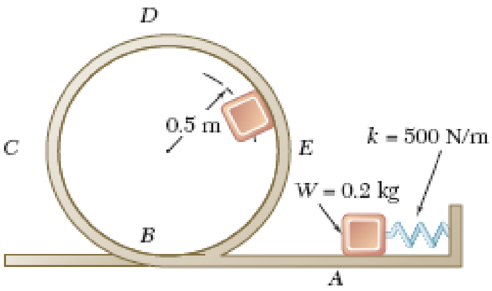
ERRATA INFORMATION BEER DYNAMICS

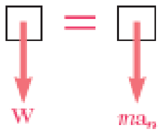
Sr. No.	Pg No.	Nature of error	Error in line	Correction
1	809	answers of sample problem 13.10	11	value of t changed from 347 to 2661.68
2	1059	change in unit in sample problem 16.10	last 4 lines	kgf to change to N
3	1059	change in answer sample problem 16.10	5th last line	Alpha=.075 and not .73
4	1059	change in answer sample problem 16.10	5th last line	R_BSin45 =7.6 and not 73.9
5	1059	change in answer sample problem 16.10	5th last line	R_A= 167.49 and not 141.7
6	765	change in answer of sample problem 13.1	25	value of N changed from 490 to 490.5
7	767	change in answer of sample problem 13.4	24	value of N changed from 44488.4 to 45081.53
8	937	change in answer sample problem 15.3	last line	value of Omega BD changed from 75.18 to 61.83
9	960	change in answer sample problem 15.7	line 14	Omega BD changed from 75.18 to 61.83
10	960	change in answer sample problem 15.7	line 18	answer of $(a_{D/B})$n is changed from 1147.36 to 776.06
11	960	change in answer sample problem 15.7	Second last line	answer of Alpha BD is changed from 94.68 to120
12	960	change in answer sample problem 15.7	last line	answer of a_D is changed from 3202.97 to 3889.68
13	1088	change in answer sample problem 17.1	line 19	answer of T1 is changed from 179.8 to 192.4 and then T1=1888.3 and not 1763.8



SAMPLE PROBLEM 13.7

The 0.2 kg pellet is pushed against the spring at A and released from rest. Neglecting friction, determine the smallest deflection of the spring for which the pellet will travel around the loop $ABCDE$ and remain at all times in contact with the loop.

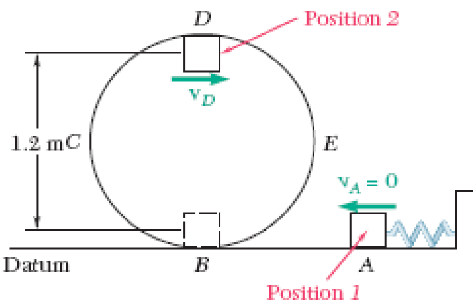
SOLUTION



Required Speed at Point D. As the pellet passes through the highest point D, its potential energy with respect to gravity is maximum and thus, its kinetic energy and speed are minimum. Since the pellet must remain in contact with the loop, the force **N** exerted on the pellet by the loop must be equal to or greater than zero. Setting **N** = 0, we compute the smallest possible speed v_D.

$$+\downarrow\Sigma F_n = ma_n: \qquad W = ma_n \qquad mg = ma_n \qquad a_n = g$$

$$a_n = \frac{v_D^2}{r}: \qquad v_D^2 = ra_n = rg = (0.5\ \text{m})(9.81\ \text{m/s}^2) = 4.905\ \text{m}^2/\text{s}^2$$



Position 1. ***Potential Energy.*** Denoting by x the deflection of the spring and noting that k = 500 N/m, we write

$$V_e = \tfrac{1}{2}kx^2 = \tfrac{1}{2} \times 500\ x^2 = 250\ x^2$$

Choosing the datum at A, we have $V_g = 0$; therefore

$$V_1 = V_e + V_g = 250\ x^2$$

Kinetic Energy. Since the pellet is released from rest, $v_A = 0$ and we have $T_1 = 0$.

Position 2. ***Potential Energy.*** The spring is now undeformed; thus $V_e = 0$. Since the pellet is 1 m above the datum, we have

$$V_g = Wy = 0.2 \times 9.81 \times 1 = 1.962\ \text{Nm}$$

$$V_2 = V_e + V_g = 1.962\ \text{Nm}$$

Kinetic Energy. Using the value of v_D^2 obtained above, we write

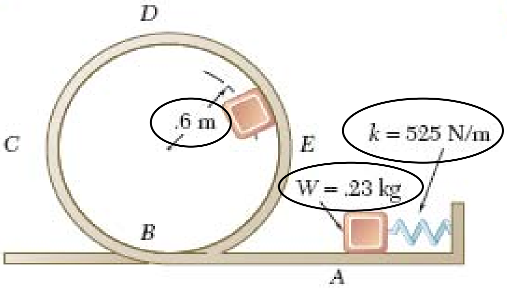
$$T_2 = \tfrac{1}{2}mv_D^2 = \tfrac{1}{2} \times 0.2 \times 4.905 = 0.4905\ \text{Nm}$$

Conservation of Energy. Applying the principle of conservation of energy between positions 1 and 2, we write

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 250\ x^2 = 0.4905\ \text{Nm} + 1.962\ \text{Nm}$$

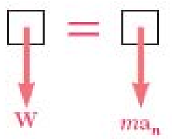
$$x = 0.099\ \text{m} \qquad\qquad x = 0.099\ \text{m} \blacktriangleleft$$



SAMPLE PROBLEM 13.7

The 0.23 kg pellet is pushed against the spring at A and released from rest. Neglecting friction, determine the smallest deflection of the spring for which the pellet will travel around the loop $ABCDE$ and remain at all times in contact with the loop.

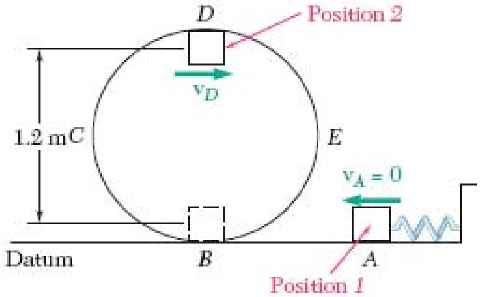
SOLUTION



Required Speed at Point *D*. As the pellet passes through the highest point D, its potential energy with respect to gravity is maximum and thus, its kinetic energy and speed are minimum. Since the pellet must remain in contact with the loop, the force **N** exerted on the pellet by the loop must be equal to or greater than zero. Setting **N** = 0, we compute the smallest possible speed v_D.

$$+\downarrow \Sigma F_n = ma_n: \qquad W = ma_n \qquad mg = ma_n \qquad a_n = g$$

$$a_n = \frac{v_D^2}{r}: \qquad v_D^2 = ra_n = rg = (.6 \text{ m})(9.81 \text{ m/s}^2) = 5.886 \text{ ft}^2/\text{s}^2$$



Position 1. ***Potential Energy.*** Denoting by x the deflection of the spring and noting that k = 525 N/m = 525 N/m, we write

$$V_e = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(525 \text{ N/m})x^2 = 262.5\ x^2$$

Choosing the datum at A, we have $V_g = 0$; therefore

$$V_1 = V_e + V_g = 262.5\ x^2$$

Kinetic Energy. Since the pellet is released from rest, $v_A = 0$ and we have $T_1 = 0$.

Position 2. ***Potential Energy.*** The spring is now undeformed; thus $V_e = 0$. Since the pellet is 1.2 m above the datum, we have

$$V_g = Wy = (2.26 \text{ N})(1.2 \text{ m}) = 2.7 \text{ N} \cdot \text{m}$$
$$V_2 = V_e + V_g = 2.7 \text{ N} \cdot \text{m}$$

Kinetic Energy. Using the value of v_D^2 obtained above, we write

$$T_2 = \tfrac{1}{2}mv_D^2 = \frac{1}{2} \times (0.23 \text{ kg}) \times (5.886 \text{ ft}^2/\text{s}^2) = 0.68 \text{ N} \cdot \text{m}$$

Conservation of Energy. Applying the principle of conservation of energy between positions 1 and 2, we write

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 262.5\ x^2 = 0.68 + 2.7$$
$$x = .1135 \text{ m} \qquad\qquad x = 0.1135 \text{ m} \blacktriangleleft$$

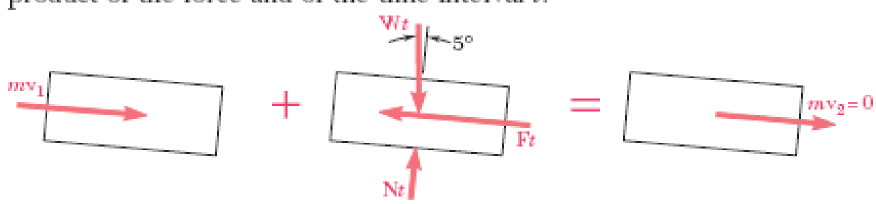
Corrected Version

SAMPLE PROBLEM 13.10

An automobile weighing 800 kg is driven down a 5° incline at a speed of 100 km/h when the brakes are applied, causing a constant total braking force (applied by the road on the tires) of 6000 N. Determine the time required for the automobile to come to a stop.

SOLUTION

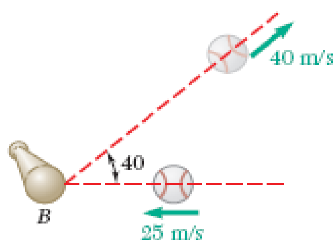
We apply the principle of impulse and momentum. Since each force is constant in magnitude and direction, each corresponding impulse is equal to the product of the force and of the time interval t.



$$m\mathbf{v}_1 + \Sigma\,\mathbf{Imp}_{1\rightarrow 2} = m\mathbf{v}_2$$

$+\searrow$ components: $\quad mv_1 + (W \sin 5°)t - Ft = 0$

$$800 \times 100\,\frac{1000}{3600}\left(\frac{\mathrm{m}}{\mathrm{s}}\right) + (800 \times 9.81 \sin 5°)t - 6000t = 0 \qquad t = 4.18\ \mathrm{s}$$

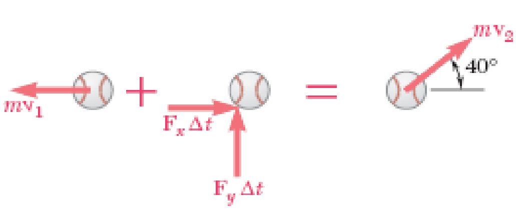


SAMPLE PROBLEM 13.11

A 0.1 kg baseball is pitched with a velocity of 25 m/s toward a batter. After the ball is hit by the bat B, it has a velocity of 40 m/s in the direction shown. If the bat and ball are in contact 0.015 s, determine the average impulsive force exerted on the ball during the impact.

