ADDITIONAL READING MATERIAL
SAMPLE SECTIONS FROM THE BOOK

Applications of DSP

Some selected applications of digital signal processing that are often encountered in daily life are listed as follows:

1. *Telecommunication* Echo cancellation in telephone networks, adaptive equalisation, ADPCM transcoders, telephone dialing application, modems, line repeaters, channel multiplexing, data communication, data encryption, video conferencing, cellular phone and FAX.
2. *Military* Radar signal processing, sonar signal processing, navigation, secure communications and missile guidance.
3. *Consumer electronics* Digital Audio/TV, electronic music synthesiser, educational toys, FM stereo applications and sound recording applications.
4. *Instrumentation and control* Spectrum analysis, position and rate control, noise reduction, data compression, digital filter, PLL, function generator, servo control, robot control and process control.
5. *Image processing* Image representation, image compression, image enhancement, image restoration, image reconstruction, image analysis and recognition, pattern recognition, robotic vision, satellite weather map and animation.
6. *Speech processing* Speech analysis methods are used in automatic speech recognition, speaker verification and speaker identification. Speech synthesis techniques includes conversion of written text into speech, digital audio and equalisation.
7. *Medicine* Medical diagnostic instrumentation such as computerised tomography (CT), X-ray scanning, magnetic resonance imaging, spectrum analysis of ECG and EEG signals to detect various disorders in heart and brain, scanners, patient monitoring and X-ray storage/enhancement.
8. *Seismology* DSP techniques are employed in geophysical exploration for oil and gas, detection of underground nuclear explosion and earthquake monitoring.
9. *Signal filtering* Removal of unwanted background noise, removal of interference, separation of frequency bands and shaping of the signal spectrum.

Table 2.1 *Important properties of the Fourier transform*

Operation	$f(t)$	$F(jv)$
Transform	$f(t)$	$F(j\omega)\ e^{-j\omega t_0}$
Inverse transform	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}\mathrm{d}\omega$	$F(j\omega)$
Linearity	$af_1(t)+bf_2(t)$	$aF_1(j\omega)+bF_2(j\omega)$
Time-reversal	$f(-t)$	$F(-j\omega)=F^*(j\omega)$, $f(t)$ real
Time-shifting (Delay)	$f(t-t_0)$	$F(j\omega)\ e^{-j\omega t_0}$
Time-Scaling	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{j\omega}{a}\right)$
Time-differentiation	$\frac{\mathrm{d}^n}{\mathrm{d}t^n}f(t)$	$(j\omega)^n F(j\omega)$
Frequency differentiation	$(-jt)\,f(t)$	$\frac{\mathrm{d}F(j\omega)}{\mathrm{d}\omega}$
Time-integration	$\int_{-\infty}^{t} f(\tau)\,\mathrm{d}\tau$	$\frac{1}{j\omega}F(j\omega)+\pi F(0)\delta(\omega)$
Frequency-integration	$\frac{1}{(-jt)}f(t)$	$\int^{\omega} f(j\omega')\,\mathrm{d}\omega'$
Time convolution	$f_1(t)*f_2(t)=\int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)\,\mathrm{d}\tau$	$F_1(j\omega)\,F_2(j\omega)$
Frequency convolution (Multiplication)	$f_1(t)\cdot f_2(t)$	$\frac{1}{2\pi}[F_1(j\omega)*F_2(j\omega)]$
Frequency shifting (Modulation)	$f(t)\,e^{j\omega_0 t}$	$F(j\omega-j\omega_0)$
Symmetry	$F(jt)$	$2\pi f(-\omega)$
Real-time function	$f(t)$	$F(j\omega)=F^*(-j\omega)$ $\mathrm{Re}\,[F(j\omega)]=\mathrm{Re}\,[F(-j\omega)]$ $\mathrm{Im}\,[F(j\omega)]=-\mathrm{Im}\,[F(-j\omega)]$ $\lvert F(j\omega)\rvert=\lvert F(-j\omega)\rvert$ $\Phi f(j\omega)=-\Phi f(j\omega)$
Parseval's theorem	$E=\int_{-\infty}^{\infty}\lvert f(t)\rvert^2\,\mathrm{d}t$	$E=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert F(j\omega)\rvert^2\,\mathrm{d}\omega$
Duality	If $f(t)\Rightarrow g(j\omega)$, then $g(t)\Rightarrow 2\pi f(-j\omega)$	

Definition of Applications of Laplace Transform to System Analysis

For a periodic or non-periodic time function $f(t)$, which is zero for $t \leq 0$ and defined for $t > 0$, the Laplace transform of $f(t)$, denoted as $\mathcal{L}\{f(t)\}$, is defined by

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-(\sigma + j\omega)t}\mathrm{d}t$$

Putting $s = \sigma + j\omega$, we have

$$F(s) = \int_0^{\infty} f(t)e^{-st} \quad t \tag{3.1}$$

The condition for the Laplace transform to exist is $\int_{-\infty}^{\infty} |f(t)e^{-st}|\mathrm{d}t < \infty$ for some finite σ. Laplace transform thus converts the time domain function $f(t)$ to the frequency domain function $F(s)$. This transform defined to the positive-time functions is called single-sided or unilateral, because it does not depend on the history of $x(t)$ prior to $t = 0$. In the double-sided or bilateral Laplace transform, the lower limit of integration is $t = -\infty$, where $x(t)$ covers over all time.

