

# Project Scheduling: PERT/CPM

## CHAPTER 8

### LEARNING OBJECTIVES

*After completing this chapter, you should be able to:*

1. Describe the role and application of PERT/CPM for project scheduling.
2. Define a project in terms of activities such that a network representation can be developed.
3. Develop a complete project schedule.
4. Compute the critical path, the project completion time and its variance.
5. Convert optimistic, most likely, and pessimistic time estimates into expected activity time estimates.
6. Compute the probability of the project being completed by a specific time.
7. Compute the project completion time given a certain level of probability.
8. Find the least expensive way to shorten the duration of a project to meet a target completion date.
9. Formulate the crashing problem as a linear programming model.
10. Formulate project scheduling as a linear programming model.
11. Know some of the specialised software available in the market for scheduling and tracking project activities.

### CHAPTER OUTLINE

- 8.1 Introduction 360
- 8.2 A Project Example: Replacement of an Airport Gate-Management System 361
- 8.3 Project Network Representation 361
- 8.4 Project Scheduling with Deterministic Activity Durations 364
- 8.5 Project Scheduling with Probabilistic Activity Durations 368
- 8.6 Uses of Simulation in Project Scheduling 372
- 8.7 Project Crashing 372
- 8.8 Using Linear Programming in Project Scheduling 378
- 8.9 Project Scheduling Software 378
- Summary 379
- Glossary 379
- Solved Problems 380
- Discussion and Review Questions 384
- Problems 385
- End of Chapter Case 390



## 8.1 INTRODUCTION

Project scheduling is at the heart of many decision problems that are, in nature, non-repetitive, unique, and clearly defined in terms of scope, objectives, and time frame. These decisions are referred to as a **project**; an interrelated set of activities directed towards the accomplishment of a unique, often major outcome and that have a definite starting and ending point. Every organization may be involved at any time in a project of any size, duration, and complexity level. Examples of typical projects are setting up of a stage for a rock concert, the construction of a new plant or facility, the design and the marketing of a new product or service, the redesign of a business process, the construction (or repair) of a bridge, the development of a new drug, and the acquisition and installation of an enterprise planning system. All these projects consist of several activities that have to be completed and some of them are interdependent, that is, they cannot start before the completion of some other activities. In many situations, managers face the challenges of planning, coordinating, and monitoring these activities so that the project of interests is completed on time and within the allocated budget. Project management provides a number of approaches to cope with these challenges.

This chapter introduces the **critical path method (CPM)** and the **program evaluation review technique (PERT)**, two management science techniques developed in the late 1950s to plan, schedule, and control large, complex projects with many activities. These approaches differ primarily on how the duration and the cost of activities are processed. In the case of CPM, it is assumed that details about these inputs are known with certainty, whereas for PERT, these details are not known with certainty. Both approaches use a network representation (see Chapter 5) to display the relationships between project activities and to help managers to address questions such as:

1. What is the total time required to complete the project (the expected total time for PERT)?
2. What are the start and the completion times for individual activities?
3. Which critical activities must be completed as scheduled to meet the estimated project completion time?
4. How much delay can be tolerated for non-critical activities without incurring a delay in the estimated project completion time?
5. What is the least expensive way to speed up a project to meet a targeted completion time?

PERT also provides answers to the following additional questions:

6. What is the probability of completing a project within a given time frame?
7. What is the variability in the project completion time?

We will first introduce a project example that will be used in the chapter to illustrate the project graphical representation and to discuss both approaches, CPM and PERT. The chapter ends with a discussion on time–cost trade-offs in project acceleration and on the use of linear programming in project acceleration as well as in project scheduling. It is worthwhile to note that project scheduling, the focus of this chapter, is only one of the major phases involved in the management of a project. Project planning and project control processes, as well as their related challenges, are two other phases, not discussed in this chapter, that are required to ensure the effective use of resources to deliver the project objectives on time and within cost constraints. Other management variables that also matter include the senior managers' support, a clear definition of roles and responsibilities, the communication systems, and the human resource management practices.



## 8.2 A PROJECT EXAMPLE: REPLACEMENT OF AN AIRPORT GATE-MANAGEMENT SYSTEM

AC Inc. is a full-service, major airline company in the international market. It relies on a user-friendly gate management software system at its hub airport to decide how to assign aircrafts to gates. Based on a flight schedule and various business criteria, the system serves for long-, medium-, and short-term planning of gate requirements. It also serves for real-time allocation of gates during the day of operation. The company faces the challenge of replacing its current system because its hardware platform is obsolete and no longer supported. As well, a considerable increase of activities at its hub facility is making the current environment less suitable. The company is concerned with the overall cost involved if a manual process would have to take place in case of a breakdown of the current system. Therefore, a project has been set-up with a mandate of acquiring and implementing a new gate management system to replace the current system. Soad El-Taji is project manager at AC Inc., and has many years of experience. Her exemplary performance in similar projects in the past has earned her the confidence of senior management. It was with unanimity that she was called once again to manage this acquisition project to ensure that it was successfully completed on time. As in the past, she resorted to PERT/CPM approaches. The first step she completed was to accurately establish a list of **activities** that needed to be undertaken, their precedence relationships and the time estimates for each activity. Table 8-1 shows the list of activities. The **immediate predecessors** of an activity refer to those activities that must be completed prior to the starting time of a given activity. Similarly, **immediate successors** of an activity refer to those that follow the completion of a given activity. The “—” in this table indicates an activity without a predecessor.

After the completion of the project activities, Soad El-Taji wants to know how to better visualize these activities and how to develop answers to questions listed at the end of the previous section.



## 8.3 PROJECT NETWORK REPRESENTATION

A **project network** representation is used to depict the project activities and their relationships. As discussed in Chapter 5, a network consists of a set of circles referred to as nodes and lines connecting nodes together referred to as arcs. The two common approaches of a project network representation are **activity on node (AON)** or **activity on arc (AOA)**. In the first approach, the nodes of the network represent the project activities and the arcs show their precedence relationships. In the second approach, the project activities are reported on arcs and nodes represent the starting or the completion of activities. We focus in this textbook solely on the activity on node representation, given its large adoption in many software packages. It is common in this approach to add one dummy source activity node (referred to as Start) and connect it to all activities nodes with no immediate predecessors as well as one dummy destination activity node (referred to as Finish) and connect it to the project network with arcs from activities with no immediate successors. This ensures that there is one starting point and one finish point in the project network. Figure 8-1 shows the network representation of the airport gate management system acquisition project example. Note that a dummy Finish node is added to ensure that the project network has one starting node and one finish node. Since there is only one node without a predecessor (A), no dummy Start node is added in this network.

TABLE 8-1

List of Activities for the Airport Gate Management System (AGMS) Acquisition Project

Activity	Description	Immediate Predecessors	Estimated Time (weeks)
A	Set up the project acquisition team	—	2
B	Write down the software requirements	A	2
C	Develop a contractor evaluation grid that will be used to evaluate proposals	B	1
D	Identify and select potential contractors	A	1
E	Develop and send out a request for proposal to potential contractors	B, D	4
F	Audit candidate contractors, select one contractor, negotiate and sign an agreement contract with the selected contractor	C, E	2
G	Prepare the definition of functional specifications	F	5
H	Develop a software testing plan	G	2
I	Software customization phase I	G	12
J	Purchase and install the hardware	G	2
K	Test the first release	H, I, J	1
L	Develop a training plan for key users	K	1
O	Train key users	L, N	2
M	Software customization phase II	K	6
N	Test the second release	M	1
P	Software customization phase III	N	3
Q	Test the final release	P	2
R	Software deployment and project sign-off	Q	4

For a small project network, one convenient way to determine its duration and critical activities (questions 1 and 3 above) is through the enumeration of all the different paths in the network. A **path** is a sequence of connected nodes in the network from the start node to the finish node. The **length of the path** is given by the sum of the durations of the activities on the path. For the network shown in Figure 8-1, the corresponding paths are shown in Table 8-2. In this list, path # 1 is the longest, while paths # 23 and # 24 are the shortest. Path # 1 is critical, because any delay in the duration of an activity located in this path will delay the entire project. For instance, increasing the duration of activity A by 2 weeks will increase the length of path # 1 by 2 weeks for a total duration of 46 weeks. Whereas, for path # 23 and # 24, the length will only increase to 20 weeks, respectively. Hence, in any project network, the path with the longest duration is called a **critical path** and the corresponding activities are called **critical activities** in that they must be completed as scheduled to meet the scheduled project completion time. The estimated duration of the project is

FIGURE 8-1

Network Representation for the AGMS Acquisition Project

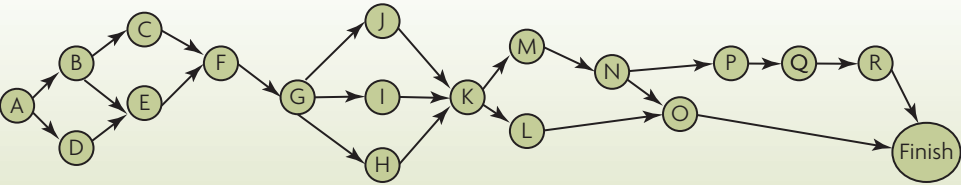


TABLE 8-2

List of Paths for the AGMS Acquisition Project

Path #	Sequence of Nodes	Length (total time in weeks)
1	A-B-E-F-G-I-K-M-N-P-Q-R-Final	44
2	A-B-E-F-G-J-K-M-N-P-Q-R-Final	34
3	A-B-E-F-G-H-K-M-N-P-Q-R-Final	34
4	A-B-C-F-G-I-K-M-N-P-Q-R-Final	41
5	A-B-C-F-G-J-K-M-N-P-Q-R-Final	31
6	A-B-C-F-G-H-K-M-N-P-Q-R-Final	31
7	A-D-E-F-G-I-K-M-N-P-Q-R-Final	43
8	A-D-E-F-G-J-K-M-N-P-Q-R-Final	33
9	A-D-E-F-G-H-K-M-N-P-Q-R-Final	33
10	A-B-E-F-G-I-K-M-N-O-Final	37
11	A-B-E-F-G-J-K-M-N-O-Final	27
12	A-B-E-F-G-H-K-M-N-O-Final	27
13	A-B-C-F-G-I-K-M-N-O-Final	34
14	A-B-C-F-G-J-K-M-N-O-Final	24
15	A-B-C-F-G-H-K-M-N-O-Final	24
16	A-D-E-F-G-I-K-M-N-O-Final	36
17	A-D-E-F-G-J-K-M-N-O-Final	26
18	A-D-E-F-G-H-K-M-N-O-Final	26
19	A-B-E-F-G-I-K-L-O-Final	31
20	A-B-E-F-G-J-K-L-O-Final	21
21	A-B-E-F-G-H-K-L-O-Final	21
22	A-B-C-F-G-I-K-L-O-Final	28
23	A-B-C-F-G-J-K-L-O-Final	18
24	A-B-C-F-G-H-K-L-O-Final	18
25	A-D-E-F-G-I-K-L-O-Final	30
26	A-D-E-F-G-J-K-L-O-Final	20
27	A-D-E-F-G-H-K-L-O-Final	20

therefore given by the length of the critical path. It is possible to find more than one critical path in a network project, but all critical paths will have the same length. For the network shown in Figure 8-1, the critical path is A-B-E-F-G-I-K-M-N-P-Q-R and its corresponding length is 44 weeks. Therefore, the project's estimated duration is 44 weeks. All activities in the critical path are called critical activities, whereas the remaining activities (C, D, H, L, and O) are non-critical activities.