SOLUTION

We apply the principle of impulse and momentum to the ball. Since the weight of the ball is a nonimpulsive force, it can be neglected.



$$m\mathbf{v}_1 + \Sigma\,\mathbf{Imp}_{1\rightarrow 2} = m\mathbf{v}_2$$

$\xrightarrow{+}$ x components: $\quad -mv_1 + F_x\,\Delta t = mv_2 \cos 40°$

$$-0.1 \times \left(25\,\frac{\mathrm{m}}{\mathrm{s}}\right) + F_x(0.015) = 0.1 \times \left(40\,\frac{\mathrm{m}}{\mathrm{s}}\right) \cos 40°$$

$$F_x = +370.94\ \mathrm{N}$$

$+\uparrow y$ components: $\quad 0 + F_y\,\Delta t = mv_2 \sin 40°$

$$F_y(0.015) = 0.1 \times 40 \sin 40°$$

$$F_y = +171.41\ \mathrm{N}$$

From its components F_x and F_y we determine the magnitude and direction of the force $\mathbf{F}$:

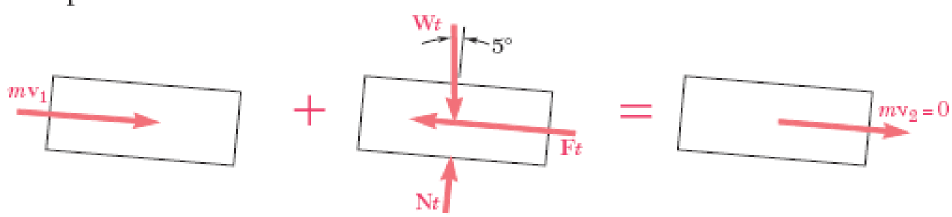
$$\mathbf{F} = 408.63\ \mathrm{N} \measuredangle 24.8°$$

SAMPLE PROBLEM 13.10

An automobile weighing 8818.5 kg is driven down a 5° incline at a speed of 96 km/h when the brakes are applied, causing a constant total braking force (applied by the road on the tires) of 6672 N. Determine the time required for the automobile to come to a stop.

SOLUTION

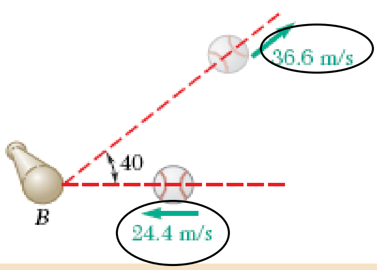
We apply the principle of impulse and momentum. Since each force is constant in magnitude and direction, each corresponding impulse is equal to the product of the force and of the time interval t.



$$m\mathbf{v}_1 + \Sigma\,\mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$

$+\searrow$ components: $\quad mv_1 + (W \sin 5°)t - Ft = 0$

$$(8818.5 \times 9.81)\,(26.7 \text{ m/s}) + (8818.5 \times 9.81 \sin 5°)t - 6672t = 0 \quad t = 2661.68 \text{ s}$$

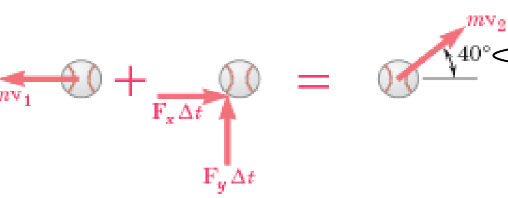


SAMPLE PROBLEM 13.11

A .12 kg baseball is pitched with a velocity of 24.4 m/s toward a batter. After the ball is hit by the bat B, it has a velocity of 36.6 m/s in the direction shown. If the bat and ball are in contact 0.015 s, determine the average impulsive force exerted on the ball during the impact.

SOLUTION

We apply the principle of impulse and momentum to the ball. Since the weight of the ball is a nonimpulsive force, it can be neglected.



$$m\mathbf{v}_1 + \Sigma\,\mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$

$\xrightarrow{+}$ x components: $\quad -mv_1 + F_x\,\Delta t = mv_2 \cos 40°$

$$-(.12 \times 9.81 \text{ N})\,(24.4 \text{ m/s}) + F_x(0.015 \text{ s}) = (.12 \times 9.81 \text{ N})(36.6 \text{ m/s}) \cos 40°$$

$$F_x = +4115.3 \text{ N}$$

$+\uparrow y$ components: $\quad 0 + F_y\,\Delta t = mv_2 \sin 40°$

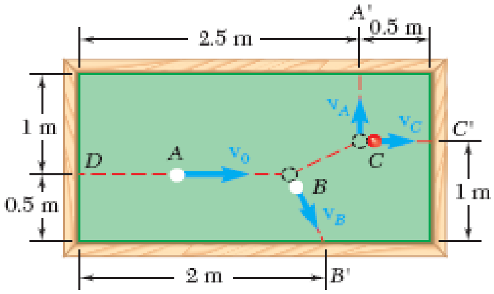
$$F_y(0.015 \text{ s}) = (.12 \times 9.81 \text{ N})(36.6 \text{ m/s}) \sin 40°$$

$$F_y = +1846.3 \text{ N}$$

From its components F_x and F_y we determine the magnitude and direction of the force $\mathbf{F}$:

$$\mathbf{F} = 4510.5 \text{ N} \measuredangle\, 24.2°$$

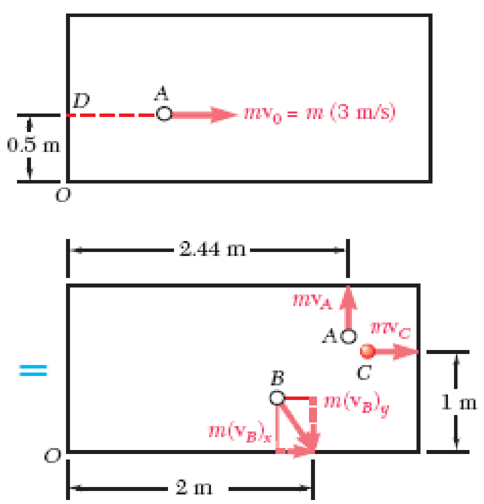
Corrected Version



SAMPLE PROBLEM 14.5

In a game of billiards, ball A is given an initial velocity $\mathbf{v}_0$ of magnitude $v_0 = 3$ m/s along line DA parallel to the axis of the table. It hits ball B and then ball C, which are both at rest. Knowing that A and C hit the sides of the table squarely at points A' and C', respectively, that B hits the side obliquely at B', and assuming frictionless surfaces and perfectly elastic impacts, determine the velocities $\mathbf{v}_A$, $\mathbf{v}_B$, and $\mathbf{v}_C$ with which the balls hit the sides of the table. (*Remark.* In this sample problem and in several of the problems which follow, the billiard balls are assumed to be particles moving freely in a horizontal plane, rather than the rolling and sliding spheres they actually are.)

SOLUTION



Conservation of Momentum. Since there is no external force, the initial momentum $m\mathbf{v}_0$ is equipollent to the system of momenta after the two collisions (and before any of the balls hits the side of the table). Referring to the adjoining sketch, we write

$\xrightarrow{+}$ x components: $\quad m(3\text{ m/s}) = m(v_B)_x + mv_C \qquad (1)$

$+\uparrow y$ components: $\quad 0 = mv_A - m(v_B)_y \qquad (2)$

$+\circlearrowleft$ moments about O: $\quad -(0.5\text{ m})m(3\text{ m/s}) = (2.5\text{ m})mv_A - (2\text{ m})m(v_B)_y - (1\text{ m})mv_C \qquad (3)$

Solving the three equations for v_A, $(v_B)_x$, and $(v_B)_y$ in terms of v_C,

$$v_A = (v_B)_y = 2v_C - 3 \qquad (v_B)_x = 3 - v_C \qquad (4)$$

Conservation of Energy. Since the surfaces are frictionless and the impacts are perfectly elastic, the initial kinetic energy $\frac{1}{2}mv_0^2$ is equal to the final kinetic energy of the system:

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m_Av_A^2 + \tfrac{1}{2}m_Bv_B^2 + \tfrac{1}{2}m_Cv_C^2$$

$$v_A^2 + (v_B)_x^2 + (v_B)_y^2 + v_C^2 = \left(3\ \frac{\text{m}}{\text{s}}\right)^2 \qquad (5)$$

Substituting for v_A, $(v_B)_x$, and $(v_B)_y$ from (4) into (5), we have

$$2(2v_C - 3)^2 + (3 - v_C)^2 + v_C^2 = 9$$

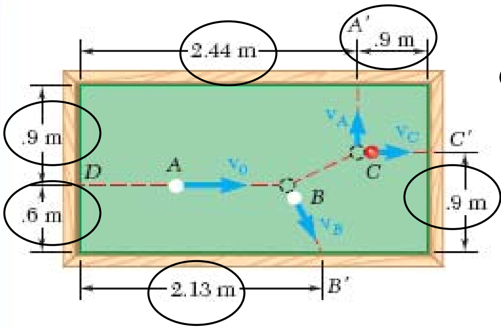
$$10v_C^2 - 30v_C + 18 = 0$$

Solving for v_C, we find $v_C = 0.83$ m/s and $v_C = 2.17$ m/s. Since only the first root yields a positive value for v_A after substitution into Eqs. (4), we conclude that $v_C = 2.17$ m/s and

$$v_A = (v_B)_y = 1.38\text{ m/s} \qquad (v_B)_x = 0.83\text{ m/s}$$

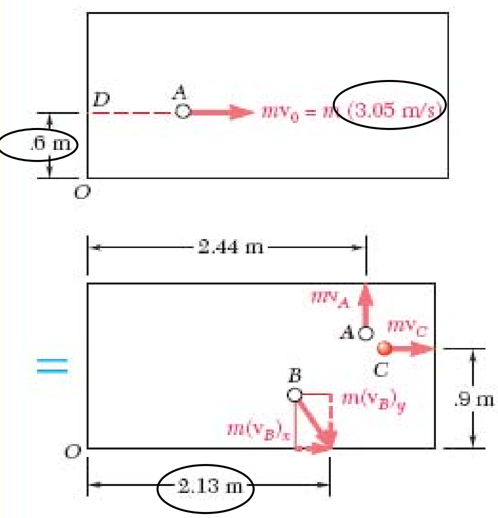
$$\mathbf{v}_A = 1.38\ \frac{\text{m}}{\text{s}}\uparrow \qquad \mathbf{v}_B = 4.20\ \frac{\text{m}}{\text{s}} \measuredangle 58.98^\circ \qquad \mathbf{v}_C = 2.17\ \frac{\text{m}}{\text{s}}\rightarrow \quad \blacktriangleleft$$

SAMPLE PROBLEM 14.5



In a game of billiards, ball A is given an initial velocity $\mathbf{v}_0$ of magnitude $v_0 =$ 3.05 m/s along line DA parallel to the axis of the table. It hits ball B and then ball C, which are both at rest. Knowing that A and C hit the sides of the table squarely at points A' and C', respectively, that B hits the side obliquely at B', and assuming frictionless surfaces and perfectly elastic impacts, determine the velocities $\mathbf{v}_A$, $\mathbf{v}_B$, and $\mathbf{v}_C$ with which the balls hit the sides of the table. (*Remark.* In this sample problem and in several of the problems which follow, the billiard balls are assumed to be particles moving freely in a horizontal plane, rather than the rolling and sliding spheres they actually are.)