Due to the convergence factor $e^{-\sigma t}$, the ramp, parabolic functions, etc. are Laplace transformable. In transient problems, the Laplace transform is preferred to the Fourier transform, as the Laplace transform directly takes into account the initial conditions at $t = 0$, due to the lower limit of integration.

Inverse Laplace Transform

The inverse Laplace transform is used to convert a frequency domain function $F(s)$ to the time domain function $f(t)$, as defined by

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j}\int_{\sigma_0 - j\omega}^{\sigma_0 + j\omega} F(s)e^{st}\mathrm{d}s \tag{3.2}$$

Here, the path of integration is a straight line parallel to the $j\omega$-axis, such that all the poles of $F(s)$ lie to the left of the line.

In practice, it is not necessary to carry out this complicated integration. With the help of partial fractions and existing tables of transform pairs, we can obtain the solution of different expressions.

4.2 DEFINITION OF THE z-TRANSFORM

The z-transform of a discrete-time signal $x(n)$ is defined as the power series

$$Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.2}$$

where z is a complex variable. This expression is generally referred to as the *two-sided z-transform*.

If $x(n)$ is a causal sequence, $x(n) = 0$ for $n < 0$, then its z-transform is

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n}$$

This expression is called a *one-sided z-transform*.

This causal sequence produces negative powers of z in $X(z)$. Generally we assume that $x(n)$ is a causal sequence, unless it is otherwise stated.

If $x(n)$ is a non-causal sequence, $x(n) = 0$ for $n \geq 0$, then its z-transform is

$$X(z) = \sum_{n=-\infty}^{-1} x(n)z^{-n} \tag{4.3}$$

This expression is also called a one-sided z-transform. This non-causal sequence produces positive powers of z in $X(z)$.

4.2.1 Definition of the Inverse z-Transform

The inverse z-transform is computed or derived to recover the original time domain discrete signal sequence $x(n)$ from its frequency domain signal $X(z)$. The operation can be expressed mathematically as

$$x(n) = Z^{-1}[X(z)] \tag{4.4}$$

Discrete Fourier Transform (DFT)

The Discrete Fourier Transform (DFT) computes the values of the z-transform for evenly spaced points around the unit circle for a given sequence.

If the sequence to be represented is of finite duration, i.e. has only a finite number of non-zero values, the transform used is Discrete Fourier Transform (DFT). DFT finds its applications in digital signal processing including linear filtering, correlation analysis and spectrum analysis.

Definition

Let $x(n)$ be a finite duration sequence. The N-point DFT of the sequence $x(n)$ is expressed by

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N},\ k=0,1,\ldots\ N-1. \tag{6.12}$$

and the corresponding IDFT is

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi nk/N},\ n=0,1,\ldots\ N-1.$$

Advantages of Digital Filters compared with Analog Filters :

(i) Digital filters can have characteristics which are not possible with analog filters such as linear phase response.
(ii) The performance of digital filters does not vary with environmental changes, for example, thermal variations.
(iii) The frequency response of a digital filter can be adjusted if it is implemented using a programmable processor.
(iv) Several input signals can be filtered by one digital filter without the need to replicate the hardware.
(v) Digital filters can be used at very low frequencies.

Disadvantages of Digital Filters compared with Analog Filters :

(i) Speed limitation
(ii) Finite word length effects
(iii) Long design and development times

Advantages of DSP Processors

DSP processors are often tuned to meet the needs of specific digital signal processing applications. This makes DSP processors strong in terms of performance. DSP processors are also very power efficient, especially when the DSP platforms are designed specifically for low-power, handheld applications such as Texas Instrument (TI) TMS320C5000. DSP processors are typically complemented by powerful and easy-to-use programming and debug tools. This allows for faster development cycles for the desired function and features. The development time can be reduced significantly with proper use of high level programming modules. A DSP processor offers shorter time to market due it its software programmability. For real-time signal processing, DSP processors are rated the best among programmable processors (DSP, RISC) because they have the best and most relevant tool sets to achieve real-time signal-processing functions.