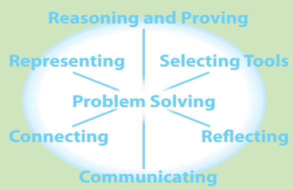
6 Geometry in Design

Why is knowledge of geometry important for so many activities? Think about a concert that you attended. How did geometry influence the band who wrote the music, the craftspeople who made the instruments, the artists who decorated the concert hall, the architect who designed the hall, the engineers who planned the hall's construction, and the tradespeople whose skills and talents made it all possible?

In this chapter, you will

- identify real-world applications of geometric shapes and figures, through investigation in a variety of contexts, and explain these applications
- represent three-dimensional objects, using concrete materials and design or drawing software, in a variety of ways
- create nets, plans, and patterns from physical models arising from a variety of real-world applications, by applying the metric and imperial systems and using design or drawing software
- solve design problems that satisfy given constraints, using physical models or drawings, and state any assumptions made



Key Terms

constraint	orthographic drawings
golden ratio	orthographic projection
golden rectangle	pattern
isometric perspective drawing	plan
net	scale model
	tessellation

Arqum studied graphic design at Fanshawe College and now creates signs for companies selling a product or service. He discusses each project with his customers, including size, materials, budget, and regulations, and designs layouts for billboards and business signs. Arqum uses geometric shapes and scale when he creates his designs on the computer.

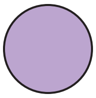
Prerequisite Skills

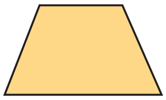
Geometric Shapes

1. Identify the two-dimensional geometric shapes.

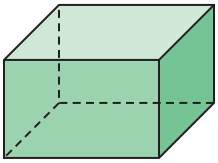
a) 

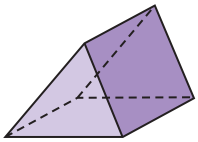
b) 

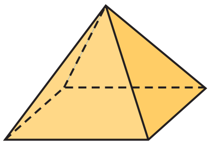
c) 

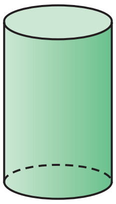
d) 

2. Identify the three-dimensional geometric shapes.

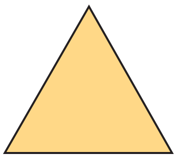
a) 

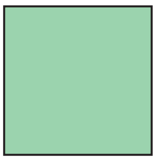
b) 

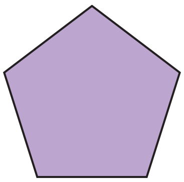
c) 

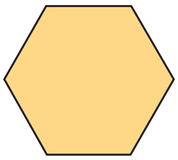
d) 

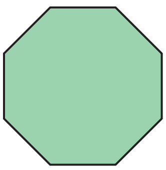
3. Identify the polygons.

a) 

b) 

c) 

d) 

e) 

Perimeter and Area

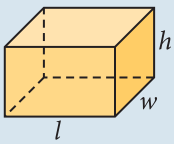
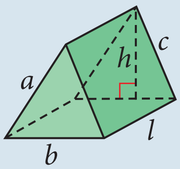
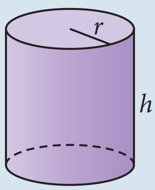
Figure	Perimeter/Circumference	Area
rectangle	$P = 2l + 2w$	$A = lw$
triangle	$P = a + b + c$	$A = \frac{1}{2}bh$
circle	$C = \pi d$, $C = 2\pi r$	$A = \pi r^2$

4. Find the perimeter and the area of a rectangular sports field that measures 120 m by 100 m.

5. Find the perimeter and the area of a school crest in the shape of an equilateral triangle with a side length of 15 cm. The height of the triangle is approximately 13 cm.

6. Find the perimeter the and area of the base of a circular wading pool with a radius of 12 m.

Surface Area and Volume

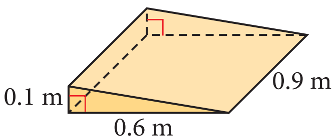
Figure	Surface Area	Volume
	$SA = 2lw + 2wh + 2lh$	$V = lwh$
	$SA = al + bl + cl + bh$	$V = \frac{1}{2}blh$
	$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$

7. Wilhelm built a closed room in his garage to use as a paint shop. The room measured 8 m by 6 m by 2.5 m.

a) Wilhelm painted the interior walls and ceiling. Find the area that was painted.

b) Wilhelm plans to install a blower to pump fresh air into the room. The size of blower required depends on the volume of the room. Find the volume of the room.

8. Sunita built a wheelchair ramp in the shape of a triangular prism, as shown. Each surface of the ramp is to be painted—with the exception of the base. Find the surface area to be painted.



9. A cylindrical water tank a radius of 5 ft and a height of 12 ft. The interior is to be painted with waterproof paint. Find the area to be painted, and the volume of water that the tank can hold.

Angles in a Polygon

10. The sum of the angles in a polygon with n sides, in degrees, is given by the formula $S = 180(n - 2)$. If the polygon is regular, the measure of each angle is given by the sum divided by n. Find the sum of the angles, and the measure of each angle, for a regular pentagon.

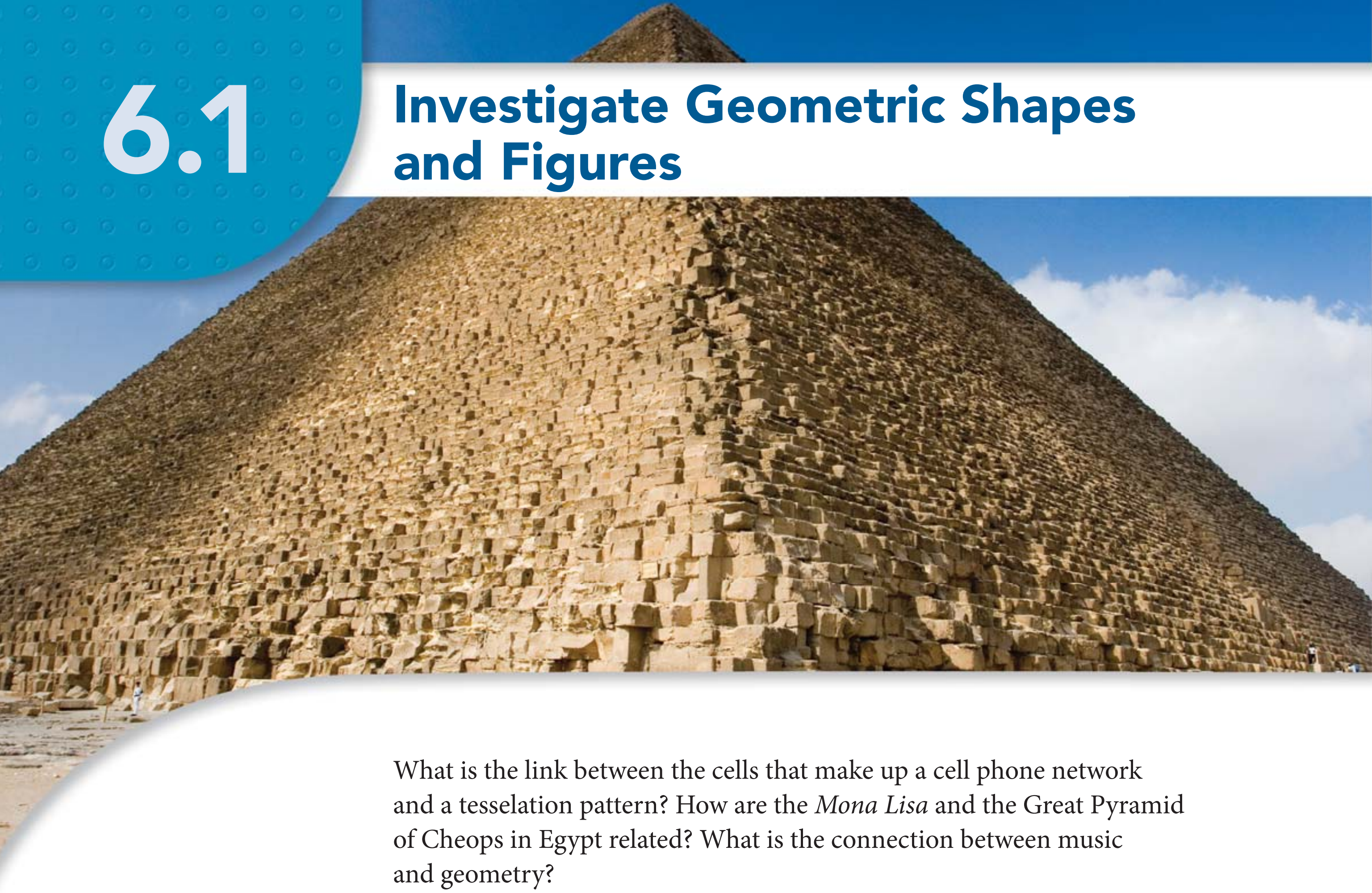
Scale Factors

11. Anil is planning to grow vegetables in a market garden measuring 24 m by 36 m. To plan the garden, he will draw a diagram using a scale of 1 cm to represent 2 m. Find the dimensions of the diagram.

12. Ted has made a scale model of a sailboat that he is planning to build. The model has a length of 10 in. Ted used a scale of 1 in. to represent 4 ft. What is the length of the sailboat?

Chapter Problem

Paul has secured a co-op position for one semester with a firm of architects and designers. The firm provides customers with designs, drawings, scale models, plans, patterns, and cost estimates for a variety of projects, such as prefabricated outbuildings, houses, and industrial products. Paul will learn how to provide customers with the services that the firm offers. As he works through the co-op position, Paul will be given design tasks that will require the knowledge and skills that he has learned during his placement.



6.1 Investigate Geometric Shapes and Figures

What is the link between the cells that make up a cell phone network and a tesselation pattern? How are the *Mona Lisa* and the Great Pyramid of Cheops in Egypt related? What is the connection between music and geometry?

In this section, you will investigate some of these relations, and begin to learn how to use geometric principles in art, architecture, and fashion.

Investigate 1

Tools

- copy of Leonardo da Vinci's painting *Mona Lisa*
- ruler

or

- computer
- *The Geometer's Sketchpad®*

Technology Tip

If the picture appears too large or too small, you can resize it using any drawing program, such as Microsoft® Paint.

The *Mona Lisa*

Uncover one of the secrets of the *Mona Lisa*.

1. Draw a rectangle that encloses the face of the *Mona Lisa*.

Method 1: Use Pencil and Paper

Use your ruler and pencil to draw the rectangle. Measure the length and width of the rectangle. Divide the length by the width.

Method 2: Use *The Geometer's Sketchpad*®

Open *The Geometer's Sketchpad*®. Search the Internet for a picture of the *Mona Lisa*. Right-click on the picture, and select **Copy**. Return to *The Geometer's Sketchpad*®. Select **Paste Picture** from the **Edit** menu. Draw a rectangle around the face of the *Mona Lisa*. Measure the length and width of the rectangle. Divide the length by the width.

golden ratio
- the ratio of two lengths is approximately 1.618:1

golden rectangle
- a rectangle that is pleasing to the eye
- the ratio of the length to the width is approximately 1.618:1

2. The ratio of the length to the width on the actual painting is 1.618:1. This is known as the **golden ratio**. A rectangle that is drawn using the golden ratio is known as a **golden rectangle**. The proportions of golden rectangles are pleasing to the eye. The golden ratio is often used in art and architecture.

3. **Reflect** Think about windows that you have seen on buildings in your neighbourhood. Which windows appear to be golden rectangles?

To learn more about the golden ratio in art and architecture, go to *www.mcgrawhill.ca/links/foundations11* and follow the links.

Investigate 2

Tools

- centimetre grid paper
- coloured pencils or markers
- pattern block template or pattern blocks

tessellation
- a pattern that covers a plane without overlapping or leaving gaps
- also called a tiling pattern

Tessellations

Certain shapes can be used to fill a plane completely, with no gaps. This is known as a **tessellation**, or "tiling the plane." As you can see from a piece of grid paper, a square is one of these shapes. Clothing is often made from fabrics with a checkerboard pattern. If colours are carefully chosen and placed, the pattern becomes more attractive.

Method 1: Use Pencil and Paper

1. Draw a square. Then, draw additional squares identical to the first square such that they form a tessellation. Use at least four different colours to make a pleasing pattern for clothing.

2. Triangles can also be used to form tessellations. Draw a triangle. Then, draw additional triangles identical to the first triangle such that they form a tessellation. Colour the triangles to create a pleasing pattern.

3. **Reflect** Can you use any triangle to tile the plane? Try several examples.

4. You can use a combination of triangles and rectangles to form a tessellation. Find a combination of a rectangle and several triangles that can be used to tile the plane. Draw your tessellation. Colour your pattern to create a pleasing design.

5. You can also use other regular geometric shapes to construct tessellations. Pattern blocks include many different regular geometric shapes. Use the blocks to experiment with different shapes that can tile the plane. Examine a vertex that is common to several polygons. What is the measure of the angle around the vertex?

Tools

- computer
- *The Geometer's Sketchpad®*
- pattern block applet

Technology Tip

Once you have a basic pattern, you can select all elements of the pattern. Then, use the **Copy** command from the **Edit** menu, and **Paste** it repeatedly. This is a fast way of tiling the plane.

Method 2: Use Computer Software

1. Open *The Geometer's Sketchpad®*. From the **Graph** menu, choose **Show Grid**. Select both axes, the origin, and the unit point. Choose **Hide Objects** from the **Display** menu.

2. Start in the upper left corner, and draw four points to enclose one of the squares. Select the four points in order. From the **Construct** menu, choose **Quadrilateral Interior**. Select the four points, and the interior. From the **Edit** menu, choose **Copy**, and then **Paste**. Drag the image to another square.

3. Right-click on one of the interiors. Choose **Color**, and change the colour of the square. You can continue this process to tile the screen with different coloured squares. Use at least four different colours to design a tessellation that would make a pleasing pattern for clothing.

4. Triangles can also be used to form tessellations. Start a new sketch. Draw a triangle. Then, draw additional triangles identical to the first triangle such that they form a tessellation. Colour the triangles to form a pleasing pattern.

5. **Reflect** Can you use any triangle to tile the plane? Try several examples.

6. You can use a combination of triangles and rectangles to form a tessellation. Start with a grid of squares. Find a combination of a rectangle and several triangles that can be used to tile the plane. Draw your tesselation. Colour your pattern to form a pleasing design.

7. You can also use other regular geometric shapes to construct tessellations. Go to *www.mcgrawhill.ca/links/foundations11* and follow the links to the Pattern Block Applet. Use the applet to experiment with different shapes that can tile the plane. Examine a vertex that is common to several polygons. What is the measure of the angle around the vertex?

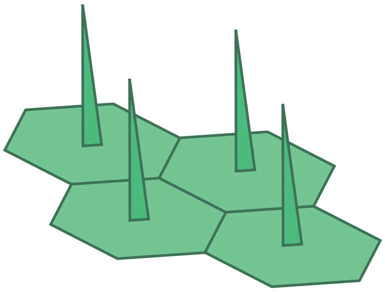
The only regular polygons (triangle, square, pentagon, and so on) that can tile the plane are those whose interior angles divide evenly into 360°. These are the triangle (60°), the square (90°), and the hexagon (120°). A pentagon, for example, will not tile a plane because its interior angles (108°) do not divide evenly into 360°.

