P2.110*

2.110 A load of lumber is lifted using a triple leg sling. Knowing that at the instant shown the lumber is at rest and that the tension in leg AD is 220 N, determine the weight of the lumber.

2.111 A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex A of the cone. Knowing that $P = 0$ and that the tension in cord BE is 0.2 N, determine the weight W of the cone.

2.112 A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex A of the cone. Knowing that the cone weighs 1.6 N, determine the range of values of P for which cord CF is taut.

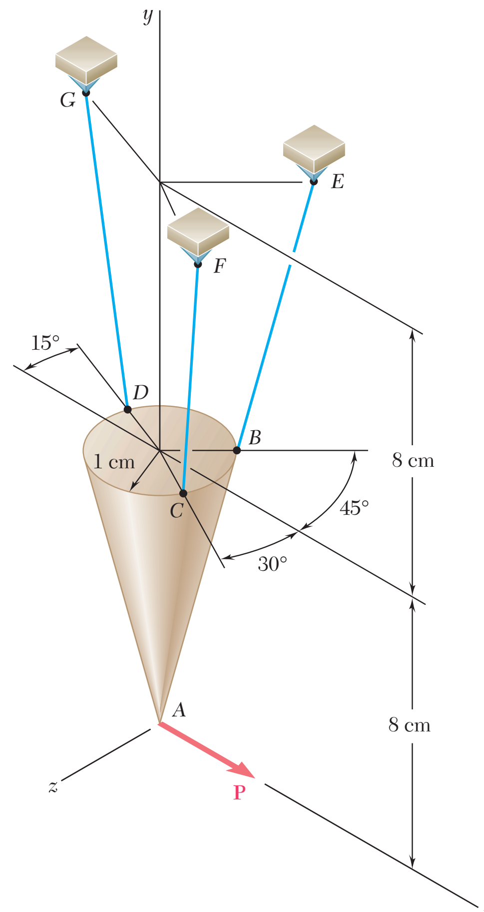


Fig. *P2.111* and P2.112

2.113 A 16-kg triangular plate is supported by three wires as shown. Knowing that $a = 150$ mm, determine the tension in each wire.

2.114 A 16-kg triangular plate is supported by three wires as shown. Knowing that $a = 200$ mm, determine the tension in each wire.

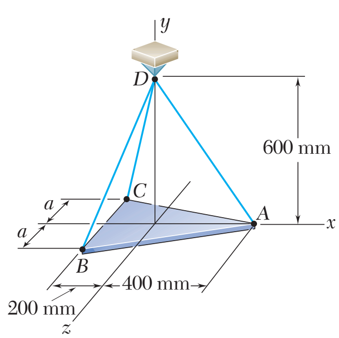


Fig. P2.113 and P2.114

2.115 A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. Knowing that the tower exerts on the pin at A an upward vertical force of 8 kN, determine the tension in each wire.

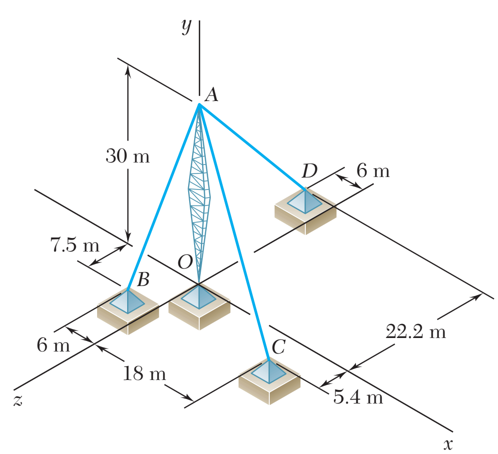


Fig. P2.115

bee76985_ch02_15-72 5/9/07 12:06 PM Page 62

2.116 A derrick boom is guyed by cables *AC* and *AD*. A worker is lifting a 20-kg block by pulling on a rope that passes through the pulley at *A*. Knowing that the boom *AB* exerts a force at *A* that is directed from *B* to *A*, determine this force and the force in each of the two cables.

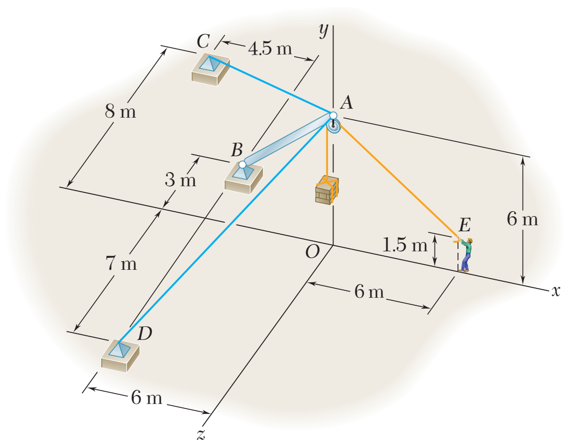


Fig. P2.116

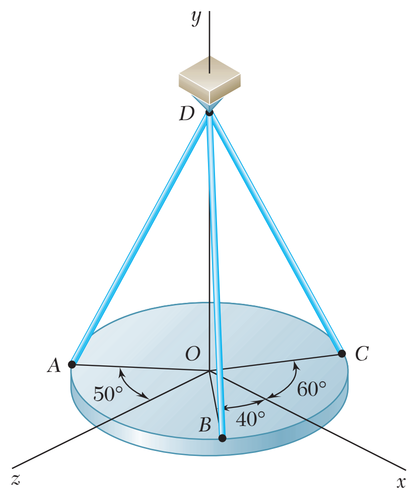


Fig. *P2.117*

2.117 A horizontal circular plate weighing 62 N is suspended as shown from three wires that are attached to a support *D* and that form 30° angles with the vertical. Determine the tension in each wire.