The procedure described above is prohibitive for large project networks. In addition, it does not provide answers to the scheduling time of individual activities and the delay that can be tolerated for non-critical activities without incurring a delay in the scheduled project completion time. In the next two sections, we discuss how techniques such as CPM and PERT help managers to address these questions.



## 8.4 PROJECT SCHEDULING WITH DETERMINISTIC ACTIVITY DURATIONS

For larger project networks, CPM provides a most efficient approach for project scheduling when the duration of activities are known with certainty. The approach consists of finding the earliest and the latest schedules to avoid delays in project completion. Therefore, for all activities in the network the following information is computed:

1. The **earliest start time (EST)**: the earliest time at which an activity can start if no delays occur in the project;
2. The **earliest finish time (EFT)**: The earliest time at which an activity can finish if no delays occur in the project;
3. The **latest start time (LST)**: The latest time at which an activity can start without delaying the completion of the project;
4. The **latest finish time (LFT)**: The latest time at which an activity can finish without delaying the completion of the project.

A **forward pass** (from the starting node to the finish node) is used to compute the EST and EFT, whereas a **backward pass** (from the finish node to the starting node) is used for the LST and LFT. To determine the critical path and the project schedule, the approach consists of calculating, respectively, the starting time and the completion time for each activity as well as identifying the corresponding slack.

The basis of the forward pass is the EST rule that states that all immediate predecessors must be completed before an activity can begin. Let  $t$  be the duration (estimated) of an activity. If no delay occurs anywhere in the project, the earliest finish time for an activity is

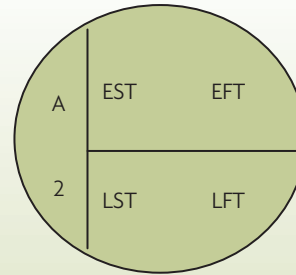
$$EFT = EST + t \quad (8-1)$$

It results from the EST rule that the earliest start time for each activity is equal to the largest earliest finish times of the immediate predecessors.

For example, consider the network shown in Figure 8-1. The project-starting activity A has no predecessor. Therefore, it can start as soon as the project starts, which we assume to be time 0. The earliest finish time of activity A can now be computed as  $0 + 2 = 2$ . Activities B and D have activity A as their immediate predecessor. The EST for B is 2 and the EFT is  $2 + 2 = 4$ . The EST for D is 2 and EFT is  $2 + 1 = 3$ . Activity B is the only immediate predecessor of activity C and its EFT is known. Therefore, the EST for C is 4 and the EFT is  $4 + 1 = 5$ . Activity E has two predecessor activities, B and D, with known EFT (4 and 3).

FIGURE 8-2

Nodes Notation



The EST for D is 4, or the maximum between 4 and 3. The EFT of activity D is  $4 + 4 = 8$ . If we continue this forward pass, the EST and the EFT for all activities can be computed as summarized in Figure 8-3, in which the node is expanded to include additional information (as shown in Figure 8-2). We show in the top left side of the node, the activity name (e.g., A); in the bottom left side, the activity duration; and in the top right side, the activity's EST and EFT. Later in our discussion we will place the activity's LST and LFT on the bottom right of the node.

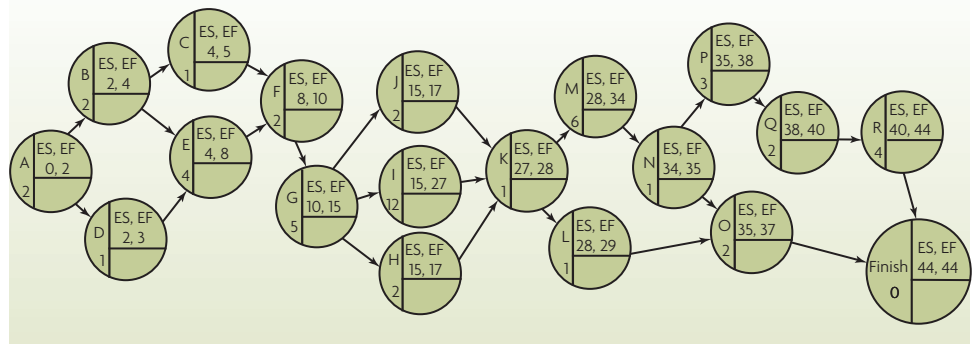
The forward pass to obtain the EST and the EFT can be summarized as follows:

1. For each activity with no predecessor, set  $EST = 0$ .
2. For each activity with known EST, calculate EFT using Equation 8-1, that is,  $EFT = EST + t$ .
3. For each new activity where all immediate predecessors have known EFT values, apply the EST rule to obtain the EST and step 2 to calculate EFT.
4. Repeat step 3 until EST and EFT have been obtained for all activities.

The basis of the backward pass is the LFT rule which states that an activity can start at the latest time if and only if all its immediate predecessors are completed. Hence, the latest finish time for each activity is equal to the smallest latest start times of the immediate successors'

FIGURE 8-3

EST and EFT Computation for the AGMS Acquisition Project





activities. Therefore, if no delay occurs anywhere in the project, the latest start time for an activity that will result on not delaying the completion of the project is:

$$LST = LFT - t \quad (8-2)$$

The backward pass starts by setting up the LFT of all activities without successors (including the finish node) equal to the maximum EFT and then works backward from the finish node to the starting node. The procedure can be summarized as follows:

1. For each of the activities without successors (including the finish node), set LFT equal to EFT of the finish node.
2. For each activity with known LFT value, calculate LST using Equation 8-2, that is,  $LST = LFT - t$ .
3. For each new activity where immediate successors have known LST values, apply, respectively, the LFT rule to obtain the corresponding LFT and step 2 to calculate LST.
4. Repeat step 3 until LFT and LST have been obtained for all activities.

For the network shown in Figure 8-4, we set the LFT and the LST of the finish node equal to its EFT = 44 weeks. The immediate successor of activities R and O is the finish node. Hence, the LFT is 44 weeks and  $LST = 44 - 4 = 40$  weeks for activity R. For activity O, LFT is 44 weeks and  $LST = 44 - 2 = 42$  weeks. Activity R is the immediate successor for activities Q. Hence the corresponding LFT for activity Q is 40 weeks, whereas the  $LST = 40 - 2 = 38$  weeks. Activity Q is the immediate successor for activity P. The LFT = 38 for activity P and the  $LST = 38 - 3 = 35$  weeks. Activity P and O are immediate successors for activity N. Hence,  $LFT = \text{minimum } (35, 42) = 35$  weeks for activity N. If we proceed backward until the starting node, the LFT and the LST for all activities can be computed as summarized in Figure 8-4. For example, Activity G has activities J, I, and H as immediate successors. Its corresponding  $LFT = \text{min } (25, 15, 25) = 15$  weeks and  $LST = 15 - 5 = 10$  weeks.

The **slack time** for an activity refers to the length of time that can be tolerated without incurring a delay in the scheduled project completion time. The slack time per activity needs to be calculated first to identify the critical path(s), by considering either the start times or the finish times. Hence, for each of the activities in the project network, the slack time can be calculated as follows:

$$Slack = LST - EST \text{ or } LFT - EFT \quad (8-3)$$

**FIGURE 8-4**

LST and LFT Computation for the AGMS Acquisition Project

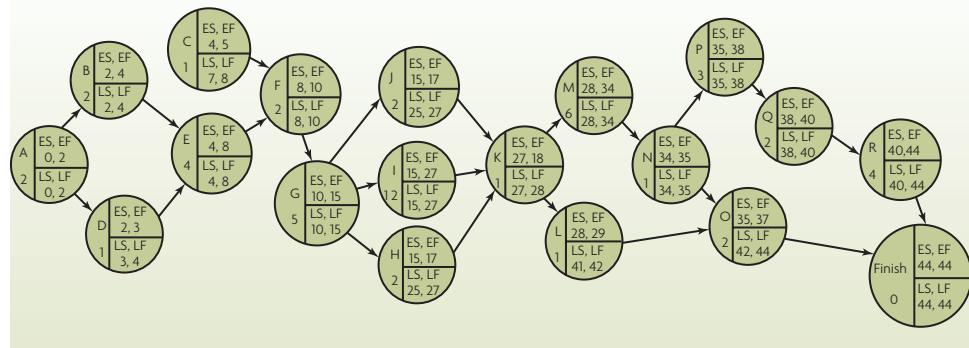




Table 8-3 summarizes the activities' slack times for the airport gate management system acquisition project example. Activities A, B, E, F, G, I, K, M, N, P, Q, and R have zero or no slack, meaning that these activities cannot be delayed without delaying the entire project. Alternately, a reduction in the duration of one these activities will result in the reduction of the entire project's length. They are called critical activities and belong to the critical path(s). The remaining activities (C, D, H, J, L, and O) are non-critical activities since they provide managers with some degree of freedom about when to start or complete them without delaying the entire project. Therefore, Table 8-3 provides the project manager with answers to some of the questions raised at the end of the introduction section.

For the airport gate management system acquisition project example, the critical path is A-B-E-F-G-I-K-M-N-P-Q-R-Finish, the estimated project completion time is 44 weeks, and each activity has to be completed according to the schedule shown in Table 8-3. As stated earlier, note that it is possible for a project to have multiple critical path(s). The slack times provide valuable information for the overall management of the project. For example, an activity with smaller slack time (e.g., activity D with 1 week) will need tighter control than an activity with a larger slack time (e.g. activities H and J with 10 weeks).

**TABLE 8-3**

Summary of Activities' Start, Finish, and Slack Times

Activity	EST	EFT	LST	LFT	Slack	Critical Activity
A	0	2	0	2	0	Yes
B	2	4	2	4	0	Yes
C	4	5	7	8	3	
D	2	3	3	4	1	
E	4	8	4	8	0	Yes
F	8	10	8	10	0	Yes
G	10	15	10	15	0	Yes
H	15	17	25	27	10	
I	15	27	15	27	0	Yes
J	15	17	25	27	10	
K	27	28	27	28	0	Yes
L	28	29	41	42	13	
M	28	34	28	34	0	Yes
N	34	35	34	35	0	Yes
O	35	37	42	44	7	
P	35	38	35	38	0	Yes
Q	38	40	38	40	0	Yes
R	40	44	40	44	0	Yes
Finish	44	44	44	44	0	Yes



## 8.5 PROJECT SCHEDULING WITH PROBABILISTIC ACTIVITY DURATIONS

The CPM approach assumes that the duration of activities are known with certainty and the actual duration will turn out to be exactly as estimated. However, in practice this is not always possible and many projects involve variability in activity times due to factors such as lack of prior experience, equipment breakdown, unpredictable weather conditions, late delivery of supplies, and others. PERT analysis is used when the duration of activities are not known with certainty. It involves three types of estimates of the duration of an activity instead of one single value as in the case of CPM:

1. The optimistic duration  $a$  = the time an activity will take under the most favourable conditions.
2. The pessimistic duration  $b$  = the time an activity will take under the most unfavourable conditions.
3. The most likely duration  $m$  = the most realistic time an activity will require to be completed, that is, the time an activity will take under normal conditions.

The duration of an activity is therefore assumed to have a **beta probability distribution** in PERT analysis. Following this distribution, the expected activity time,  $t_e$ , and the variance of the activity completion time,  $\sigma^2$  can be obtained as follows:

$$t_e = \frac{a + 4m + b}{6} \quad (8-4)$$

$$\sigma^2 = \left( \frac{b - a}{6} \right)^2 \quad (8-5)$$

Figure 8-5 illustrates the shape of the beta distribution, where at the two extremes we have the two estimates  $a$  and  $b$  with a very small probability, whereas the third estimate  $m$  provides the highest point (mode) of the probability distribution.