To learn more about tessellations, go to *www.mcgrawhill.ca/links/foundations11* and follow the links.

Investigate 3

Cell Phone Tower Networks

Cell phones are really small radios that communicate with towers connected to the telephone system. The towers in an urban area must be placed close together because cell phones are low-powered, typically about 3 W, and have a limited range. Also, each tower only offers a limited number of frequencies (about 800), restricting the number of calls that a tower can handle at one time.



In an urban area, towers are placed about 5 km apart. Each tower is thought of as the centre of a hexagonal cell, as shown. Consider an urban area that measures 20 km on a side. Draw a tessellation using hexagons, and show how many towers are needed to service the cell phone network.

Tools

- ruler
- grid paper

Method 1: Use Pencil and Paper

1. Let 1 cm represent 1 km. Draw a square to represent an area 20 km by 20 km.
2. Lightly draw a hexagon, with a dot in the centre to represent the tower. Draw another hexagon next to the first one, leaving no gaps. Measure the distance between the two towers. If it is not close to 5 cm, adjust your hexagons. Once you think you have the correct size of hexagon, continue your diagram to cover the entire area.
3. How many towers are required to service the urban area?
4. **Reflect** What are the similarities between tiling the plane with triangles, designing a pattern for clothing, and ensuring that cell phone users have good service?

Tools

- computer
- *The Geometer's Sketchpad®*

Technology Tip

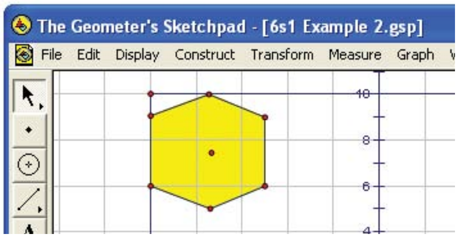
Do not worry if your hexagons look a bit irregular. You will learn how to draw regular geometric shapes accurately later in this chapter.

Method 2: Use *The Geometer's Sketchpad®*

1. Open *The Geometer's Sketchpad®*. From the **Graph** menu, choose **Show Grid**. Drag the unit point on the horizontal axis until you can see +10 and −10 on your screen, on both the horizontal and vertical axes. Use the **Segment Tool** to draw a rectangle measuring 20 squares by 20 squares. This will represent an area 20 km by 20 km.

2. Use the **Segment Tool** to construct a hexagon in the upper left hand corner of the 20 km by 20 km grid. Draw a point in the centre of your hexagon to represent the tower.

3. Draw another hexagon next to the first one, leaving no gaps. Determine the distance between the two towers. If it is not close to 5 km, adjust your hexagons by dragging vertices. Once you think you have the correct size of hexagon, continue your diagram to cover the entire area.



4. How many towers are required to service the urban area?

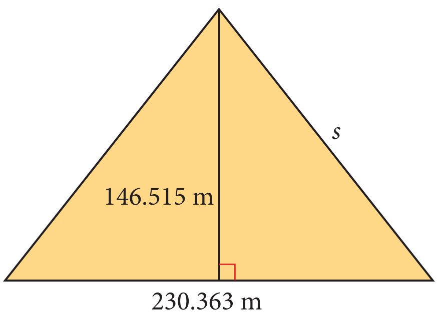
5. **Reflect** What are the similarities between tiling the plane with triangles, designing a pattern for clothing, and ensuring that cell phone users have good service?

To learn more about cell phones and how they work, go to *www.mcgrawhill.ca/links/foundations11* and follow the links.

Example

The Golden Ratio in Ancient Architecture

A cross-section of the Great Pyramid of Cheops on the Giza plateau in Egypt is shown. Find the ratio of the slant height s to half of the base. Express your answer to three decimal places.



Solution

Half of the base $= 230.363 \div 2$

$\doteq 115.182$

Use the Pythagorean theorem to find the slant height.

$s^2 \doteq (115.182)^2 + (146.515)^2$

$s^2 \doteq 34\,733.538$

$s \doteq \sqrt{34\,733.538}$

$s \doteq 186.369$

The slant height is approximately 186.369 m.

Slant height:half of the base $\doteq$ 186.369:115.182

$\doteq$ 1.618:1

Math Connect

The number given for the golden ratio, 1.618, is an approximation. The exact value is $\frac{1+\sqrt{5}}{2}$. Use a calculator to verify that this expression gives the correct approximation.

The ratio in the Example is the golden ratio once again. The golden ratio is a connection between the Mona Lisa and the Great Pyramid. A triangle that has this property, such as the cross-section of the Pyramid of Cheops, is known as an Egyptian triangle.

To learn more about the Pyramids, go to *www.mcgrawhill.ca/links/foundations11* and follow the links.

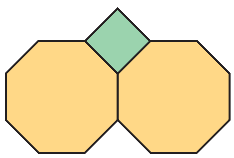
Key Concepts

- Geometric shapes are used in architecture, art, and fashion. These shapes often use the golden ratio, which is approximately 1.618:1. The golden ratio is used in design because its proportions are pleasing to the eye.
- Some geometric shapes can be used to "tile a plane" such that there are no spaces. These tiling patterns are known as tessellations. Some polygons that tile a plane are squares, rectangles, equilateral triangles, and regular hexagons. Combinations of regular polygons can also tile a plane.

Discuss the Concepts

D1. How are geometric shapes used in the construction of your school? Make a list of the shapes that you notice, and where they are used.

D2. Can the shape shown be used to tile the plane? Use a sketch to explain.



Practise A

1. Which of these regular polygons cannot be used to tile a plane?

   A triangle
   B square
   C hexagon
   D octagon

2. Sandor claims that the screen on his new HD television is a golden rectangle. It measures 32 in. by 18 in. Is he correct? Justify your answer.

3. Look at the buildings in your neighbourhood. Identify geometric shapes that you see in the construction of these buildings. In particular, describe the use of shapes other than squares or rectangles.

4. Think about musical instruments that you have seen. Geometric shapes are present in the design of many musical instruments. List several instruments, and identify the geometric figures used in their design.

5. The old railroad station in Petrolia, ON, is now the public library. Identify the different two-dimensional and three-dimensional geometric figures that were used in its design.

Apply B

Literacy Connect

6. Uploading personal digital pictures to the Internet is a popular activity. If you do this, and identify people in your pictures, the software that operates the posting site can "learn" to identify faces. It uses measurements and ratios similar to those you found for the *Mona Lisa* in Investigate 1. Once a face is in the database, other users can employ the software to identify individuals in other pictures. Write a short paragraph discussing the advantages and disadvantages of having such a software tool freely available on the Internet.

For help with questions 7 and 8, refer to Investigate A.

7. Sports such as soccer or lacrosse are often played on rectangular fields. Investigate the dimensions of several of sports fields. Find the dimensions of an Olympic swimming pool. Are any dimensions close to a golden rectangle?

Technology Tip

You can search for images directly using most search engines, rather than searching for web sites and then looking for images.

8. Use the Internet to find a picture of each of these famous buildings. Copy each picture into *The Geometer's Sketchpad*®, and use the software to identify any golden rectangles that were used in the design.

a) The Parthenon, in Athens, Greece. Use a front view.

b) The Taj Mahal, in Agra, India. Use a front view, taken down the reflecting pool.

c) The United Nations building in New York, NY. Use a view from the river.

d) The Buddhist Temple in Niagara Falls, ON. Use a front view.

9. The actor Drew Barrymore is considered as a classic example of a "round face," while actor Fred Gwynne (who played a Frankenstein-like character on a popular television show) is considered to be an example of a "long face." Search for pictures of each actor. Use pencil and paper or *The Geometer's Sketchpad*® to determine how the actors' faces compare to the golden ratio.

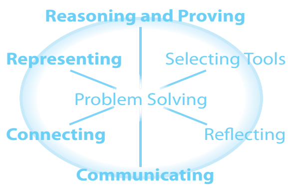
10. The faces of some animals appear to have a greater width than height. One example is a house cat. Use pencil and paper or *The Geometer's Sketchpad*® to determine the facial ratio of a typical cat.

Chapter Problem

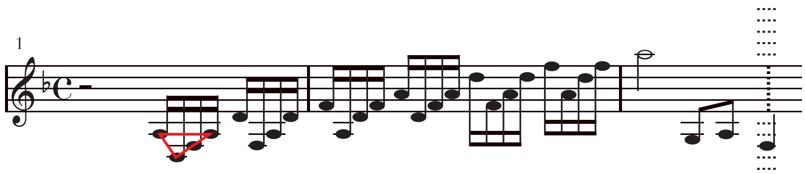
11. Paul has been reading about the history of architecture. Ancient Egyptian builders often needed to lay out large right angles for their constructions. They did this using a "12-knot rope." Obtain a piece of string about 1 m long. Tie 12 knots in the string such that the knots are 5 cm apart. Tie the ends of the string together such that you have 12 equal sections. Arrange the string to form a triangle with sides of 3, 4, and 5 sections.

a) Measure the angle opposite the longest side.

b) Write a short explanation of the mathematics behind this method.

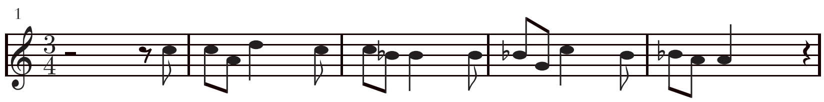


12. Where is the geometry in music? The first few bars of the classic hard rock piece "In-A-Gadda-Da-Vida" by Iron Butterfly, widely acknowledged as the first heavy metal band, are shown.

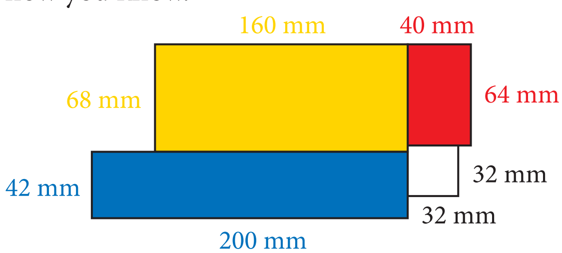


a) Notice the groupings of four notes. Draw lines joining adjacent notes in each group, and then a single line returning to the first note. The first triangle has been drawn for you. Describe the kinds of triangles that are formed.

b) Triangles are often present in music composition. Find two three-note congruent triangles in this short excerpt from the popular holiday song "O Tannenbaum."



13. A stained glass panel contains several rectangles. Which rectangles are golden rectangles or close to being golden rectangles? Explain how you know.



Achievement Check

14. Anwar claims that a hexagon does not need to be regular to tile the plane. He says that any hexagon that has opposite sides equal will work.

a) Draw a hexagon with opposite sides equal. Be sure to use three different lengths in your drawing.

b) Determine whether your hexagon can be used to tile the plane. If it can, show how it can be done. If it cannot, explain why not.

Extend C

15. An architect is designing a window using the golden ratio.

a) The longer side measures 2.5 m. To the nearest tenth of a metre, what should be the length of the shorter side?

b) Draw a sketch of the window. Cut the window in half using the midpoints of the lengths. Compare the ratios of sides of one of the smaller rectangles. What do you notice?

16. Many countries use international paper sizes, such as ISO A4. A sheet of A4 paper measures 29.7 cm by 21.0 cm. This size is sold in Canada at office supply stores. Obtain a piece of A4 paper, or make one from a larger piece of paper.

a) Find the ratio of the length to the width, and record it.

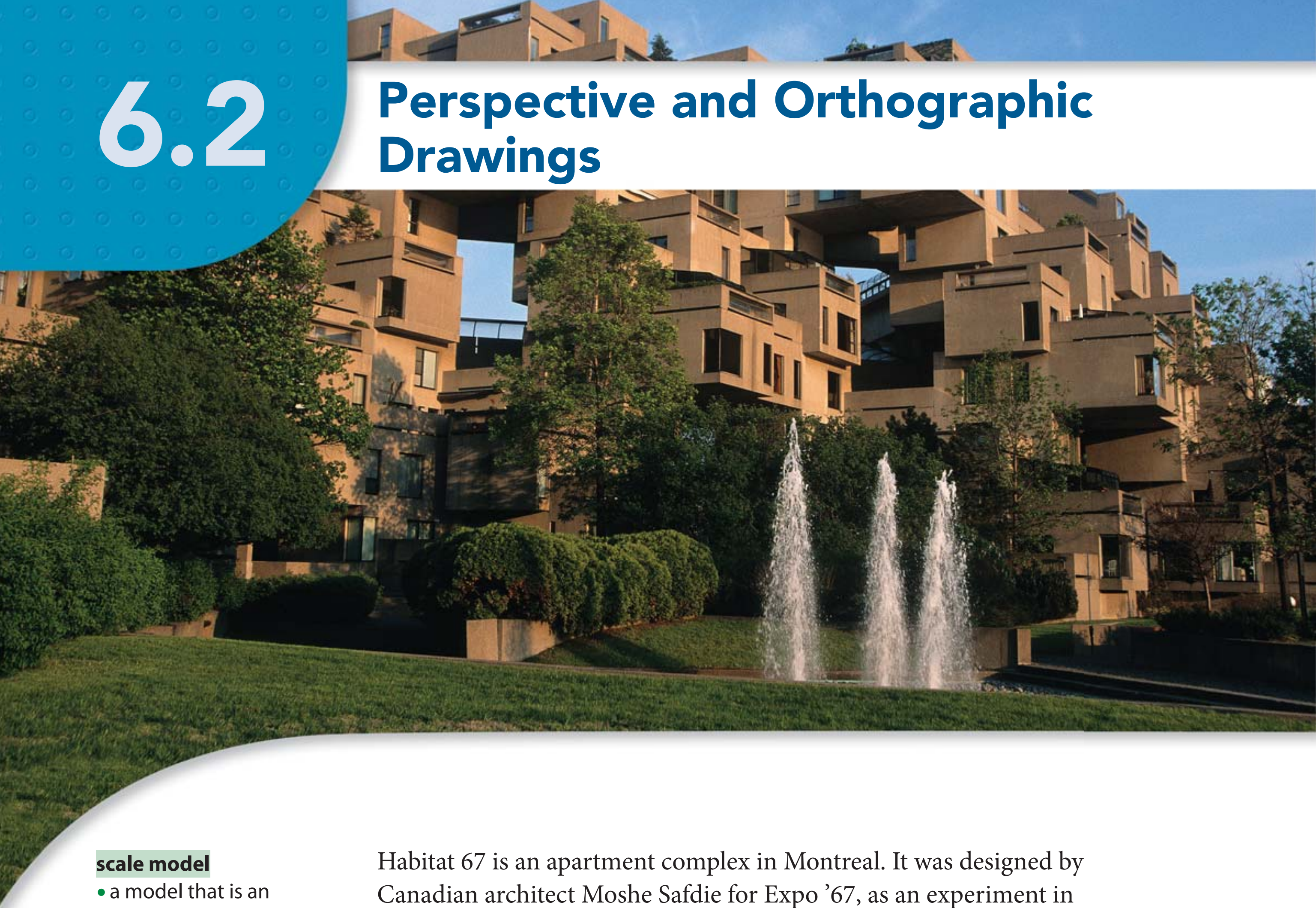
b) Fold the paper in half across a line joining the midpoints of the lengths. Measure the length and width of the folded paper. Find the ratio of the length to the width.

c) Fold the paper again. Find the ratio of the length to the width of the folded piece. What do you notice?

d) The ratio of length to width for the ISO standard is always $\sqrt{2} : 1$. Use a calculator to compare this ratio to the ratios that you found.