2.118 For the cone of Prob. 2.112, determine the range of values of *P* for which cord *DG* is taut if **P** is directed in the −*x* direction.

2.119 In trying to move across a slippery icy surface, a 175-N man uses two ropes *AB* and *AC*. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

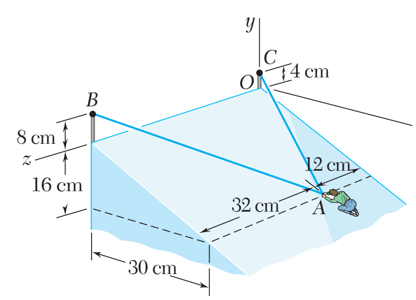


Fig. *P2.119*

2.120 Solve Prob. 2.119 assuming that a friend is helping the man at *A* by pulling on him with a force $\mathbf{P} = -(45 \text{ N})\mathbf{k}$.

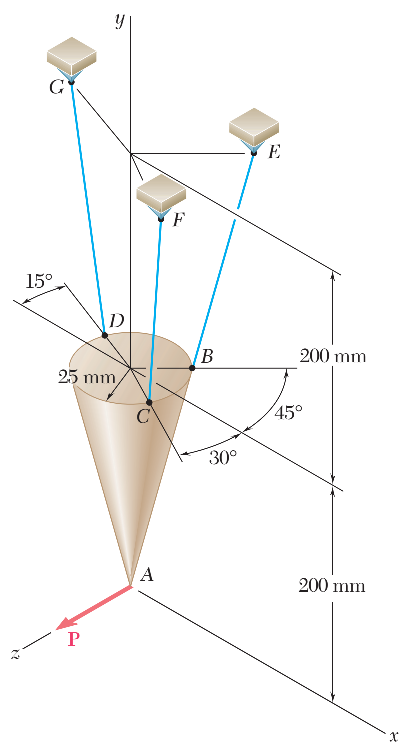


Fig. P2.121 and P2.122

2.121 A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex *A* of the cone. Knowing that the cone weighs 10.5 N and that $P = 0$, determine the tension in each cord.

2.122 A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex *A* of the cone. Knowing that the cone weighs 10.5 N and that $P = 0.5$ N, determine the tension in each cord.

bee76985_ch02_15-72 5/9/07 12:06 PM Page 63

2.123 To lower a pack of weight W to the hiker at C, the hikers at A and B have passed a rope ADB through a ring attached to the pack at D. The hiker at C is 64 m below A and guides the pack using rope CD. Knowing that at the instant shown the pack is at rest and that the tension in rope CD is 17 N, determine the tension in rope ADB and the weight of the pack. (*Hint:* The tension is the same in both portions of rope ADB.)

2.124 To lower a pack of weight W to the hiker at C, the hikers at A and B have passed a rope ADB through a ring attached to the pack at D. The hiker at C is 64 m below A and guides the pack using rope CD. Knowing that $W = 120$ N and that at the instant shown the pack is at rest, determine the tension in each rope. (*Hint:* The tension is the same in both portions of rope ADB.)

2.125 A piece of machinery of weight W is temporarily supported by cables AB, AC, and ADE. Cable ADE is attached to the ring at A, passes over the pulley at D and back through the ring, and is attached to the support at E. Knowing that $W = 1400$ N, determine the tension in each cable. (*Hint:* The tension is the same in all portions of cable ADE.)

2.126 A piece of machinery of weight W is temporarily supported by cables AB, AC, and ADE. Cable ADE is attached to the ring at A, passes over the pulley at D and back through the ring, and is attached to the support at E. Knowing that the tension in cable AB is 300 N, determine (*a*) the tension in AC, (*b*) the tension in ADE, (*c*) the weight W. (*Hint:* The tension is the same in all portions of cable ADE.)