In PERT analysis, the project completion time is computed in a similar manner as in the CPM approach, but by substituting the three estimates of the activity duration with the expected activity time,  $t_e$ , as obtained according to Equation 8-4, and by the variance of the activity completion time,  $\sigma^2$ , as obtained according to Equation 8-5. Therefore, the expected completion time of the project ( $\mu_p$ ) can be derived as well as the variability in the project completion time ( $\sigma_p$ ), as follows:

$$\begin{aligned} \mu_p &= \max(\text{EFT}) = \max(\text{LFT}) \\ &= \text{sum of the expected duration for the activities in the critical path} \end{aligned} \quad (8-6)$$

$$\sigma_p = \sqrt{\text{sum of the variances of the duration for the activities in the critical path}} \quad (8-7)$$

Assume that, due to variability in activity times, the three estimates of activities duration for the airport gate management system acquisition project example are as shown in Table 8-4. (see columns 2–4). Therefore, by applying Equations 8-4 and 8-5 the mean and the variance of the activity duration shown in the last two columns of Table 8-4 can be computed.

The estimated start and finish times for all activities according to the forward pass and the backward pass were summarized in Figure 8-4. The critical path is A-B-E-F-G-I-K-M-N-P-Q-R-Finish. The expected project completion time ( $\mu_p$ ) is 44 weeks (given by the

FIGURE 8-5

Illustration of the Shape of the Beta Probability Distribution

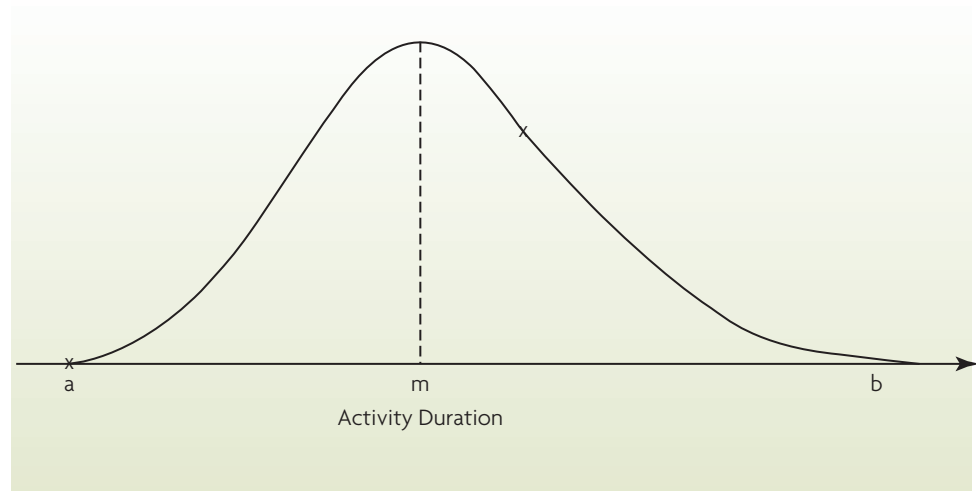


TABLE 8-4

Activities Expected Times and Variances for the AGMS Acquisition Project

ACTIVITY	$a$	$m$	$b$	Mean, $t_e = \frac{a + 4m + b}{6}$	Variance, $\sigma^2 = \left(\frac{b - a}{6}\right)^2$
A	1	2	3	2	0.111
B	1	2	3	2	0.111
C	0.5	1	1.5	1	0.028
D	0.5	1	1.5	1	0.028
E	3	4	5	4	0.111
F	1	2	3	2	0.111
G	3	5	7	5	0.444
H	1	2	3	2	0.111
I	10	12	14	12	0.444
J	1	2	3	2	0.111
K	0.5	1	1.5	1	0.028
L	0.5	1	1.5	1	0.028
M	4	6	8	6	0.444
N	0.5	1	1.5	1	0.028
O	1	2	3	2	0.111
P	1.5	3	4.5	3	0.250
Q	1	2	3	2	0.111
R	2	4	6	4	0.444

maximum EFT or LFT). Given the data in Figure 8-4, the standard deviation for the project equals 1.62 weeks, computed as follows:

$$\sigma_p = \sqrt{\text{var(A)} + \text{var(B)} + \text{var(E)} + \text{var(F)} + \text{var(G)} + \text{var(I)} + \text{var(K)} + \text{var(M)} + \text{var(N)} + \text{var(P)} + \text{var(Q)} + \text{var(R)}}$$

Using the numerical values in Table 8-4, we get:

$$\sigma_p = \sqrt{(0.111) + (0.111) + (0.111) + (0.111) + (0.444) + (0.444) + (0.028) + (0.444) + (0.028) + (0.250) + (0.111) + (0.444)}$$

$$\sigma_p = \sqrt{(2.64)} = 1.62$$

In addition to providing answers to questions about the project's critical activities, the start and completion times of activities, the expected completion time of the project, and the variability in the project completion time, PERT analysis also answers questions such as the probability of whether or not the project will be completed on time or, conversely, what the project completion time will be, given a certain probability.

### Probability of Whether a Project Can Be Completed or Not by a Specific Deadline

Let  $c$  and  $d$  denote the possible project specified deadlines (assuming  $c < d$ ) and  $X$  the total time required to complete the project. Due to the central limit theorem, which indicates that the sum of independent random variables can be approximately represented by a normal distribution as the number of random variables becomes larger, the project completion is approximated by a normal distribution with mean  $\mu_p$  and standard deviation  $\sigma_p$ . Without loss of generality, the three possible situations of interests are illustrated in Figure 8-6 and refer, respectively, to the probability that the project duration ( $X$ ):

- does not exceed the deadline  $b$ , i.e.,  $P(X \leq d)$ ;
- does exceed the deadline  $b$ , i.e.,  $P(X \geq d)$ ;
- falls between  $c$  and  $d$ , i.e.,  $P(c \leq X \leq d)$ .

The approach consists of converting  $X$  into a standard normal distribution and determining the area under the normal curve using Table A in Appendix B. To that end the  $z$  value is computed as follows,

$$z = \frac{x - \mu_p}{\sigma_p} \quad (8-8)$$

where

$$x = c \text{ or } d$$

For the airport gate management system acquisition project, what is the probability of completing the project within 46 weeks? The  $Z$  value for the normal probability distribution at  $x = 46$  is

$$z = \frac{46 - 44}{1.62} = 1.23. \text{ Therefore from Table A in Appendix B,}$$

$P(X \leq 1.23) = 0.50 + 0.3907 = 0.8907$  or 89.07%. Hence, there is about 89% probability that the project will be completed on time. Alternatively the Excel function NORMSDIST( $z$ ) can be used to seek the probability corresponding to the  $z$  value.

What is the probability that the project lasts more than 46 weeks?

$P(X > 46) = 1 - P(X \leq 46) = 1 - P(X \leq 46) = 1 - 0.8907 = 0.1093$  or 10.93%. There is a 11% probability that the project will not be completed on time.

FIGURE 8-6

Illustration of Possible Situations of Interest for Probability Computation

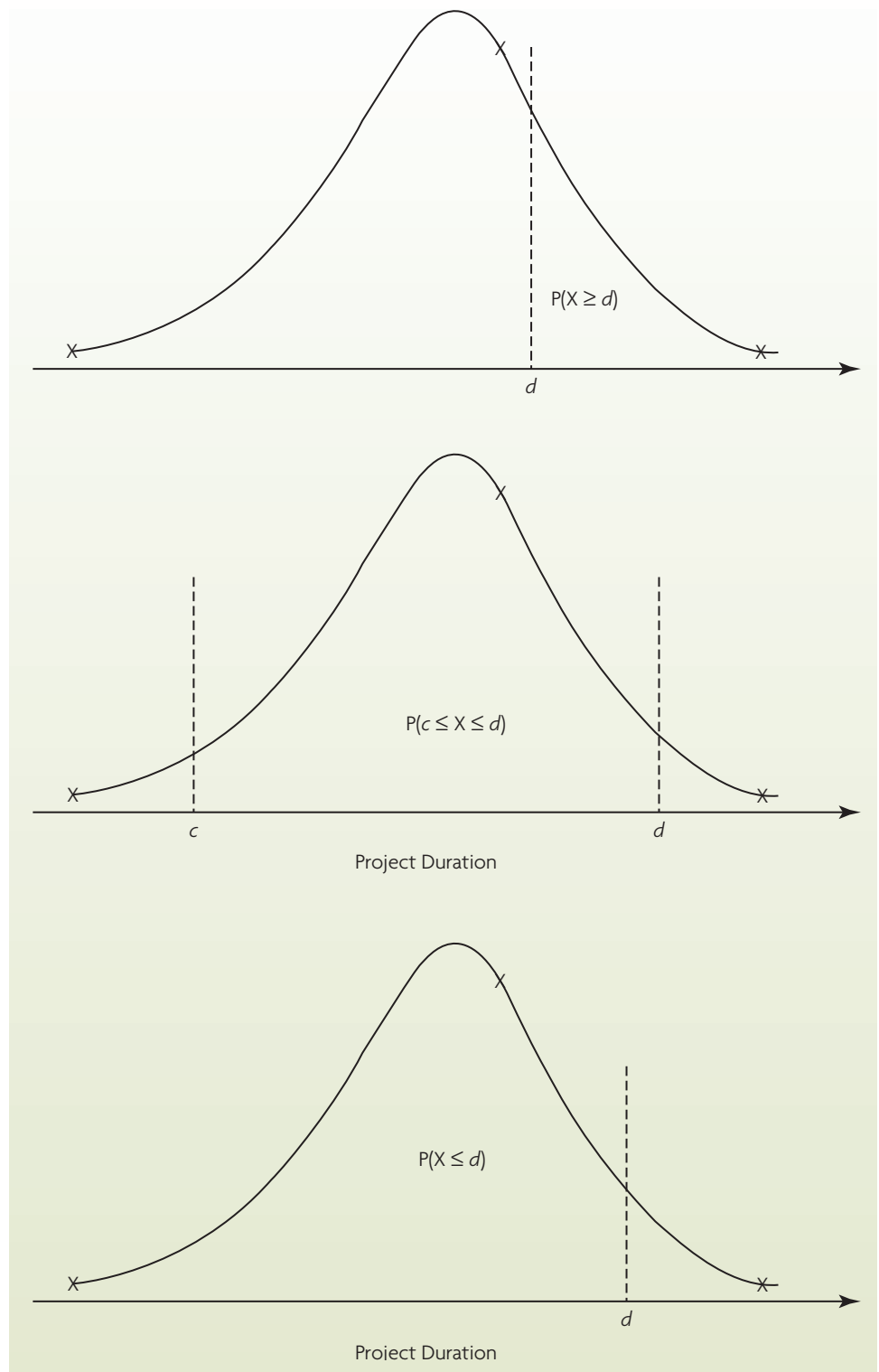
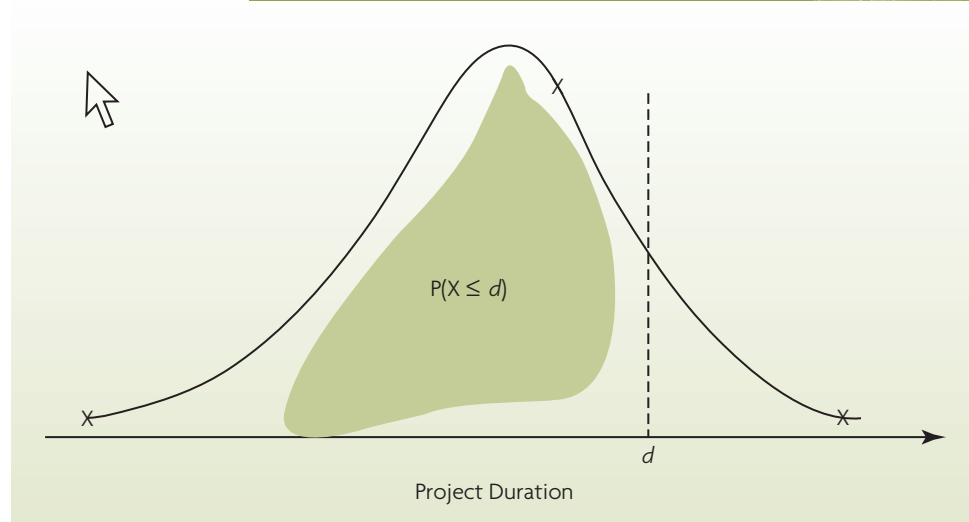


FIGURE 8-7

Project Completion Time Given a Certain Probability



### Project Completion Time Given a Certain Probability

In this case one must find the target project completion time (value of  $d$ ) that corresponds to a specified probability level, as shown in the shaded area of the standard normal curve in Figure 8-7.