17. Use pencil and paper or software, such as *The Geometer's Sketchpad*®, to design fabric with a tessellation print. The fabric should appeal to a person with a particular career or hobby, such as a musician, a golfer, a pilot, a nurse, etc. Be sure to use basic geometric shapes to form a more complex shape. Be creative in your selection of colours.

6.2 Perspective and Orthographic Drawings



scale model
- a model that is an enlargement of a small object or a reduction of a large object

isometric perspective drawing
- visual representation of three-dimensional objects in two dimensions

orthographic projection
- a set of drawings that show up to six views of an object
- usually front, side, and top views are given

orthographic drawing
- a drawing that uses orthographic projection

Habitat 67 is an apartment complex in Montreal. It was designed by Canadian architect Moshe Safdie for Expo '67, as an experiment in building apartments that allowed urban dwellers to enjoy both privacy and outdoor space. A **scale model** was used to represent what the complex would look like when finished.

Another means of representing three-dimensional objects is an **isometric perspective drawing**, which creates the illusion of three dimensions on a two-dimensional plane. A third representation is a series of **orthographic projections**, which illustrate front, side, and top views of the object.

How could you draw the isometric perspective and the orthographic representations of Habitat 67? How can drawing software be used to create these representations?

To learn more about Habitat 67, go to *www.mcgrawhill.ca/links/foundations11* and follow the links.

Investigate 1 Perspective Drawings of Cubes and Rectangular Prisms

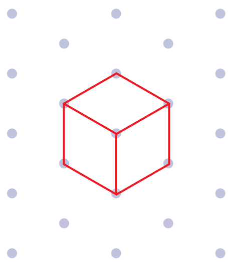
Use a single linking cube or a model of a cube. Turn the cube until an edge is facing you. Then, tip the cube downwards, as shown. This is the perspective that you will draw.

Tools

- isometric dot paper
- linking cubes or other physical models of cubes
- ruler
- coloured pencils

Method 1: Use Pencil and Paper

1. a) Inspect the isometric dot paper. Find the lines of dots that are vertical. Draw a short vertical line segment in blue.
   b) Find the lines of dots that slope down to the right at 30°. Draw a short line segment sloping down to the right at 30° in red.
   c) Find the lines of dots that slope down to the left at 30°. Draw a short line segment sloping down to the left at 30° in green.
   d) Use the dot pattern to draw the isometric perspective, as shown.



2. **Reflect** Compare the isometric perspective drawing with the view of the cube. On an isometric perspective drawing, the lengths on the drawing are the same as the lengths on the real cube. Check that the sides of the cube on the drawing are of equal length.

3. Use linking cubes to construct a model of a rectangular prism that measures 1 cube by 2 cubes by 3 cubes. Draw an isometric perspective drawing of the prism.

Math Connect

The word "isometric" comes from the Greek words "iso," meaning "the same," and "metric," meaning "measure."

Tools

- computer
- *The Geometer's Sketchpad®*

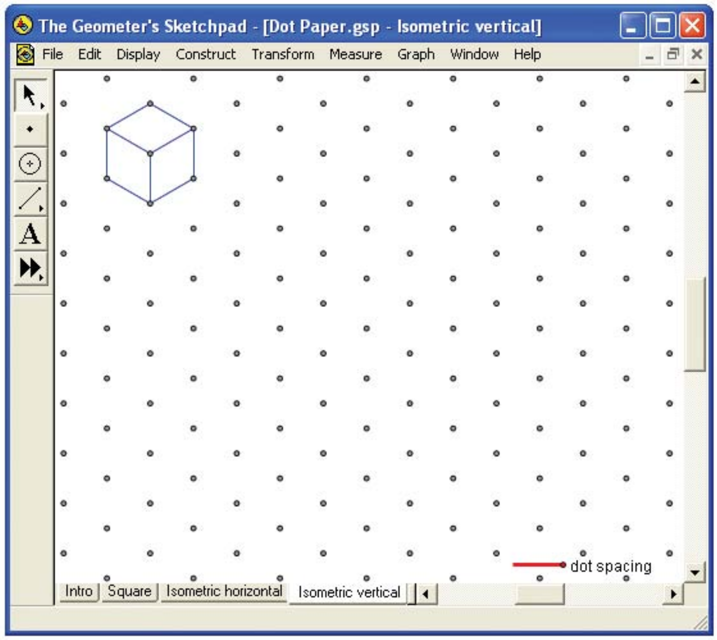
Technology Tip

You can change the spacing of the dots using the slider marked "dot spacing."

Method 2: Use *The Geometer's Sketchpad®*

1. Open *The Geometer's Sketchpad®*. Choose **Open** from the **File** menu. Navigate to the directory that stores *The Geometer's Sketchpad®* files on your computer. This will likely be called **Sketchpad**, and be located in the **Program Files** directory. Choose **Samples**, then **Sketches**, and then **Geometry**. Select and open the file Dot Paper.gsp. Select the button **Isometric vertical**.

2. Inspect the isometric vertical dot pattern. Find the lines of dots that are vertical. Find the lines of dots that slope down to the right at 30°. Find the lines of dots that slope down to the left at 30°. Use the dot pattern to create the isometric perspective drawing of the cube, as shown.



3. **Reflect** Compare the isometric perspective drawing with the view of the cube. On an isometric perspective drawing, the lengths on the drawing are the same as the lengths on the real cube. Check that the sides of the cube on the drawing are of equal length.

4. Use linking cubes to construct a model of a rectangular prism that measures 1 cube by 2 cubes by 3 cubes. Draw an isometric perspective drawing of the prism.

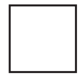
Investigate 2 Orthographic Drawings of Cubes and Rectangular Prisms

Tools

- square dot paper
- linking cubes or other physical models of cubes
- ruler

Method 1: Use Pencil and Paper

1. Use a single linking cube or a model of the cube. Turn the cube until the front is facing you. Draw the front of the cube. Label the drawing.



Front View

2. Turn the cube until the right side is facing you. Draw and label the side view. Similarly, draw and label the top view.

3. **Reflect** What do you notice about the three orthographic drawings of the cube? A cube is a simple shape. However, the same method can be used to represent more complex shapes.

4. Use linking cubes to construct a model of a rectangular prism that measures 1 cube by 2 cubes by 3 cubes. Create three orthographic drawings of the prism: front view, side view, and top view.

Tools

- computer
- *The Geometer's Sketchpad®*

Method 2: *Use The Geometer's Sketchpad*®

1. Open *The Geometer's Sketchpad*®. Open the file Dot Paper.gsp. Select the button **Square dot paper**.

2. Use a single linking cube or a model of a cube. Turn the cube until the front is facing you. Draw the front of the cube. Label the drawing.

3. Turn the cube until the side is facing you. Draw and label the side view. Similarly, draw and label the top view. Refer to the diagrams in Method 1.

4. **Reflect** What do you notice about the three orthographic drawings of the cube? A cube is a simple shape. However, the same method can be used to represent more complex shapes.

5. Use linking cubes to construct a model of a rectangular prism that measures 1 cube by 2 cubes by 3 cubes. Create three orthographic drawings of the prism: front view, side view, and top view.

You have been using *The Geometer's Sketchpad*® to draw perspective diagrams. Technical designers use more sophisticated programs, such as *AutoCAD*®. The science-fiction television series *Babylon 5* was created using *AutoCAD*® to render three-dimensional spaceships, space stations, and even some aliens. Amateur filmmakers often use commonly available drawing and design software to create inexpensive special effects for their productions.

To learn more about drawing software, go to *www.mcgrawhill.ca/links/foundations11* and follow the links.

Example 1

Modelling a Flight of Stairs

a) Use linking cubes to create a model for a flight of stairs. There are three steps. The steps are 30 cm deep and 120 cm wide. Each riser measures 30 cm. Let the side of a cube represent 30 cm.

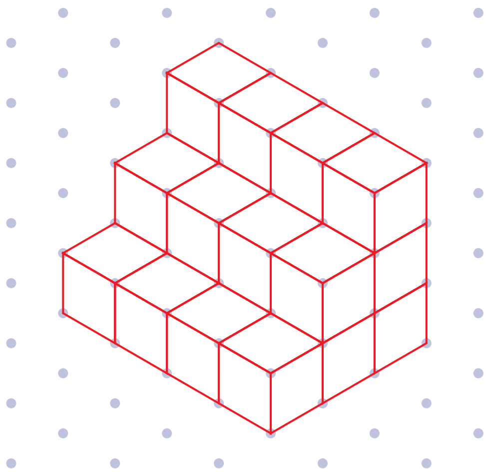
b) Draw an isometric perspective drawing of the model.

c) Sketch a set of orthographic drawings of the model.

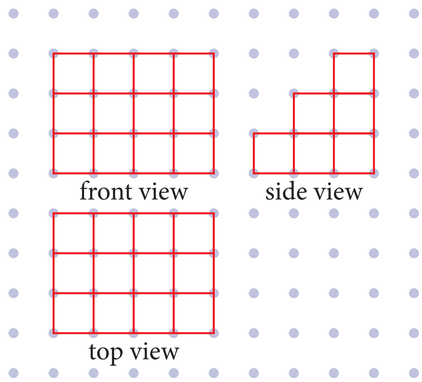
Solution

a) The side of a cube represents 30 cm. The stairs will be 4 cubes wide, and 3 cubes deep at the bottom layer.

b) Use isometric dot paper or *The Geometer's Sketchpad*® to create the isometric perspective drawing of the stairs.



c) Use square dot paper or *The Geometer's Sketchpad®* to draw the orthographic drawings for the stairs.

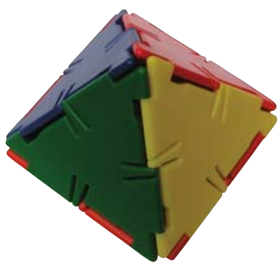


Example 2

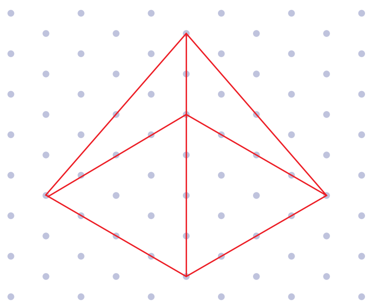
Isometric Drawing of a Square-Based Pyramid

Construct a model of a square-based pyramid using drinking straws and masking tape or other materials. Use isometric dot paper or *The Geometer's Sketchpad®* to sketch an isometric perspective drawing of the pyramid.

Solution



First, draw the base. The base will appear as a diamond shape. Select a point in line with two opposite vertices to be the apex of the pyramid. Complete the pyramid by drawing the sides.



Example 3

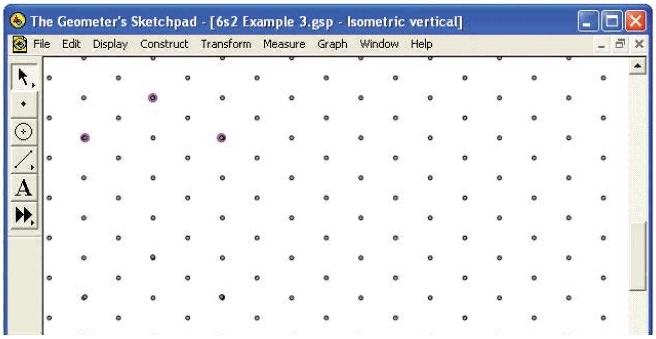
Isometric Drawings With Curved Edges

Consider a concrete model of a cylinder, such as a juice or soup can. Use *The Geometer's Sketchpad*® to sketch an isometric perspective drawing of the can.

Solution

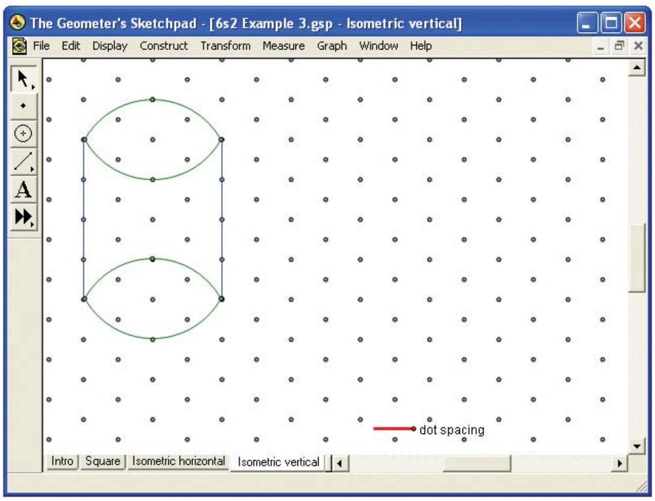
Curved edges are represented by arcs of circles in isometric perspective drawings.

- Open *The Geometer's Sketchpad*®. Open the file Dot Paper.gsp. Press the button **Isometric vertical**.
- Use the **Point Tool** to draw three points just to the left of three of the points on the paper, as shown.



Technology Tip
The dots in the dot paper sketch in *The Geometer's Sketchpad*® are drawn using transformations. If you try to draw an arc through three of these dots, it will not work. Try it!

- Select the three points. Select **Arc Through 3 Points** from the **Construct** menu. Drag the arc slightly to the right such that it falls on three points on the dot paper. In a similar manner, construct three more arcs. Then, use the **Segment** tool to complete the sides of the cylinder, as shown.



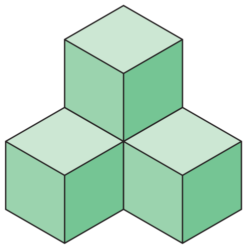
Note: You can also draw an isometric perspective drawing of a cylinder on paper using a ruler and a compass.

Key Concepts

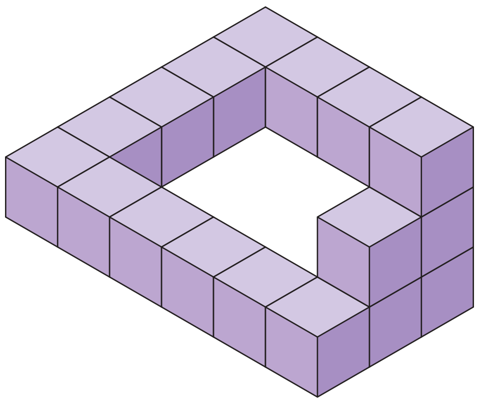
- A three-dimensional object can be represented in three ways.
  - as a scale model using concrete materials
  - as an isometric perspective drawing
  - as a set of orthographic drawings (usually the front view, side view, and top view)

Discuss the Concepts

D1. Consider the isometric perspective drawing shown. Indira claims that all three orthographic drawings of this object will look the same. Laszlo says that only the front and the side will look the same. Soon-Tek says that all three will be different. Who is correct? Give reasons.



D2. Consider the isometric perspective drawing shown. Discuss whether the drawing could represent a real object. Give reasons for your answer.