SOLUTION



Conservation of Momentum. Since there is no external force, the initial momentum $m\mathbf{v}_0$ is equipollent to the system of momenta after the two collisions (and before any of the balls hits the side of the table). Referring to the adjoining sketch, we write

$\xrightarrow{+}$ x components: $\quad m(3.05 \text{ m/s}) = m(v_B)_x + mv_C \qquad (1)$

$+\uparrow y$ components: $\quad 0 = mv_A - m(v_B)_y \qquad (2)$

$+\circlearrowleft$ moments about O: $\quad -(.6 \text{ m})m(3.05 \text{ m/s}) = (2.44 \text{ m})mv_A - (2.13 \text{ m})m(v_B)_y - (.9 \text{ m})mv_C \qquad (3)$

Solving the three equations for v_A, $(v_B)_x$, and $(v_B)_y$ in terms of v_C,

$$v_A = (v_B)_y = 2.9v_C - 5.9 \qquad (v_B)_x = 3.05 - v_C \qquad (4)$$

Conservation of Energy. Since the surfaces are frictionless and the impacts are perfectly elastic, the initial kinetic energy $\frac{1}{2}mv_0^2$ is equal to the final kinetic energy of the system:

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m_Av_A^2 + \tfrac{1}{2}m_Bv_B^2 + \tfrac{1}{2}m_Cv_C^2$$
$$v_A^2 + (v_B)_x^2 + (v_B)_y^2 + v_C^2 = (3.05 \text{ m/s})^2 \qquad (5)$$

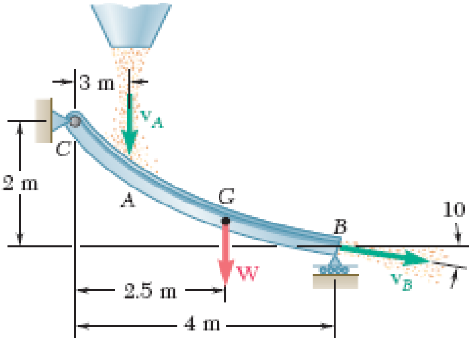
Substituting for v_A, $(v_B)_x$, and $(v_B)_y$ from (4) into (5), we have

$$2(2.9v_C - 5.9)^2 + (3.05 - v_C)^2 + v_C^2 = 9.3$$
$$18.82v_C^2 - 74.54v_C + 69.62 = 0$$

Solving for v_C, we find $v_C = 2.45$ m/s and $v_C = 1.5$ m. Since only the first root yields a positive value for v_A after substitution into Eqs. (4), we conclude that $v_C =$ 2.44 m/s and

$$v_A = (v_B)_y = 2.9(2.45) - 5.9 = 1.205 \text{ m/s} \qquad (v_B)_x = 3.05 - 2.45 = 0.6 \text{ m/s}$$

$\mathbf{v}_A = 1.205 \text{ m/s}\uparrow \qquad \mathbf{v}_B = 1.346 \text{ m/s} \measuredangle 63.4° \qquad \mathbf{v}_C = 2.45 \text{ m/s} \rightarrow$ ◀

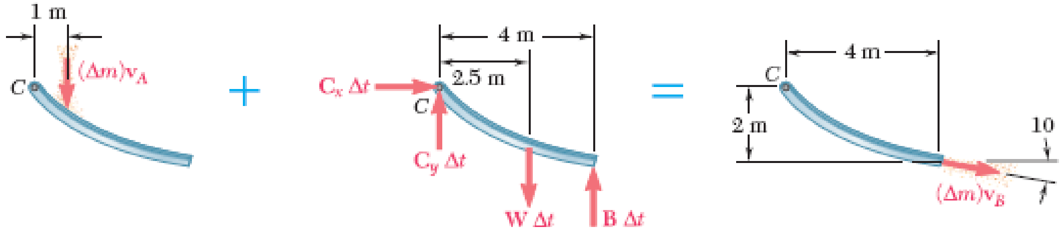


SAMPLE PROBLEM 14.6

Grain falls from a hopper onto a chute CB at the rate of 100 kg/s. It hits the chute at A with a velocity of 6.0 m/s and leaves at B with a velocity of 4.5 m/s, forming an angle of 10° with the horizontal. Knowing that the combined weight of the chute and of the grain it supports is a force $\mathbf{W}$ of magnitude 2500 N applied at G, determine the reaction at the roller support B and the components of the reaction at the hinge C.

SOLUTION

We apply the principle of impulse and momentum for the time interval Δt to the system consisting of the chute, the grain it supports, and the amount of grain which hits the chute in the interval Δt. Since the chute does not move, it has no momentum. We also note that the sum $\Sigma m_i \mathbf{v}_i$ of the momenta of the particles supported by the chute is the same at t and $t + \Delta t$ and can thus be omitted.



Since the system formed by the momentum $(\Delta m)\mathbf{v}_A$ and the impulses is equipollent to the momentum $(\Delta m)\mathbf{v}_B$, we write

$\xrightarrow{+}$ x components: $\quad C_x\,\Delta t = (\Delta m)v_B \cos 10° \quad (1)$

$+\uparrow$ y components: $\quad -(\Delta m)v_A + C_y\,\Delta t - W\,\Delta t + B\,\Delta t = -(\Delta m)v_B \sin 10° \quad (2)$

$+\circlearrowleft$ moments about C: $\quad -1(\Delta m)v_A - 2.5(W\,\Delta t) + 4(B\,\Delta t) = 2(\Delta m)v_B \cos 10° - 4(\Delta m)v_B \sin 10° \quad (3)$

Using the given data, $W = 2500$ N, $v_A = 6$ m/s, $v_B = 4.5$ m/s, and $\Delta m/\Delta t = 100$ kg/s and solving Eq. (3) for B and Eq. (1) for C_x,

$$4B = 2.5 \times 2500 + (400)(6) + 2(100)(4.5)(\cos 10° - 2 \sin 10°)$$

$$B = 1855.94 \text{ N} \qquad \mathbf{B} = 1855.94 \text{ N} \uparrow \blacktriangleleft$$

$$C_x = (100)(4.5) \cos 10° = 443.16 \text{ N} \qquad \mathbf{C}_x = 443.16 \text{ N} \rightarrow \blacktriangleleft$$

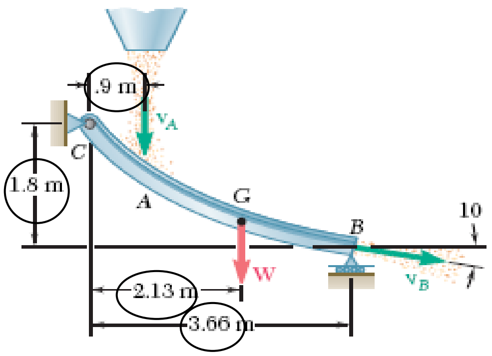
Substituting for B and solving Eq. (2) for C_y,

$$C_y = 2500 - 1855.94 + (100)(6 - 4.5 \sin 10°)$$

$$\mathbf{C}_y = 1165.92 \text{ N} \uparrow \blacktriangleleft$$

Current Version

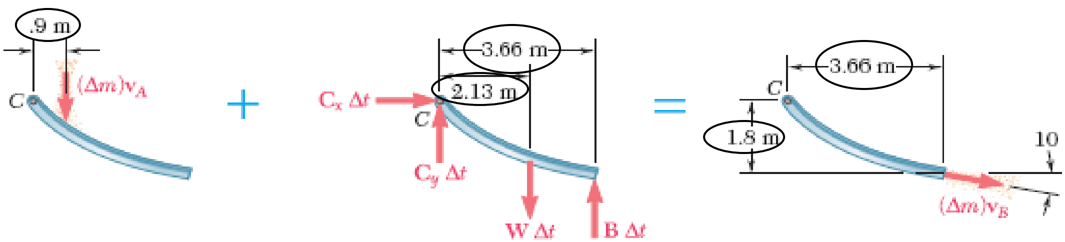
SAMPLE PROBLEM 14.6



Grain falls from a hopper onto a chute *CB* at the rate of 108.86 kg/s. It hits the chute at *A* with a velocity of 6.09 m/s and leaves at *B* with a velocity of 4.57 m/s, forming an angle of 10° with the horizontal. Knowing that the combined weight of the chute and of the grain it supports is a force **W** of magnitude 2668.9 N applied at *G*, determine the reaction at the roller support *B* and the components of the reaction at the hinge *C*.

SOLUTION

We apply the principle of impulse and momentum for the time interval Δt to the system consisting of the chute, the grain it supports, and the amount of grain which hits the chute in the interval Δt. Since the chute does not move, it has no momentum. We also note that the sum $\Sigma m_i \mathbf{v}_i$ of the momenta of the particles supported by the chute is the same at t and $t + \Delta t$ and can thus be omitted.



Since the system formed by the momentum $(\Delta m)\mathbf{v}_A$ and the impulses is equipollent to the momentum $(\Delta m)\mathbf{v}_B$, we write

$\xrightarrow{+}$ x components: $\quad C_x\,\Delta t = (\Delta m)v_B \cos 10° \qquad (1)$

$+\uparrow$ y components: $\quad -(\Delta m)v_A + C_y\,\Delta t - W\,\Delta t + B\,\Delta t = -(\Delta m)v_B \sin 10° \qquad (2)$

$+\circlearrowleft$ moments about C: $\quad -.9(\Delta m)v_A - 2.13(W\,\Delta t) + 3.66(B\,\Delta t) = 1.8(\Delta m)v_B \cos 10° - 3.66(\Delta m)v_B \sin 10° \qquad (3)$

Using the given data, $W = 2668.9$ N, $v_A = 6.09$ m/s, $v_B = 4.57$ m/s, and $\Delta m/\Delta t = 108.86$ kg/s and solving Eq. (3) for B and Eq. (1) for C_x,

$$3.66B = 2.13(2668.9) + .9(108.86)(6.09) + 1.8(108.86)(4.57)(\cos 10° - 2 \sin 10°)$$

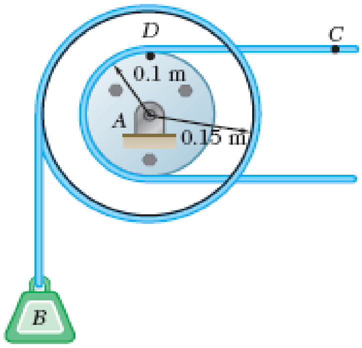
$$3.66B = 6852.296 \qquad B = 1872.2 \text{ N} \qquad \mathbf{B} = 1872.2 \text{ N} \uparrow$$

$$C_x = (108.86)(4.57) \cos 10° = 489.9 \text{ N} \qquad \mathbf{C}_x = 489.9 \text{ N} \rightarrow$$

Substituting for B and solving Eq. (2) for C_y,

$$C_y = 2668.9 - 1872.2 + (108.86)(6.09 - 4.57 \sin 10°) = 1373 \text{ N}$$

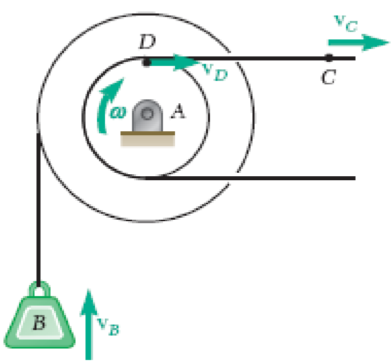
$$\mathbf{C}_y = 1373.3 \text{ N} \uparrow$$



SAMPLE PROBLEM 15.1

Load B is connected to a double pulley by one of the two inextensible cables shown. The motion of the pulley is controlled by cable C, which has a constant acceleration of 0.25 m/s^2 and an initial velocity of 0.3 m/s, both directed to the right. Determine (*a*) the number of revolutions executed by the pulley in 2 s, (*b*) the velocity and change in position of the load B after 2 s, and (*c*) the acceleration of point D on the rim of the inner pulley at $t = 0$.

