CHAPTER 20

Statistical Quality Control

GOALS

When you have completed this chapter, you will be able to:

- Discuss the role of quality control in production and service operations
- Define and understand the terms *chance cause*, *assignable cause*, *in control*, *out of control*, *attribute*, and *variable*
- Construct and interpret a *Pareto chart*
- Construct and interpret a *fishbone diagram*
- Construct and interpret a *mean* and *range chart*
- Construct and interpret a *percent defective* and a *c-bar chart*
- Discuss *acceptance sampling*
- Construct an *operating characteristic curve* for various sampling plans.

With the advent of industrial revolution in the 19th century, mass production replaced manufacturing in small shops by skilled craftsman and artisans. While in the small shops the individual worker was completely responsible for the quality of the work, this was no longer true in mass production where each individual's contribution to the finished product constituted only an insignificant part in the total process. The quality control by the large companies was achieved with the help of quality inspectors responsible for checking a 100 percent inspection of all the important characteristics.

Dr. Walter A. Shewhart, called the father of quality control analysis, developed the concepts of statistical quality control. For the purpose of controlling quality, Shewhart developed charting techniques and statistical procedures for controlling in-process manufacturing operations. His statistical procedures are based on the concept of independent and identically distributed random variables. Based on these concepts, he distinguished between chance causes, producing random variation intrinsic to the process, and assignable causes, one should look for and take requiring corrective actions. In order to identify the assignable causes, he recommended 3σ (standard deviation) limits above and below the mean value of the variable. Observations beyond these limits were then used to identify the assignable causes.

Shewharts' other contributions to statistics include his emphasis on providing full evidence in reporting research results and applications of measurement processes in science, particularly physics.

Shewhart served as consultant to several organizations including the War Department, the United Nations, and the Indian government. He also taught at the universities of Illinois and California, and was involved with various other universities such as Harvard, Rutgers, and Princeton.

W. Edwards Deming, one of Shewharts's students—now more popular than his mentor in the arena of quality control, carried on Shewhart's work on statistical quality control to new heights. Deming's contributions include not only a further development of procedures, but also a new philosophy, popularly known as Deming's 14 points in modern statistical quality control literature.

INTRODUCTION

Throughout this text we have presented many applications of hypothesis testing. In Chapter 10 we described methods for testing a hypothesis regarding a single population value. In Chapter 11 we described methods for testing a hypothesis about two populations. In this chapter we present another, somewhat different application of hypothesis testing, called **statistical process control** or **SPC**.

Statistical process control is a collection of strategies, techniques, and actions taken by an organization to ensure they are producing a quality product or providing a quality service. It begins at the product planning stage, when we specify the attributes of the product or service. It continues through the production stage. Each attribute throughout the process contributes to the overall quality of the product. To effectively use quality control, measurable attributes and specifications must be developed against which the actual attributes of the product or service can be compared.

20.1 STATISTICAL QUALITY CONTROL AND EDWARDS DEMING'S 14 POINTS PROGRAM

Statistical quality control really came into its own during World War II. The need for mass-produced war-related items, such as bomb sights, accurate radar, and other electronic equipment, at the lowest possible cost hastened the use of statistical sampling and quality control charts. Since World War II these statistical techniques have been refined and sharpened. The use of computers in the last decade has also widened the use of these techniques.

World War II virtually destroyed the Japanese production capability. Rather than retool their old production methods, the Japanese enlisted the aid of the late Dr. W. Edwards Deming, of the United States Department of Agriculture, to help them develop an overall plan. In a series of seminars with Japanese planners he stressed a philosophy that is known today as Deming's 14 points. These 14 points are listed on the following page. He emphasized that quality originates from improving the process, not from inspection, and that quality is determined by the customers. The manufacturer must be able, via market research, to anticipate the needs of customers. Upper management has the responsibility for long-term improvement. Another of his points, and one that the Japanese strongly endorsed, is that every member of the company must contribute to the long-term improvement. To achieve this improvement, ongoing education and training are necessary.

Deming had some ideas that did not mesh with contemporary management philosophies in the West. Two areas where Deming's ideas differed from Western management philosophy were with production quotas and merit ratings. He claimed these two practices, which are both common in the West, are not productive and should be eliminated. He also pointed out that Western managers are mostly interested in good news. Good news, however, does not provide an opportunity for improvement. On the other hand, bad news opens the door for new products and allows for company improvement.

Listed below, in a condensed form, are Dr. Deming's 14 points. He was adamant that the 14 points needed to be adopted as a package in order to be successful. The underlying theme is cooperation, teamwork, and the belief that workers want to do their jobs in a quality fashion.

1. Create constancy of purpose for the continual improvement of products and service to society.
2. Adopt a philosophy that we can no longer live with commonly accepted levels of delays, mistakes, defective materials, and defective workmanship.
3. Eliminate the need for mass inspection as the way to achieve quality. Instead achieve quality by building the product correctly in the first place.
4. End the practice of awarding business solely on the basis of price. Instead, require meaningful measures of quality along with the price.
5. Improve constantly and forever every process for planning, production, and service.
6. Institute modern methods of training on the job for all employees, including managers. This will lead to better utilization of each employee.
7. Adopt and institute leadership aimed at helping people do a better job.
8. Encourage effective two-way communication and other means to drive out fear throughout the organization so that everyone may work more effectively and more productively for the company.
9. Break down barriers between departments and staff areas.
10. Eliminate the use of slogans, posters, and exhortations demanding zero defects and new levels of productivity without providing methods.
11. Eliminate work standards that prescribe quotas for the workforce and numerical goals for people in management. Substitute aids and helpful leadership in order to achieve continual improvement in quality and productivity.
12. Remove the barriers that rob hourly workers and the people in management of their right to pride of workmanship.
13. Institute a vigorous program of education and encourage self-improvement for everyone. What an organization needs is good people and people who are improving with education. Advancement to a competitive position will have its roots in knowledge.
14. Define clearly management's permanent commitment to ever-improving quality and productivity to implement all of these principles.