Practise A

1. Which of these representations is not suitable for representing a three-dimensional object? Explain why.

A a scale model

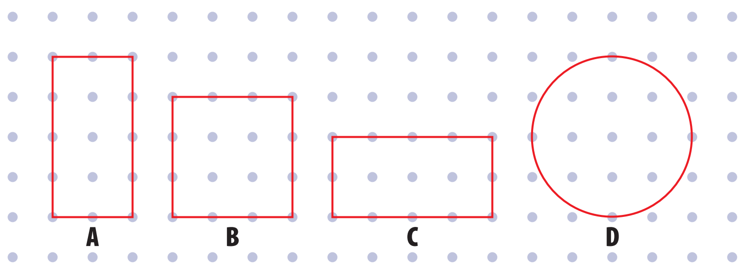
B an isometric perspective drawing

C a blueprint showing front and side views

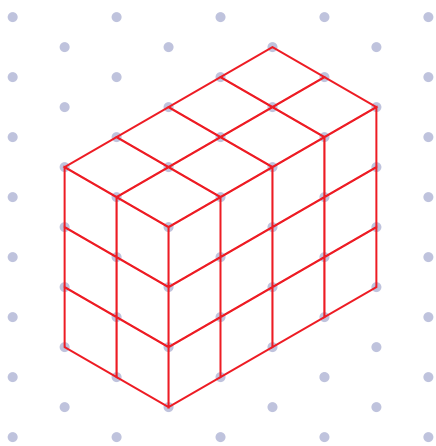
D a set of orthographic drawings

2. Marcie is building a scale model of an office building using linking cubes. The building is a rectangular prism with a length of 49 m, a width of 35 m, and a height of 56 m. Suggest a suitable scale, and calculate the dimensions for the model.

3. Farmer Jacques is building a new silo in the shape of a cylinder. To maximize the storage volume, he is planning to make the diameter the same as the height. Which of these diagrams matches the side view?



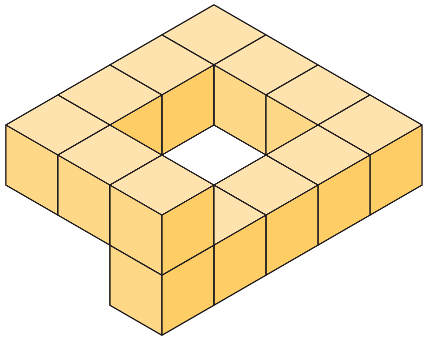
4. Ivanka is using linking cubes to make a scale model of the isometric perspective drawing shown. Describe the physical model in terms of length, width, and height.



Apply B

Literacy Connect

5. Study the isometric perspective drawing shown. Write a short paragraph explaining why this drawing cannot represent a real object.



For help with question 6, refer to Example 1.

6. Central courtyards are featured in the architecture of many cultures. The Romans often built their houses, or villas, with a central open space surrounded by the rooms of the house.

a) Use linking cubes or other concrete materials to build a scale model of a Roman villa. Include a set of rooms surrounding a central open space.

b) Use isometric dot paper or *The Geometer's Sketchpad*® to create an isometric perspective drawing of the model.

c) Use isometric dot paper or *The Geometer's Sketchpad*® to create a set of orthographic drawings for the model.

For help with question 7, refer to Examples 2 and 3.

7. Salvatore needs an isometric perspective drawing of a cone such that the height of the cone appears equal to the diameter.

a) Use isometric dot paper or *The Geometer's Sketchpad*® to create an isometric perspective drawing of a cone that closely matches Salvatore's requirements.

b) Is it possible to match the height to the diameter exactly, if the diameter and the tip of the cone must land on one of the dots? Explain.

8. The artist M. C. Escher often used false perspective in his drawings to depict impossible situations. Study the Escher work *Convex and Concave* carefully to see how the illusions work. Consider the "stairway" at the centre right of the drawing.

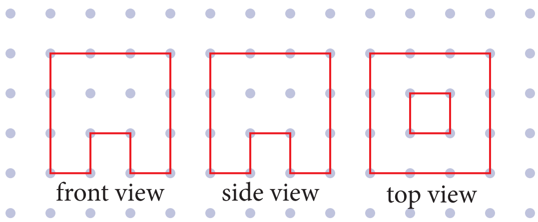
a) Approximate the stairway with a triangle (i.e., do not show the individual steps). Create a set of orthographic drawings in which you see this as a stairway going up.

b) Create a set of orthographic drawings in which you see this as the bottom support of an arch.

c) Describe the triangles that occur in your orthographic drawings.

Chapter Problem

9. Paul has been given a set of orthographic drawings for an inn, to be built near a tropical beach. The design incorporates open spaces throughout the building for air circulation. Paul's task is to create a scale model and an isometric perspective drawing of the inn.

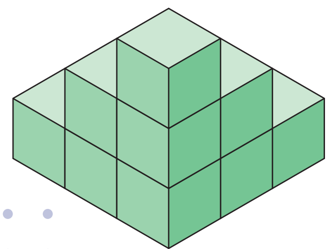


a) Use the orthographic drawings to construct a scale model using linking cubes or other concrete materials.

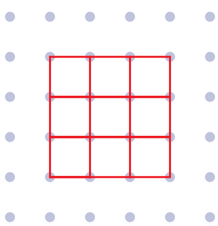
b) Use isometric dot paper or *The Geometer's Sketchpad*® to make an isometric perspective drawing of the inn.

Achievement Check

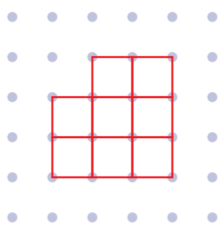
10. Sara used linking cubes to build the structure shown in isometric perspective. Which drawing is a possible orthographic projection of the top view of the structure? Explain.



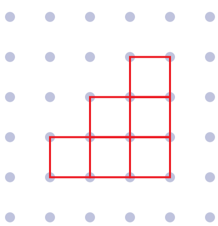
A



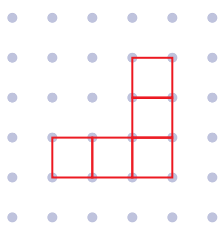
B



C



D

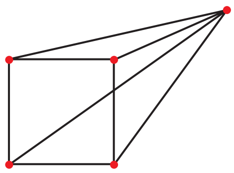


Extend C

11. Use isometric dot paper or *The Geometer's Sketchpad*® to create an "impossible" isometric perspective drawing different from the ones in question D2 and question 5.

12. In isometric perspective drawings the lengths in the drawing of a cube match the side lengths of the real cube. In real life, the apparent size of an object decreases as the object moves farther away from an observer. Artists use the concept of a "vanishing point" to achieve the effect of perspective in their art.

To see how this works, draw a square using square dot paper or *The Geometer's Sketchpad*®. Select a point far away from the square, as shown. Join the vertices of the square to this "vanishing point."



Use these lines to draw the rest of the cube in perspective. Then, hide or erase the parts of the lines that extend outside of the cube.

13. Advertisements for buildings that have not yet been constructed often make use of two-point perspective. In the example shown, each street leads to a vanishing point. The left side view of the building uses the left vanishing point, while the right side view uses the right vanishing point.

Use square dot paper or *The Geometer's Sketchpad*® to make a two-point perspective drawing of the model in question 4.

6.3 Create Nets, Plans, and Patterns

net
- a two-dimensional diagram that can be cut out, and folded to form a three-dimensional object

plan
- a scale drawing of a structure or object
- a design or arrangement scheme

pattern
- a form, template, or model from which an object can be created

Commercial products are packaged in various shapes of containers. These containers are often manufactured as a two-dimensional **net** and then folded or bent into a three-dimensional shape. One example is a cereal box, which is a rectangular prism.

Other three-dimensional objects, such as houses or items of furniture, are constructed from a **plan**. Plans are usually parts of nets, drawn to scale, that give enough information to construct the three-dimensional object.

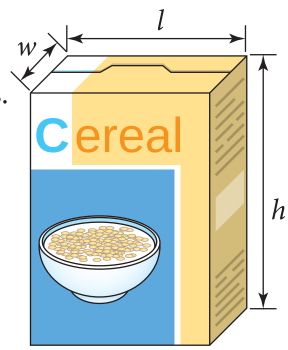
Clothing is made by using a **pattern** to cut the cloth. The cloth pieces are then sewn together to make the clothing item. A pattern is usually a net that is drawn in separate pieces.

Investigate 1

Tools
- empty cereal box
- scissors
- ruler
- tape

Design a Box

Many products are packaged in rectangular prisms.



1. Measure and record the dimensions of the cereal box.

Math Connect

The first of the modern packaged cereals was toasted corn flakes, introduced in 1906. To learn more about packaging, go to *http://www.mcgrawhill.ca/links/foundations11* and follow the links.

2. Find the edge on the cereal box that was used to join the cardboard. Cut along this edge. Cut any other joins that you find.

3. Unfold the cereal box to form a net. Measure the edges of the net. Identify each edge as one of the dimensions of the box.

4. **Reflect** Compare the measures of the net to the measures of the cereal box. How are the dimensions of the net found to produce a box of the desired shape and size?

5. Sketch a net for a box that will measure 4 cm by 3 cm by 2 cm. Label the edges. Test your net by cutting it out and folding it into a box. Tape the box together. Measure the dimensions to check that the box matches the dimensions given. Suggest a product that would be suitable for a package of this size.

Investigate 2

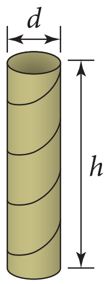
Tools

- empty tissue roll
- ruler
- tape

Design a Cylinder

Some products are packaged in cylindrical containers.

1. Measure and record the height and diameter of the tissue roll.



2. Cut the roll along its height, and spread it into a rectangle. Trace the rectangle onto a piece of paper. Measure the dimensions of the rectangle. Which dimension is equal to the height of the roll? What is the name of the other dimension in relation to the cylinder? Divide the other dimension by the diameter of the tissue roll. What do you notice?

3. To draw two congruent circles for the top and bottom of the cylinder, what dimension do you need to consider? Draw two circles for the top and bottom using this dimension. Cut out the net, and bend it to form a cylinder with a top and a bottom.

4. **Reflect** Compare the measures of the net to the measures of the cylinder. How are the dimensions of the net related to the height and diameter of the cylinder?

5. Sketch a net that can be bent into a cylinder with a diameter of 6 cm and a height of 10 cm. Test your net by cutting it out and bending it into a cylinder with a top and a bottom. Tape the model together.

Investigate 3

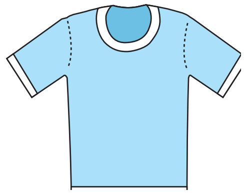
Tools

- simple item of clothing (old T-shirt, etc.)
- scissors
- newsprint

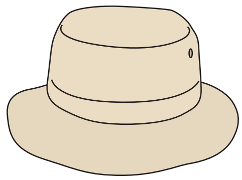
Patterns

Patterns for clothing are nets that are used to cut cloth. The pieces of cloth are sewn to form the three-dimensional garment.

1. Identify the seams on the article of clothing. Use scissors to carefully cut the threads along each seam. Lay out the pieces.



2. Sketch the pattern on newsprint.
3. **Reflect** How can clothing manufacturers use a pattern to cut out hundreds of pieces at one time? Research clothing manufacturing on the Internet to help you explain.
4. Sketch a pattern that could be used to cut out the pieces for a bucket hat.



Example

Cabin Plan

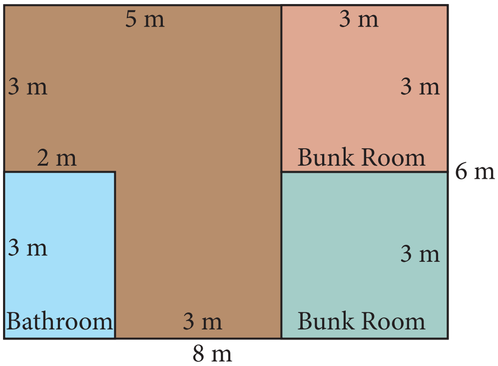
a) Draw a floor plan for a vacation cabin. The cabin will measure 8 m along the side by 6 m across the front. There will be two congruent square bunkrooms at the back, with side length 3 m. There will be a bathroom measuring 2 m by 3 m along the front wall. The remaining space will be living area. Label the measurements on your plan.

b) The cabin will be of the A-frame type, measuring 5 m tall.

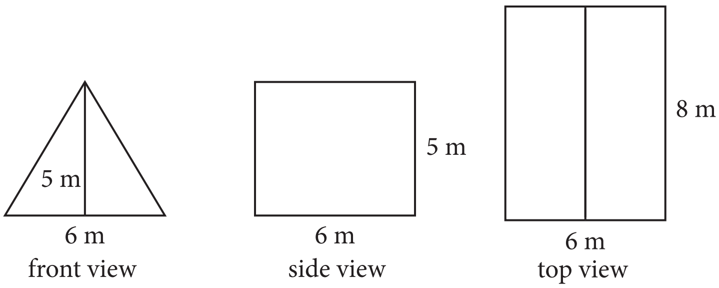
Sketch orthographic drawings of the cabin. Label all measurements.

Solution

a) Select a suitable scale. For example, let 1 cm represent 1 m. Draw a rectangle to represent the floor of the cabin. Draw walls for the two bunkrooms and the bathroom. Label all dimensions.



b) Draw the front, side, and top views of the cabin.



Key Concepts

- A three-dimensional object can be constructed by folding or bending a two-dimensional net.
- Parts of a net drawn to scale can be used as a plan to construct a three-dimensional object.
- A net can be separated into pieces to form a pattern. The pattern can be used to make pieces of a three-dimensional object, which can be assembled into the object.

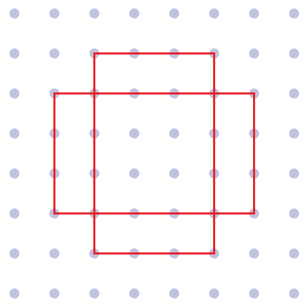
Discuss the Concepts

D1. Sketch a net for a cube. Explain how it must be folded to make the cube.

D2. A hobby club builds and flies model airplanes. To make such an airplane, would you use a net, a plan, or a pattern? Discuss the advantages and disadvantages of each representation.

Practise A

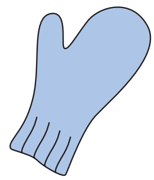
1. Consider the net shown. The distance between two dots represents 5 cm. When folded, what three-dimensional object will the net make?



   A an open box 5 cm high, with a square base with side length 15 cm
   B a closed box 5 cm high, with a square base with side length 15 cm
   C an open box 15 cm high, with a square base with side length 5 cm

2. Draw a net that can be used to make a cube-shaped box with an open top.

3. Sketch a pattern that could be used to make a mitten.



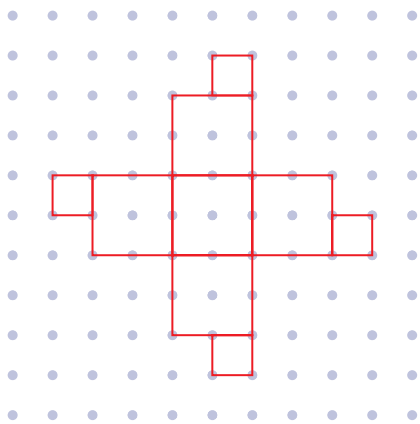
4. Woodworking magazines often include diagrams in their articles about building furniture. If you were writing an article about building a bookcase, would you include a net, a plan, or a pattern? Explain.