SOLUTION



a. Motion of Pulley. Since the cable is inextensible, the velocity of point D is equal to the velocity of point C and the tangential component of the acceleration of D is equal to the acceleration of C.

$$(\mathbf{v}_D)_0 = (\mathbf{v}_C)_0 = 0.3 \text{ m/s} \rightarrow \qquad (\mathbf{a}_D)_t = \mathbf{a}_C = 0.25 \text{ m/s}^2 \rightarrow$$

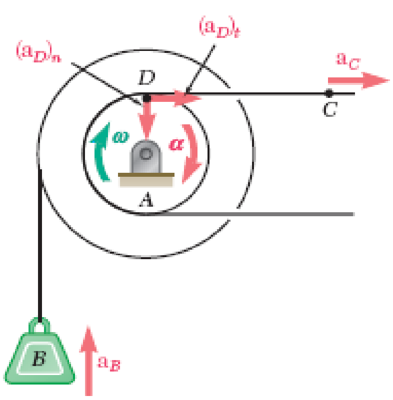
Noting that the distance from D to the center of the pulley is 3 in., we write

$$(v_D)_0 = r\omega_0 \qquad 0.3 \text{ m/s} = (0.1 \text{ m})\omega_0 \qquad \boldsymbol{\omega}_0 = 3 \text{ rad/s} \downarrow$$
$$(a_D)_t = r\alpha \qquad 0.25 \text{ m/s}^2 = (0.1 \text{ m})\alpha \qquad \boldsymbol{\alpha} = 2.5 \text{ rad/s}^2 \downarrow$$

Using the equations of uniformly accelerated motion, we obtain, for $t = 2$ s,

$$\omega = \omega_0 + \alpha t = 3 \text{ rad/s} + (2.5 \text{ rad/s}^2)(2 \text{ s}) = 8 \text{ rad/s}$$
$$\boldsymbol{\omega} = 8 \text{ rad/s} \downarrow$$
$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (3 \text{ rad/s})(2 \text{ s}) + \tfrac{1}{2}(2.5 \text{ rad/s}^2)(2 \text{ s})^2 = 11 \text{ rad}$$
$$\boldsymbol{\theta} = 11 \text{ rad} \downarrow$$

$$\text{Number of revolutions} = (11 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 1.75 \text{ rev} \quad ◀$$



b. Motion of Load B. Using the following relations between linear and angular motion, with $r = 0.15$ in., we write

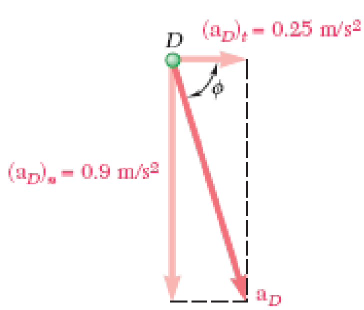
$$v_B = r\omega = (0.15 \text{ m})(8 \text{ rad/s}) = 12 \text{ m/s} \qquad \mathbf{v}_B = 12 \text{ m/s} \uparrow \quad ◀$$
$$\Delta y_B = r\theta = (0.15 \text{ m})(11 \text{ rad}) = 1.65 \text{ cm} \qquad \Delta y_B = 1.65 \text{ m upward} \quad ◀$$

c. Acceleration of Point D at t = 0. The tangential component of the acceleration is

$$(\mathbf{a}_D)_t = \mathbf{a}_C = 0.25 \text{ m/s}^2 \rightarrow$$

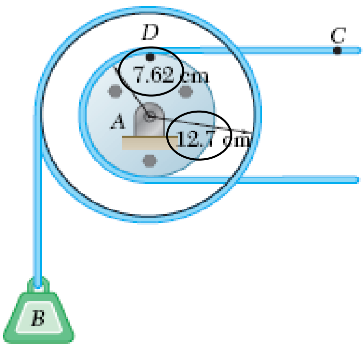
Since, at $t = 0$, $\omega_0 = 4$ rad/s, the normal component of the acceleration is

$$(a_D)_n = r_D\omega_0^2 = (0.1 \text{ m})(3 \text{ rad/s})^2 = 0.9 \text{ m/s}^2 \qquad (\mathbf{a}_D)_n = 0.9 \text{ m/s}^2 \downarrow$$



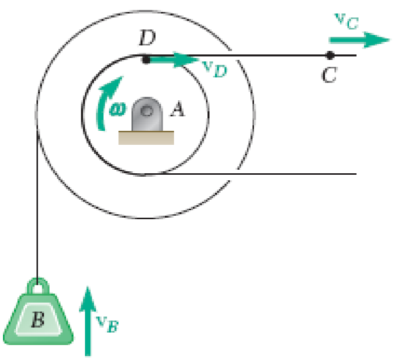
The magnitude and direction of the total acceleration can be obtained by writing

$$\tan \phi = (0.9 \text{ m/s}^2)/(0.25 \text{ m/s}^2) \qquad \phi = 74.48°$$
$$a_D \sin 74.48° = 0.9 \text{ m/s}^2 \qquad a_D = 0.93 \text{ m/s}^2$$
$$\mathbf{a}_D = 0.93 \text{ m/s}^2 \searrow 74.48° \quad ◀$$



SAMPLE PROBLEM 15.1

Load B is connected to a double pulley by one of the two inextensible cables shown. The motion of the pulley is controlled by cable C, which has a constant acceleration of 22.86 cm/s^2 and an initial velocity of 30.48 cm/s, both directed to the right. Determine (*a*) the number of revolutions executed by the pulley in 2 s, (*b*) the velocity and change in position of the load B after 2 s, and (*c*) the acceleration of point D on the rim of the inner pulley at $t = 0$.



SOLUTION

***a.* Motion of Pulley.** Since the cable is inextensible, the velocity of point D is equal to the velocity of point C and the tangential component of the acceleration of D is equal to the acceleration of C.

$$(\mathbf{v}_D)_0 = (\mathbf{v}_C)_0 = 30.48 \text{ cm/s} \rightarrow \qquad (\mathbf{a}_D)_t = \mathbf{a}_C = 22.8 \text{ cm/s}^2 \rightarrow$$

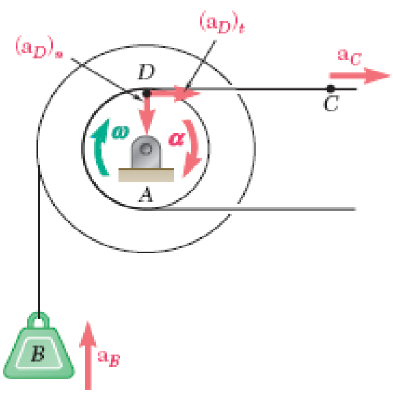
Noting that the distance from D to the center of the pulley is 3 in., we write

$$(v_D)_0 = r\omega_0 \qquad 30.48 \text{ cm/s} = (7.62 \text{ cm})\omega_0 \qquad \boldsymbol{\omega}_0 = 4 \text{ rad/s} \downarrow$$
$$(a_D)_t = r\alpha \qquad 22.8 \text{ cm/s}^2 = (7.62 \text{ cm})\alpha \qquad \boldsymbol{\alpha} = 3 \text{ rad/s}^2 \downarrow$$

Using the equations of uniformly accelerated motion, we obtain, for $t = 2$ s,

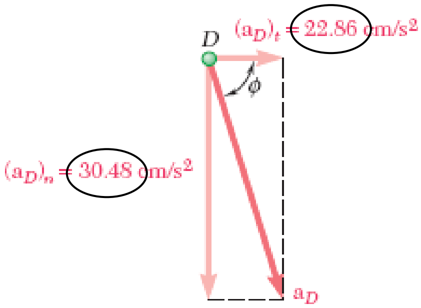
$$\omega = \omega_0 + \alpha t = 4 \text{ rad/s} + (3 \text{ rad/s}^2)(2 \text{ s}) = 10 \text{ rad/s}$$
$$\boldsymbol{\omega} = 10 \text{ rad/s} \downarrow$$
$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (4 \text{ rad/s})(2 \text{ s}) + \tfrac{1}{2}(3 \text{ rad/s}^2)(2 \text{ s})^2 = 14 \text{ rad}$$
$$\boldsymbol{\theta} = 14 \text{ rad} \downarrow$$

$$\text{Number of revolutions} = (14 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 2.23 \text{ rev} \quad ◀$$



***b.* Motion of Load *B*.** Using the following relations between linear and angular motion, with $r = 5$ in., we write

$$v_B = r\omega = (12.7 \text{ cm})(10 \text{ rad/s}) = 127 \text{ cm/s} \qquad \mathbf{v}_B = 127 \text{ cm/s} \uparrow \quad ◀$$
$$\Delta y_B = r\theta = (12.7 \text{ cm})(14 \text{ rad}) = 177.8 \text{ cm} \qquad \Delta y_B = 177.8 \text{ cm upward} \quad ◀$$



***c.* Acceleration of Point *D* at $t = 0$.** The tangential component of the acceleration is

$$(\mathbf{a}_D)_t = \mathbf{a}_C = 22.8 \text{ cm/s}^2 \rightarrow$$

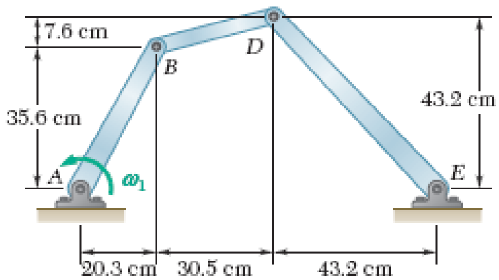
Since, at $t = 0$, $\omega_0 = 4$ rad/s, the normal component of the acceleration is

$$(a_D)_n = r_D\omega_0^2 = (7.62 \text{ cm})(4 \text{ rad/s})^2 = 30.48 \text{ cm/s}^2 \qquad (\mathbf{a}_D)_n = 30.48 \text{ cm/s}^2 \downarrow$$

The magnitude and direction of the total acceleration can be obtained by writing

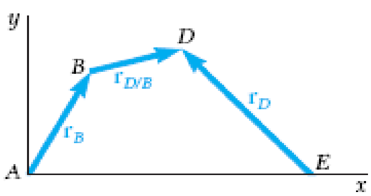
$$\tan\phi = (30.48 \text{ cm/s}^2)/(22.8 \text{ cm/s}^2) \qquad \phi = 53.2°$$
$$a_D \sin 53.2° = 30.48 \text{ cm/s}^2 \qquad a_D = 38 \text{ cm/s}^2$$
$$\mathbf{a}_D = 38 \text{ cm/s}^2 \measuredangle 53.2° \quad ◀$$

Corrected Version



SAMPLE PROBLEM 15.8

The linkage *ABDE* moves in the vertical plane. Knowing that in the position shown crank *AB* has a constant angular velocity $\boldsymbol{\omega}_1$ of 20 rad/s counterclockwise, determine the angular velocities and angular accelerations of the connecting rod *BD* and of the crank *DE*.