Deming's 14 points did not ignore statistical quality control, which is often abbreviated as SQC, TQC, or just QC. The objective of statistical quality control is to monitor production through many stages of manufacturing. We use the tools of statistical quality control, such as X-bar and R charts, to monitor the quality of many processes and services. Control charts allow us to identify when a process or service is "out of control," that is, when the point is reached where an excessive number of defective units are being produced.

Interest in quality has accelerated dramatically since the late 1980s. Turn on the television and watch the commercials sponsored by Ford, Nissan, and GM to verify the emphasis on quality control on the assembly line. It is now one of the "in" topics in all facets of business. V. Daniel Hunt, president of Technology Research Corporation, wrote in his book *Quality in America* that in the United States, 20 to 25 percent of the cost of production is currently spent finding and correcting mistakes. And, he added, the additional cost incurred in repairing or replacing faulty products in the field drives the total cost of poor quality to nearly 30 percent. In Japan, he indicates, this cost is about 3 percent!

In recent years companies have been motivated to improve quality by the challenge of being recognized for their quality achievements. The Malcolm Baldrige National Quality Award, established in 1988, is awarded annually to U.S. companies that demonstrate excellence in quality achievement and management. The award categories include manufacturing, service, and small business. Past winners include Motorola, Xerox, IBM, Federal Express, and Cadillac. The 2002 award winners were Motorola Inc., Commercial, Government and Industrial Solutions Sector from Schaumberg, Illinois (manufacturing category); Branch-Smith Printing Division, Fort Worth, Texas (small business category); and SSM Health Care, St. Louis, Missouri (health care category). You can obtain more information on the 2002 winners and other winners by visiting the Web site: www.quality.nist.gov.

Canada's National Quality Institute (NQI) similarly awards organizations that demonstrate sustainable measures of continuous improvement. For the past 19 years, Canada Awards for Excellence have been awarded for Quality. (An new category, "Healthy Workplace" has been awarded since 1999.) The 2002 recipients of the Canada Awards for Excellence—Quality Award Trophy were Dana Canada Inc., Spicer Driveshaft Group, Magog, Quebec; Canada Post, Saskatoon Operations; Homewood Health Centre, Guelph Ontario; and Mullen Trucking, Aldersyde, Alberta. To find out more about these and other winners, visit the Web site www.nqi.ca.

What is quality? There is no commonly agreed upon definition of quality. To cite a few diverse definitions: From Westinghouse, "Total quality is performance leadership in meeting the customer requirements by doing the right things right the first time." From AT&T, "Quality is meeting customer expectations." Historian Barbara W. Tuchman says, "Quality is achieving or reaching the highest standard as against being satisfied with the sloppy or fraudulent."

20.2 CAUSES OF VARIATION

No two parts are *exactly* the same. There is always some variation. The weight of each McDonald's Quarter Pounder is not exactly 0.25 pounds. Some will weigh more than 0.25 pounds, others less. The standard time for the bus run from downtown Toronto to airport is 45 minutes. Each run does not take *exactly* 45 minutes. Some runs take longer. In some cases there is a reason for the bus being late, an accident on the highway or a snowstorm, for example. In other cases the driver may not "hit" the green lights or the traffic is unusually heavy and slow for no apparent reason. There are two general causes of variation in a process—chance and assignable.

> **CHANCE VARIATION** Variation that is random in nature. This type of variation cannot be completely eliminated unless there is a major change in the equipment or material used in the process.

Internal machine friction, slight variations in material or process conditions (such as the temperature of the mold being used to make glass bottles), atmospheric conditions (such as temperature, humidity, and the dust content of the air), and vibrations transmitted to a machine from a passing forklift are a few examples of sources of chance variation.

If the hole drilled in a piece of steel is too large due to a dull drill, the drill may be sharpened or a new drill inserted. An operator who continually sets up the machine incorrectly can be replaced or retained. If the roll of steel to be used in the process does not have the correct tensile strength, it can be rejected. These are examples of assignable variation.

> **Assignable Variation** Variation that is not random. It can be eliminated or reduced by investigating the problem and finding the cause.

There are several reasons we should be concerned with variation.

1. It will change the shape, dispersion, and central tendency of the distribution of the product characteristic being measured.
2. Assignable variation is usually correctable, whereas chance variation usually cannot be corrected or stabilized economically.

20.3 DIAGNOSTIC CHARTS

There are a variety of diagnostic techniques available to investigate quality problems. Two of the more prominent of these techniques are *Pareto charts* and *fishbone diagrams*.

PARETO CHARTS

Pareto analysis is a technique for tallying the number and type of defects that happen within a product or service. The chart is named after a 19th-century Italian scientist, Vilfredo Pareto. He noted that most of the "activity" in a process is caused by relatively few of the "factors." His concept, often called the 80–20 rule, is that 80 percent of the activity is caused by 20 percent of the factors. By concentrating on 20 percent of the factors, managers can attack 80 percent of the problem. For example, Emily's Family Restaurant is investigating "customer complaints." The five complaints heard most frequently are: discourteous service, cold food, long wait for seating, few menu choices, and unruly young children. Suppose discourteous service was mentioned most frequently and cold food second. These two factors total more than 85 percent of the complaints and hence are the two that should be addressed first because this will yield the largest reduction in complaints.

To develop a Pareto chart, we begin by tallying the type of defects. Next, we rank the defects in terms of frequency of occurrence from largest to smallest. Finally, we produce a vertical bar chart, with the height of the bars corresponding to the frequency of each defect. The following example illustrates these ideas.