Apply B

For help with question 5, refer to Investigate 1.

Literacy Connect

5. Neela submitted an alternative net for a cube, as shown.



   a) Write a short paragraph explaining why Neela's net will not work.
   b) Explain how the net can be modified so it will work.

For help with question 6, refer to Investigate 2.

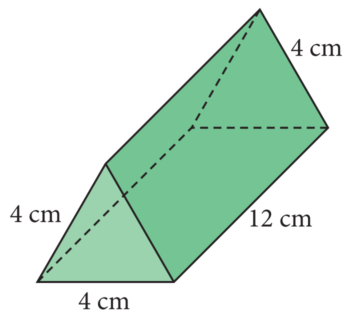
6. A bakery needs a new cylindrical package for their oatmeal cookies. Each cookie is 1 cm thick and has a diameter of 8 cm. Each package should hold a dozen cookies.

a) Draw a net that can be used to make a model of the package.

b) Cut out your net, and bend it to form the package. Tape the model together.

c) Check the dimensions of the model by measuring. Do they meet the requirements?

7. An imported cheese is sold in the shape of a triangular prism. The ends of the prism are congruent equilateral triangles. Each triangle has a side length of 4 cm, and the prism is 12 cm long.



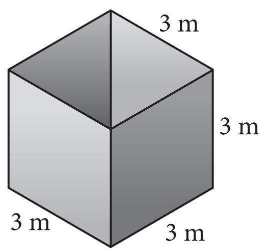
a) Draw a net that can be used to make a model of the package.

b) Cut out your net, and bend it to form the model. Tape the model together.

c) Check the dimensions of the model by measuring. Do they meet the requirements?

Chapter Problem

8. Paul's company has been asked to design a grain storage shed that will be exported to developing countries. The shed will be made of galvanized sheet metal with four congruent square walls, with side length 3 m. The roof will be four congruent equilateral triangles with side length 3 m. The bottom will be left open so that the shed can be attached to a poured concrete base. The sheds will be shipped as nets to conserve shipping space. Paul's task is to design the net for the shed, and to estimate the cost of the metal required.

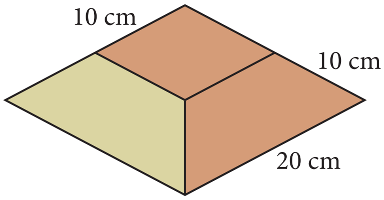


a) Draw a net that can be used to make a model of the shed.

b) Find the total area of sheet metal required.

c) Sheet metal costs $\$6.50/\text{m}^2$. Find the cost of the metal for the shed.

9. The owner of a garden centre needs a pedestal for displaying a prize-winning flowering plant. The pedestal is to be made of painted sheet metal, with dimensions as shown. The bottom of the pedestal will be left open. Draw a net that could be used to make the pedestal. Show all measurements.



Achievement Check

10. Refer to question 9. The pedestal will be modified. Now, the top will have a circular hole to accommodate a 10-cm diameter metal cylinder. The height of the cylinder will be 30 cm. Draw the net for the modified pedestal, and the net for the cylinder. Show all measurements.

Extend C

11. A regular tetrahedron is a three-dimensional shape with four congruent equilateral triangular faces. Consider a tetrahedron made of equilateral triangles with side length 3 cm.
   a) Draw a net that can be used to make the tetrahedron.
   b) Cut out your net, and fold it into a tetrahedron. Tape the tetrahedron together.

12. Lydia made a square shape using eight linking cubes, leaving the centre space empty. Draw a net that could be used to make a paper model of the shape. If you think it is not possible to make a net, explain why.

13. A regular octahedron has eight congruent equilateral triangular faces. It looks like two square-based pyramids glued together at their bases. Draw a net that can be used to make a regular octahedron. Cut out your net, and fold it into an octahedron. Tape the octohedron together.

Use Technology

Tools

- computer
- *The Geometer's Sketchpad®*

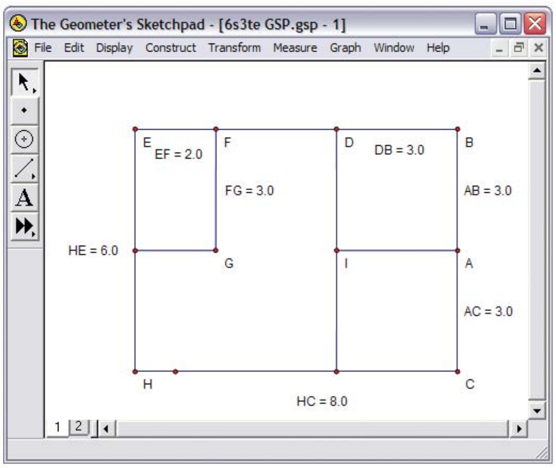
Use *The Geometer's Sketchpad®* to Draw Plans With Dynamic Measurements

Use *The Geometer's Sketchpad®* to draw a scale floor plan for the vacation cabin in Example 1 of section 6.3. The cabin will measure 8 m along the side by 6 m across the front. There will be two congruent square bunk rooms at the back, with side length 3 m. There will be a bathroom measuring 2 m by 3 m along the front wall. The remaining space will be living area.

1. Start by constructing the fixed boundaries of the cabin. Open *The Geometer's Sketchpad®*. Choose **Preferences** from the **Edit** menu, and then the **Units** tab. Choose **Show Grid** from the **Graph** menu. Drag the origin to the lower left area of the workspace. If necessary, adjust the unit point until you can see at least 10 gridlines horizontally and 8 gridlines vertically.

2. Choose **Plot Points** from the **Graph** menu. Plot the point (8, 6). In the same way, plot the points (0, 6) and (8, 0). Select the origin and the three plotted points, moving clockwise. Choose **Segments** from the **Construct** menu. You now have a rectangle that represents the boundaries of the cabin. Try to move one of the corners. What happens?

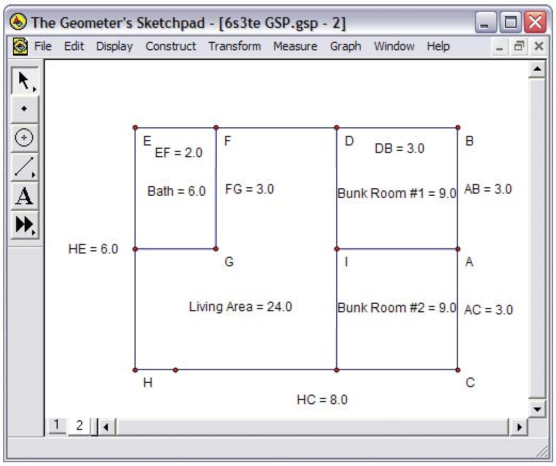
3. Use the **Segment Tool** to sketch the bunk rooms and the bathroom. Select the endpoints of the width of a bunk room. Choose **Coordinate Distance** from the **Graph** menu. Move the measurement to a convenient location. Measure and label dimensions for remaining the rooms. Include the outside dimensions of the cabin. Hide the axes and the grid.



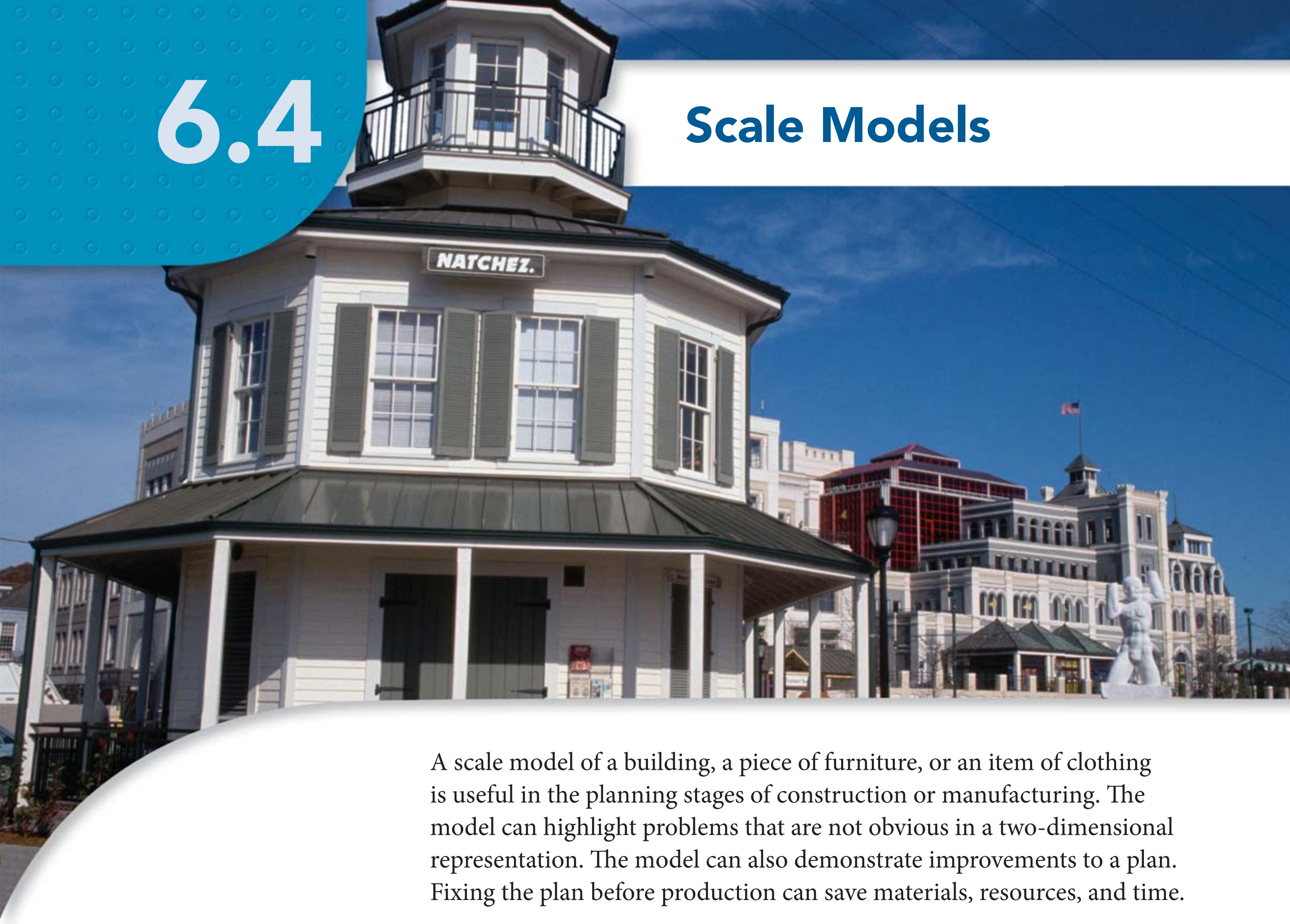
4. Reflect Select the line segment that forms the wall between the bunk rooms. Move it up and down. Note how the measurements change as you move the line segment. Try changing the dimensions of the other rooms.

5. Calculate the area of each room. Choose **Calculate** from the **Measure** menu. A calculator will appear. Select the length of one bunk room. Press the * button. Select the width of the bunk room. Move the area measurement to a convenient location. Change the label to Bunk Room #1. Measure the areas of the other rooms. To find the living area, calculate the area of the entire cabin, and subtract the areas of the rooms.

6. Reflect Change the dimensions of one room by dragging a wall. Notice how the measurements, including the areas, change.



6.4 Scale Models



A scale model of a building, a piece of furniture, or an item of clothing is useful in the planning stages of construction or manufacturing. The model can highlight problems that are not obvious in a two-dimensional representation. The model can also demonstrate improvements to a plan. Fixing the plan before production can save materials, resources, and time.

Example 1

Design a Cylindrical Tank

An engineer is designing a cylindrical tank for a tanker truck that will hold about 800 ft^3 of liquid. The length of the cylinder should be twice the diameter and should be a whole number of feet.

a) Use the formula for the volume of a cylinder to determine the dimensions of the tank required.

b) Select a suitable scale, and draw a net for a paper model of the tank.

c) Cut out your net, and bend it into the cylindrical shape. Tape the model together.

Solution

a) The formula for the volume of a cylinder is $V = \pi r^2 h$. In this case h is twice the diameter, or four times the radius.

The volume formula becomes

$$V = \pi r^2(4r)$$
$$= 4\pi r^3$$

Try different values for the radius.

Radius (ft)	Volume (ft^3)
1	12.6
2	100.5
3	339.3
4	804.2
5	1570.8

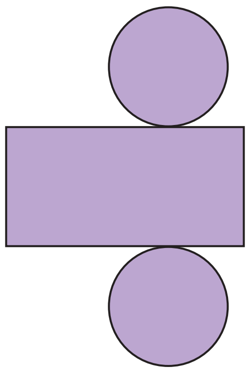
A radius of 4 ft gives a volume of just over 800 ft^3. Using a radius of 4 ft, the diameter is 2×4 ft, or 8 ft, and the length is 2×8 ft, or 16 ft.

b) One dimension of the rectangle for the net is the length of the cylinder, or 16 ft. The other dimension is the circumference of the cylinder.

$$C = \pi d$$
$$= \pi(8)$$
$$\doteq 25.1$$

The circumference is approximately 25.1 ft.

Select a suitable scale (for example, 1 in. represents 4 ft). Draw the rectangle for the with a net. Add two circles, each diameter of 2 in., for the ends.



c)

Example 2

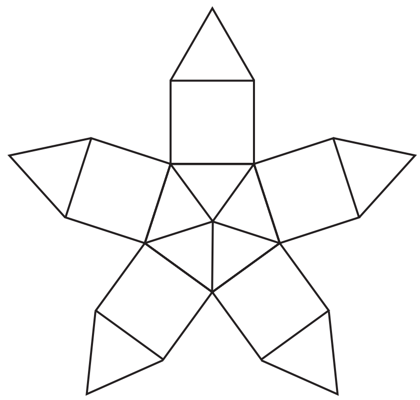
Design a Pentagonal House

An architect has designed a house in the shape of a regular pentagonal prism. The floor forms a pentagon with a side length of 6 m. The five walls are congruent squares. The peaked roof is made of five congruent equilateral triangles. Select a suitable scale. Draw a net that can be used to make a scale model of this house. Then, cut out your net and fold it into a model of the house. Tape the model together.

Solution

Method 1: Use Pencil and Paper

- Choose an appropriate scale. For example, 1 cm represents 1 m. Draw a regular pentagon. A regular pentagon consists of five congruent isosceles triangles whose apexes meet at the centre of the pentagon. The apex angle for each triangle is $360° \div 5 = 72°$. Draw a dot for the centre, and a horizontal line segment to the first vertex. Draw an angle of 72° from the horizontal line segment. Use compasses to help you create an isosceles triangle that includes the second vertex. Continue this process until you have 5 vertices. Join the vertices.
- Use a ruler and compasses to draw a square on each side of the pentagon. Then, draw an equilateral triangle on each square.
- Use your net to make a scale model of the house.