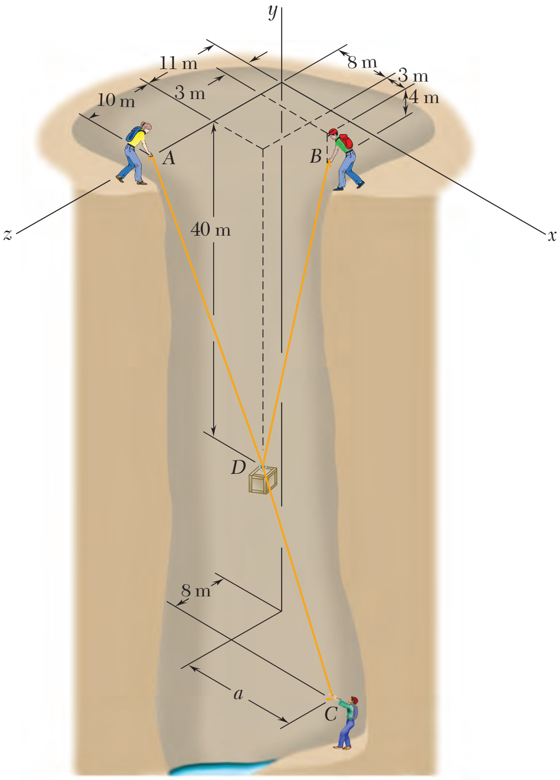


Fig. P2.123 and P2.124

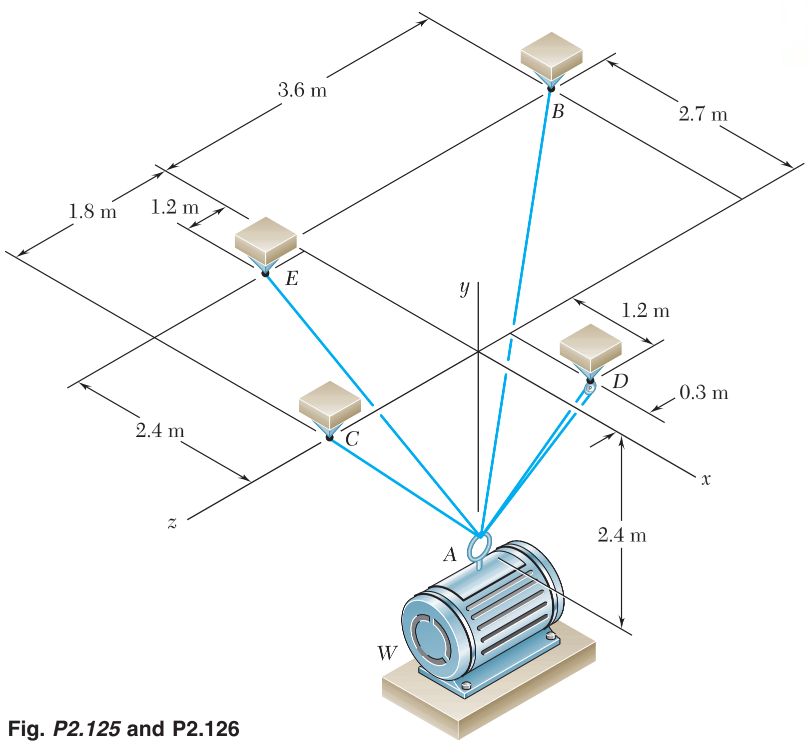


Fig. *P2.125* and P2.126

2.127 Collars A and B are connected by a 1-m-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (680\text{ N})\mathbf{j}$ is applied at A, determine (*a*) the tension in the wire when $y = 300$ mm, (*b*) the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the system.

2.128 Solve Prob. 2.127 assuming $y = 550$ mm.

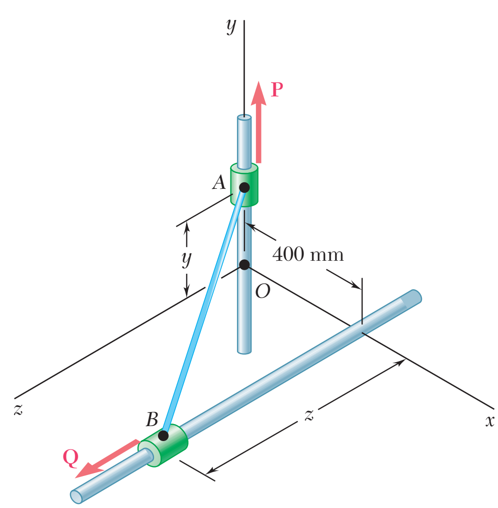


Fig. P2.127

REVIEW AND SUMMARY FOR CHAPTER 2

In this chapter we have studied the effect of forces on particles, that is, on bodies of such shape and size that all forces acting on them may be assumed applied at the same point.

Resultant of two forces

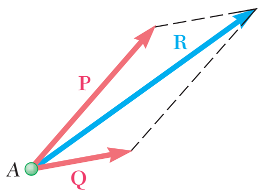


Fig. 2.35

Forces are *vector quantities*; they are characterized by a *point of application*, a *magnitude*, and a *direction*, and they add according to the *parallelogram law* (Fig. 2.35). The magnitude and direction of the resultant **R** of two forces **P** and **Q** can be determined either graphically or by trigonometry, using successively the law of cosines and the law of sines [Sample Prob. 2.1].

Components of a force

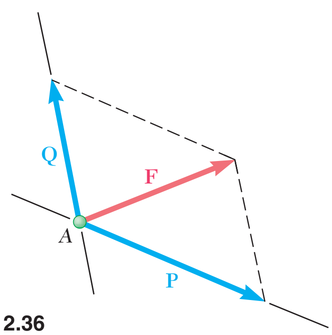


Fig. 2.36

Any given force acting on a particle can be resolved into two or more *components*, that is, it can be replaced by two or more forces which have the same effect on the particle. A force **F** can be resolved into two components **P** and **Q** by drawing a parallelogram which has **F** for its diagonal; the components **P** and **Q** are then represented by the two adjacent sides of the parallelogram (Fig. 2.36) and can be determined either graphically or by trigonometry [Sec. 2.6].

Rectangular components
Unit vectors

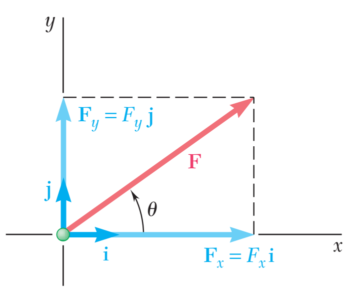


Fig. 2.37

A force **F** is said to have been resolved into two *rectangular components* if its components $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular to each other and are directed along the coordinate axes (Fig. 2.37). Introducing the *unit vectors* **i** and **j** along the x and y axes, respectively, we write [Sec. 2.7]