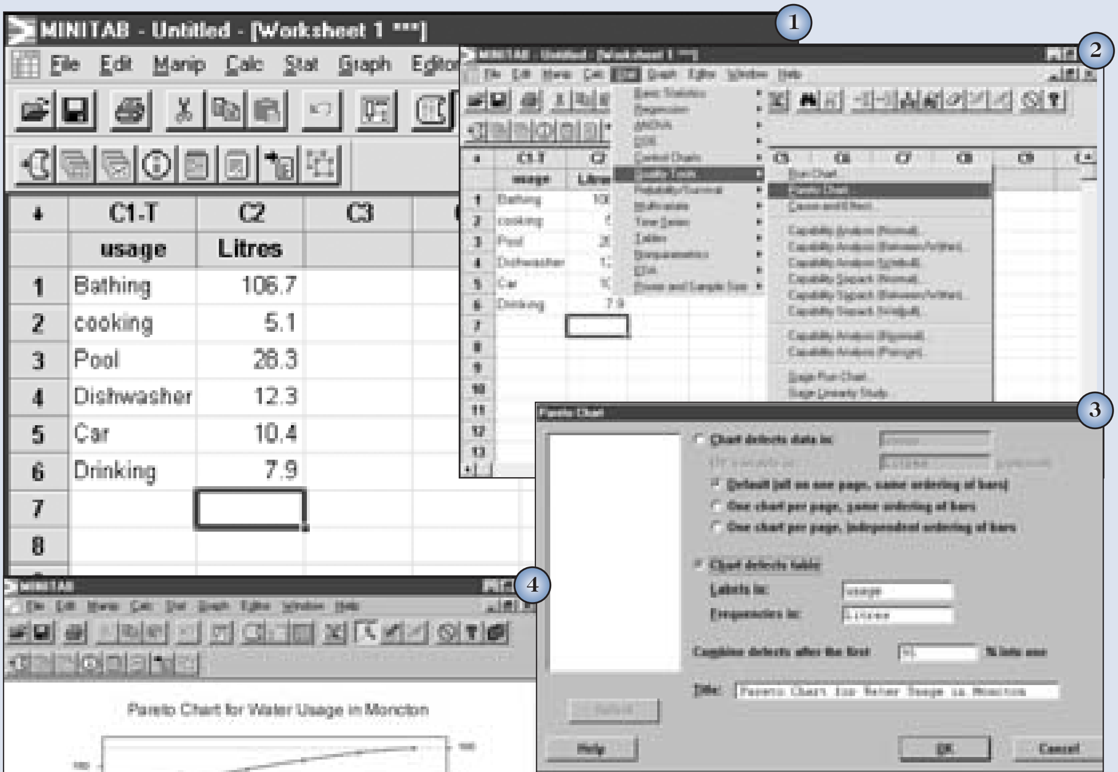
Example 20-1

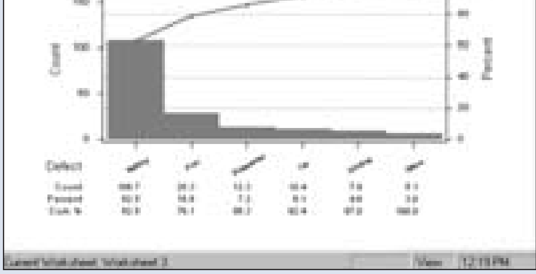
The Mayor of Moncton, NB, is concerned with water usage, particularly in single family homes. He would like to develop a plan to reduce the water usage in Moncton. To investigate, he selects a sample of 100 homes and determines the typical daily water usage for various purposes. These sample results are as follows.

Reasons for Water Usage	Litres per Day	Reasons for Water Usage	Litres per Day
Laundering	24.9	Swimming pool	28.3
Watering lawn	143.7	Dishwashing	12.3
Personal bathing	106.7	Car washing	10.4
Cooking	5.1	Drinking	7.9

What is the area of greatest usage? Where should he concentrate his efforts to reduce the water usage?

MINITAB CHART 20-1: Pareto Chart for Water Usage in Moncton, NB





MINITAB INSTRUCTIONS

1. Enter the reasons for water usage in C1 and the litres used in C2.
2. Click Stat, Quality Tools, Pareto Chart, and Enter.
3. In the dialogue box, select Chart defects table, indicate the location of the labels and frequencies, type a chart title, and click OK.

Solution A Pareto chart is useful for identifying the major areas of water usage and focusing on those areas where the greatest reduction can be achieved. The first step is to convert each of the activities to a percent and then to order them from largest to smallest. The total water usage per day is 339.3 litres, found by totaling the litres used in the eight activities. The activity with the largest use is watering lawns. It accounts for 143.7 litres of water per day, or 42.4 percent of the amount of water used. The next largest category is personal bathing, which accounts for 31.4 percent of the water used. These two activities account for 73.8 percent of the water usage.

Reasons for Water Usage	Litres per Day	Percent
Laundering	24.9	7.3
Watering lawn	143.7	42.4
Personal bathing	106.7	31.4
Cooking	5.1	1.5
Swimming pool usage	28.3	8.3
Dishwashing	12.3	3.6
Car washing	10.4	3.1
Drinking	7.9	2.3
Total	339.3	100.0

To draw the Pareto chart, we begin by scaling the number of litres used on the left vertical axis and the corresponding percent on the right vertical axis. Next we draw a vertical bar with the height of the bar corresponding to the activity with the largest number of occurrences. In the Moncton example, we draw a vertical bar for the activity watering lawns to a height of 143.7 litres. (We call this the count.) We continue this procedure for the other activities, as shown in the Minitab Chart 20-1.

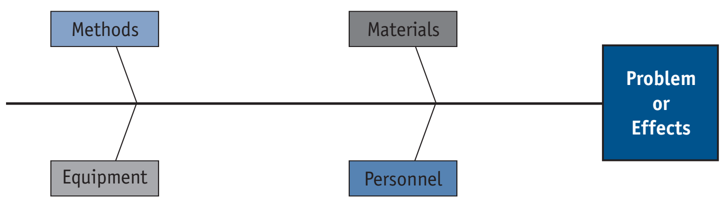
Below the chart we list the activities, their frequency of occurrence, and the percent of the time each activity occurs. In the last row we list the cumulative percentage. This cumulative row will allow us to quickly determine which set of activities account for most of the activity. These cumulative percents are plotted above the vertical bars. In the Moncton example, the activities of watering lawn, personal bathing, and pools account for 82.1 percent of the water usage. The city manager can attain the greatest gain by looking to reduce the water usage in these three areas.

FISHBONE DIAGRAM

Another diagnostic chart is a **cause-and-effect diagram** or a **fishbone diagram**. It is called a cause-and-effect diagram to emphasize the relationship between an effect and a set of possible causes that produce the particular effect. This diagram is useful to help organize ideas and to identify relationships. It is a tool that encourages open "brainstorming" for ideas. By identifying these relationships we can determine factors that are the cause of variability in our process. The name *fishbone* comes from the manner in which the various causes and effects are organized on the diagram. The effect is usually a particular problem, or perhaps a goal, and it is shown on the right-hand side of the diagram. The major causes are listed on the left-hand side of the diagram.