Method 2: Use *The Geometer's Sketchpad*®

- Draw the pentagonal base. Turn on automatic labelling of points. Sketch a horizontal line segment AB. Select point A, and choose **Mark Center** from the **Transform** menu. Select the segment and point B. Choose **Rotate** from the **Transform** menu. Change the fixed angle to 72°. Select **Rotate**. Continue until you have five vertices. Join the vertices.
- Draw the square walls. Select one endpoint of a side. Choose **Mark Center** from the **Transform** menu. Select the segment and the other endpoint. Choose **Rotate** from the **Transform** menu. Change the fixed angle to 90°. Select **Rotate**. Repeat for each side. Then, use the other

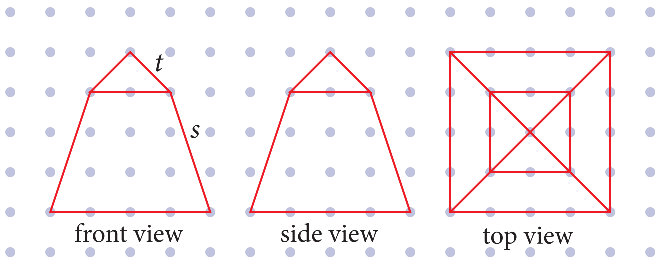
endpoint of the side as the centre, and rotate through an angle of 270°. Complete the five squares.

- Draw the triangular roof faces. Select one outside corner of a square. Choose **Mark Center** from the **Transform** menu. Select the outside segment and the other endpoint. Choose **Rotate** from the **Transform** menu. Change the fixed angle to 60°. Select **Rotate**. Repeat for the other squares. Complete the five triangles. Hide the points and labels.
- Print your net, and use it to make a scale model of the house.

Example 3

Design a Tent

An outfitting company plans to manufacture a new tent. Orthographic drawings for the tent are shown. The distance between pairs of horizontal or vertical dots represents 1 m.



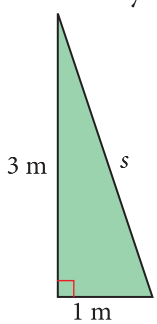
Use paper or drinking straws and masking tape to construct a scale model of the tent. Select a suitable scale. Make any necessary calculations.

Solution

Let s represent the slant length of the side of the tent and t represent the slant length of the roof. Use the Pythagorean theorem to find the lengths of s and t.

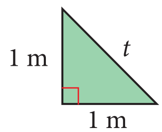
$s^2 = 3^2 + 1^2$
$s^2 = 10$
$s = \sqrt{10}$
$s \doteq 3.2$

The side slant length is approximately 3.2 m.



$t^2 = 1^2 + 1^2$
$t^2 = 2$
$t = \sqrt{2}$
$t \doteq 1.4$

The top slant length is approximately 1.4 m.



Select a scale that is suitable for your materials. For example, let 4 cm represent 1 m. The base of the model will measure 4 × 4 cm, or 16 cm. The side slant length will measure 4 × 3.2 cm, or 12.8 cm. The top slant length will measure 4 × 1.4 cm, or 5.6 cm.

Key Concepts

- Scale models can be constructed using nets, orthographic drawings, or isometric perspective drawings.
- Models are useful in the planning stages of a project.
- Models can identify problems in a plan, or suggest improvements.

Discuss the Concepts

D1. Can a model of any three-dimensional object be constructed using a net? Explain.

D2. To obtain a building permit for a house, the owner must submit a set of orthographic drawings, known as elevations. Suggest reasons why orthographic drawing are required rather than a scale model or an isometric perspective drawing.

Practise A

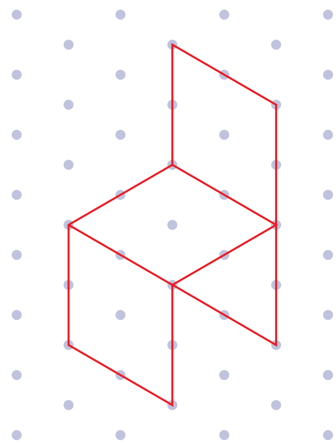
1. Maria is building a scale model of a garden shed. She will let 1 in. represent 2 ft. If the base of the shed measures 10 ft by 12 ft, what measurements will Maria need for the model?

2. The most famous Canadian-designed and -built aircraft was the Avro CF-105 Arrow. A number of $\frac{1}{8}$ scale models were made for testing in a wind tunnel. The length of the model was 10 ft 8 in. What was the length of the full-size aircraft?

3. The town of Port Colborne is planning to construct a new recreational centre, including a skating arena and a swimming pool. Suggest reasons why an architect bidding on the contract would provide a scale model in addition to drawings.

4. Freda is planning a new silo for her farm, made of a cylinder topped with a hemisphere. The height of the cylinder will form a golden ratio with its diameter. Freda would like to make a scale model of the silo to see if her design works. Describe a procedure and suggest materials that Freda could use to make the scale model.

5. Suleiman plans to construct a "minimalist" chair as shown in the isometric perspective drawing. He will use three pieces of 1 in.-thick plywood. The distance between a pair of dots represents 1 ft. Draw plans that show the pieces of wood required, with measurements.



Apply B

Literacy Connect

6. A standard golf ball has a diameter of 1.68 in. You have been contracted to design a package to hold 12 golf balls. Sketch a set of orthographic drawings for your chosen package. Write a brief explanation of why you selected this design.

7. Draw an isometric perspective drawing for the tent in Example 3.

8. A local artist wants to include a rectangular prism in her newest sculpture. The prism should have a length of 1 m, a width of 2 m, and a height of 3 m.

a) Select a suitable scale. Use pencil and paper or *The Geometer's Sketchpad*® to draw a net that can be used to make a scale model of the prism.

b) Cut out your net and fold it to make the model. Tape the model together.

c) The artist has decided to paint all of the sides of the prism except one, which will be placed on the ground. She has not, however, decided which side will be placed on the ground. If paint costs $\$2/m^2$, what are the possible costs for painting the prism?

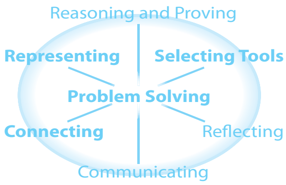
For help with question 8, refer to Example 1.

9. A new soup can must hold at least 500 mL of soup. Its height must be equal to its diameter. The height and the diameter must be whole numbers of centimetres.

a) Determine the minimum height and width of the can.

b) Select a suitable scale. Draw a net that can be used to make a scale model of the can.

c) Cut out your net and bend it to make a scale model of the can. Tape the model together.



For help with question 9, refer to Example 2.

10. A band council has decided to commission a new community centre to be used for various events such as the annual pow-wow. The centre will be in the shape of a regular hexagon with each side measuring 10 m. Each congruent rectangular wall will be 3 m high. The building will have a flat roof.

a) Select a suitable scale. Choose measurements that ensure you have a regular hexagon. Use pencil and paper or *The Geometer's Sketchpad*® to draw a net that can be used to make a scale model of the centre.

b) Cut out your net and fold it to make the model. Tape the model together.

c) The base of the centre will be poured concrete, 10 cm thick. Take measurements from your net to find the area of the hexagonal base. Multiply the area by the thickness to determine how many cubic metres of concrete will be needed.

d) Concrete costs $75/m^3, delivered to the site. Estimate the cost of the concrete for the base.

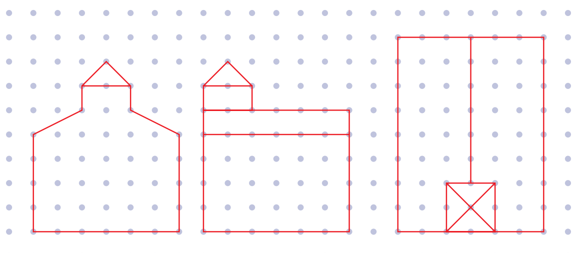
11. After studying the scale model in question 10, the band council decided to change the flat roof to one made of six congruent isosceles triangles. This will ensure that snow and rain will not accumulate on the roof. It will also make the building more attractive. The central peak of the roof will be 5 m above the floor.

a) How will this change affect your net? Explain.

b) How will this change affect the cost of concrete for the base? Explain.

Chapter Problem

12. A customer has seen the post office in Gorrie, ON, and would like a similar building for his business. He has supplied Paul's company with sample orthographic drawings, as shown. The distance between pairs of horizontal or vertical dots represents 2 m.



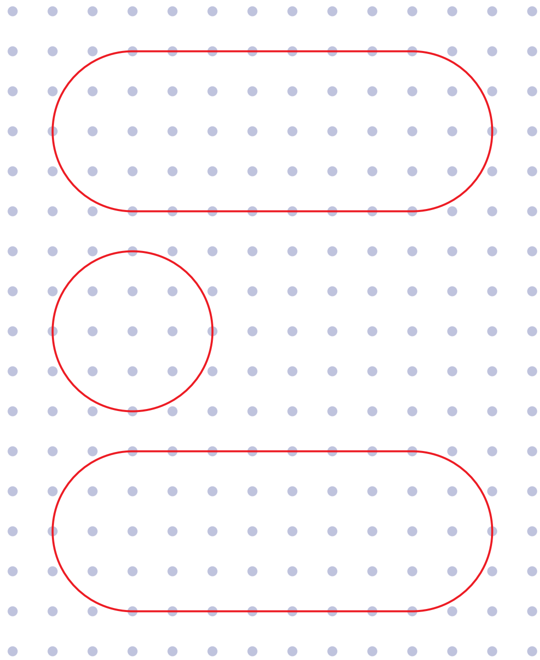
a) Draw a net that can be used to make a scale model of the building.

b) Cut out the net, and construct the model. Tape the model together.

c) The cost of the building is estimated to be $1200/m^2 of floor area. Estimate the cost of the building.

Achievement Check

13. A propane storage tank is to be built from the orthographic drawings shown. The distance between pairs of horizontal or vertical dots represents 1 m.



Which description best fits the propane tank? Explain.

A A cylinder 11 m long, with a hemisphere of radius 2 m at each end.

B A cylinder 7 m long, with a hemisphere of radius 2 m at each end.

C A cylinder 11 m long, with a hemisphere of radius 4 m at each end.

D A cylinder 7 m long, with a hemisphere of radius 4 m at each end.

Extend C

14. How can you make a net for a cone? Roll a piece of notebook paper to form a horn. Cut the horn to form a cone. Continue cutting until you have the minimum amount of paper necessary to form the cone. Unroll the paper, and sketch the net.

15. Use trial and error to make a net for a cone with a radius of 5 cm and a height of 3 cm. Cut out your net and use it to make the cone. Measure the dimensions to check that they are correct.

6.5 Solve Problems With Given Constraints

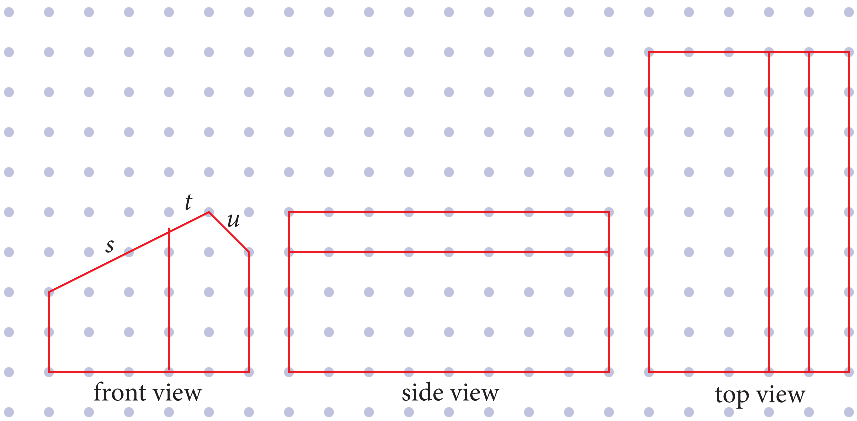
constraint
- a condition that limits the acceptable range of values for a variable

Designers such as architects, engineers, or fashion designers often must ensure their designs satisfy given **constraints**. Examples of constraints include a maximum cost, a minimum or maximum size, or regulations governing safety, energy efficiency, or performance.

Example 1

Modular House

Orthographic drawings for a modular house are shown.



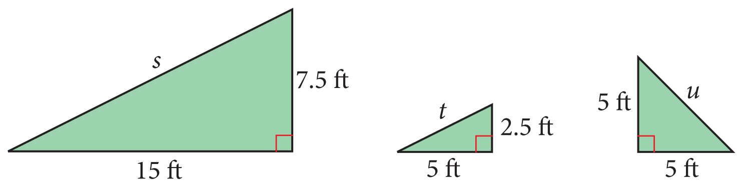
The distance between pairs of vertical or horizontal dots represents 5 ft. The house will be built in two sections and then transported to the building site for assembly. Highway traffic rules limit the width of each section to 15 ft or less. The centre wall will be located 15 ft from the left front wall. It will be doubly thick, so that each half of the house will have four walls during transport.

a) Select a suitable scale. Draw a net for each section of the house.

b) Cut out your nets, and make scale models for the two sections of the house. Tape your models together. Ensure that the models fit together properly for assembly.

Solution

a) From the orthographic drawings, the centre wall is 17.5 ft high. Use the Pythagorean theorem to determine the slant lengths of the roof.



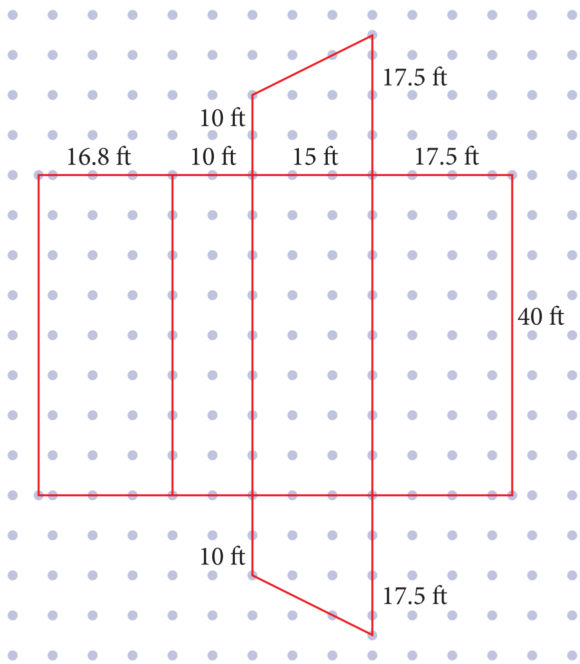
$s^2 = 15^2 + 7.5^2$
$s^2 = 281.25$
$s \doteq 16.8$
Slant length s is approximately 16.8 ft.

$t^2 = 5^2 + 2.5^2$
$t^2 = 31.25$
$t \doteq 5.6$
Slant length t is approximately 5.6 ft.

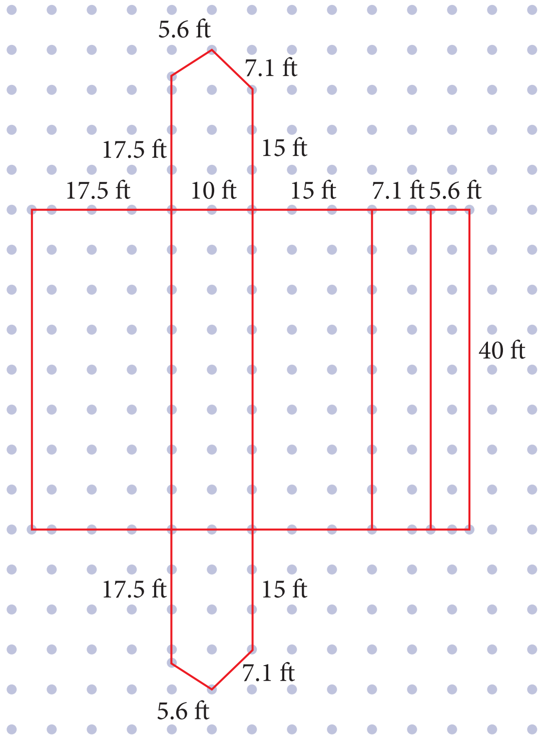
$u^2 = 5^2 + 5^2$
$u^2 = 50$
$u \doteq 7.1$
Slant length u is approximately 7.1 ft.