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j} \tag{2.6}$$

and

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{2.7}$$

where F_x and F_y are the *scalar components* of **F**. These components, which can be positive or negative, are defined by the relations

$$F_x = F\cos\theta \qquad F_y = F\sin\theta \tag{2.8}$$

When the rectangular components F_x and F_y of a force **F** are given, the angle θ defining the direction of the force can be obtained by writing

$$\tan\theta = \frac{F_y}{F_x} \tag{2.9}$$

The magnitude F of the force can then be obtained by solving one of the equations (2.8) for F or by applying the Pythagorean theorem and writing

$$F = \sqrt{F_x^2 + F_y^2} \tag{2.10}$$

bee76985_ch02_15-72 5/9/07 12:06 PM Page 65

Resultant of several coplanar forces

When *three or more coplanar forces* act on a particle, the rectangular components of their resultant **R** can be obtained by adding algebraically the corresponding components of the given forces [Sec. 2.8]. We have

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \tag{2.13}$$

The magnitude and direction of **R** can then be determined from relations similar to Eqs. (2.9) and (2.10) [Sample Prob. 2.3].

Forces in space

A force **F** in *three-dimensional space* can be resolved into rectangular components $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$ [Sec. 2.12]. Denoting by θ_x, θ_y, and θ_z, respectively, the angles that **F** forms with the x, y, and z axes (Fig. 2.38), we have

$$F_x = F \cos \theta_x \qquad F_y = F \cos \theta_y \qquad F_z = F \cos \theta_z \tag{2.19}$$

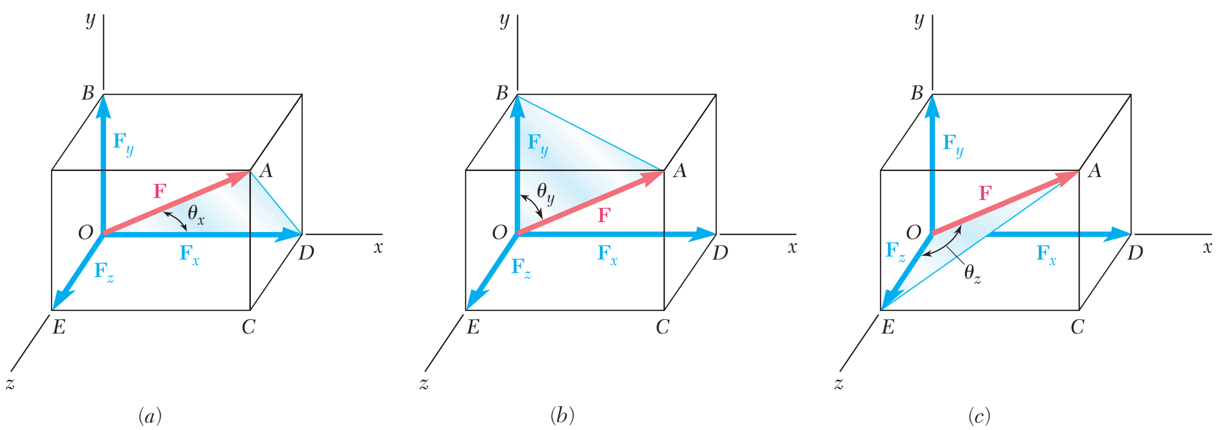


Fig. 2.38

Direction cosines

The cosines of θ_x, θ_y, and θ_z are known as the *direction cosines* of the force **F**. Introducing the unit vectors **i**, **j**, and **k** along the coordinate axes, we write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \tag{2.20}$$

or

$$\mathbf{F} = F(\cos \theta_x\mathbf{i} + \cos \theta_y\mathbf{j} + \cos \theta_z\mathbf{k}) \tag{2.21}$$

which shows (Fig. 2.39) that **F** is the product of its magnitude F and the unit vector

$$\boldsymbol{\lambda} = \cos \theta_x\mathbf{i} + \cos \theta_y\mathbf{j} + \cos \theta_z\mathbf{k}$$

Since the magnitude of $\boldsymbol{\lambda}$ is equal to unity, we must have

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1 \tag{2.24}$$

When the rectangular components F_x, F_y, and F_z of a force **F** are given, the magnitude F of the force is found by writing

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \tag{2.18}$$

and the direction cosines of **F** are obtained from Eqs. (2.19). We have

$$\cos \theta_x = \frac{F_x}{F} \qquad \cos \theta_y = \frac{F_y}{F} \qquad \cos \theta_z = \frac{F_z}{F} \tag{2.25}$$

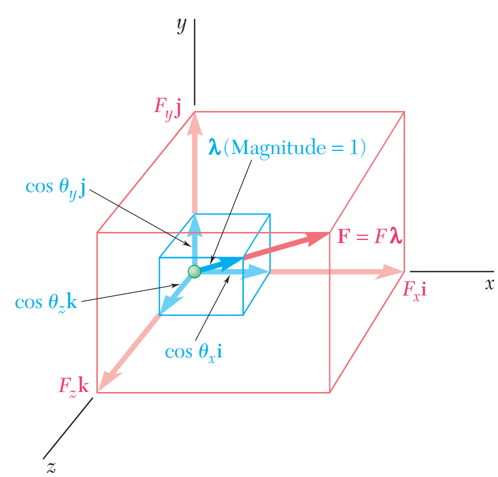


Fig. 2.39

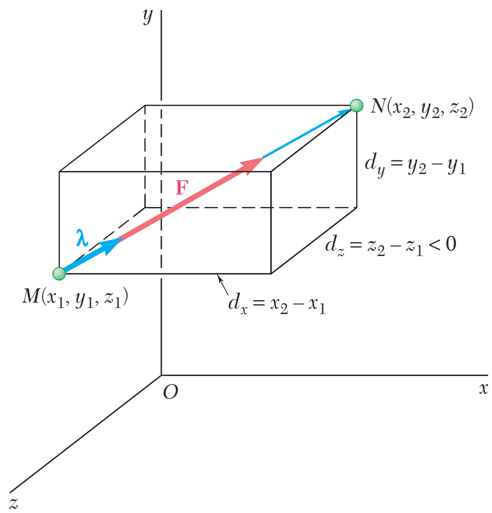


Fig. 2.40

When a force **F** is defined in three-dimensional space by its magnitude F and two points M and N on its line of action [Sec. 2.13], its rectangular components can be obtained as follows. We first express the vector $\overrightarrow{MN}$ joining points M and N in terms of its components d_x, d_y, and d_z (Fig. 2.40); we write

$$\overrightarrow{MN} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k} \tag{2.26}$$