The approach lies on finding the  $z$  value using Table A in Appendix B. Therefore from Equation 8-8 the following relation can be obtained:

$$x = \mu_p + z \times \sigma_p \quad (8-9)$$

For the airport gate management system acquisition project, what would be the completion time under which for the project has 95% probability of completing?

From Table A in Appendix B,  $z = 1.65$  for  $P(X \leq b) = 0.95$ . Alternatively, the Excel function NORMINV(probability) can be used to seek the corresponding  $z$  value. Hence, using Equation 8-9,  $x = 44 + 1.65 \times 1.62 = 46.67$  weeks, that is, if the project completion time is 46.67 weeks, the company will complete it on time with a 95% probability.

## 8.6 USES OF SIMULATION IN PROJECT SCHEDULING

Resorting to PERT analysis to cope with uncertainty presents a number of difficulties in practice. The underlying assumption that the activity durations are independent is sometimes difficult to justify. In addition, the activity durations may not follow a beta distribution. One of the approaches to use when other distribution functions are involved is simulation, the topic discussed in Chapter 13.

## 8.7 PROJECT CRASHING

It is common in project management that additional resources are used to either speed up some activities to get the project back on schedule or to reduce the project completion time. Late penalty costs, monetary incentives, cost savings, or strategic benefits are some of the reasons for shortening a project completion time. **Crashing** an activity refers to the

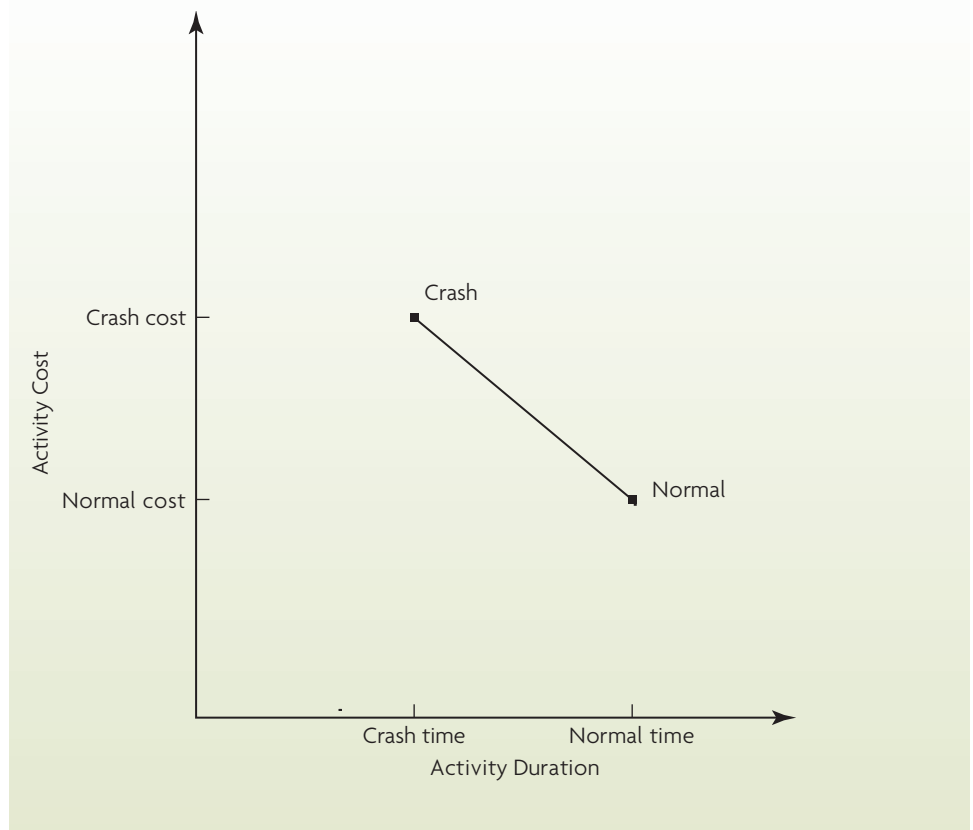
speeding up or shortening of the duration of an activity by using additional resources. These include overtime, hiring temporary staff, renting more efficient equipment, and other measures. **Project crashing** refers to the process of shortening the duration of the project by crashing the duration of a number of activities. Since it generally results in an increase of the overall project costs, the challenge faced by the project manager is to identify the activities to crash and the duration reduction for each activity such that as the project crashing is done in the least expensive manner possible. This section discusses a procedure that can be used for small-sized projects. We also discuss how linear programming (Chapter 3) can be used to investigate project crashing decisions, especially for larger size projects. Figure 8-8 illustrates the activity costs and activity duration relationships, where the **normal time** refers to the estimated activity duration used with CPM or PERT in the computation of earliest (latest) start or finish times. The **normal cost** refers to the activity cost under the normal activity time. The **crash time** refers to the shortest possible time to complete an activity with additional resources. The **crashing cost** refers to the activity cost under the crashing activity time.

This relationship is assumed to be linear. Hence, for each activity a crash cost per period (e.g., per week) can be derived as follows:

$$\text{crash cost per period} = \frac{(\text{Crash cost} - \text{Normal Cost})}{(\text{Normal time} - \text{Crash time})} \quad (8-10)$$

FIGURE 8-8

Activity Cost and Activity Time Relationship





The general procedure for project crashing involves the following four steps:

1. Compute the crash cost per period for all activities using Equation 8-10.
2. Find critical path(s) in the project network using the normal times and identify critical activities.
3. Select a critical activity with the smallest crash cost per week that can still be crashed, in the case that there exists only one critical path. Otherwise, select one activity from each critical path that can be still crashed and yield the smallest total crash cost per period (including a common activity among critical paths). Crash the selected activity or activities by one period. Update the length of paths.
4. Stop the procedure if the completion deadline is reached. Otherwise, check to ensure current critical path(s) are still critical and find the new ones, if any. Return to Step 3.

To illustrate the procedure above, consider the network shown in Figure 8-9, which shows the activities for a new product development project and their precedence relationships. Start is a dummy activity with zero duration added to ensure that all activities have one starting node.

Table 8-5 provides the information about the activities normal times and costs, crash times and costs, the maximum crashing reduction in time as obtained by the difference between the normal time and the crash time, and the crash cost per week as obtained using Equation 8-10. For example, the normal time for activity C is 10 weeks and its cost is \$45 000. It can be shortened by up to 4 weeks at an additional cost of \$36 000 or \$9000 per week of reduction.

The project critical path obtained by using normal times is Start-A-C-E-H. The estimated project completion time is 28 weeks. Suppose that management wants to shorten the project to 24 weeks to beat competition. Which activities should be crashed, and for each crashed activity provide the total number of weeks crashed and the total cost. What is the overall project crashing cost?

To reduce the project completion time from 28 weeks to 27 weeks, one of the activities on the critical path needs to be reduced. Activity A has the lowest crash cost per week among all critical activities (\$3000). It is therefore selected and crashed by one week, that is, the duration for activity is 4 weeks instead of 5. This shortens the project completion time

**FIGURE 8-9**

Project Network for the New Product Development Example

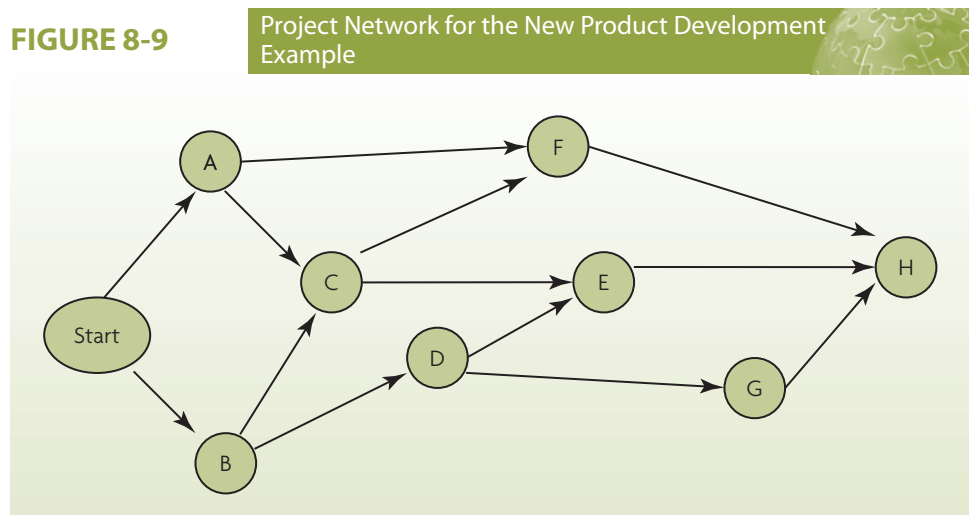


TABLE 8-5

Time and Cost Information for the New Product Development Project

Activity	Time (weeks)		Cost (\$)		Maximum Reduction in Time (weeks)	Crash Cost per Week
	Normal	Crash	Normal	Crash		
A	5	2	25 000	34 000	3	3000
B	4	2	30 000	40 000	2	5000
C	10	6	45 000	81 000	4	9000
D	5	3	30 000	38 000	2	4000
E	7	6	30 000	37 000	1	7000
F	5	3	20 000	26 000	2	3000
G	4	2	35 000	44 000	2	4500
H	6	3	35 000	65 000	3	10 000

to 27 weeks and Start-A-C-E-H is still the single critical path. To shorten the project completion to 26 weeks, activity A is once again selected, given that it can still be crashed and it has the lowest crash cost per week among all critical activities (\$3000). The project completion time is now 26 weeks. Start-A-C-E-H is still a critical path but another path, Start-B-C-E-H, has also become critical. To reduce the entire project to 25 weeks, one activity from each critical path needs to be crashed. One option is to choose activity A and B, respectively, but the total cost would be \$8000 (\$3000 + \$5000). Activity E is common to both critical paths, has a crash cost per week of \$7000, which is lower than \$8000 (if A and B are crashed), and will reduce both path simultaneously if selected. We crash activity E by one week to reduce the project completion time to 25 weeks.

At this stage, we have 3 critical paths: Start-A-C-E-H, Start-B-C-E-H, and Start-B-C-F-H. Activity E cannot be crashed further. To reduce the project completion time to 24 weeks, Activities A and B will be crashed for a total cost of \$8000 (\$3000 + \$5000), any other combination being more costly. Note that the option of shortening A and F is cheaper, but it does not reduce the length of the critical path Start-B-C-E-H.