Let the distance between pair of horizontal or vertical dots represent 5 ft. Calculate the measures for the scale drawing by dividing each actual measure by 5.

The net for the left side of the house.



The net for the right side of the house.



b) Use the nets to make scale models of the two halves of the house.

To learn more about modular homes, go to *www.mcgrawhill.ca/links/foundations11* and follow the links.

Example 2

Oil Tank

Sometimes, basic safety considerations depend on understanding nets and volumes. Consider an engineer who needs to design a square-based berm, a shallow container to prevent the spread of oil from a leaking oil tank. The cylindrical oil tank is 20 m in diameter, and has a height of 20 m. The berm must have a height of 5 m.

a) Make appropriate calculations, and find the minimum dimensions of the berm.

b) Select a suitable scale. Draw a net for the berm and a net for the oil tank. Use your nets to construct a scale model of the arrangement required.

Solution

a) Start by calculating the maximum volume of oil that must be contained if the tank is full.

$$V = \pi r^2 h$$
$$= \pi \times 10^2 \times 20$$
$$\doteq 6283$$

The berm must be able to hold 6283 m^3 of oil.

Math Connect

A square-based berm has the shortest side lengths required to contain a given volume. Minimizing the berm's perimeter cuts down on the cost of materials.

Let the side length of the square base of the berm be represented by s.

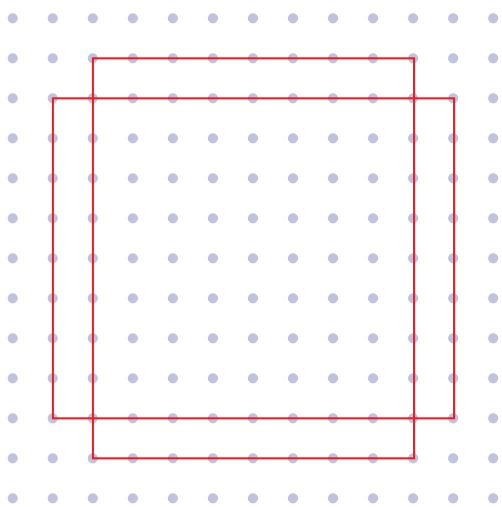
Volume of berm = area of base × height

$$= s^2 \times 5$$
$$= 5s^2$$

$$5s^2 = 6283$$
$$s^2 = \frac{6283}{5}$$
$$s^2 = 1256.6$$
$$s = \sqrt{1256.6}$$
$$s \doteq 35.4$$

It is safer to overestimate. Round 35.4 m up to 40 m. Suitable dimensions for the berm would be 40 m by 40 m by 5 m.

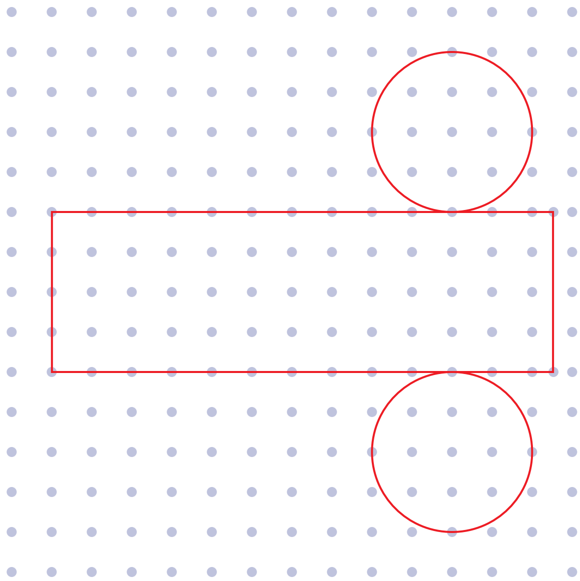
b) Draw a net for the berm. Let the space between pairs of horizontal or vertical dots represents 5 m. The base of the berm measures 8 dots by 8 dots, and the height measures 1 dot.



Draw a net for the oil tank. The net consists of two circles of diameter 20 m for the top and bottom, plus a rectangle for the wall of the cylinder. One dimension of the rectangle is the same as the height of the tank, or 20 m. The other dimension is the same as the circumference of the tank.

$$\begin{aligned} C &= \pi d \\ &= \pi \times 20 \\ &\doteq 63 \end{aligned}$$

The circumference is approximately 63 m.



Use the nets to construct the scale models for the berm and the oil tank.

Key Concepts

- Design problems are often subject to constraints or conditions.
- Constraints may include cost considerations, safety regulations, maximum or minimum sizes, or performance requirements.

Discuss the Concepts

D1. A fashion designer is to design a new line of sunglasses to complement a clothing line. Discuss some of the constraints that she might face during the design stage.

D2. The highest mountain in Africa is Mt. Kilimanjaro in Tanzania. On the Marangu hiking route to the top, climbers spend the night in huts designed by Norwegian architects. There is no road. The trail is the only route to the huts.

Discuss some of the constraints that the architects faced when designing the huts.

Practise A

1. A new laundry detergent will be sold in a cubical box. The box holds 1 L of detergent. Find the side length of the box.

2. A factory temporarily stores liquid wastes in a circular cement-lined pond that has a base area of 500 m^2. Find the length of berm needed to enclose the pond.

3. Santo's Pizzeria is introducing a new giant-size pizza with a diameter of 24 in. Give possible reasons why the pizza will be packaged in an octagonal box rather than a square box.

4. Prefabricated housing for developing countries with warm climates is often made of galvanized sheet metal. Discuss some of the constraints that might affect the designers of this type of housing.

Apply B

5. A one-story bungalow is to be designed for a small lot. The floor plan of the house must fit into a rectangle measuring 30 ft by 40 ft. The buyer has made the following specifications.

Room	Minimum Floor Area (ft^2)
bedroom #1	200
bedroom #2	150
bedroom #3	100
bathroom	100
eat-in kitchen	200
living room	300

There must be a clear path to all rooms. Use pencil and paper or drawing software to design such a house. **Design Tip:** Lay out the rooms such that you minimize the area required for hallways.

6. A flexible floating pipe is used to contain an oil spill. The pipe can contain a depth of 50 cm of oil. It is placed around a leaking tanker carrying 200 000 m^3 of oil. The pipe is laid such that it forms a circle around the leaking tanker. What length of pipe is needed to contain all of the oil?

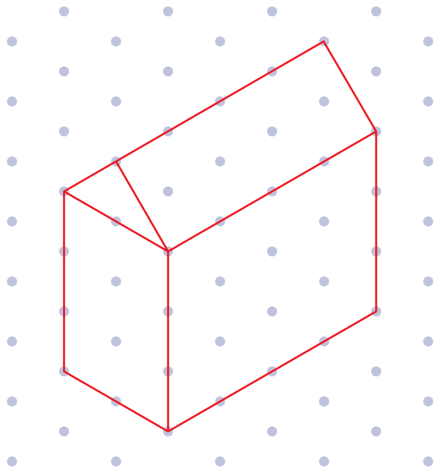
7. Build a boat using one piece of water-resistant cardboard or plastic the same size as notebook paper, up to six popsicle sticks, glue as required, and enough tape to join the edges. The boat must hold the maximum load possible without sinking. Test the boat using standard weights, coins, or other materials. Record your plan.

Math Connect

Many bridges use triangles in their construction. Triangles are rigid and add to the strength of the bridge.

8. Build a bridge using 12 plastic drinking straws, one sheet of notebook paper, and duct tape to join the straws. The bridge must span 15 cm and hold the maximum load possible. **Note:** The duct tape may be used only to join the straws. It may not be used to reinforce the bridge. Place the bridge over a gap of 15 cm, and test it with standard weights, coins, or other materials. Record your plan.

9. Farmer Macdonald sketched an isometric perspective drawing of a new chicken coop that he would like to build. The space between pairs of dots represents 1 m.

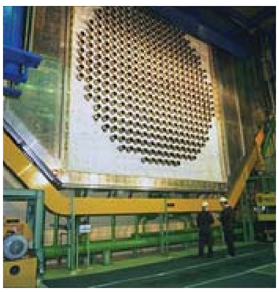


a) Draw a set of orthographic drawings for the coop.

b) Draw a net that can be used to make a scale model of the coop. Cut out the net, and construct the model. Tape the model together.

c) All sides of the coop except the floor will be made of sheet metal, which sells for $\$15/m^2$. Estimate the cost of buying the sheet metal for the coop.

Chapter Problem



10. The uranium fuel for a nuclear reactor is placed in a cylinder. The cylinder lies horizontally on its side with a diameter of 8 m and a length of 6 m. It must be surrounded by a containment building in the shape of a vertical cylinder. Paul has been given the task of designing the building and estimating the cost of the buildings concrete exterior.

a) Use pencil and paper or drawing software to determine the minimum dimensions of the containment building required.

b) Find the area of the walls and roof of the building. The concrete exterior must be 80 cm thick. Multiply the area by the thickness to determine the approximate volume of concrete required, in cubic metres.

c) Concrete can be delivered to the site and poured for $\$90/m^3$. Estimate the cost of the concrete for the building.

Achievement Check

11. Refer to the oil tank problem from Example 2. Hartmut suggests using a circular berm that is also 5 m high. He says that the circular berm would be smaller than the square berm. Is he correct? Show calculations to support your answer.

Extend C

12. The inside of a new high-pressure scuba tank has a volume of 1 ft^3. The walls, top, and bottom must be 0.5 in. thick to withstand the pressure. The ratio of height to diameter will be 3:1. Model the tank using a cylinder. Determine the dimensions required inside and outside. Prepare a set of orthographic drawings for the tank.

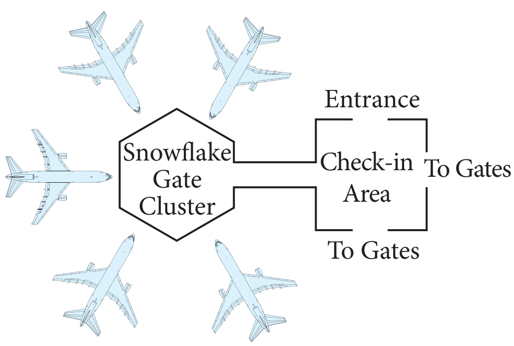
13. An electric furnace can produce a maximum of 12 000 W of heat. A concrete building requires a heat source of 10 W/m^2 on its exterior walls and roof.

a) Design a small office building in the shape of a rectangular prism that could be heated by this furnace, running at maximum. **Note:** For multiple storeys, the building code requires that each storey be a minimum of 3 m in height.

b) Construct a scale model of your building.

14. The layout for a new terminal at the international airport will use the "snowflake" design. Passengers will arrive through one face of a square-based reception building. Once they have checked in, they will follow a passageway through one of the other faces to a hexagonal "snowflake" gate cluster. Five of the faces will be gates for aircraft, while the sixth will house the passageway. The aircraft using these gates need a clear, square area with a side length of at least 50 m in front of the gate.

Use pencil and paper or drawing software to design the terminal. The passageways should be kept as short as possible to minimize the distance that passengers need to walk to get from one "snowflake" to another.



Use Technology

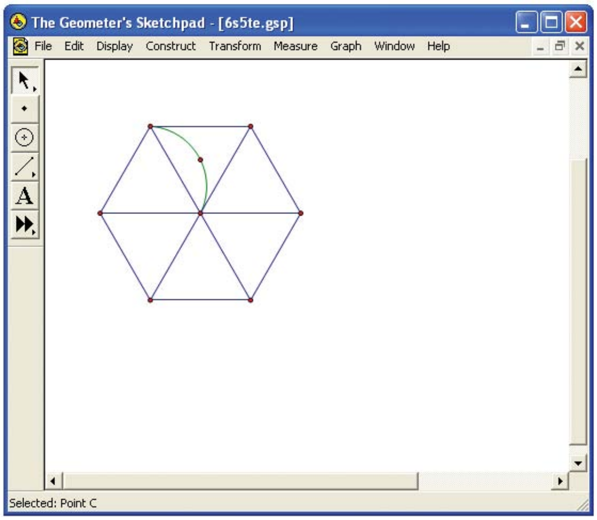
Use *The Geometer's Sketchpad®* to Create Tessellations Using Rotations

Tools

- computer
- *The Geometer's Sketchpad®*

Use *The Geometer's Sketchpad®* to draw a complex tessellation using rotations. Begin with a regular hexagon. Then, alter the interior of the hexagon using the **Transform** menu.

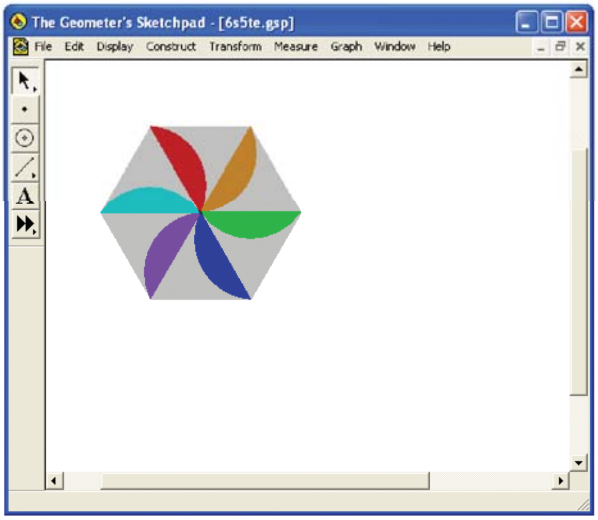
1. Open *The Geometer's Sketchpad®*. Use the method shown in Section 6.4, Example 2, Method 2, to draw a regular hexagon. All the angles in each triangle will be 60°. Draw a point approximately in the centre of one of the triangles that make up the hexagon. Select this point, as well as the centre and a vertex of the hexagon, as shown. Choose **Arc Through 3 Points** from the **Construct** menu.



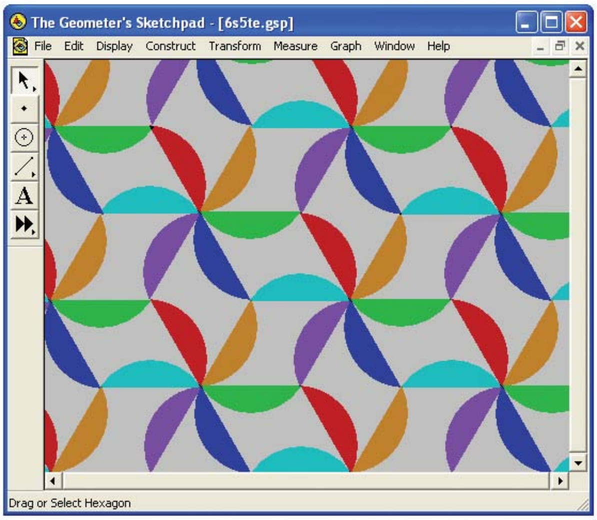
2. Select the centre of the hexagon. Choose **Mark Center** from the **Transform** menu. Select the arc. Choose **Rotate** from the **Transform** menu. Adjust the angle of rotation to 60°. Repeat the rotation until you have six arcs, one in each triangle.