$\mathbf{r}_B = 20.3\mathbf{i} + 35.6\mathbf{j}$
$\mathbf{r}_D = -43.2\mathbf{i} + 43.2\mathbf{j}$
$\mathbf{r}_{D/B} = 30.5\mathbf{i} + 7.6\mathbf{j}$

SOLUTION

This problem could be solved by the method used in Sample Prob. 15.7. In this case, however, the vector approach will be used. The position vectors $\mathbf{r}_B$, $\mathbf{r}_D$, and $\mathbf{r}_{D/B}$ are chosen as shown in the sketch.

Velocities. Since the motion of each element of the linkage is contained in the plane of the figure, we have

$$\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} = (20 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$$

where $\mathbf{k}$ is a unit vector pointing out of the paper. We now write

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$
$$\omega_{DE}\mathbf{k} \times \mathbf{r}_D = \omega_{AB}\mathbf{k} \times \mathbf{r}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{D/B}$$
$$\omega_{DE}\mathbf{k} \times (-43.2\mathbf{i} + 43.2\mathbf{j}) = 20\mathbf{k} \times (20.3\mathbf{i} + 35.1\mathbf{j}) + \omega_{BD}\mathbf{k} \times (30.5\mathbf{i} + 7.6\mathbf{j})$$
$$-43.2\omega_{DE}\mathbf{j} - 43.2\omega_{DE}\mathbf{i} = 406\mathbf{j} - 712\mathbf{i} + 30.5\omega_{BD}\mathbf{j} - 7.6\omega_{BD}\mathbf{i}$$

Equating the coefficients of the unit vectors $\mathbf{i}$ and $\mathbf{j}$, we obtain the following two scalar equations:

$$-43.2\omega_{DE} = -712 - 7.6\omega_{BD}$$
$$-43.2\omega_{DE} = +406 + 30.5\omega_{BD}$$
$$\boldsymbol{\omega}_{BD} = -(29.34 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\omega}_{DE} = (11.32 \text{ rad/s})\mathbf{k} \quad ◀$$

Accelerations. Noting that at the instant considered crank *AB* has a constant angular velocity, we write

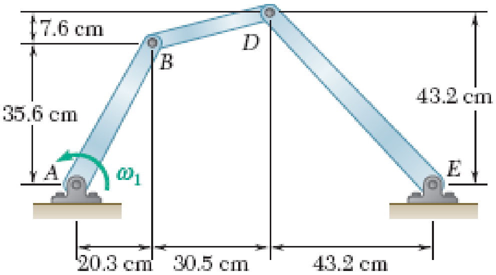
$$\boldsymbol{\alpha}_{AB} = 0 \qquad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k} \qquad \boldsymbol{\alpha}_{DE} = \alpha_{DE}\mathbf{k}$$
$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \tag{1}$$

Each term of Eq. (1) is evaluated separately:

$$\mathbf{a}_D = \alpha_{DE}\mathbf{k} \times \mathbf{r}_D - \omega_{DE}^2\mathbf{r}_D$$
$$= \alpha_{DE}\mathbf{k} \times (-43.2\mathbf{i} + 43.2\mathbf{j}) - (11.32)^2(-43.2\mathbf{i} + 43.2\mathbf{j})$$
$$= -43.2\alpha_{DE}\mathbf{j} - 43.2\alpha_{DE}\mathbf{i} + 5535.75\mathbf{i} - 5535.75\mathbf{j}$$
$$\mathbf{a}_B = \alpha_{AB}\mathbf{k} \times \mathbf{r}_B - \omega_{AB}^2\mathbf{r}_B = 0 - (20)^2(20.3\mathbf{i} + 35.6\mathbf{j})$$
$$= -8120\mathbf{i} - 14240\mathbf{j}$$
$$\mathbf{a}_{D/B} = \alpha_{BD}\mathbf{k} \times \mathbf{r}_{D/B} - \omega_{BD}^2\mathbf{r}_{D/B}$$
$$= \alpha_{BD}\mathbf{k} \times (30.5\mathbf{i} + 7.6\mathbf{j}) - (29.34)^2(30.5\mathbf{i} + 7.6\mathbf{j})$$
$$= 30.5\alpha_{BD}\mathbf{j} - 7.6\alpha_{BD}\mathbf{i} - 26255.5\mathbf{i} - 6542.35\mathbf{j}$$

Substituting into Eq. (1) and equating the coefficients of $\mathbf{i}$ and $\mathbf{j}$, we obtain

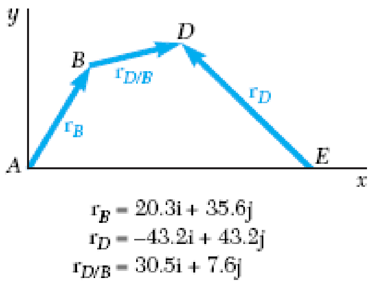
$$-43.2\alpha_{DE} + 7.6\alpha_{BD} = -39911.25$$
$$-43.2\alpha_{DE} - 30.5\alpha_{BD} = -15246.6$$
$$\boldsymbol{\alpha}_{BD} = -(647.37 \text{ rad/s}^2)\mathbf{k} \qquad \boldsymbol{\alpha}_{DE} = (810 \text{ rad/s}^2)\mathbf{k} \quad ◀$$



SAMPLE PROBLEM 15.8

The linkage *ABDE* moves in the vertical plane. Knowing that in the position shown crank *AB* has a constant angular velocity $\boldsymbol{\omega}_1$ of 20 rad/s counterclockwise, determine the angular velocities and angular accelerations of the connecting rod *BD* and of the crank *DE*.

SOLUTION



This problem could be solved by the method used in Sample Prob. 15.7. In this case, however, the vector approach will be used. The position vectors $\mathbf{r}_B$, $\mathbf{r}_D$, and $\mathbf{r}_{D/B}$ are chosen as shown in the sketch.

Velocities. Since the motion of each element of the linkage is contained in the plane of the figure, we have

$$\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} = (20 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$$

where $\mathbf{k}$ is a unit vector pointing out of the paper. We now write

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$
$$\omega_{DE}\mathbf{k} \times \mathbf{r}_D = \omega_{AB}\mathbf{k} \times \mathbf{r}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{D/B}$$
$$\omega_{DE}\mathbf{k} \times (-43.2\mathbf{i} + 43.2\mathbf{j}) = 20\mathbf{k} \times (20.3\mathbf{i} + 35.8\mathbf{j}) + \omega_{BD}\mathbf{k} \times (30.5\mathbf{i} + 7.6\mathbf{j})$$
$$-43.2\omega_{DE}\mathbf{j} - 43.2\omega_{DE}\mathbf{i} = 406\mathbf{j} - 712\mathbf{i} + 30.5\omega_{BD}\mathbf{j} - 7.6\omega_{BD}\mathbf{i}$$

Equating the coefficients of the unit vectors $\mathbf{i}$ and $\mathbf{j}$, we obtain the following two scalar equations:

$$-43.2\omega_{DE} = -712 - 7.6\omega_{BD}$$
$$-43.2\omega_{DE} = +406 + 30.5\omega_{BD}$$
$$\boldsymbol{\omega}_{BD} = -(29.34 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\omega}_{DE} = (21.64 \text{ rad/s})\mathbf{k} \quad ◀$$

Accelerations. Noting that at the instant considered crank *AB* has a constant angular velocity, we write

$$\boldsymbol{\alpha}_{AB} = 0 \qquad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k} \qquad \boldsymbol{\alpha}_{DE} = \alpha_{DE}\mathbf{k}$$
$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \qquad (1)$$

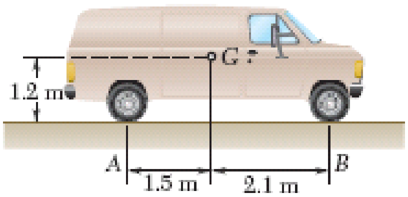
Each term of Eq. (1) is evaluated separately:

$$\mathbf{a}_D = \alpha_{DE}\mathbf{k} \times \mathbf{r}_D - \omega_{DE}^2\mathbf{r}_D$$
$$= \alpha_{DE}\mathbf{k} \times (-43.2\mathbf{i} + 43.2\mathbf{j}) - (21.64)^2(-43.2\mathbf{i} + 43.2\mathbf{j})$$
$$= -43.2\alpha_{DE}\mathbf{j} - 43.2\alpha_{DE}\mathbf{i} + 20230.1\mathbf{i} - 20230.1\mathbf{j}$$
$$\mathbf{a}_B = \alpha_{AB}\mathbf{k} \times \mathbf{r}_B - \omega_{AB}^2\mathbf{r}_B = 0 - (20)^2(20.3\mathbf{i} + 35.6\mathbf{j})$$
$$= -8120\mathbf{i} - 14240\mathbf{j}$$
$$\mathbf{a}_{D/B} = \alpha_{BD}\mathbf{k} \times \mathbf{r}_{D/B} - \omega_{BD}^2\mathbf{r}_{D/B}$$
$$= \alpha_{BD}\mathbf{k} \times (30.5\mathbf{i} + 7.6\mathbf{j}) - (29.34)^2(30.5\mathbf{i} + 7.6\mathbf{j})$$
$$= 30.5\alpha_{BD}\mathbf{j} - 7.6\alpha_{BD}\mathbf{i} - 26255.5\mathbf{i} - 6542.35\mathbf{j}$$

Substituting into Eq. (1) and equating the coefficients of $\mathbf{i}$ and $\mathbf{j}$, we obtain

$$-43.2\alpha_{DE} + 7.6\alpha_{BD} = -14145.5$$
$$-43.2\alpha_{DE} - 30.5\alpha_{BD} = -41012.45$$
$$\boxed{\boldsymbol{\alpha}_{BD} = +(705.2 \text{ rad/s}^2)\mathbf{k} \qquad \boldsymbol{\alpha}_{DE} = (451.5 \text{ rad/s}^2)\mathbf{k}} \quad ◀$$

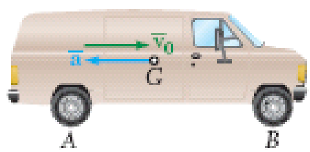
Corrected Version



SAMPLE PROBLEM 16.1

When the forward speed of the truck shown was 9.1 m/s, the brakes were suddenly applied, causing all four wheels to stop rotating. It was observed that the truck skidded to rest in 6 m. Determine the magnitude of the normal reaction and of the friction force at each wheel as the truck skidded to rest.

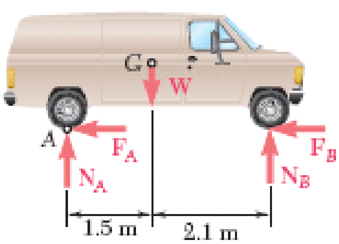
SOLUTION



Kinematics of Motion. Choosing the positive sense to the right and using the equations of uniformly accelerated motion, we write

$$\bar{v}_0 = +9.1 \text{ m/s} \qquad \bar{v}^2 = \bar{v}_0^2 + 2\bar{a}\bar{x} \qquad 0 = (9.1)^2 + 2\bar{a}(6)$$
$$\bar{a} = -6.9 \text{ m/s}^2 \qquad \bar{\mathbf{a}} = 6.9 \text{ m/s}^2 \leftarrow$$

Equations of Motion. The external forces consist of the weight **W** of the truck and of the normal reactions and friction forces at the wheels. (The vectors $\mathbf{N}_A$ and $\mathbf{F}_A$ represent the sum of the reactions at the rear wheels, while $\mathbf{N}_B$ and $\mathbf{F}_B$ represent the sum of the reactions at the front wheels.) Since the truck is in translation, the effective forces reduce to the vector $m\bar{\mathbf{a}}$ attached at G. Three equations of motion are obtained by expressing that the system of the external forces is equivalent to the system of the effective forces.