In conclusion, to shorten the completion project duration to 24 weeks, activity A should be crashed by 3 weeks at a cost of \$9000 ( $3 \times \$3000$ ), activity E by 1 week at a cost of \$7000, and activity B by one week at a cost of \$5000. The total project crashing cost is \$21 000 (\$9000 + \$7000 + \$5000). Table 8-6 provides a summary of the procedure after 4 weeks of project crashing, where \* indicates a critical path.

### Using Linear Programming to Make Crashing Decisions

The manual procedure described above is suitable for projects involving a small number of activities. For large-scale projects, linear programming can be used to make crashing decisions. The problem consists of minimizing the total cost of the project (including the extra cost of crashing activities), subject to the constraint that project duration must be less than or equal to the desired deadline. The decision variables are the start time of each activity, the reduction in the duration of each activity due to crashing, and the finish time of the project. The constraints are that the maximum reduction in time for each activity cannot be exceeded, the project finish time must be less or equal to the desired finish time; and the precedence relationships of all activities must be respected.

TABLE 8-6

Summary Output from the Crashing Procedure

Paths	Length After Crashing $x$ Weeks				
	0	1	2	3	4
Start-A-F-H	15	14	13	13	12
Start-A-C-E-H	28*	27*	26*	25*	24
Start-A-C-F-H	21	20	19	19	18
Start-B-C-F-H	25	25	25	25*	24
Start-B-C-E-H	26	26	26*	25*	24
Start-B-D-E-H	22	22	22	21	20
Start-B-D-G-H	19	19	19	19	18
Activity crashed		A	A	E	A,B
Crashing cost		\$3000	\$3000	\$7000	\$8000

For the new product development project, the linear programming will be formulated as follows, where  $X_j$  = starting time for activity  $j$  and  $Y_j$  = number of weeks by which activity  $i$  is crashed.

The cost for completing the project using normal times is fixed. Hence, the objective function is formulated as minimizing the project crashing:

$$\text{minimize } Z = 3000Y_A + 5000Y_B + 9000Y_C + 4000Y_D + 7000Y_E + 3000Y_F + 4500Y_G + 10\,000Y_H$$

**Precedence Relationship Constraints** To express the precedence relationships between activities, the following three relations are used for each activity: (1) Earliest start time  $\geq$  Finish time of preceding activity(ies); (2) Finish time = Earliest start time + activity duration; and (3) Activity duration = Normal activity time – number of weeks by which an activity is crashed. Therefore, the following constraints are developed:

$$\begin{aligned} X_A &= 0 && \text{(Earliest starting time for activity A)} \\ X_B &= 0 && \text{(Earliest starting time for activity B)} \\ X_C &\geq X_A + 5 - Y_A && \text{(C cannot start earlier than the completion of activity A)} \\ X_C &\geq X_B + 4 - Y_B && \text{(C cannot start earlier than the completion of activity B)} \\ X_D &\geq X_B + 4 - Y_B && \text{(D cannot start earlier than the completion of activity B)} \\ X_E &\geq X_C + 10 - Y_C && \text{(E cannot start earlier than the completion of activity C)} \\ X_E &\geq X_D + 5 - Y_D && \text{(E cannot start earlier than the completion of activity D)} \\ X_F &\geq X_A + 5 - Y_A && \text{(F cannot start earlier than the completion of activity A)} \\ X_F &\geq X_C + 10 - Y_C && \text{(F cannot start earlier than the completion of activity C)} \\ X_G &\geq X_D + 5 - Y_D && \text{(G cannot start earlier than the completion of activity D)} \\ X_H &\geq X_E + 7 - Y_E && \text{(H cannot start earlier than the completion of activity E)} \\ X_H &\geq X_F + 5 - Y_F && \text{(H cannot start earlier than the completion of activity F)} \\ X_H &\geq X_G + 4 - Y_G && \text{(H cannot start earlier than the completion of activity G)} \end{aligned}$$

**Maximum Reduction Constraints** This set of constraints refers to the limits by which each activity can be crashed.

$$Y_A \leq 3 \text{ (activity A can be crashed up to 3 weeks)}$$

$$Y_B \leq 2 \text{ (activity B can be crashed up to 2 weeks)}$$

$$Y_C \leq 4 \text{ (activity C can be crashed up to 4 weeks)}$$

$$Y_D \leq 2 \text{ (activity D can be crashed up to 2 weeks)}$$

$$Y_E \leq 1 \text{ (activity E can be crashed up to 1 week)}$$

$$Y_F \leq 2 \text{ (activity F can be crashed up to 2 weeks)}$$

$$Y_G \leq 2 \text{ (activity G can be crashed up to 2 weeks)}$$

$$Y_H \leq 3 \text{ (activity H can be crashed up to 3 weeks)}$$

**Project Completion Deadline** Since H is the finish activity of the project, its finishing time must be no longer than the project deadline, that is,  $X_H + 6 - Y_H \leq 24$  weeks.

**Non-negativity Constraints** This set of constraints stipulates that all variables must be non-negative, that is, all  $X_j$  and  $Y_j \geq 0$ ,  $j = A, B, C, D, E, F, G, H$ .

Exhibit 8-1 shows the Excel Solver layout for the new product development crashing project. The Excel's function SUMPRODUCT is used in the LHS column as well as in the Z value cell. The solution is shown in the bottom section of the Exhibit. It shows that activity A should be crashed by 2.5 weeks and activity C should be crashed by 1.5 week for a total crashing cost of \$21 000. Note that the Solver's solution has the same objective function as the manual procedure, but with a different solution. This indicates that the problem has multiple solutions.

## EXHIBIT 8-1

### Excel Solver Output for the Crashing Example

Exhibit 8-1 [Compatibility Mode] - Microsoft Excel

HomeInsertPage LayoutFormulasDataReviewView

PasteClipboard

Arial10

**B***I*U<sup>A</sup><sub>A</sub>

Font

Alignment

Accounting

\$- %

%0.00

Number

Conditional Formatting

Format as Table

Cell Styles

Styles

Insert

Delete

Format

Cells

Sort & Find & Filter & Select

Editing

B4

<



## 8.8 USING LINEAR PROGRAMMING IN PROJECT SCHEDULING

The project scheduling problem can also be formulated as a linear programming problem that seeks to determine the project completion time subject to meeting the precedence relationships between activities. To determine the EST and the EFT for activities the following model can be developed. Let  $X_j$  = earliest start time for activity  $j$ ,  $t_j$  = the duration for activity  $j$ , and  $P(j)$  = set of immediate predecessors of activity  $j$ . Therefore, the linear programming model consists of:

$$\text{minimize } Z = \sum_j X_j \quad (8-11)$$

Subject to

$$X_j \geq X_i + t_i, \text{ for all } j, \text{ for all } i \in P(j) \quad (8-12)$$

$$X_j \geq 0, \text{ for all } j \quad (8-13)$$

The objective function minimizes the sum of earliest start times of activities. Its value is only to ensure that each activity starts at the earliest time. Constraint 8-12 states that an activity cannot start unless all its immediate predecessors are completed. Constraint 8-13 is the non-negativity constraint.

To determine the LST and LFT for activities, the linear programming model can be written as follows, where  $W_j$  = latest start time for activity  $j$ :

$$\text{maximize } Z = \sum_j W_j \quad (8-14)$$

Subject to

$$W_j \geq W_i + t_i, \text{ for all } j, \text{ for all } i \in P(j) \quad (8-15)$$

$$W_j + t_j = EFT_j, \quad j = \text{finish activity} \quad (8-16)$$

$$W_j \geq 0, \text{ for all } j \quad (8-17)$$

The objective function to maximize the sum of all activity start times to ensure the latest activity start times. Constraint 8-15 defines the activity precedence relationships. Constraint 8-16 states the completion time of the finish activity in the project network.



## 8.9 PROJECT SCHEDULING SOFTWARE

For efficient management of large scale projects, many specialized project management software packages have been developed for scheduling and tracking project activities. Examples include Microsoft Project ([www.microsoft.com/office/project](http://www.microsoft.com/office/project)), Primavera (<http://www.primavera.com/products/p6/index.asp>), and Artemis Project (<http://www.aisc.com/>). Microsoft Project, a project management software program developed and sold by Microsoft, is one of the most-used products of this type in the market. It allows users to draw the project network, develop the project schedule, assign resources, track the project's progress, manage the budget, and analyze the project workload. As the focus of this chapter is how to develop a basic project schedule, the capabilities of these software packages will not be discussed further.

## Summary

This chapter introduced the critical path method (CPM) and the program evaluation review technique (PERT), two management science techniques developed in the late 1950s to plan, schedule, and control large, complex projects with many activities. We showed when and how these approaches can be used to help managers address questions such as what is the expected total time required to complete a project; what are the start and the completion times for individual activities; which critical activities must be completed as scheduled; how much delay can be tolerated for non-critical activities; what is the least expensive way to speed up a project; and in the case where the durations of activities are not known with certainty,

the probability to complete a project according to a given time frame and the variability in the project completion time. The chapter also showed how the network representation can be used to depict the project activities and their relationships, how to handle crashing decisions heuristically or using a linear programming model, and how the project scheduling problem can be formulated as a linear programming problem that seeks to determine the project completion time subject to meeting the precedence relationships between activities. Finally, a brief list was provided of specialized project management software packages that have been developed for scheduling and tracking project activities.

## Glossary

**Activity** A task that needs to be completed within a project and consumes both time and resources.

**Activity on arc (AOA)** A project network representation in which the project activities are reported on arcs and the nodes represent the starting or the completion of activities.

**Activity on node (AON)** A project network representation in which the project activities are reported on nodes and the arcs represent their precedence relationships.

**Backward pass** The process used to determine the latest start time and the latest finish time of an activity, which consists of moving backward through the project network from the finish node to the starting node.

**Beta probability distribution** A form of distribution used to represent the duration of an activity in PERT analysis.

**Critical path method (CPM)** An analysis approach used in project scheduling when the project inputs (e.g., activities duration and costs) are assumed to be known with certainty.

**Crashing** Speeding up or shortening the duration of an activity by using additional resources.

**Crash time** The shortest possible time to complete an activity with additional resources.

**Crash cost** The cost to crash an activity.

**Critical path(s)** The path(s) with the longest length in the project network.

**Critical activities** Activities in the critical path. These activities must be completed as scheduled to prevent delaying the project completion.

**Earliest start time (EST)** The earliest time at which an activity can start if no delays occur in the project.

**Earliest finish time (EFT)** The earliest time at which an activity can finish if no delays occur in the project.

**Forward pass** A process used to determine the earliest start time and the earliest finish time of an activity, which consists of moving forward through the project network from the starting node to the finish node.

**Immediate predecessors** Activities that must be completed prior to the starting time of a given activity.

**Immediate successors** Activities that follow the completion of a given activity.

**Latest start time (LST)** The latest time at which an activity can start without delaying the completion of the project.

**Latest finish time (LFT)** The latest time at which an activity can finish without delaying the completion of the project.

**Length of path** The sum of the durations of the activities on the path.

**Normal time** The estimated activity duration used with CPM or PERT in the computation of earliest (latest) start or finish times.

**Normal cost** The activity cost under the normal activity time.

**Path** A sequence of connected nodes in the network from the start node to the finish node.

**Program evaluation review technique (PERT)** An analysis approach used in project scheduling when some of the project inputs (e.g., activities duration and costs) are not known with certainty.

**Project** A set of activities interrelated by their precedence relationships that need to be undertaken in the accomplishment of a unique, often major outcome.

**Project crashing** The process of shortening the duration of a project by crashing the duration of a number of activities.

**Project network** A representation used to depict the project activities.

**Slack time** The length of time that can be tolerated for an activity without incurring a delay in the estimated project completion time.