We next determine the unit vector $\boldsymbol{\lambda}$ along the line of action of **F** by dividing $\overrightarrow{MN}$ by its magnitude $MN = d$:

$$\boldsymbol{\lambda} = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \tag{2.27}$$

Recalling that **F** is equal to the product of F and $\boldsymbol{\lambda}$, we have

$$\mathbf{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \tag{2.28}$$

from which it follows [Sample Probs. 2.7 and 2.8] that the scalar components of **F** are, respectively,

$$F_x = \frac{Fd_x}{d} \qquad F_y = \frac{Fd_y}{d} \qquad F_z = \frac{Fd_z}{d} \tag{2.29}$$

Resultant of forces in space

When *two or more forces* act on a particle in *three-dimensional space,* the rectangular components of their resultant **R** can be obtained by adding algebraically the corresponding components of the given forces [Sec. 2.14]. We have

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z \tag{2.31}$$

The magnitude and direction of **R** can then be determined from relations similar to Eqs. (2.18) and (2.25) [Sample Prob. 2.8].

Equilibrium of a particle

A particle is said to be in *equilibrium* when the resultant of all the forces acting on it is zero [Sec. 2.9]. The particle will then remain at rest (if originally at rest) or move with constant speed in a straight line (if originally in motion) [Sec. 2.10].

Free-body diagram

To solve a problem involving a particle in equilibrium, one first should draw a *free-body diagram* of the particle showing all the forces acting on it [Sec. 2.11]. If *only three coplanar forces* act on the particle, a *force triangle* may be drawn to express that the particle is in equilibrium. Using graphical methods or trigonometry, this triangle can be solved for no more than two unknowns [Sample Prob. 2.4]. If *more than three coplanar forces* are involved, the equations of equilibrium

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \tag{2.15}$$

should be used. These equations can be solved for no more than two unknowns [Sample Prob. 2.6].

Equilibrium in space

When a particle is in *equilibrium in three-dimensional space* [Sec. 2.15], the three equations of equilibrium

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{2.34}$$

should be used. These equations can be solved for no more than three unknowns [Sample Prob. 2.9].

Review Problems

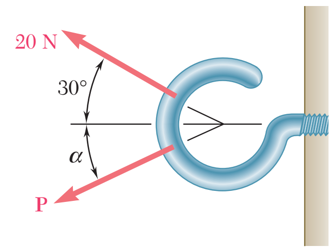


Fig. P2.129

2.129 Two forces are applied as shown to a hook support. Using trigonometry and knowing that the magnitude of **P** is 14 N, determine (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

2.130 Determine the x and y components of each of the forces shown.

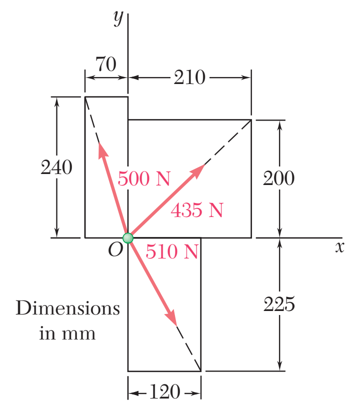


Fig. *P2.130*

2.131 The guy wire BD exerts on the telephone pole AC a force **P** directed along BD. Knowing that **P** has a 450-N component along line AC, determine (*a*) the magnitude of the force **P**, (*b*) its component in a direction perpendicular to AC.

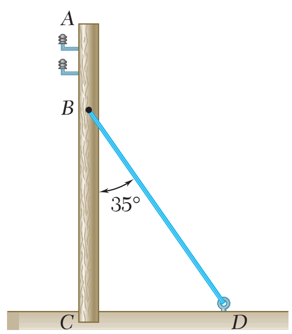


Fig. P2.131

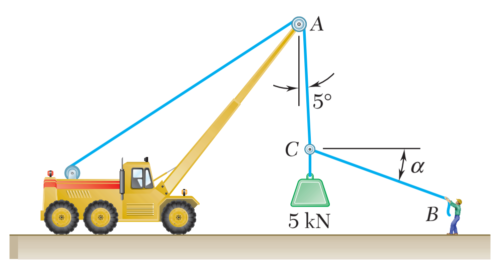


Fig. P2.132

2.132 Knowing that $\alpha = 25°$, determine the tension (*a*) in cable AC, (*b*) in rope BC.

2.133 The cabin of an aerial tramway is suspended from a set of wheels that can roll freely on the support cable ACB and is being pulled at a constant speed by cable DE. Knowing that $\alpha = 42°$ and $\beta = 32°$, that the tension in cable DE is 20 kN, and assuming the tension in cable DF to be negligible, determine (*a*) the combined weight of the cabin, its support system, and its passengers, (*b*) the tension in the support cable ACB.