The usual approach to a fishbone diagram is to consider four problem areas, namely, methods, materials, equipment, and personnel. The problem, or the effect, is the head of the fish. See Chart 20-2.

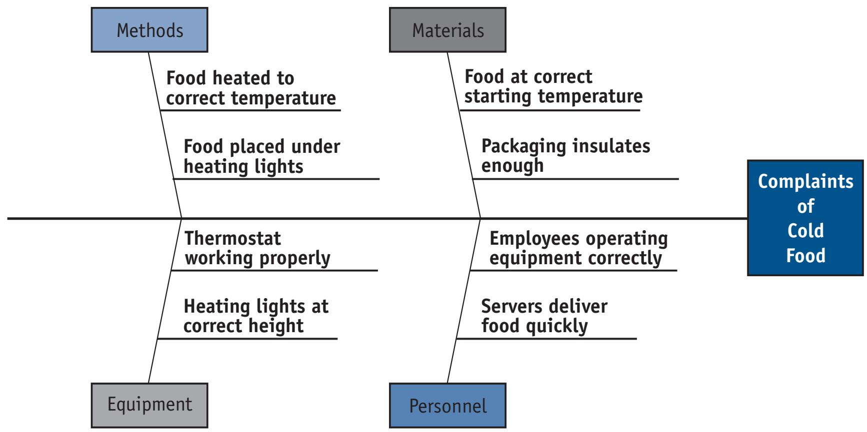
CHART 20-2: Fishbone Diagram



Under each of the possible causes are subcauses that are identified and investigated. The subcauses are factors that may be producing the particular effect. Information is gathered about the problem and used to fill in the fishbone diagram. Each of the subcauses is investigated and those that are not important eliminated, until the real cause of the problem is identified.

Chart 20-3 illustrates the details of a fishbone diagram. Suppose a family restaurant, such as those found along a highway, has recently been experiencing complaints from customers that the food being served is cold. Notice each of the subcauses are listed as assumptions. Each of these subcauses must be investigated to find the real problem regarding the cold food. In a fishbone diagram there is no weighting of the subcauses.

CHART 20-3: Fishbone Diagram for a Restaurant Investigation of Cold Food Complaints



Source: Adapted from M.A. Vonderembse and G.P. White, *Operations Management*, 3rd Ed. (South Western College Publishing, 1996), p. 489.

SELF-REVIEW 20-1

Patients at the Rouse Home have been complaining recently about the conditions at the home. The administrator would like to use a Pareto chart to investigate. When a patient or patient's relative has a complaint, they are asked to complete a complaint form. Listed below is a summary of the complaint forms received during the last 12 months.

Complaint	Number	Complaint	Number
Nothing to do	45	Dirty conditions	63
Poor care by staff	71	Poor quality food	84
Medication error	2	Lack of respect by staff	35

Develop a Pareto chart. What complaints would you suggest the administrator work on first to achieve the most significant improvement?

EXERCISES 20-1 TO 20-2

20-1. Tom Sharkey is the owner of Sharkey Chevy. At the start of the year Tom instituted a customer opinion program to find ways to improve service. One week after the service is performed, Tom's administrative assistant calls the customer to find out whether the service was performed satisfactorily and how the service might be improved. Listed below is a summary of the complaints for the first six months. Develop a Pareto chart. What complaints would you suggest that Tom work on to improve the quality of service?

Complaint	Frequency	Complaint	Frequency
Problem not corrected	38	Price too high	23
Error on invoice	8	Wait too long for service	10
Unfriendly atmosphere	12		

20-2. Out of 110 diesel engines tested, a rework and repair facility found 9 had leaky water pumps, 15 had faulty cylinders, 4 had ignition problems, 52 had oil leaks, and 30 had cracked blocks. Draw a Pareto chart to identify the key problem in the production process.

20.4 PURPOSE AND TYPES OF QUALITY CONTROL CHARTS

Control charts identify when assignable causes of variation or changes have entered the process. For example, the Wheeling Company makes vinyl-coated aluminum replacement windows for older homes. The vinyl coating must have a thickness between certain limits. If the coating becomes too thick, it will cause the windows to jam. On the other hand, if the coating become too thin, the window will not seal properly. The mechanism that determines how much coating is put on each window becomes worn and begins making the coating too thick. Thus, a change has occurred in the process. Control charts are useful for detecting the change in process conditions. It is important to know when changes have entered the process, so that the cause may be identified and corrected before a large number of unacceptable items are produced.

Control charts may be compared to the scoreboard in a baseball game. By looking at the scoreboard, the fans, coaches, and players can tell which team is winning the game. However, the scoreboard can do nothing to win or lose the game. Control charts provide a similar function. These charts indicate to the workers, group leaders, quality control engineers, production supervisor, and management whether the production of the part or service is "in control" or "out of control." If the production is "out of control," the control chart will not fix the situation; it is just a piece of paper with figures and dots on it. Instead, the person responsible will adjust the machine manufacturing the part or do what is necessary to return production to "in control."

There are two types of control charts. A **variable control chart** portrays measurements, such as the amount of cola in a two-litre bottle or the time it takes a nurse at General Hospital to respond to a patient's call. A variable control chart requires the interval or the ratio scale of measurement. **An attribute control chart** classifies a product or service as either acceptable or unacceptable. It is based on the nominal scale of measurement. Patients in a hospital are asked to rate the meals served as acceptable or unacceptable; bank loans are either repaid or they are defaulted.