Solved Problems

Problem 1

Expected durations (in weeks) and variances for the major activities of an R&D project are depicted in the precedence network diagram chart. Determine the probability that project completion time in Figure 8-10 will be

- a. Less than 50 weeks.
- b. More than 50 weeks.

Solution

Because S and End have zero durations, we can ignore them in the following calculations. The mean and standard

deviation for each path are shown in Table 8-7. Path S-1-5-8-End is the critical path. Therefore, the project expected completion time is 51 weeks and the corresponding variance is 1.488 weeks.

- a. The  $z$  value for the normal distribution at  $x = 50$  is  $z = \frac{50 - t_{S1}}{1.22} = -.82$ . Hence,  $P(x \leq 50) = .2061$ .
- b.  $P(x > 50) = 1 - P(x \leq 50) = .7939$ .

FIGURE 8-10

Precedence Network Diagram Chart for Problem 1

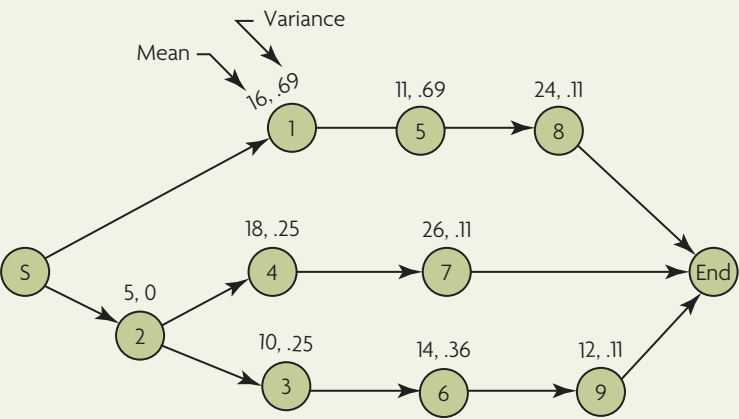


Table 8-7 Probabilities for Problem 1

Path	Expected Time (weeks)	Standard Deviation (weeks)
S-1-5-8-End	$16 + 11 + 24 = 51$	$\sqrt{.69 + .69 + .11} = 1.22$
S-2-4-7-End	$5 + 18 + 26 = 49$	$\sqrt{.00 + .25 + .11} = .60$
S-2-3-6-9-End	$5 + 10 + 14 + 12 = 41$	$\sqrt{.00 + .25 + .36 + .11} = .85$



**Problem 2**

Table 8-8 shows the information related to a project that involves the merger of two marketing firms (in days).

**Table 8-8** Data for Solved Problem 2

Activity	Immediate predecessor(s)	Estimated duration(days)
A	—	10
B	—	15
C	A	5
D	B	12
E	C, D	14
F	B	8
G	D, F	15
H	E	10
I	E, G	6
J	F, I	9

**Solution**

- a. The project network is shown in Figure 8-11.  
b. The project schedule is as follows:

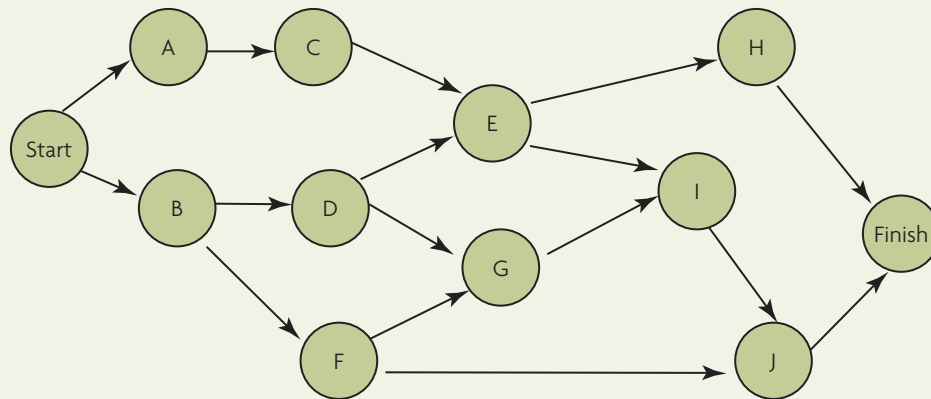
Activity	Earliest Start	Latest Start	Earliest Finish	Latest Finish	Slack
Start	0	0	0	0	0
A	0	13	10	23	13
B	0	0	15	15	0
C	10	23	15	28	13
D	15	15	27	27	0
E	27	28	41	42	1
F	15	19	23	27	4
G	27	27	42	42	0
H	41	47	51	57	6
I	42	42	48	48	0
J	48	48	57	57	0
Finish	57	57	57	57	0

- a. Draw the project network.  
b. Develop the project schedule (EST, EFT, LST, LFT).  
c. What are the critical activities?  
d. What is the project completion duration?  
e. If there is an option to delay one activity without delaying the entire merge project, which activity would you delay and why?  
f. Formulate the LP model that would determine the earliest start times of activities.
- c. The critical activities are: B-D-G-I-J  
d. The project completion duration is 57 days.  
e. Activity A or C since they have the largest slack times.  
f. The LP formulation of the model is as follows, where  $X_j$  = earliest start time for activity  $j$  = A, B, C, D, E, F, G, H, I, and J.

$$\text{minimize } Z = X_A + X_B + X_C + X_D + X_E + X_F + X_G + X_H + X_I + X_J$$

**FIGURE 8-11**

Network for Solved Problem 2



subject to

$$\begin{aligned} X_C &\geq X_A + 10 \\ X_D &\geq X_B + 15 \\ X_E &\geq X_C + 5 \\ X_E &\geq X_D + 12 \\ X_F &\geq X_B + 15 \\ X_G &\geq X_D + 12 \\ X_G &\geq X_F + 8 \\ X_H &\geq X_E + 14 \\ X_I &\geq X_E + 14 \\ X_I &\geq X_G + 15 \\ X_J &\geq X_F + 8 \\ X_J &\geq X_I + 6 \\ X_A &= X_B = 0 \end{aligned}$$

All variables  $\geq 0$

Problem 3

Consider the network information shown in the previous problem (Problem 2) and assume that the duration of some activities is not known with certainty. The estimates of these activities are shown below, assuming that the duration for the other activities remains unchanged.

Activity	Optimistic	Most Likely	Pessimistic
A	8	10	12
C	3	5	7
D	10	12	14
G	13	15	17
H	8	10	12

- a. What is the critical path?
- b. What is the project’s expected completion time and its variance?
- c. What is the probability that the project will be completed in 60 days or more? In no more than 55 days?
- d. If the company wants a 96% probability of completing the project on time, state the latest time each activity should have started and completed.

Solution

- a. The mean and variance for activities for which the duration is not known with certainty:

Activity	a	m	b	Mean, $t_e = \frac{a+4m+b}{6}$	Variance, $\sigma^2 = \left(\frac{b-a}{6}\right)^2$
A	8	10	12	10	0.44
C	3	5	7	5	0.44
D	10	12	14	12	0.44
G	13	15	17	15	0.44
H	8	10	12	10	0.44

The critical path is Start – B – D – G – I – J – Finish.

- b. The expected project completion time is 57 days and its variance is  $.44 + .44 = .88$ .

- c. The probability that the project will be completed in 60 days or more:  $P(X \geq 60) = P(X \geq z)$ , where

$$z = \frac{60-57}{\sqrt{0.88}} = 3.198.$$
 Therefore from Table A in

Appendix B,  $P(X \geq 3.198) = 0.5 - 0.4990 = 0.001$  or 0.1 percent.

The probability that the project will be completed in 55 days or less:  $P(X \leq 50) = P(X \leq z)$ , where

$$z = \frac{55-57}{\sqrt{0.88}} = -2.13.$$
 Therefore from Table A in

Appendix B,  $P(X \geq -2.13) = 0.5 - .4788 = .0212$  or 2.12 percent.

- d.  $z_{0.96} = 1.76$ . Using Equation 8-9, the project completion time  $X = 57 + 1.76 \times .938 = 58.65$  days. Therefore to meet this completion time, the latest start and finish times for each activity should be as follows:

Activity	Latest Start	Latest Finish
Start	1.65	1.65
A	14.65	24.65
B	1.65	16.65
C	24.65	29.65
D	16.65	28.65
E	29.65	43.65
F	20.65	28.65
G	28.65	43.65
H	48.65	58.65
I	43.65	49.65
J	49.65	58.65
Finish	58.65	58.65

Problem 4

Indirect cost for a project is \$12 000 per week for as long as the project lasts. The project manager has supplied the cost and time information and precedence network diagram shown in Figure 8-12.

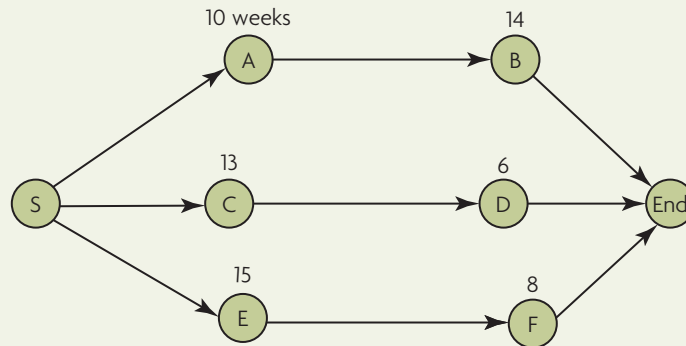
Activity	Crashing Potential (weeks)	Cost per Week to Crash
A	3	\$11 000
B	3	3000 first week, \$4000 after that
C	2	6000
D	1	1000
E	3	6000
F	1	2000

Use the information to:

- a. Determine an optimum crashing plan.
- b. Graph the total costs for the plan.

FIGURE 8-12

Network Diagram for Solved Problem 4

**Solution**

- a. 1. Calculate path lengths and identify the critical path:

Path	Duration (weeks)
S-A-B-End	24 (critical path)
S-C-D-End	19
S-E-F-End	23

2. Rank critical activities according to crash costs:

Activity	Cost per Week to Crash
B .....	\$3000 first week, \$4000 after that
A .....	11 000

Activity B should be shortened one week since it has the lower crashing cost per week. This would reduce indirect costs by \$12 000 at a cost of \$3000, for a net savings of \$9000. At this point, paths S-A-B-End and S-E-F-End would both have a length of 23 weeks, so both would be critical.

3. Rank activities by crashing cost on the two critical paths:

Path	Activity	Cost per Week to Crash
S-A-B-End .....	B	\$4000
.....	A	11 000
S-E-F-End .....	F	2000
.....	E	6000

Choose one activity (the least costly) on each path to crash: B on S-A-B-End and F on S-E-F-End, for a total cost of \$4000 + \$2000 = \$6000 and a net savings of \$12 000 - \$6000 = \$6000.

Note: There is no activity common to the two critical paths.

4. Check to see which path(s) might be critical: S-A-B-End and S-E-F-End would be 22 weeks in length, and S-C-D-End would still be 19 weeks.
5. Rank activities on the critical paths:

Path	Activity	Cost per Week to Crash
A-B .....	B	\$4000
.....	A	11 000
E-F .....	E	6000
.....	F	(no further crashing possible)

Crash B on path S-A-B-End and E on S-E-F-End for a cost of \$4000 + \$6000 = \$10 000, for a net savings of \$12 000 - \$10 000 = \$2000.

6. At this point, no further reduction is cost-effective: paths S-A-B-End and S-E-F-End would be 21 weeks in length, and one activity from each path would have to be shortened. This would mean activity A at \$11 000 and E at \$6000 for a total of \$17 000, which exceeds the \$12 000 potential savings in indirect costs. Note that no further crashing for activity B is possible.