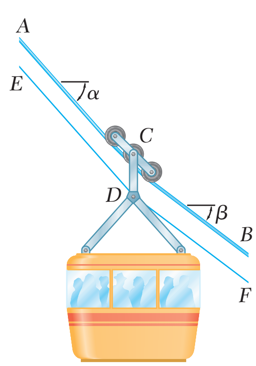


Fig. *P2.133*

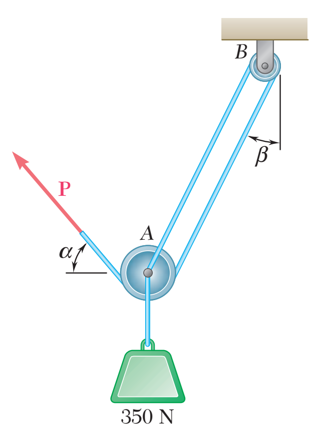


Fig. P2.134

2.134 A 350-N load is supported by the rope-and-pulley arrangement shown. Knowing that $\beta = 25°$, determine the magnitude and direction of the force **P** which should be exerted on the free end of the rope to maintain equilibrium. (See the hint of Prob. 2.68.)

bee76985_ch02_15-72 5/9/07 12:06 PM Page 68

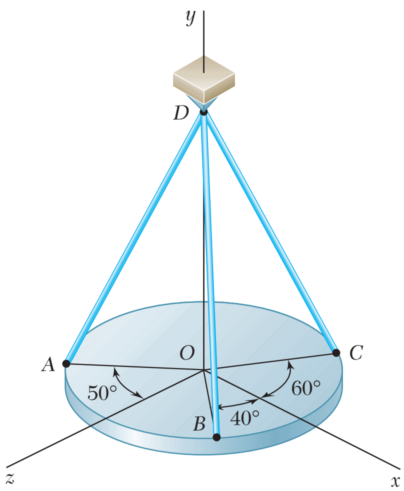


Fig. P2.135

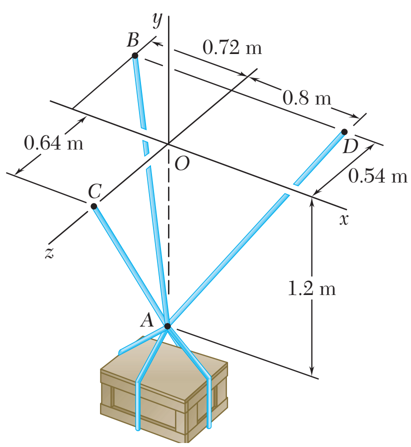
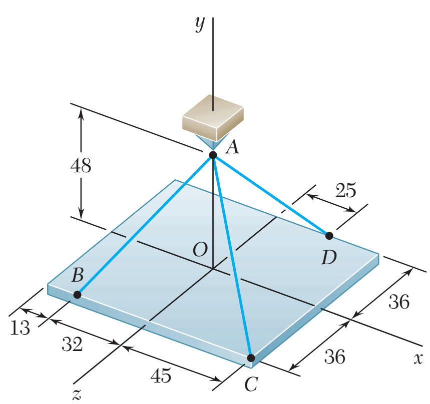


Fig. *P2.138*



Dimensions in m

Fig. P2.139

2.135 A horizontal circular plate is suspended as shown from three wires that are attached to a support at D and that form 30° angles with the vertical. Knowing that the x component of the force exerted by wire AD on the plate is 220.6 N, determine (a) the tension in wire AD, (b) the angles θ_x, θ_y, and θ_z that the force exerted at A forms with the coordinate axes.

2.136 A force **F** of magnitude 600 N acts at the origin of a coordinate system. Knowing that $F_x = 200$ N, $\theta_z = 136.8°$, and $F_y < 0$, determine (a) the components F_y and F_z, (b) the angles θ_x and θ_y.

2.137 Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 500$ N and $Q = 600$ N.

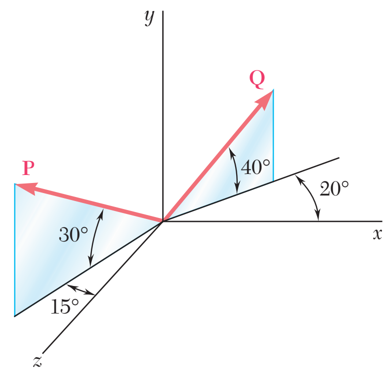


Fig. P2.137

2.138 The crate shown is supported by three cables. Determine the weight of the crate knowing that the tension in cable AB is 3 kN.

2.139 A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 120 N, determine the weight of the plate.

2.140 A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 1200$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

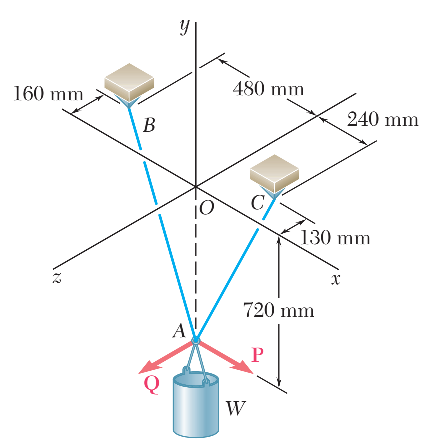


Fig. *P2.140*

bee76985_ch02_15-72 5/9/07 12:06 PM Page 69

Computer Problems

2.C1 Using computational software, determine the magnitude and direction of the resultant of n coplanar forces applied at a point A. Use this software to solve Probs. 2.31, 2.32, 2.33, and 2.34.