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \qquad N_A + N_B - W = 0$$

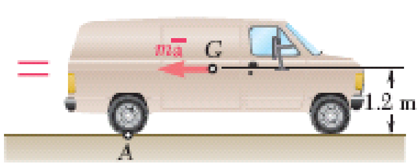
Since $F_A = \mu_k N_A$ and $F_B = \mu_k N_B$, where μ_k is the coefficient of kinetic friction, we find that

$$F_A + F_B = \mu_k (N_A + N_B) = \mu_k W$$

$$\overset{+}{\rightarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \qquad -(F_A + F_B) = -m\bar{a}$$

$$-\mu_k W = -\frac{W}{9.81 \text{ m/s}^2}(6.9 \text{ m/s}^2)$$

$$\mu_k = 0.703$$



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \qquad -W(1.5 \text{ m}) + N_B(3.6 \text{ m}) = m\bar{a}(1.2 \text{ m})$$

$$-W(1.5 \text{ m}) + N_B(3.6 \text{ m}) = \frac{W}{9.81 \text{ m/s}^2}(6.9 \text{ m/s}^2)(1.2 \text{ m})$$

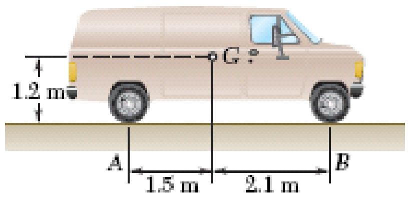
$$N_B = 0.651W$$

$$F_B = \mu_k N_B = (0.703)(0.650W) \qquad F_B = 0.458W$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \qquad N_A + N_B - W = 0$$
$$N_A + 0.650W - W = 0$$
$$N_A = 0.350W$$
$$F_A = \mu_k N_A = (0.703)(0.350W) \qquad F_A = 0.246W$$

Reactions at Each Wheel. Recalling that the values computed above represent the sum of the reactions at the two front wheels or the two rear wheels, we obtain the magnitude of the reactions at each wheel by writing

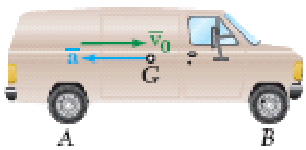
$$N_{\text{front}} = \tfrac{1}{2}N_B = 0.325W \qquad N_{\text{rear}} = \tfrac{1}{2}N_A = 0.175W \quad \blacktriangleleft$$
$$F_{\text{front}} = \tfrac{1}{2}F_B = 0.229W \qquad F_{\text{rear}} = \tfrac{1}{2}F_A = 0.123W \quad \blacktriangleleft$$



SAMPLE PROBLEM 16.1

When the forward speed of the truck shown was 9.1 m/s, the brakes were suddenly applied, causing all four wheels to stop rotating. It was observed that the truck skidded to rest in 6 m/s. Determine the magnitude of the normal reaction and of the friction force at each wheel as the truck skidded to rest.

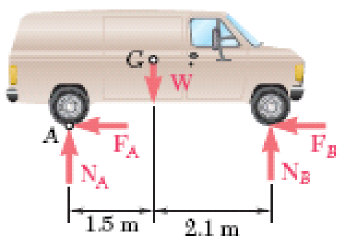
SOLUTION



Kinematics of Motion. Choosing the positive sense to the right and using the equations of uniformly accelerated motion, we write

$$\bar{v}_0 = +9.1 \text{ m/s} \qquad \bar{v}^2 = \bar{v}_0^2 + 2\bar{a}\bar{x} \qquad 0 = (9.1)^2 + 2\bar{a}(6)$$
$$\bar{a} = -6.9 \text{ m/s}^2 \qquad \bar{\mathbf{a}} = 6.9 \text{ m/s}^2 \leftarrow$$

Equations of Motion. The external forces consist of the weight **W** of the truck and of the normal reactions and friction forces at the wheels. (The vectors $\mathbf{N}_A$ and $\mathbf{F}_A$ represent the sum of the reactions at the rear wheels, while $\mathbf{N}_B$ and $\mathbf{F}_B$ represent the sum of the reactions at the front wheels.) Since the truck is in translation, the effective forces reduce to the vector $m\bar{\mathbf{a}}$ attached at G. Three equations of motion are obtained by expressing that the system of the external forces is equivalent to the system of the effective forces.

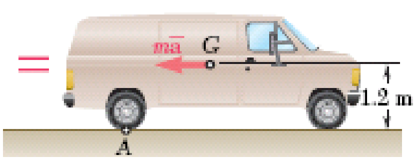


$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \qquad N_A + N_B - W = 0$$

Since $F_A = \mu_k N_A$ and $F_B = \mu_k N_B$, where μ_k is the coefficient of kinetic friction, we find that

$$F_A + F_B = \mu_k(N_A + N_B) = \mu_k W$$

$$\overset{+}{\rightarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \qquad -(F_A + F_B) = -m\bar{a}$$
$$-\mu_k W = -\frac{W}{32.2 \text{ ft/s}^2}(6.9 \text{ m/s}^2)$$
$$\mu_k = 0.703$$

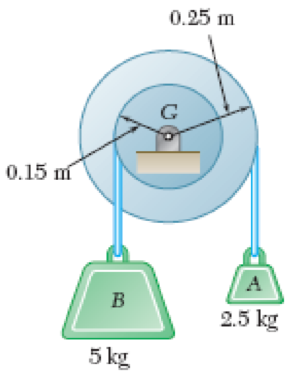


$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \qquad -W(1.5 \text{ m}) + N_B(3.6 \text{ m}) = m\bar{a}(1.2 \text{ m})$$
$$-W(1.5 \text{ m}) + N_B(3.6 \text{ m}) = \frac{W}{32.2 \text{ ft/s}^2}(6.9 \text{ m/s}^2)(1.2 \text{ m})$$
$$N_B = 0.650W$$
$$F_B = \mu_k N_B = (0.703)(0.650W) \qquad F_B = 0.456W$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \qquad N_A + N_B - W = 0$$
$$N_A + 0.650W - W = 0$$
$$N_A = 0.350W$$
$$F_A = \mu_k N_A = (0.703)(0.350W) \qquad F_A = 0.246W$$

Reactions at Each Wheel. Recalling that the values computed above represent the sum of the reactions at the two front wheels or the two rear wheels, we obtain the magnitude of the reactions at each wheel by writing

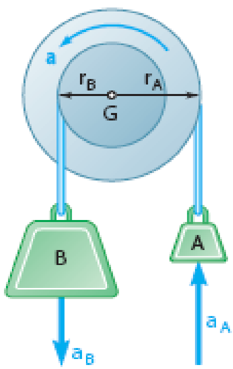
$$N_{\text{front}} = \tfrac{1}{2}N_B = 0.325W \qquad N_{\text{rear}} = \tfrac{1}{2}N_A = 0.175W \quad \blacktriangleleft$$
$$F_{\text{front}} = \tfrac{1}{2}F_B = 0.228W \qquad F_{\text{rear}} = \tfrac{1}{2}F_A = 0.123W \quad \blacktriangleleft$$



SAMPLE PROBLEM 16.3

A pulley weighing 6 kg and having a radius of gyration of 0.2 m is connected to two blocks as shown. Assuming no axle friction, determine the angular acceleration of the pulley and the acceleration of each block.

SOLUTION



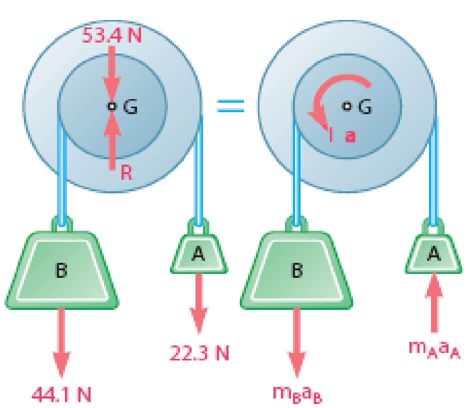
Sense of Motion. Although an arbitrary sense of motion can be assumed (since no friction forces are involved) and later checked by the sign of the answer, we may prefer to determine the actual sense of rotation of the pulley first. The weight of block B required to maintain the equilibrium of the pulley when it is acted upon by the 2.5 kg block A is first determined. We write

$$+\circlearrowleft \Sigma M_G = 0: \quad W_B(0.15 \text{ m}) - (2.5 \text{ g})(0.25 \text{ m}) = 0 \qquad W_B = 4.16 \text{ gN or}$$
$$m_B = 4.16 \text{ kg}$$

Since block B actually weighs 5 kg, the pulley will rotate counterclockwise.

Kinematics of Motion. Assuming $\boldsymbol{\alpha}$ counterclockwise and noting that $a_A = r_A\alpha$ and $a_B = r_B\alpha$, we obtain

$$\mathbf{a}_A = (0.25 \text{ m})\alpha \uparrow \qquad \mathbf{a}_B = (0.15 \text{ m})\alpha \downarrow$$



Equations of Motion. A single system consisting of the pulley and the two blocks is considered. Forces external to this system consist of the weights of the pulley and the two blocks and of the reaction at G. (The forces exerted by the cables on the pulley and on the blocks are internal to the system considered and cancel out.) Since the motion of the pulley is a centroidal rotation and the motion of each block is a translation, the effective forces reduce to the couple $\bar{I}\boldsymbol{\alpha}$ and the two vectors $m\mathbf{a}_A$ and $m\mathbf{a}_B$. The centroidal moment of inertia of the pulley is

$$\bar{I} = m\bar{k}^2 = 6 \text{ kg } (0.2 \text{ m})^2 = 0.24 \text{ kgm}^2$$

Since the system of the external forces is equipollent to the system of the effective forces, we write

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}:$$

$$5 \text{ g } (0.15 \text{ m}) - (2.5 \text{ g})(0.25 \text{ m}) = +\bar{I}\alpha + m_B a_B(0.15 \text{ m}) + m_A a_A(0.25 \text{ m})$$

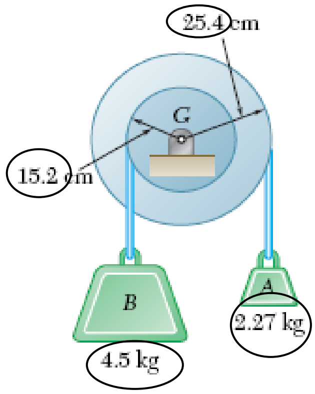
$$5(9.81)(0.15) - (2.5)(9.81)(0.25) = 0.24\alpha + 5(0.15\alpha)(0.15) + 2.5(0.25\alpha)(0.25)$$

$$= \alpha\,(0.24 + 0.1125 + 0.15625)$$

$$\alpha = 2.41 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 2.4 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

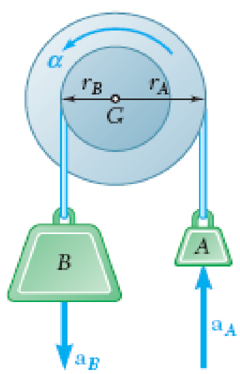
$$a_A = r_A\alpha = 0.25 \times (2.41 \text{ rad/s}^2) \qquad \mathbf{a}_A = 0.6 \text{ m/s}^2 \uparrow \blacktriangleleft$$

$$a_B = r_B\alpha = (0.15\text{m})\,(2.41 \text{ rad/s}^2) \qquad \mathbf{a}_B = 0.36 \text{ m/s}^2 \downarrow \blacktriangleleft$$



SAMPLE PROBLEM 16.3

A pulley weighing 5.44 kg and having a radius of gyration of 20.3 kg is connected to two blocks as shown. Assuming no axle friction, determine the angular acceleration of the pulley and the acceleration of each block.