3. Select the six vertices of the hexagon. Choose **Hexagon Interior** from the **Construct** menu. You can change the colour by choosing **Color** from the **Display** menu. Select one of the arcs. Choose **Arc Interior** from the **Construct** menu. Then, choose **Arc Segment**. While the arc interior is selected, change the colour by choosing **Color** from the **Display** menu. Repeat this procedure until you have six arc segments of different colours.

4. Hide the lines, the points, and the arcs.



5. Select the hexagonal pattern, and then choose **Copy** from the **Edit** menu. Choose **Paste** from the **Edit** menu. Move the copy beside the original pattern. Continue pasting copies until you have formed a tessellation.



6. **Reflect** Think of other ways to use rotations. For example, you can use a polygon in place of an arc. Consider other colour variations. What other regular polygons can be used for tessellations? What combinations of polygons can be used?

6 Review

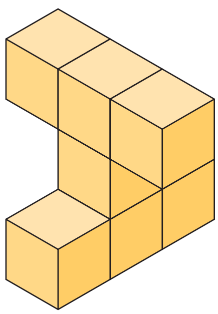
6.1 Investigate Geometric Shapes and Figures, pages XX–XX

1. After using the "12-knot rope" to lay out a square or rectangle on the ground, ancient builders then measured the two diagonals. Why did they do this? Why does this method work?

2. Basiruddin has sewn a blanket for his sister's new baby. It measures 130 cm by 80 cm. Basiruddin claims that the blanket is a golden rectangle, and that, if it is folded in half, the half-rectangle is also a golden rectangle. Use calculations to check his claims.

6.2 Perspective and Orthographic Drawings, pages XX–XX

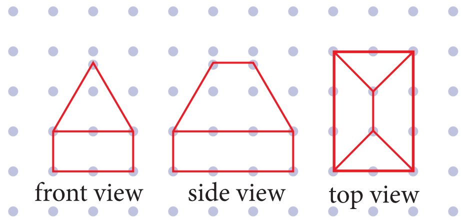
3. Shawna is designing an unusual seven-room house for a client. Each room will have the shape of a cube. Shawna has represented the basic plan as an isometric perspective drawing, as shown.



a) Use linking cubes or other materials to build a scale model of the house.

b) Draw a set of orthographic drawings for the house.

4. Jorge drew orthographic drawings of his family home in Spain, as shown. The distance between horizontal or vertical pairs of dots represents 2 m. Draw an isometric perspective drawing of the house.



6.3 Create Nets, Plans, and Patterns, pages XX–XX

5. You are designing a rondaval, a circular hut. The hut will have a diameter of 24 ft. In the centre will be a square washroom block with side length 8 ft. The hut's diameter will divide the washroom block and the hut into two units, each with a washroom and a bedroom. Select a suitable scale, and draw a floor plan for the rondaval. Include all measurements.

6. A new line of bath crystals will be marketed in a decorative cylindrical container with a diameter of 6 cm and a height of 18 cm. A cork will be used to seal the container, so no top is required.

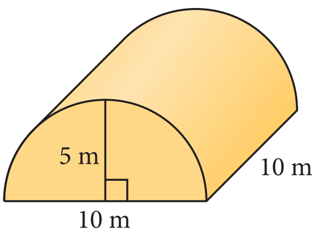
Draw a net that could be used to make a model of the container. Cut out your net and construct the container. Check the dimensions of the model by measuring.

7. David is building an airplane. The frame is made of welded steel tubing, and will be covered with fabric. Each wing is in the shape of a rectangle measuring 5 m long by 1 m wide, with a semicircle at one end. The wing will be covered with fabric. Sketch a pattern that could be used to sew an envelope to slip over the wing frame. Show all measurements.

6.4 Scale Models, pages XX–XX

8. Model airplane kits are commonly sold in $\frac{1}{72}$ scale. A Boeing 777 airliner has a wingspan of 60.9 m and a length of 73.9 m. Find the wingspan and length of a $\frac{1}{72}$ model of the Boeing 777.

9. Quonset huts are often used for storage of farm machinery, aircraft, or other large machines.



Sandrine is planning to build a Quonset hut to shelter the ultralight aircraft that she is building. It will have a square floor with side length 10 m. The curved roof will have a maximum height of 5 m. Draw a net that can be used to make a scale model of the Quonset hut. Cut out your net, and construct the scale model.

10. A new office building will have a square base with side length 40 m. It will be 60 m high. A revolving restaurant in the shape of a cylinder with a diameter of 80 m and a height of 4 m will be built on top. Draw nets that can be used to make scale models of the building and the restaurant. Cut out your nets, and construct the models.

6.5 Solve Problems With Given Constraints, pages XX–XX

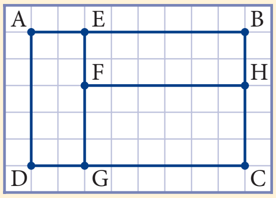
11. Martin is designing a hexagonal bunkhouse containing six triangular rooms that meet at the centre. The base of the bunkhouse will be poured concrete 10 cm thick. Concrete can be delivered to the site for $\$75/m^3$. The budget for the concrete base is a maximum of \$600. The walls will be congruent squares, and the roof will be flat. The cost of wood for the walls and the roof is $\$20/m^2$.

a) Design the base such that the hexagon is as large as possible without exceeding the cost constraint.

b) Continue the design for the walls and roof. Estimate the total cost of materials for the base, walls, and roof.

6 Practice Test

1. Which rectangle is closest to being a golden rectangle?

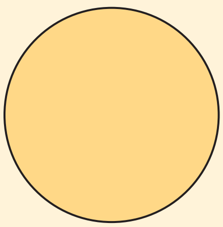


   A ABCD **B** AEGD
   C EBHF **D** FHCG

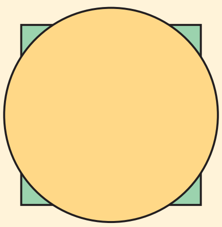
2. The measures of the interior angles of several regular polygons are shown. Which of these polygons can be used to tile the plane?
   A 108° pentagon **B** 120° hexagon
   C 135° octagon **D** all of them

3. An ornament is a square-based pyramid with a sphere on top. The pyramid has a base of side length 4 cm and a height of 6 cm. The diameter of the sphere is 5 cm. Which of these represents the top view of the ornament?

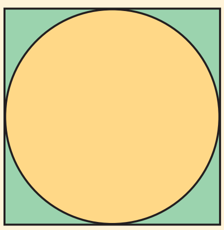
A



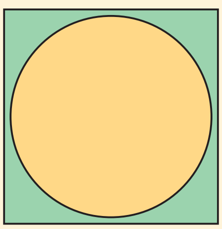
B



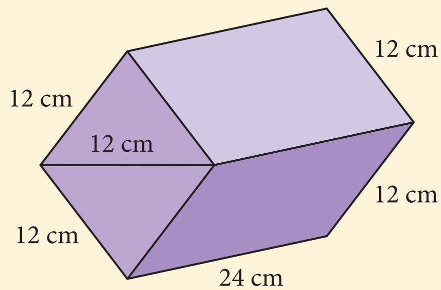
C



D



4. A new specialty fruit and nut bar is being made. The bar has a diamond cross-section made up of two congruent equilateral triangles with side length 12 cm. The bar is 24 cm long. Draw a set of orthographic drawings for the bar. Include all measurements.



5. Select a suitable scale, and draw a net that could be used to make a scale model of a package for the fruit and nut bar in question 4.

6. Yasmini received a pet budgie in a cylindrical cage with a diameter of 60 cm and a height of 50 cm. Yasmini needs a night cover for the cage. The cover requires a top and sides, but no bottom. Select a suitable scale, and draw a pattern that could be used to cut the pieces of cloth needed to make the cover.

Chapter Problem Wrap-Up

Paul has learned a lot about geometry and architecture during his coop placement. Choose a design project of your own, similar to the projects you have worked on throughout this chapter. It could be a building or structure, a package, or an item of clothing.

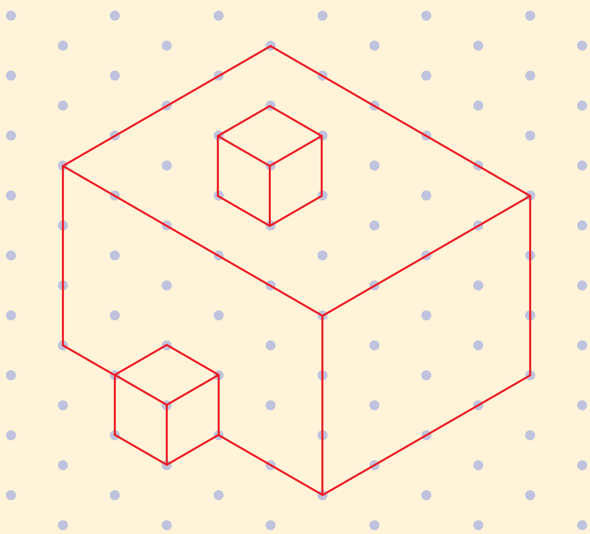
a) Write a plan for your project. Include at least four geometric elements in the design. Describe the constraints on your project and how you can meet them.

b) Draw an isometric perspective drawing or a set of orthographic drawings for your project.

c) Calculate the project's dimensions. Draw a pattern or a net. Build a scale model of your project.

d) Write a summary of your project. Include your drawings and model. Show all your calculations and measurements.

7. An isometric perspective drawing for a new building is shown. The distance between pairs of dots represents 5 m. Draw orthographic drawings for the building.



8. A gift box for a travel mug will have a square base with side length 4 in. and a square lid with side length 6 in. The box must be 5 in. high. Select a scale, and sketch nets that can be used to make a model of the box and the lid. Show all measurements on the net.

9. A new tomato juice can is being designed such that the ratio of the height to the diameter is equal to the golden ratio. The diameter will be 12 cm.

a) Select a scale, and draw a net that could be used to make a scale model of the can.

b) Determine the volume of tomato juice that the can will hold.

Chapters 4 to 6 Review

Chapter 4: Quadratic Relations I

1. a) Which of these relations are quadratic? Explain your reasoning.

i)

x	y
0	17
1	13
2	9
3	5
4	1
5	−3
6	−7

ii)

x	y
1	0
3	1
5	3
7	6
9	10
11	15
13	21

iii) $y = 2x + 1$ **iv)** $y = 12 - 2x^2$

b) Check your predictions by graphing each relation.

2. Describe the graph of each parabola relative to the graph of $y = x^2$.

a) $y = (x + 4)^2$ **b)** $y = -(x - 1)^2$

c) $y = -0.3(x + 7)^2$ **d)** $y = 2(x + 9)^2$

e) $y = 0.9(x + 32)^2$ **f)** $y = -4(x - 18)^2$

3. For each relation:

i) identify the coordinates of the vertex

ii) determine if the parabola opens upward or downward

iii) determine if the parabola is vertically stretched or vertically compressed

iv) sketch the graph

a) $y = (x - 1)^2 + 9$ **b)** $y = -2(x + 8)^2 - 5$

c) $y = 0.1x^2 - 1$ **d)** $y = 9.1x^2$

e) $y = 0.5(x - 2)^2 + 2$ **f)** $y = 8(x + 1)^2 + 13$

4. A flare is sent up as a distress signal. The path is modelled by the relation $h = -4.9(t - 6)^2 + 177.4$, where h is the flare's distance above the water, in metres, and t is the time, in seconds.

a) How long will the flare take to reach its maximum height? What is the maximum height?

b) A typical flare will stay lit for 7 s. How high above the water will the flare be 7 s after it is launched?

c) Graph this relation.

d) After how many seconds will the flare hit the water?

Chapter 5: Quadratic Relations II

5. Expand and simplify.

a) $(3x + 4)(10x + 1)$ **b)** $(x - 2)(4x + 15)$

c) $(12x - 8)(2x + 0.5)$ **d)** $(6 + 2x)(6 - 2x)$

6. a) Write an expression, in simplified form, for the area of an apartment with dimensions $s - 1$ by $4s - 7$.

b) If $s = 15$ m, find the actual area of the apartment.

7. Write each relation in standard form.

a) $y = (x - 4)^2 + 0.5$

b) $y = (x + 10)^2 - 3$

c) $y = 8(x + 2)^2 + 27$

d) $y = -3.2(x - 4)^2 - 0.8$

8. Determine the y-intercept for $y = (7 - 3x)(2x - 6)$.

9. Factor each trinomial.

a) $x^2 + 11x + 24$ b) $x^2 + x - 30$

c) $x^2 - 8x + 7$ d) $x^2 + 8x + 16$

e) $3x^2 + 39x + 108$ f) $-10x^2 - 110x - 100$

10. Which pairs contain equivalent expressions? How do you know?

a) $x^2 + 4x - 5$ $(5 + x)(x - 1)$

b) $x^2 + 3.5x + 3$ $(x - 2)(x - 1.5)$

c) $x^2 + x - 2$ $(x - 1)(x + 2)$

11. Find the zeros by factoring. Check by substituting your answers for x.

a) $y = x^2 - x - 20$

b) $y = 10x^2 - 360$

c) $y = -2x^2 + 4x + 70$

d) $y = 3.5x^2 + 21x + 31.5$

12. The height of a rectangular prism is 16 cm more than the length of the base. The width of the base if half the length.

a) Sketch and label a diagram to represent the rectangular prism.

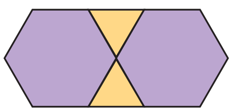
b) Write an expression for the surface area of the prism.

c) What is the surface area of the prism if the width of the base is 5 cm?

d) What are the dimensions of the prism if the surface area is 640 cm^2?

Chapter 6: Geometry in Design

13. Can the shape shown be used to tile the plane? Provide evidence in the form of a sketch.

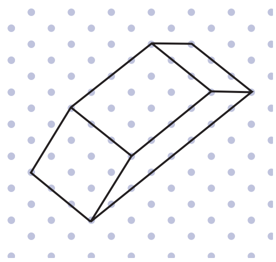


14. a) Use linking cubes to model the capital letter "L". The base of the L should be 10 cm and the height should be 20 cm. The letter should be 10 cm thick. Let the side of a cube represent 5 cm.

b) Use the model to help you draw an isometric perspective drawing.

c) Use the model to help you sketch a set of orthographic drawings.

15. A car dealership has ordered a platform to display their newest car models. An isometric perspective drawing for the display platform is shown. The distance between pairs of vertical or horizontal dots represents 2 m.



a) Draw a set of orthographic drawings for the platform.

b) Draw a net that can be used to make a scale model for the platform. The rectangles that will become the sloped part of the platform should have a length of 3 units and a width of 2.25 units. Cut out the net, and construct the model.

c) The top of the platform is to be made from a special type of thick plywood that sells for \$40/m^2. Determine how much plywood is needed and the cost of buying it.