- b. The following table summarizes the results, showing the length of the project after crashing n weeks:

Path	n = 0	1	2	3
S-A-B-End .....	24	23	22	21
S-C-D-End .....	19	19	19	19
S-E-F-End .....	23	23	22	21
Activity crashed .....		B	B,F	B,E
Crashing costs (\$000) .....		3	6	10

A summary of costs for the preceding schedule would look like Table 8-9.

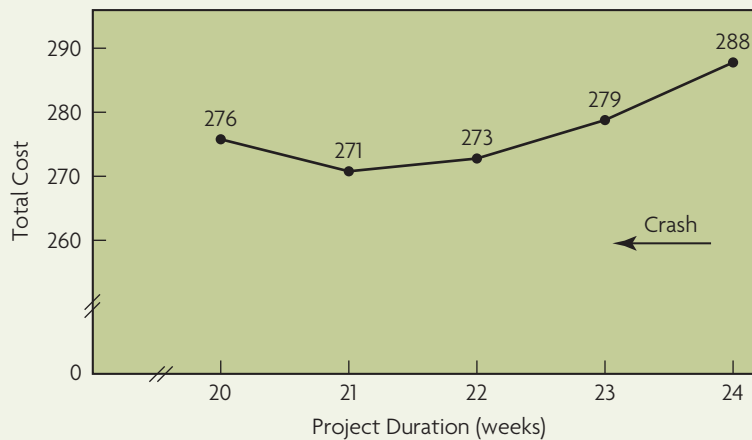
The graph of total cost is shown in Figure 8-13.

**Table 8-9** Summary of Costs for Solved Problem 4

Project Length	Cumulative Weeks Shortened	Cumulative Crashing Costs (\$000)	Indirect Costs(\$000)	Total Costs (\$000)
24	0	0	$24(12) = 288$	288
23	1	3	$23(12) = 276$	279
22	2	$3 + 6 = 9$	$22(12) = 264$	273
21	3	$9 + 10 = 19$	$21(12) = 252$	271
20	4	$19 + 17 = 36$	$20(12) = 240$	276

**FIGURE 8-13**

Total Cost for Solved Problem 4



## Discussion and Review Questions

1. Explain differences and similarities between CPM and PERT.
2. What are the different types of information that need to be gathered to depict a project network?
3. Explain briefly two approaches that can be used to determine the project critical path(s) and when each is the most suitable.
4. What is the meaning of activity slack times and how are they computed?
5. Explain why the project manager should monitor the progress of activities with small slack times.
6. Why is it important in project management to identify and monitor the critical activities?
7. When is simulation approach the most appropriate in project scheduling?
8. Why must all immediate predecessor activities be considered when determining the activity's earliest start time?
9. Why must all immediate successor activities be considered when finding the activity's latest finish time?
10. What is project crashing and how can it be done manually?
11. Provide some of the reasons for a company to consider project crashing.
12. What constraints are involved in formulating the project crashing problem as a linear programming model?
13. What constraints are involved in formulating the project scheduling problem as a linear programming model?

## Problems

- For the precedence network diagram in Figure 8-14, determine both the critical path and the project duration by determining the length of each path. The numbers above the nodes represent activity duration in days.
- Using the data in Table 8-10 construct a precedence network diagram. The project is completed when activities H, I, and J are all finished.
- Using the data in Table 8-11 construct a precedence network diagram. The project is completed when activities D, F, J, and K are all finished.
- ET+ is an entertainment group that specializes in the management of band tours around the world. The company is considering placing a bid for the management of the next 2-day concert of a very popular rock band in the city of Toronto. As part of the planning process, the company has determined that the list of activities in Table 8-12 would need to be performed to carry out the project. Construct the precedence network diagram that can be used in the scheduling of these activities.

FIGURE 8-14

Network Diagram for Problem 1

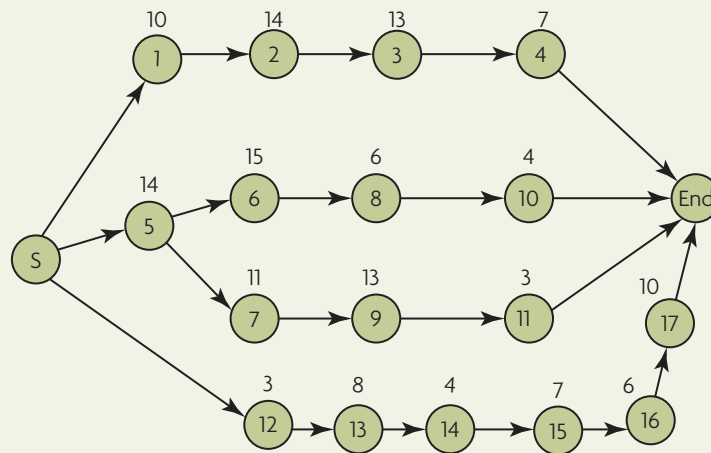


Table 8-10 Data for Problem 2

Activity	A	B	C	D	E	F	G	H	I	J
Immediate Predecessors	—	—	—	A	B	B	C	F	F, G	D, E

Table 8-11 Data for Problem 3

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Immediate Predecessors	—	—	A	C	A	E	—	B	G	H, I	J, L	—

Table 8-12 Data for Problem 4

Activity	A	B	C	D	E	F	G	H	I
Immediate Predecessors	—	A	A	B	B	C, E	D, E	F, G	H

5. Assume that the activities in Problem 4 have the duration (in days) as shown in Table 8-13.

a. What is the maximum number of working days the company would have to write in their bid to complete all activities of this project if no delays occur?

b. What are the critical activities?

c. When would each activity have to start and finish at the earliest to complete this project on time?

d. When would each activity have to start and finish at the latest to complete this project on time?

6. Softbank is a consulting company that specializes in the customization of software in the banking industry. The company has been hired to serve on a project that involves the replacement of the user interface currently in place for online banking in the website of a large Canadian bank. It has been determined that the list of activities in Table 8-14 would need to be performed to carry out the project.  
Construct the precedence network diagram that can be used in the scheduling of these activities.

7. Assume that activities in Problem 6 have the durations (in days) as shown in Table 8-15.

a. Determine the following values for each activity: the earliest start time, the earliest finish time, the latest start time, the latest finish time, and the activity slack time.

b. Identify the critical activities, and determine the expected duration of the project. What is the maximum number of working days the company would have to write in their bid to complete all project activities if no delays occur?

8. Reconsider the list of activities in Problem 6 and their corresponding durations in Problem 7. Suppose that after 12 days, activities A, B, and I have been finished, activity E is 75 percent finished, and activity J is half finished. How many days after the original start time would the project finish?

9. The following table contains information related to the major activities of a research project. Use the information to do the following:

a. Draw a precedence network diagram.

b. Find the critical path by identifying all the start to end paths and calculating their lengths.

c. What is the expected duration of the project?

10. Chris received a new word-processing software program for her birthday. She also received a cheque, with which she intends to purchase a new computer. Chris's university instructor assigned a paper due next week. Chris decided that she will prepare the paper on the new computer. She made a list of the activities and

Table 8-13 Data for Problem 5

Activity	A	B	C	D	E	F	G	H	I
Duration (in days)	5	18	13	3	10	11	4	9	2

Table 8-14 Data for Problem 6

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M
Immediate Predecessors	—	A	B	C	A	E	F	D, G	—	I	J	K	L

Table 8-15 Data for Problem 7

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M
Duration (in days)	4	9	5	2	8	7	2	3	10	6	4	5	6

their estimated durations. Chris's friend has offered to shop for, select and purchase a computer, and install the software.

- Arrange the activities into two logical sequences.
- Construct a precedence network diagram.
- Determine the critical path and its expected duration.
- What are some possible reasons for the project to take longer than the expected duration?

#### Estimated

Time (hours)	Activity (abbreviation)
.8	Install software (Install)
.4	Outline the paper (Outline)
.2	Submit paper to instructor (Submit)
.6	Choose a topic (Choose)
.5	Use grammar-checking routine and make corrections (Check)
3.0	Write the paper using the word-processing software (Write)
2.0	Shop for a new computer (Shop)
1.0	Select and purchase computer (Select)
2.0	Library research on chosen topic (Library)

- The information in the following table pertains to a project that is about to commence.

Activity	Immediate Predecessor(s)	Estimated Duration (days)
A.....	—	15
B.....	A	12
C.....	B	6
D.....	B	5
E.....	C	3
F.....	—	8
G.....	F	8
H.....	F	9
I.....	G	7
J.....	H	14
K.....	J	6

- As the project manager, which activities would you be concerned with in terms of timely project completion? Explain.
  - Determine the following values for each activity: the earliest start time, the earliest finish time, the latest start time, the latest finish time, and the activity slack time.
- Three recent university graduates have formed a partnership and have opened an advertising firm.

Their first project consists of activities listed in the following table.

- Draw the precedence network diagram.
- What is the probability that the project can be completed in 24 days or less? In 21 days or less?
- Suppose it is now the end of the seventh day and that activities A and B have been completed while activity D is 50 percent completed. Optimistic, most likely, and pessimistic estimates for the completion of activity D are now 5, 6, and 7 days. Activities C and H are ready to begin. Determine the probability of finishing the project by day 24 and the probability of finishing by day 21.

Activity	Immediate Predecessor(s)	DURATION IN DAYS		
		Optimistic	Most Likely	Pessimistic
A.....	—	5	6	7
B.....	—	8	8	11
C.....	A	6	8	11
D.....	—	9	12	15
E.....	C	5	6	9
F.....	D	5	6	7
G.....	F	2	3	7
H.....	B	4	4	5
I.....	H	5	7	8

- The new director of special events at a large university has decided to completely revamp graduation ceremonies. Toward that end, a precedence network diagram of the major activities has been developed. The chart has five paths with expected durations and variances as shown in the following table. Graduation day is 16 full weeks from now. Assuming that the project begins now, what is the probability that the project will be completed before:

- Graduation time?
- The end of week 15?
- The end of week 13?

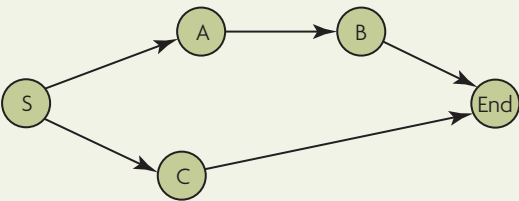
Path	Expected Duration (weeks)	Variance
A.....	10	1.21
B.....	8	2.00
C.....	12	1.00
D.....	15	2.89
E.....	14	1.44

- What is the probability that the following project will take more than 10 weeks to complete if the precedence network diagram, activity means, and



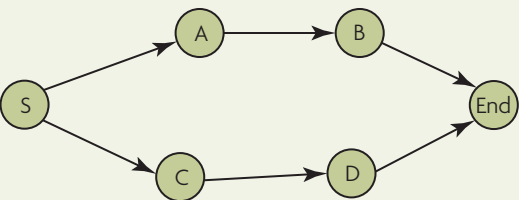
standard deviations (both in weeks) are as shown below?

Activity	Mean	Standard Deviation
A	5	1.3
B	4	1.0
C	8	1.6



15. The project described in the following table and precedence network diagram has just begun. It is scheduled to be completed in 11 weeks.
- a. If you were the manager of this project, would you be concerned? Explain.
  - b. If there is a penalty of \$5000 a week for each week the project is late, what is the probability of incurring a penalty of at least \$5000?