SOLUTION

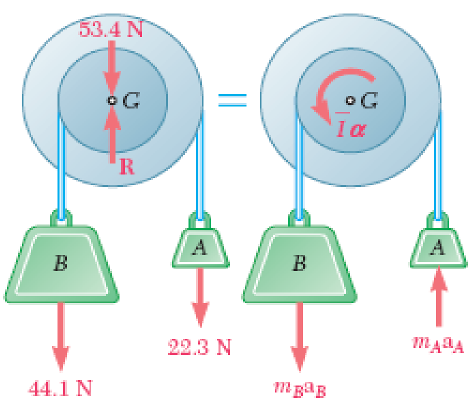
Sense of Motion. Although an arbitrary sense of motion can be assumed (since no friction forces are involved) and later checked by the sign of the answer, we may prefer to determine the actual sense of rotation of the pulley first. The weight of block B required to maintain the equilibrium of the pulley when it is acted upon by the 2.27 kg block A is first determined. We write

$$+\circlearrowleft \Sigma M_G = 0: \qquad W_B(15.2 \text{ cm}) - (2.27 \text{ kg})(25.4 \text{ cm}) = 0 \qquad W_B = 3.79 \text{ kg}$$

Since block B actually weighs 4.5 kg, the pulley will rotate counterclockwise.

Kinematics of Motion. Assuming $\boldsymbol{\alpha}$ counterclockwise and noting that $a_A = r_A\alpha$ and $a_B = r_B\alpha$, we obtain

$$\mathbf{a}_A = (25.4 \text{ cm})\alpha \uparrow \qquad \mathbf{a}_B = (15.2 \text{ cm})\alpha \downarrow$$



Equations of Motion. A single system consisting of the pulley and the two blocks is considered. Forces external to this system consist of the weights of the pulley and the two blocks and of the reaction at G. (The forces exerted by the cables on the pulley and on the blocks are internal to the system considered and cancel out.) Since the motion of the pulley is a centroidal rotation and the motion of each block is a translation, the effective forces reduce to the couple $\bar{I}\boldsymbol{\alpha}$ and the two vectors $m\mathbf{a}_A$ and $m\mathbf{a}_B$. The centroidal moment of inertia of the pulley is

$$\bar{I} = m\bar{k}^2 = 5.44 \text{ kg } (20.3 \text{ kg})^2 = 2241.77 \text{ kg} \cdot \text{cm}^2$$

Since the system of the external forces is equipollent to the system of the effective forces, we write

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}:$$

$$(4.5 \text{ kg})(15.2 \text{ cm}) - (2.27 \text{ kg})(25.4 \text{ cm}) = +\bar{I}\alpha + m_B a_B(15.2 \text{ cm}) + m_A a_A(25.4 \text{ cm})$$

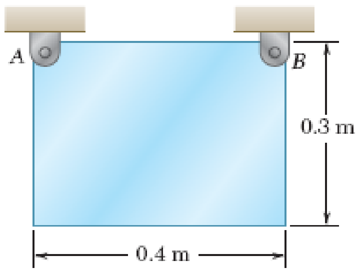
$$(4.5)(15.2) - (2.27)(25.4) = 2241.77\alpha + 4.5(15.2\alpha)(15.2) + 2.27(25.4\alpha)(25.4)$$

$$\alpha = +443.55 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 443.55 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

$$a_A = r_A\alpha = (25.4 \text{ cm})(443.55 \text{ rad/s}^2) \qquad \mathbf{a}_A = 11266.17 \text{ cm/s}^2 \uparrow \blacktriangleleft$$

$$a_B = r_B\alpha = (15.2 \text{ cm})(443.55 \text{ rad/s}^2) \qquad \mathbf{a}_B = 6741.96 \text{ cm/s}^2 \downarrow \blacktriangleleft$$

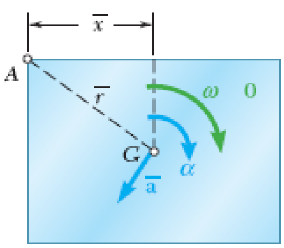
Corrected Version



SAMPLE PROBLEM 16.7

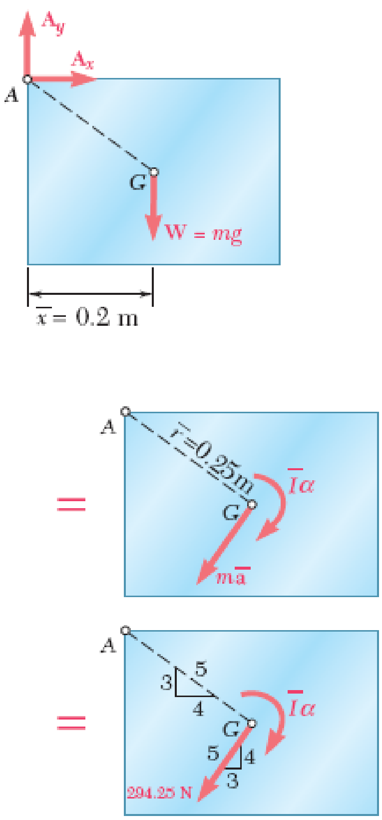
A 0.3 × 0.4 m rectangular plate weighing 50 kg is suspended from two pins A and B. If pin B is suddenly removed, determine (a) the angular acceleration of the plate, (b) the components of the reaction at pin A, immediately after pin B has been removed.

SOLUTION



a. Angular Acceleration. We observe that as the plate rotates about point A, its mass center G describes a circle of radius $\bar{r}$ with center at A.

Since the plate is released from rest ($\omega = 0$), the normal component of the acceleration of G is zero. The magnitude of the acceleration $\bar{\mathbf{a}}$ of the mass center G is thus $\bar{a} = \bar{r}\alpha$. We draw the diagram shown to express that the external forces are equivalent to the effective forces:



$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}$: $\qquad W\bar{x} = (m\bar{a})\bar{r} + \bar{I}\alpha$

Since $\bar{a} = \bar{r}\alpha$, we have

$$W\bar{x} = m(\bar{r}\alpha)\bar{r} + \bar{I}\alpha \qquad \alpha = \frac{W\bar{x}}{m\bar{r}^2 + \bar{I}} \tag{1}$$

The centroidal moment of inertia of the plate is

$$\bar{I} = \frac{m}{12}(a^2 + b^2) = \frac{50}{12}[(0.3 \text{ m})^2 + (0.4 \text{ m})^2]$$

$$= 1.042 \text{ kg m}^2$$

Substituting this value of $\bar{I}$ together with $W = (50)(9.81)$ N, $\bar{r} = 0.25$ m, and $\bar{x} = 0.2$ m into Eq. (1), we obtain

$$\alpha = +23.54 \text{ rad/s}^2 \qquad \alpha = 23.54 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

b. Reaction at A. Using the computed value of α, we determine the magnitude of the vector $m\bar{\mathbf{a}}$ attached at G.

$$m\bar{a} = m\bar{r}\alpha = (50 \text{ kg})\ (0.25 \text{ m})(23.54 \text{ rad/s}^2) = 294.25 \text{ N}$$

Showing this result on the diagram, we write the equations of motion

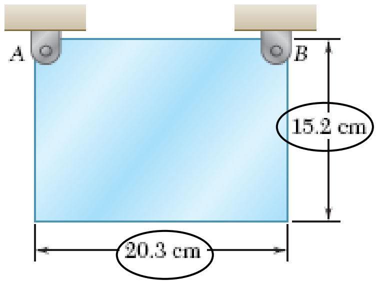
$\overset{+}{\rightarrow}\Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\qquad A_x = -\frac{3}{5}(294.25 \text{ N})$

$$= -176.55 \text{ N} \qquad \mathbf{A}_x = -176.55 \text{ N} \leftarrow \blacktriangleleft$$

$+\uparrow\Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $\quad A_y - 50(9.81) \text{ N} = -\frac{4}{5}(294.25 \text{ N})$

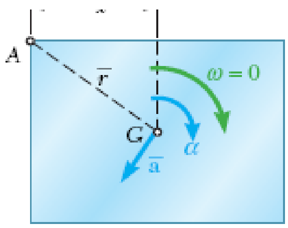
$$A_y = 255.1 \text{ N} \qquad \mathbf{A}_y = 255.1 \text{ N} \uparrow \blacktriangleleft$$

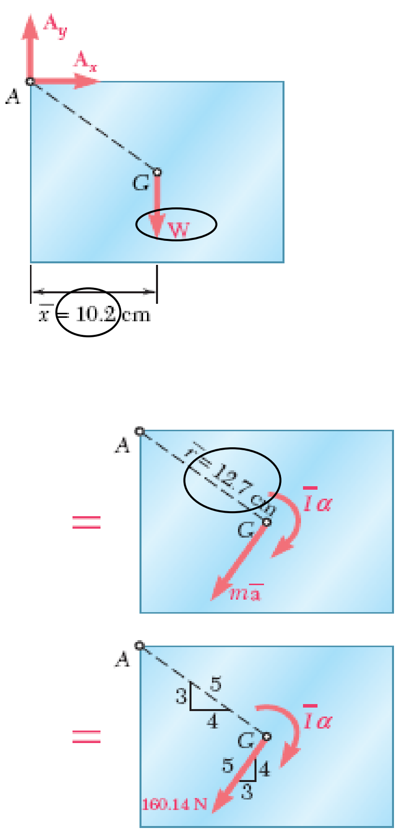
The couple $\bar{I}\alpha$ is not involved in the last two equations; nevertheless, it should be indicated on the diagram.



SAMPLE PROBLEM 16.7

A 15.2 × 20.3 cm rectangular plate weighing 27.2 kg is suspended from two pins A and B. If pin B is suddenly removed, determine (*a*) the angular acceleration of the plate, (*b*) the components of the reaction at pin A, immediately after pin B has been removed.





SOLUTION

a. Angular Acceleration. We observe that as the plate rotates about point A, its mass center G describes a circle of radius $\bar{r}$ with center at A.