CONTROL CHARTS FOR VARIABLES

To develop control charts for variables, we rely on the sampling theory discussed in connection with the central limit theorem in Chapter 8. Suppose a sample of five pieces is selected each hour from the production process and the mean of each sample computed. The sample means are $\bar{X}_1, \bar{X}_2, \bar{X}_3$, and so on. The mean of these sample means is denoted as $\bar{\bar{X}}$. We use k to indicate the number of sample means. The overall or grand mean is found by:

Grand Mean $$\bar{\bar{X}} = \frac{\Sigma \text{ of the means of the subgroups}}{\text{Number of sample means}} = \frac{\Sigma\bar{X}}{k} \qquad \textbf{20-1}$$

The standard error of the distribution of the sample means is designated by $s_{\bar{x}}$. It is found by:

Standard Error of the Mean $$S_{\bar{x}} = \frac{S}{\sqrt{n}} \qquad \textbf{20-2}$$

These relationships allow limits to be set up around the sample means to show how much variation can be expected for a given sample size. These expected limits are called the **upper control limit** (*UCL*) and the **lower control limit** (*LCL*). An example will illustrate the use of control limits and how the limits are determined.

Example 20-2

Statistical Software, Inc., offers a toll-free number where customers can call from 7 A.M. until 11 P.M. daily with problems involving the use of their products. It is impossible to have every call answered immediately by a technical representative, but it is important customers do not wait too long for a person to come on the line. Customers become upset when they hear the message “Your call is important to us. The next available representative will be with you shortly” too many times. To understand their process, Statistical Software decides to develop a control chart describing the total time from when a call is received until the representative answers the caller’s question. Yesterday, for the 16 hours of operation, five calls were sampled each hour. This information is reported on the next page, in minutes until a call was answered.

Time	Sample Number 1	2	3	4	5
A.M. 7	8	9	15	4	11
8	7	10	7	6	8
9	11	12	10	9	10
10	12	8	6	9	12
11	11	10	6	14	11
P.M. 12	7	7	10	4	11
1	10	7	4	10	10
2	8	11	11	7	7
3	8	11	8	14	12
4	12	9	12	17	11
5	7	7	9	17	13
6	9	9	4	4	11
7	10	12	12	12	12
8	8	11	9	6	8
9	10	13	9	4	9
10	9	11	8	5	11

Based on this information, develop a control chart for the mean duration of the call. Does there appear to be a trend in the calling times? Is there any period in which it appears that customers wait longer than others?

Solution

A mean chart has two limits, an upper control limit (*UCL*) and a lower control limit (*LCL*). These upper and lower control limits are computed by:

Control Limits for the Mean $$UCL = \bar{\bar{X}} + 3\frac{S}{\sqrt{n}} \quad \text{and} \quad LCL = \bar{\bar{X}} - 3\frac{S}{\sqrt{n}} \qquad \textbf{20-3}$$

where S is an estimate of the standard deviation of the population, σ. Notice that in the calculation of the upper and lower control limits the number 3 appears. It represents the 99.74 percent confidence limits. The limits are often called the 3-sigma limits. However, other levels of confidence (such as 90 or 95 percent) can be used.

This application developed before computers were widely available and computing standard deviations was difficult. Rather than calculate the standard deviation from each sample as a measure of variation, it is easier to use the range. For fixed sized samples there is a constant relationship between the range and the standard deviation, so we can use the following formulas to determine the 99.74 percent control limits for the mean. It can be demonstrated that the term $3(s/\sqrt{n})$ from Formula 20-3 is equivalent to $A_2\bar{R}$ in the following formula.

Control Limits for the Mean $$UCL = \bar{\bar{X}} + A_2\bar{R} \qquad LCL = \bar{\bar{X}} - A_2\bar{R} \qquad \textbf{20-4}$$

where:

A_2 is a constant used in computing the upper and the lower control limits. It is based on the average range, $\bar{R}$. The factors for various sample sizes can be found in the Table of Factors for Control Charts, Appendix 1 on this CD. (Note: n in this table refers to the number in the sample.) A portion of this table is shown below. To locate the A_2 factor for this problem, find the sample size for n in the left margin. It is 5. Then move horizontally to the A_2 column, and read the factor. It is 0.577.

n	A_2	D_2	D_3	D_4
2	1.880	1.128	0	3.267
3	1.023	1.693	0	2.575
4	0.729	2.059	0	2.282
5	0.577	2.326	0	2.115
6	0.483	2.534	0	2.004

$\bar{\bar{X}}$ is the mean of the sample means, computed by $\Sigma\bar{X}/k$, where k is the number of samples selected. In this problem a sample of 5 observations is taken each hour for 16 hours, so $k = 16$.

$\bar{R}$ is the mean of the ranges of the sample. It is $\Sigma R/k$. Remember the range is the difference between the largest and the smallest value in each sample. It describes the variability occurring in that particular sample. (See Table 20-1.)

TABLE 20-1 Duration of 16 Samples of Five Help Sessions

Time	1	2	3	4	5	Mean	Range
A.M. 7	8	9	15	4	11	9.4	11
8	7	10	7	6	8	7.6	4
9	11	12	10	9	10	10.4	3
10	12	8	6	9	12	9.4	6
11	11	10	6	14	11	10.4	8
P.M. 12	7	7	10	4	11	7.8	7
1	10	7	4	10	10	8.2	6
2	8	11	11	7	7	8.8	4
3	8	11	8	14	12	10.6	6
4	12	9	12	17	11	12.2	8
5	7	7	9	17	13	10.6	10
6	9	9	4	4	11	7.4	7
7	10	12	12	12	12	11.6	2
8	8	11	9	6	8	8.4	5
9	10	13	9	4	9	9.0	9
10	9	11	8	5	11	8.8	6
Total						150.60	102

The centreline for the chart is $\bar{\bar{X}}$. It is 9.4125 minutes, found by 150.60/16. The mean of the ranges ($\bar{R}$) is 6.375 minutes, found by 102/16. Thus, the upper control limit of the X bar chart is:

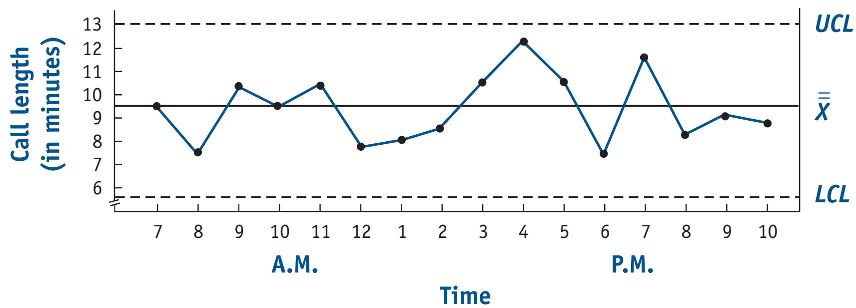
$$UCL = \bar{\bar{X}} + A_2\bar{R} = 9.4125 + 0.577(6.375) = 13.0909$$