Activity	Expected Duration (weeks)	Standard Deviation (weeks)
A	4	.70
B	6	.90
C	3	.62
D	9	1.90



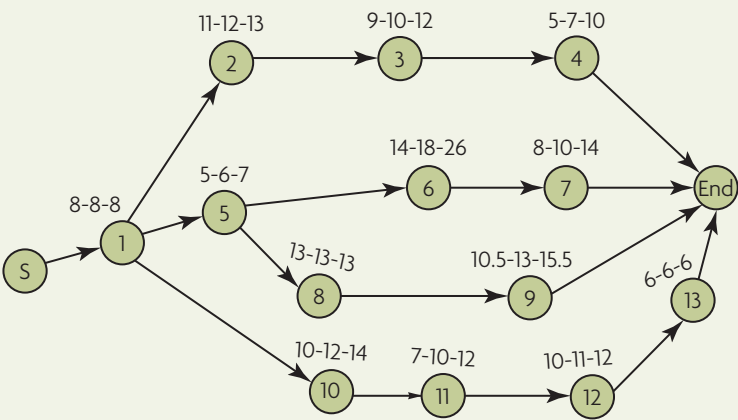
16. The precedence network diagram in Figure 8-15 reflects optimistic, most likely, and pessimistic estimates for each activity. Determine:
- a. The expected completion time for each path and its variance.
  - b. The probability that the project will require more than 49 weeks.
  - c. The probability that the project can be completed in 46 weeks or fewer.
17. A project manager has compiled a list of major activities that will be required to install a computer information system in her company. The list includes three estimates of durations (optimistic, most likely, pessimistic) for activities and precedence relationships.

Activity	Precedes	3-Point Estimates (weeks)
A	—	2-4-6
D	A	6-8-10
E	D	7-9-12
H	E	2-3-5
F	A	3-4-8
G	F	5-7-9
B	—	2-2-3
I	B	2-3-6
J	I	3-4-5
K	J	4-5-8
C	—	5-8-12
M	C	1-1-1
N	M	6-7-11
O	N	8-9-13

If the project is finished within 26 weeks of its start, the project manager will receive a bonus of \$1000; and if the project is finished within 27 weeks of its start, the bonus will be \$500. Find the probability of each bonus.

FIGURE 8-15

Precedence Network Diagram for Problem 16



18. The project manager of a task force planning the construction of a domed stadium had hoped to be able to complete construction prior to the start of the next season. After reviewing construction duration estimates, it now appears that a certain amount of crashing will be needed to ensure project completion before the season opener. Given the following information, determine a minimum-cost crashing schedule that will shave five weeks off the project length.

Activity	Precedes	Normal Duration (weeks)	CRASHING COSTS	
			First Week	Second Week
A.....	—	12	\$15 000	\$20 000
B.....	A	14	10 000	10 000
C.....	—	10	5000	5000
D.....	C	17	20 000	21 000
E.....	C	18	16 000	18 000
F.....	C	12	12 000	15 000
G.....	D	15	24 000	24 000
H.....	E	8	—	—
I.....	F	7	30 000	—
J.....	I	12	25 000	25 000
K.....	B	9	10 000	10 000
M.....	G	3	—	—
N.....	H	11	40 000	—
P.....	H, J	8	20 000	20 000

19. A construction project has indirect costs totalling \$40 000 per week. Major activities in the project and their expected durations are shown in the precedence network diagram in Figure 8-16:

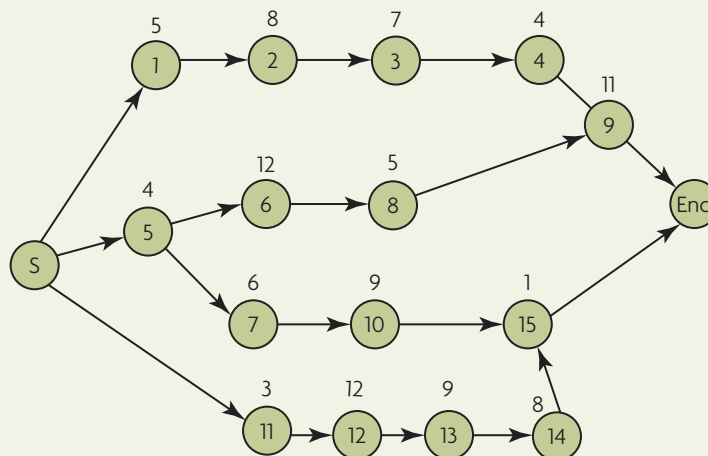
Crashing costs for each activity are:

Activity	CRASHING COSTS (\$000)		
	First Week	Second Week	Third Week
1.....	\$18	\$22	\$—
2.....	24	25	25
3.....	30	30	35
4.....	15	20	—
9.....	30	33	36
5.....	12	24	26
6.....	—	—	—
8.....	40	40	40
7.....	3	10	12
10.....	2	7	10
15.....	26	—	—
11.....	10	15	25
12.....	8	13	—
13.....	5	12	—
14.....	14	15	—

- a. Determine a minimum-cost crashing plan that will take off six weeks from the project duration.
- b. Plot the total-cost curve from part a against project duration. What is the optimum number of weeks to crash?
20. Chuck's Custom Boats (CCB) builds luxury yachts to customer order. CCB has landed a contract with a Vancouver businessman (Mr. P). Relevant data are shown below. The complication is that Mr. P wants delivery in 32 weeks or he will impose a penalty of \$375 for each week his yacht is late.

FIGURE 8-16

Precedence Network Diagram for Problem 19



Activity	Immediate Predecessor(s)	Normal Duration (weeks)	CRASHING COSTS	
			1st Week	2nd Week
K	—	9	\$410	\$415
L	K	7	125	—
N	K	5	45	45
M	L	4	300	350
J	N	6	50	—
Q	M, J	5	200	225
P	Q	8	—	—
Y	Q	7	85	90
Z	P	6	90	—

**Table 8-16** Data for Problem 21

A List of Activities in a QS-9000 Registration Project			
Activity	Description	Immediate Predecessor(s)	Estimated time*
A	Appointment of QS-9000 taskforce	none	1 week
B	Preparation of a feasible plan	A	1 week
C	Delegation of authority responsibilities	B	1 week
D	Searching for a QS-9000 registrar	C	1 week
E	Preparation of three levels of documentation	C	12 weeks
F	QS-9000 awareness training	C	6 weeks
G	QS-9000 training of auditors and quality personnel	F	6 weeks
H	Preparing the plant for QS-9000 registrar	C	24 weeks
I	Conference with lead auditor	D	1 week
J	Examination of documentation	E, I	3 weeks
K	Internal audit of plant sections	G, H, J	12 weeks
L	Corrective actions of plant sections	K	12 weeks
M	Lead auditor and audit team audit plant	L	1 week
N	Audit conference and corrective action plan	M	2 weeks
O	Implementation of corrective action plans	N	12 weeks
P	Lead auditor re-audit corrective actions	O	2 weeks
Q	Lead auditor's recommendation	P	1 week

\*These are estimates of time and may vary based upon situation, company, and registrar.

Develop a minimum cost crashing schedule.

21. Table 8-16 is a list of activities and their expected duration, used by a component supplier to automobile manufacturers, to plan for QS9000 (the auto industry version of ISO9000) certification (registration).
- Draw the precedence network diagram.
  - Determine the earliest and latest times, and identify the critical activities and the project duration.

## Case 1: Fantasy Products

### Company Background

The Fantasy Products Company (disguised name) is a manufacturer of high-quality small appliances intended for home use. Their current product line includes irons, a small hand-held vacuum, and a number of kitchen appliances such as toasters, blenders, waffle irons, and coffee makers. Fantasy Products has a strong R&D department that continually searches for ways to improve existing products as well as developing new products.

Currently, the R&D department is working on the development of a new kitchen appliance that will chill

foods quickly much as a microwave oven heats them quickly, although the technology involved is quite different. Tentatively named The Big Chill, the product will initially carry a price tag of around \$125, and the target market consists of upper-income consumers. At this price, it is expected to be a very profitable item. R&D engineers now have a working prototype and are satisfied that, with the cooperation from the production and marketing people, the product can be ready in time for the all-important Christmas buying season. A target date has been set for product introduction that is 24 weeks away.

Current Problem

Fantasy Products’ Marketing Vice-President Vera Sloan has recently learned from reliable sources that a competitor is also in the process of developing a similar product, which it intends to bring out at almost the same time. In addition, her source indicated that the competitor plans to sell its product, which will be smaller than The Big Chill, for \$99 in the hope of appealing to more customers. Vera, with the help of several of her key people who are to be involved in marketing The Big Chill, has decided that to compete, the selling price for The Big Chill will have to be lowered to within \$10 of the competitor’s price. At this price level it will still be profitable, although not nearly as profitable as originally anticipated.

However, Vera is wondering whether it would be possible to expedite the usual product introduction process to beat the competition to the market. If possible, she would like to get a six-week jump on the competition; this would put the product introduction date only 18 weeks away. During this initial period, Fantasy Products could sell The Big Chill for \$125, reducing the selling price to \$109 when the competitor’s product actually enters the market. Since forecasts based on market research show

that sales during the first six weeks will be about 400 per week, there is an opportunity for \$25 per unit profit if the early introduction can be accomplished. In addition, there is a certain amount of prestige involved in being first to the market. This should help enhance The Big Chill’s image during the anticipated battle for market share.

Data Collection

Since Fantasy Products has been through the product-introduction process a number of times, Vera has developed a list of the tasks that must be accomplished and the order in which they must be completed. Although the duration and costs vary depending on the particular product, the basic process does not. The list of activities involved and their precedence relationships are presented in Table 8-17. Duration and cost estimates for the introduction of The Big Chill are presented in Table 8-18. Note that some of the activities can be completed on a crash basis, with an associated increase in cost. For example, activity B can be crashed from 8 weeks to 6 weeks at an additional cost of \$3000 (i.e., \$12 000–\$9000). Assume that if B is crashed to 7 weeks, the additional cost will be \$1500 (i.e., \$3000/2).

TABLE 8-17

List of Activities and Precedence Relationships

Activity	Description	Immediate Predecessor(s)
A	Select and order equipment	—
B	Receive equipment from supplier	A
C	Install and set up equipment	B
D	Finalize bill of materials	—
E	Order component parts	D
F	Receive component parts	E
G	First production run	C, F
H	Finalize marketing plan	—
I	Produce magazine ads	H
J	Script for TV ads	H
K	Produce TV ads	J
L	Begin ad campaign	I, K
M	Ship product to consumers	G, L

TABLE 8-18 Duration and Cost Estimates

Activity	Normal Duration (weeks)	Normal Cost	Crash Duration (weeks)	Normal & Crash Cost
A	3	\$2000	2	\$4400
B	8	9000	6	12 000
C	4	2000	2	7000
D	2	1000	1	2000
E	2	2000	1	3000
F	5	0	5	0
G	6	12 000	3	24 000
H	4	3500	2	8000
I	4	5000	3	8000
J	3	8000	2	15000
K	4	50 000	3	70 000
L	6	10 000	6	10 000
M	1	5000	1	5000

Managerial Report

Fantasy Products needs to decide whether to bring The Big Chill to market 18 weeks from now as Vera Sloan is recommending. As the project management specialist in the R&D department, you have been asked to answer the following questions:

- 1. When would the project be completed using normal durations?

- 2. Is it possible to complete the project in 18 weeks? What would the associated additional cost be? Which activities would need to be completed on a crash basis?
- 3. Is there some time frame shorter than the 18 weeks Vera has recommended that would make more sense in terms of profits?

**Source:** Adapted from an original case by Robert J. Thieraus, Margaret Cunningham, and Melanie Blackwell, Xavier University, Cincinnati, Ohio.