Since the plate is released from rest ($\omega = 0$), the normal component of the acceleration of G is zero. The magnitude of the acceleration $\bar{\mathbf{a}}$ of the mass center G is thus $\bar{a} = \bar{r}\alpha$. We draw the diagram shown to express that the external forces are equivalent to the effective forces:

$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}$: $\qquad W\bar{x} = (m\bar{a})\bar{r} + \bar{I}\alpha$

Since $\bar{a} = \bar{r}\alpha$, we have

$$W\bar{x} = m(\bar{r}\alpha)\bar{r} + \bar{I}\alpha \qquad \alpha = \frac{W\bar{x}}{\boxed{\frac{W}{g}\bar{r}^2} + \bar{I}} \tag{1}$$

The centroidal moment of inertia of the plate is

$$\bar{I} = \frac{m}{12}(a^2 + b^2) = \boxed{\frac{27.2\text{ kg}}{12}\left[(20.3\text{ cm})^2 + (15.2\text{ cm})^2\right]}$$

$$\boxed{= 1457.76\text{ kg cm}^2}$$

Substituting this value of $\bar{I}$ together with $W = 266.8$ N, $\bar{r} = 12.7$ cm, and $\bar{x} = 10.2$ cm into Eq. (1), we obtain

$$\alpha = +.466\text{ rad/s}^2 \qquad \alpha = 466\text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

b. Reaction at A. Using the computed value of α, we determine the magnitude of the vector $m\bar{\mathbf{a}}$ attached at G.

$$\boxed{m\bar{a} = m\bar{r}\alpha = 27.2\text{ kg }(12.7\text{ cm})(.466\text{ rad/s}^2) = 1.6\text{ N}}$$

Showing this result on the diagram, we write the equations of motion

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$:

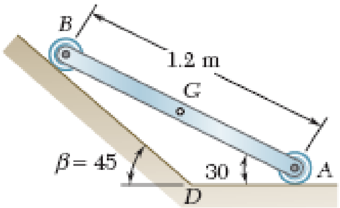
$$\boxed{\begin{aligned} A_x &= -\tfrac{3}{5}(1.6\text{ N}) = .96\text{ N} \\ &= -266.8\text{ N} \qquad \mathbf{A}_x = .96\text{ N} \leftarrow \blacktriangleleft \end{aligned}}$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$:

$$\boxed{\begin{aligned} A_y - 266.8\text{ N} &= -\tfrac{4}{5}(1.6\text{ N}) \\ A_y &= 265.5\text{ N} \qquad \mathbf{A}_y = 265.5\text{ N} \uparrow \blacktriangleleft \end{aligned}}$$

The couple $\bar{I}\alpha$ is not involved in the last two equations; nevertheless, it should be indicated on the diagram.

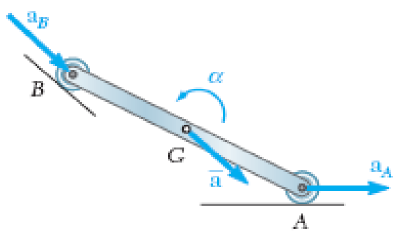
Corrected Version

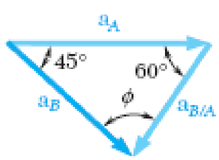


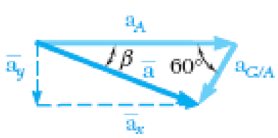
SAMPLE PROBLEM 16.10

The extremities of a 1.2 m rod weighing 22.7 kg can move freely and with no friction along two straight tracks as shown. If the rod is released with no velocity from the position shown, determine (*a*) the angular acceleration of the rod, (*b*) the reactions at *A* and *B*.

SOLUTION







Kinematics of Motion. Since the motion is constrained, the acceleration of *G* must be related to the angular acceleration $\boldsymbol{\alpha}$. To obtain this relation, we first determine the magnitude of the acceleration $\mathbf{a}_A$ of point *A* in terms of α. Assuming that $\boldsymbol{\alpha}$ is directed counterclockwise and noting that $a_{B/A} = 4\alpha$, we write

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$[a_B \searrow 45°] = [a_A \rightarrow] + [4\alpha \swarrow 60°]$$

Noting that $\phi = 75°$ and using the law of sines, we obtain

$$a_A = 5.46\alpha \qquad a_B = 4.90\alpha$$

The acceleration of *G* is now obtained by writing

$$\bar{\mathbf{a}} = \mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$$
$$\bar{\mathbf{a}} = [5.46\alpha \rightarrow] + [2\alpha \swarrow 60°]$$

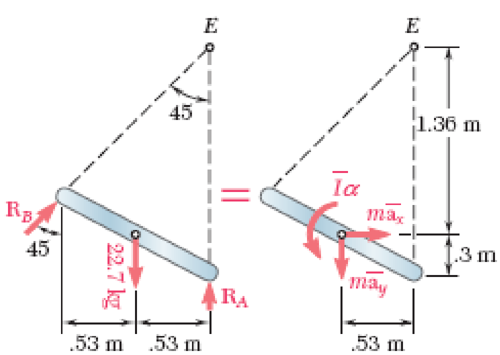
Resolving $\bar{\mathbf{a}}$ into *x* and *y* components, we obtain

$$\bar{a}_x = 5.46\alpha - 2\alpha \cos 60° = 4.46\alpha \qquad \bar{\mathbf{a}}_x = 4.46\alpha \rightarrow$$
$$\bar{a}_y = -2\alpha \sin 60° = -1.732\alpha \qquad \bar{\mathbf{a}}_y = 1.732\alpha \downarrow$$

Kinetics of Motion. We draw a free-body-diagram equation expressing that the system of the external forces is equivalent to the system of the effective forces represented by the vector of components $m\bar{\mathbf{a}}_x$ and $m\bar{\mathbf{a}}_y$ attached at *G* and the couple $\bar{I}\boldsymbol{\alpha}$. We compute the following magnitudes:

$$\bar{I} = \tfrac{1}{12}ml^2 = \frac{1}{12}22.7 \text{ kg } (1.2 \text{ m})^2 = 2.7 \text{ kg}\cdot\text{m}^2 \qquad \bar{I}\alpha = 2.7\alpha$$

$$m\bar{a}_x = 22.7 \text{ kg } (4.46\alpha) = 101.24\alpha \qquad m\bar{a}_y = -22.7\ (1.732\alpha) = -39.3\alpha$$



Equations of Motion

$+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{\text{eff}}$:

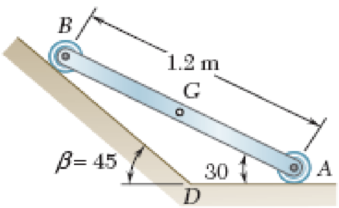
$$(22.7)(.53) = (101.24\alpha)(1.36) + (39.3\alpha)(.53) + 2.7\alpha$$
$$\alpha = 0.075 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 0.075 \text{ rad/s}^2 \circlearrowleft \quad ◀$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$:

$$R_B \sin 45° = (101.24)(.73) = 7.6 \text{ N}$$
$$R_B = 7.6 \text{ N} \qquad \mathbf{R}_B = 7.6 \text{ N} \nearrow 45° \quad ◀$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$:

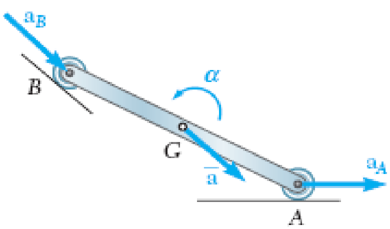
$$R_A + R_B \cos 45° - 222.70 = -(39.3)(.73)$$
$$R_A = -28.7 - 52.26 + 222.7 = 167.49 \text{ N} \qquad \mathbf{R}_A = 167.49 \text{ N} \uparrow \quad ◀$$

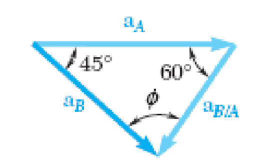


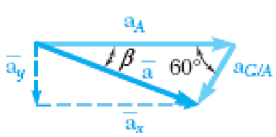
SAMPLE PROBLEM 16.10

The extremities of a 1.2 m rod weighing 22.7 kg can move freely and with no friction along two straight tracks as shown. If the rod is released with no velocity from the position shown, determine (*a*) the angular acceleration of the rod, (*b*) the reactions at *A* and *B*.

SOLUTION







Kinematics of Motion. Since the motion is constrained, the acceleration of *G* must be related to the angular acceleration $\boldsymbol{\alpha}$. To obtain this relation, we first determine the magnitude of the acceleration $\mathbf{a}_A$ of point *A* in terms of α. Assuming that $\boldsymbol{\alpha}$ is directed counterclockwise and noting that $a_{B/A} = 4\alpha$, we write

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$[a_B \searrow 45°] = [a_A \rightarrow] + [4\alpha \swarrow 60°]$$

Noting that $\phi = 75°$ and using the law of sines, we obtain

$$a_A = 5.46\alpha \qquad a_B = 4.90\alpha$$

The acceleration of *G* is now obtained by writing

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$$
$$\bar{\mathbf{a}} = [5.46\alpha \rightarrow] + [2\alpha \swarrow 60°]$$

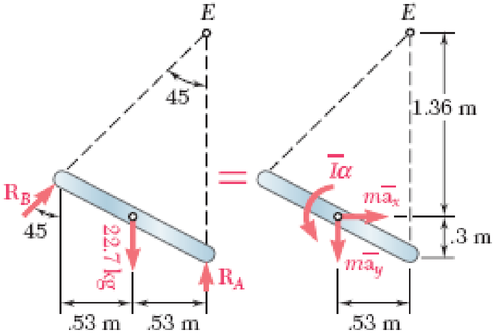
Resolving $\bar{\mathbf{a}}$ into *x* and *y* components, we obtain

$$\bar{a}_x = 5.46\alpha - 2\alpha\cos 60° = 4.46\alpha \qquad \bar{\mathbf{a}}_x = 4.46\alpha \rightarrow$$
$$\bar{a}_y = -2\alpha\sin 60° = -1.732\alpha \qquad \bar{\mathbf{a}}_y = 1.732\alpha \downarrow$$

Kinetics of Motion. We draw a free-body-diagram equation expressing that the system of the external forces is equivalent to the system of the effective forces represented by the vector of components $m\bar{\mathbf{a}}_x$ and $m\bar{\mathbf{a}}_y$ attached at *G* and the couple $\bar{I}\boldsymbol{\alpha}$. We compute the following magnitudes:

$$\bar{I} = \tfrac{1}{12}ml^2 = \frac{1}{12}22.7\text{ kg }(1.2\text{ m})^2 = 2.7\text{ kg}\cdot\text{m}^2 \qquad \bar{I}\alpha = 2.7\alpha$$

$$m\bar{a}_x = 22.7\text{ kg }(4.46\alpha) = 101.24\alpha \qquad m\bar{a}_y = -22.7\,(1.732\alpha) = -39.3\alpha$$



Equations of Motion

$+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{\text{eff}}$:

$$(22.7)(.53) = (101.24\alpha)(1.36) + (39.3\alpha)(.53) + 2.7\alpha$$
$$\alpha = 0.075\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 0.075\text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\quad R_B\sin 45° = (101.24)(.73) = 7.6\text{ kgf}$

$$R_B = 7.6\text{ kgf} \qquad \mathbf{R}_B = 7.6\text{ kgf} \nearrow 45° \blacktriangleleft$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $\quad R_A + R_B\cos 45° - 222.70 = -(39.3)(.73)$

$$R_A = -28.7 - 52.26 + 222.7 = 167.49\text{ kgf} \qquad \mathbf{R}_A = 167.49\text{ kgf}\uparrow \blacktriangleleft$$