The lower control limit of the X bar chart is:

$$LCL = \bar{\bar{X}} - A_2\bar{R} = 9.4125 - 0.577(6.375) = 5.7341$$

$\bar{\bar{X}}$, UCL, and LCL, and the sample means are portrayed in Chart 20-4. The mean, $\bar{\bar{X}}$, is 9.4125 minutes, the upper control limit is located at 13.0909 minutes, and the lower control limit is located at 5.7341. There is some variation in the duration of the calls, but all sample means are within the control limits. Thus, based on 16 samples of five calls, we conclude that 99.74 percent of the time the mean length of a sample of 5 calls will be between 5.7341 minutes and 13.0909 minutes.

CHART 20-4: Control Chart for Mean Length of Customer Calls to Statistical Software, Inc.



Because the statistical theory is based on the normality of large samples, control charts should be based on a stable process, that is, a fairly large sample, taken over a long period of time. One rule of thumb is to design the chart after at least 25 samples have been selected.

STATISTICS IN ACTION

Control charts were used to help convict a person who bribed jai alai players to lose. $\bar{X}$ and R charts showed unusual betting patterns and that some contestants did not win as much as expected when they made certain bets. A quality control expert was able to identify times when assignable variation stopped, and prosecutors were able to tie those times to the arrest of the suspect.

RANGE CHART

In addition to the central tendency in a sample, we must also monitor the amount of variation from sample to sample. A **range chart** shows the variation in the sample ranges. If the points representing the ranges fall between the upper and the lower limits, it is concluded that the operation is in control. According to chance, about 997 times out of 1,000 the range of the samples will fall within the limits. If the range should fall above the limits, we conclude that an assignable cause affected the operation and an adjustment to the process is needed. Why are we not as concerned about the lower control limit of the range? For small samples the lower limit is often zero. Actually, for any sample of six or less, the lower control limit is 0. If the range is zero, then logically all the parts are the same and there is not a problem with the variability of the operation.

The upper and lower control limits of the range chart are determined from the following equations.

Control Chart for Ranges $UCL = D_4\bar{R}$ $LCL = D_3\bar{R}$ **20-5**

The values for D_3 and D_4, which reflect the usual three σ (sigma) limits for various sample sizes, are found in the Table of Factors for Control Charts (Appendix 1 on this CD-ROM).

Example 20-3 The length of time customers of Statistical Software, Inc. waited from the time their call was answered until a technical representative answered their question or solved their problem is recorded in Table 20-1. Develop a control chart for the range. Does it appear that there is any time when there is too much variation in the operation?

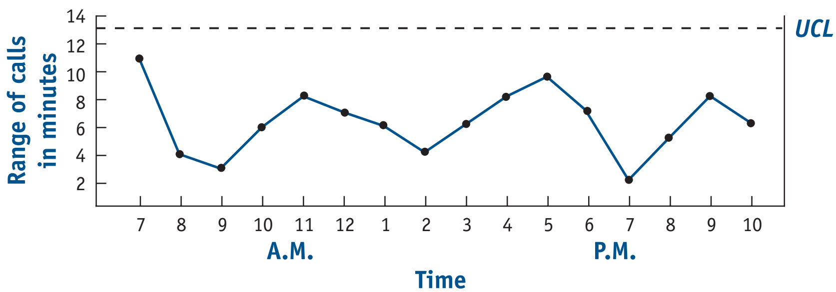
Solution The first step is to find the mean of the sample ranges. The range for the five calls sampled in the 7 A.M. hour is 11 minutes. The longest call selected from that hour was 15 minutes and the shortest 4 minutes; the difference in the lengths is 11 minutes. In the 8 A.M. hour the range is 4 minutes. The total of the 16 ranges is 102 minutes, so the average range is 6.375 minutes, found by $\bar{R} = 102/16$. Referring to the Table of Factors for Control Charts in Appendix 1 on this CD or the partial table on page 13, D_3 and D_4 are 0 and 2.115, respectively. The lower and upper control limits are 0 and 13.4831.