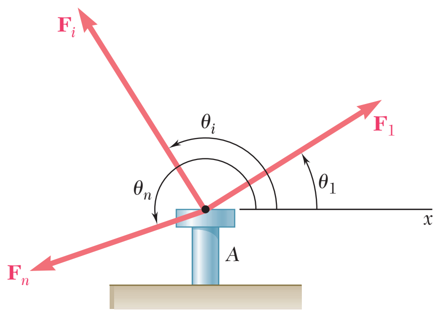


Fig. P2.C1

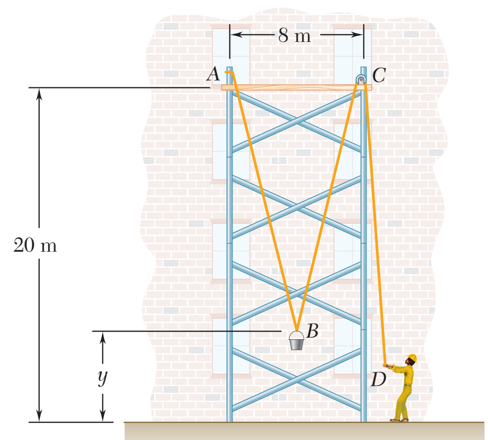


Fig. P2.C2

2.C2 A worker plans to lift a 60-N 5-m^3 bucket of paint by tying a rope to the scaffold at A and then passing the rope through the bail of the bucket at B and finally over the pulley at C. (*a*) Plot the tension in the rope as a function of the height y for 2 m $\leq y \leq$ 18 m. (*b*) Evaluate the worker's plan.

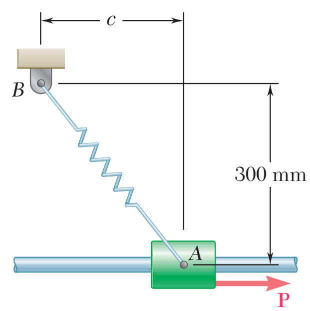


Fig. P2.C3

2.C3 The collar A can slide freely on the horizontal frictionless rod shown. The spring attached to the collar has a spring constant k and is undeformed when the collar is directly below support B. Express in terms of k and the distance c the magnitude of the force $\mathbf{P}$ required to maintain equilibrium of the system. Plot P as a function of c for values of c from 0 to 600 mm when (*a*) $k = 2$ N/mm, (*b*) $k = 3$ N/mm, (*c*) $k = 4$ N/mm.

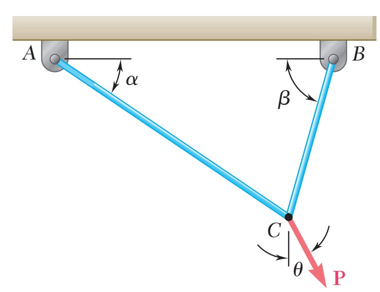


Fig. P2.C4

2.C4 A load $\mathbf{P}$ is supported by two cables as shown. Using computational software, determine the tension in each cable as a function of P and θ. For the following three sets of numerical values, plot the tensions for values of θ ranging from $\theta_1 = \beta - 90°$ to $\theta_2 = 90°$ and then determine from the graphs (*a*) the value of θ for which the tension in the two cables is as small as possible, (*b*) the corresponding value of the tension.

(1) $\alpha = 35°, \beta = 75°, P = 1.6$ kN
(2) $\alpha = 50°, \beta = 30°, P = 2.4$ kN
(3) $\alpha = 40°, \beta = 60°, P = 1.0$ kN

bee76985_ch02_15-72 5/9/07 12:06 PM Page 70

2.C5 Cables AC and BC are tied together at C and are loaded as shown. Knowing that $P = 100$ N, (a) express the tension in each cable as a function of θ. (b) Plot the tension in each cable for $0 \leq \theta \leq 90°$. (c) From the graph obtained in part a, determine the smallest value of θ for which both cables are in tension.

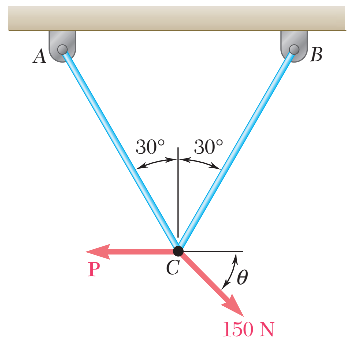


Fig. P2.C5

2.C6 A container of weight W is suspended from ring A to which cable AB of length 5 m and spring AC are attached. The constant of the spring is 100 N/m, and its unstretched length is 3 m. Determine the tension in the cable when (a) $W = 120$ N, (b) $W = 160$ N.

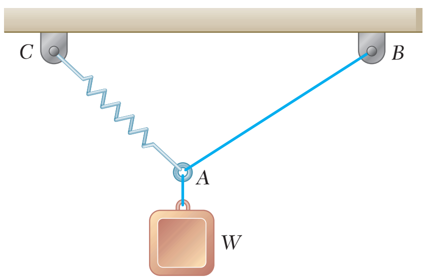


Fig. P2.C6

2.C7 An acrobat is walking on a tightrope of length $L = 80.3$ m attached to supports A and B at a distance of 80.0 m from each other. The combined weight of the acrobat and his balancing pole is 200 N, and the friction between his shoes and the rope is large enough to prevent him from slipping. Neglecting the weight of the rope and any elastic deformation, use computational software to determine the deflection y and the tension in portions AC and BC of the rope for values of x from 0.5 m to 40 m using 0.5-m increments. From the results obtained, determine (a) the maximum deflection of the rope, (b) the maximum tension in the rope, (c) the minimum values of the tension in portions AC and BC of the rope.