$$UCL = D_4\bar{R} = 2.115(6.375) = 13.4831$$

$$LCL = D_3\bar{R} = 0(6.375) = 0$$

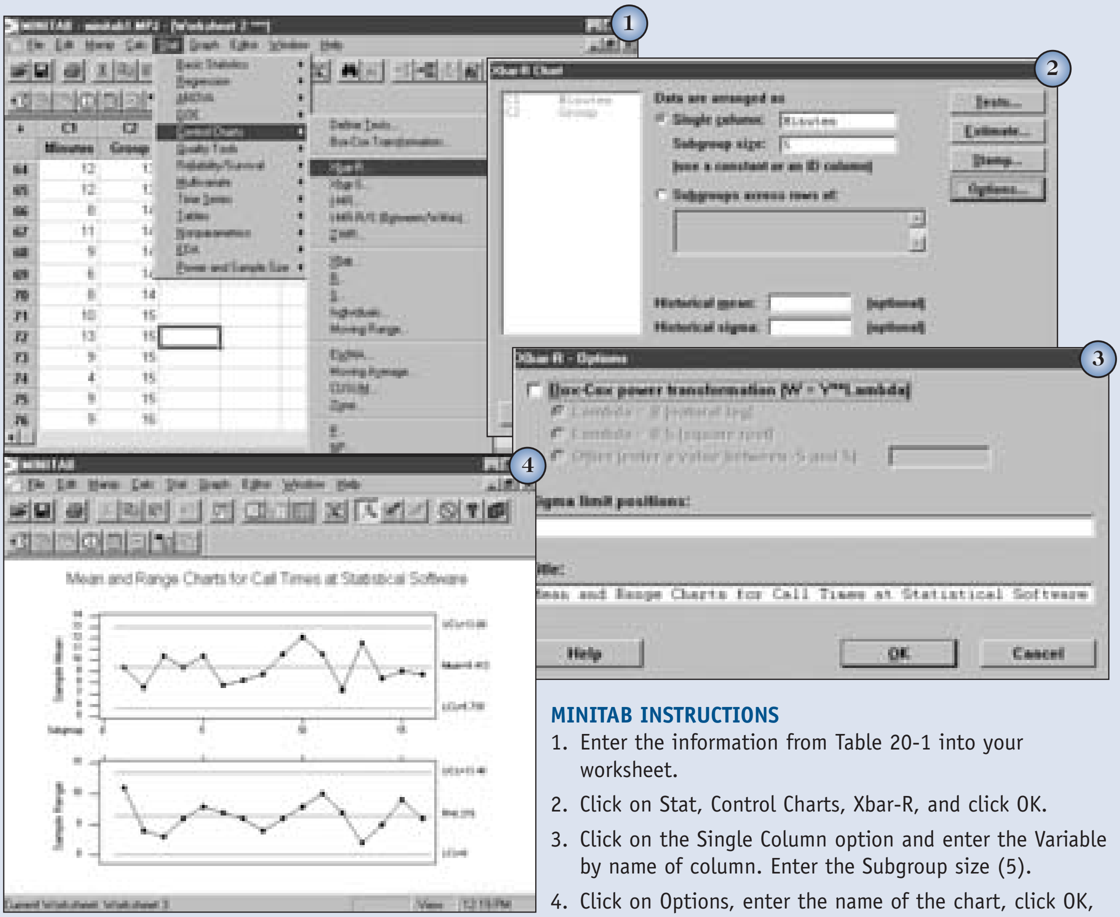
The range chart with the 16 sample ranges plotted is shown in Chart 20-5. This chart shows all the ranges are well within the control limits. Hence, we conclude the variation in the time to service the customer's calls is within normal limits, that is, "in control." Of course, we should be determining the control limits based on one set of data and then applying them to evaluate future data, not the data we already know.

CHART 20-5: Control Chart for Ranges of Length of Customer Calls to Statistical Software, Inc.



Minitab will draw a control chart for the mean and the range as shown below for the Statistical Software example. The data is in Table 20-1. The minor differences in the control limits are due to rounding.

MINITAB CHART 20-6: Mean and Range Chart for Call Times at Statistical Software



MINITAB INSTRUCTIONS

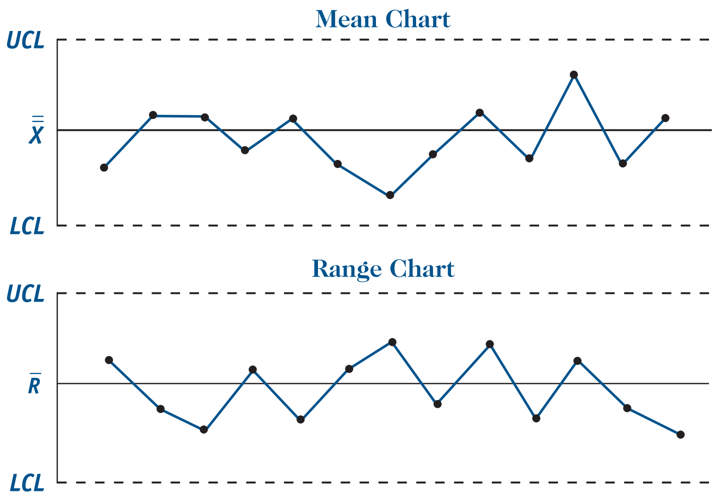
1. Enter the information from Table 20-1 into your worksheet.
2. Click on Stat, Control Charts, Xbar-R, and click OK.
3. Click on the Single Column option and enter the Variable by name of column. Enter the Subgroup size (5).
4. Click on Options, enter the name of the chart, click OK, and then OK again.

20.5 SOME IN-CONTROL AND OUT-OF-CONTROL SITUATIONS

Following are three illustrations of in-control and out-of-control processes.

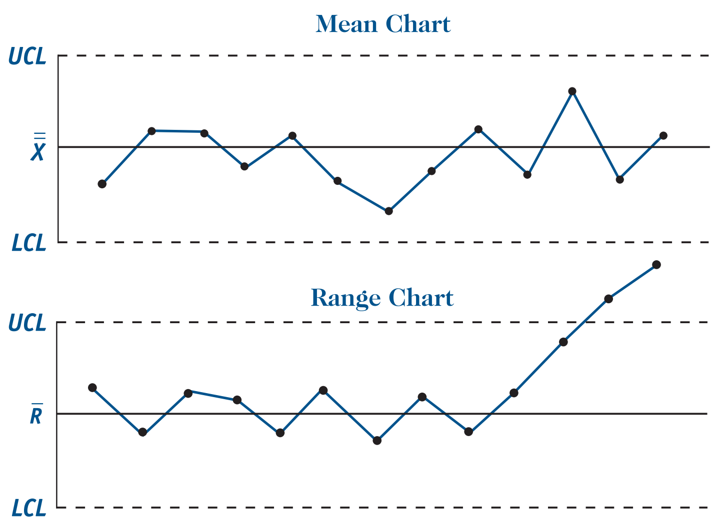
Everything OK

1. The mean chart and the range chart together indicate that the process is in control. Note the sample means and sample ranges are clustered close to the centrelines. Some are above and some below the centrelines, indicating the process is quite stable. That is, there is no visible tendency for the means and ranges to move toward the “out of control” areas.