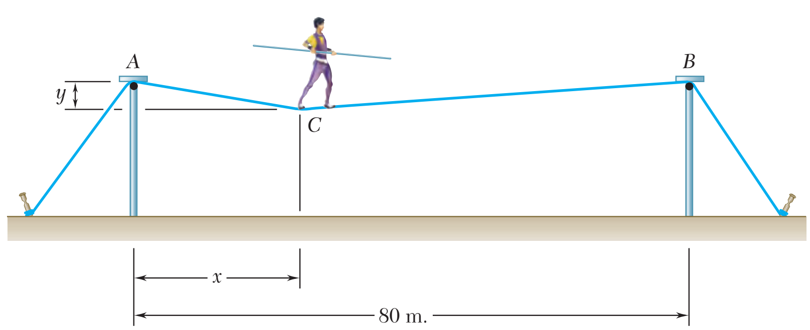


Fig. P2.C7

bee76985_ch02_15-72 5/9/07 12:06 PM Page 71

2.C8 The transmission tower shown is guyed by three cables attached to a pin at A and anchored at points B, C, and D. Cable AD is 21 m long and the tension in that cable is 20 kN. (*a*) Express the x, y, and z components of the force exerted by cable AD on the anchor at D and the corresponding angles θ_x, θ_y, and θ_z in terms of α. (*b*) Plot the components of the force and the angles θ_x, θ_y, and θ_z for $0 \le \alpha \le 60°$.

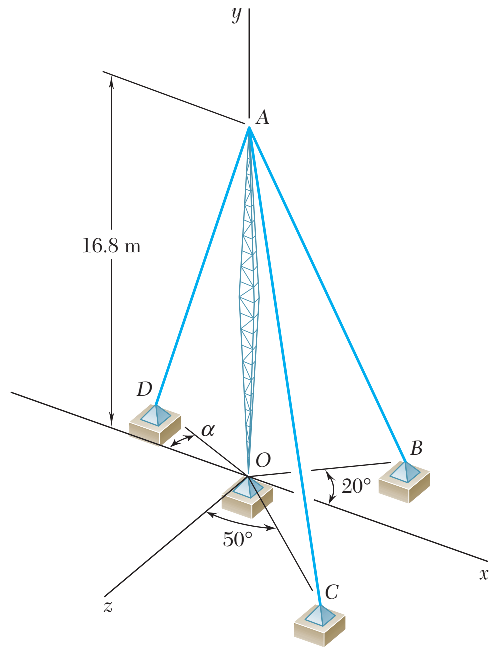


Fig. P2.C8

2.C9 A tower is guyed by cables AB and AC. A worker ties a rope of length 12 m to the tower at A and exerts a constant force of 160 N on the rope. (*a*) Express the tension in each cable as a function of θ knowing that the resultant of the tensions in the cables and the rope is directed downward. (*b*) Plot the tension in each cable as a function of θ for $0 \le \theta \le 180°$, and determine from the graph the range of values of θ for which both cables are taut.

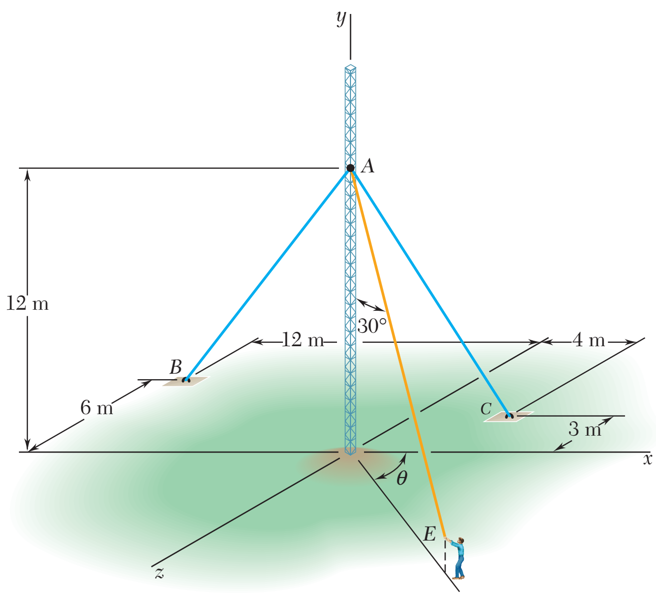


Fig. P2.C9

2.C10 Collars A and B are connected by a 10-m-long wire and can slide freely on frictionless rods. If a force **Q** of magnitude 25 N is applied to collar B as shown, determine the tension in the wire and the corresponding magnitude of the force **P** required to maintain equilibrium. Plot the tension in the wire and the magnitude of the force **P** for $0 \le x \le 5$ m.

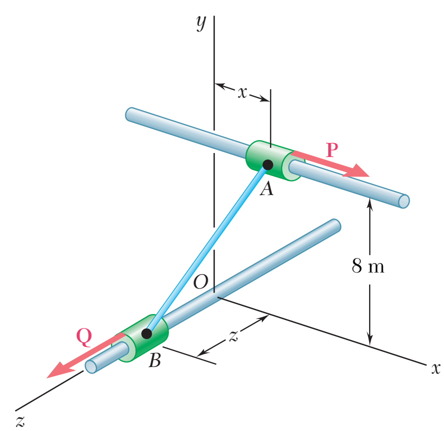


Fig. P2.C10

bee76985_ch02_15-72 5/9/07 12:06 PM Page 72