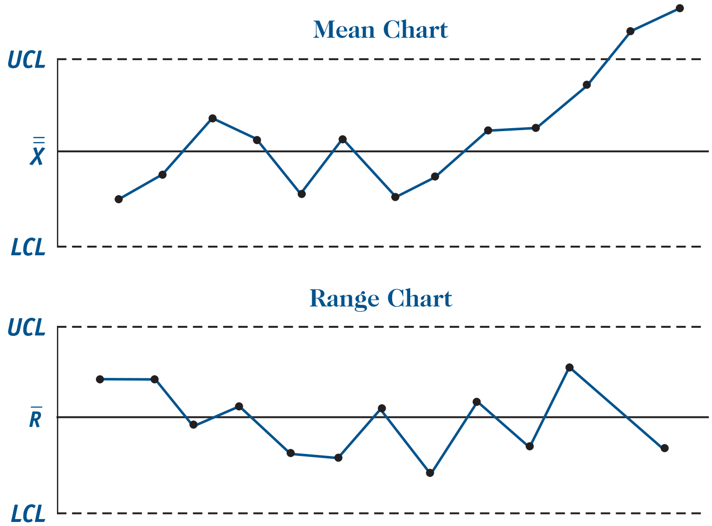
Considerable variation in ranges

2. The sample means are in control, but the ranges of the last two samples are out of control. This indicates there is considerable variation from piece to piece. Some pieces are large; others are small. An adjustment in the process is probably necessary.



3. The mean was in control for the first samples, but there is an upward trend toward *UCL*. The last two sample means were out of control. An adjustment in the process is indicated.

Mean out of control



The above chart for the mean is an example in which the control chart offers some additional information. Note the direction of the last five observations of the mean. They are all above $\bar{\bar{X}}$ and increasing, and, in fact, the last two observations are out of control. The fact that the sample means were increasing for seven consecutive observations is very improbable and another indication that the process is out of control.

SELF-REVIEW 20-2

Every hour a quality control inspector measures the outside diameter of four parts. The results of the measurements are shown below.

	Sample Piece			
Time	**1**	**2**	**3**	**4**
9 A.M.	1	4	5	2
10 A.M.	2	3	2	1
11 A.M.	1	7	3	5

(a) Compute the mean outside diameter, the mean range, and determine the control limits for the mean and the range.

(b) Are the measurements within the control limits? Interpret the chart.

EXERCISES 20-3 TO 20-8

20-3. Describe the difference between assignable variation and chance variation.

20-4. Describe the difference between an attribute control chart and a variable control chart.

20-5. Samples of size $n = 4$ are selected from a production line.
  (a) What is the value of the A_2 factor used to determine the upper and lower control limits for the mean?
  (b) What are the values of the D_3 and D_4 factors used to determine the upper and lower control limits for the range?

20-6. Samples of size 5 are selected from a manufacturing process. The mean of the sample ranges is .50. What is the estimate of the standard deviation of the population?

20-7. A new industrial oven has just been installed at the Piatt Bakery. To develop experience regarding the oven temperature, an inspector reads the temperature at four different places inside the oven each half hour. The first reading, taken at 8:00 A.M., was 340 degrees Fahrenheit. (Only the last two digits are given in the following table to make the computations easier.)

	Reading			
Time	**1**	**2**	**3**	**4**
8:00 A.M.	40	50	55	39
8:30 A.M.	44	42	38	38
9:00 A.M.	41	45	47	43
9:30 A.M.	39	39	41	41
10:00 A.M.	37	42	46	41
10:30 A.M.	39	40	39	40

  (a) Based on this initial experience, determine the control limits for the mean temperature. Determine the grand mean. Plot the experience on a chart.
  (b) Interpret the chart. Does there seem to be a time when the temperature is out of control?

20-8. Refer to Exercise 20-7.
  (a) Based on this initial experience, determine the control limits for the range. Plot the experience on a chart.
  (b) Does there seem to be a time when there is too much variation in the temperature?

20.6 ATTRIBUTE CONTROL CHARTS

Often the data we collect are the result of counting rather than measuring. That is, we observe the presence or absence of some attribute. For example, the screw top on a bottle of shampoo either fits onto the bottle and does not leak (an "acceptable" condition) or does not seal and a leak results (an "unacceptable" condition), or a bank makes a loan to a customer and the loan is either repaid or it is not repaid. In other cases we are interested in the number of defects in a sample. Air Canada might count the number of its flights arriving late per day at Dorval Airport in Montreal. In this section we discuss two types of attribute charts: the p (percent defective) and the c (number of defectives).

PERCENT DEFECTIVE CHART

If the item recorded is the fraction of unacceptable parts made in a larger batch of parts, the appropriate control chart is the percent defective chart. This chart is based on the binomial distribution, discussed in Chapter 6, and proportions, discussed in Chapter 9. The centreline is at p, the mean proportion defective. The p replaces the $\bar{\bar{X}}$ of the variable control chart. The mean proportion defective is found by:

Mean Proportion Defective $$p = \frac{\text{Total number defective}}{\text{Total number of items sampled}} \qquad \textbf{20-6}$$

The variation in the sample proportion is described by the standard error of a proportion. It is found by:

Standard Error of the Proportion $$S_p = \sqrt{\frac{p(1-p)}{n}} \qquad \textbf{20-7}$$

Hence, the upper control limit (UCL) and the lower control limit (LCL) are computed as the mean percent defective plus or minus three times the standard error of the percents (proportions). The formula for the control limits is:

Control Limits for Proportions $$LCL,\ UCL = p \pm 3\sqrt{\frac{p(1-p)}{n}} \qquad \textbf{20-8}$$

An example will show the details of the calculations and the conclusions.

Example 20-4

The Credit Department at Global National Bank is responsible for entering each transaction charged to the customer's monthly statement. Of course, accuracy is critical and errors will make the customer very unhappy! To guard against errors, each data entry clerk rekeys a sample of 1500 of their batch of work a second time and a computer program checks that the numbers match. The program also prints a report of the number and size of any discrepancy. Seven people were working last hour and here are their results:

Inspector	Number Inspected	Number Mismatched
Mullins	1500	4
Rider	1500	6
Gankowski	1500	6
Smith	1500	2
Reed	1500	15
White	1500	4
Reading	1500	4

Construct the percent defective chart for this process. What are the upper and the lower control limits? Interpret the results. Does it appear any of the data entry clerks are "out of control"?

Solution

The first step is to determine the mean proportion defective p, using Formula 20-6. It is 0.0039, found by 41/10 500.

Inspector	Number Inspected	Number Mismatched	Proportion Defective
Mullins	1500	4	0.00267
Rider	1500	6	0.00400
Gankowski	1500	6	0.00400
Smith	1500	2	0.00133
Reed	1500	15	0.01000
White	1500	4	0.00267
Reading	1500	4	0.00267
Total	10 500	41	

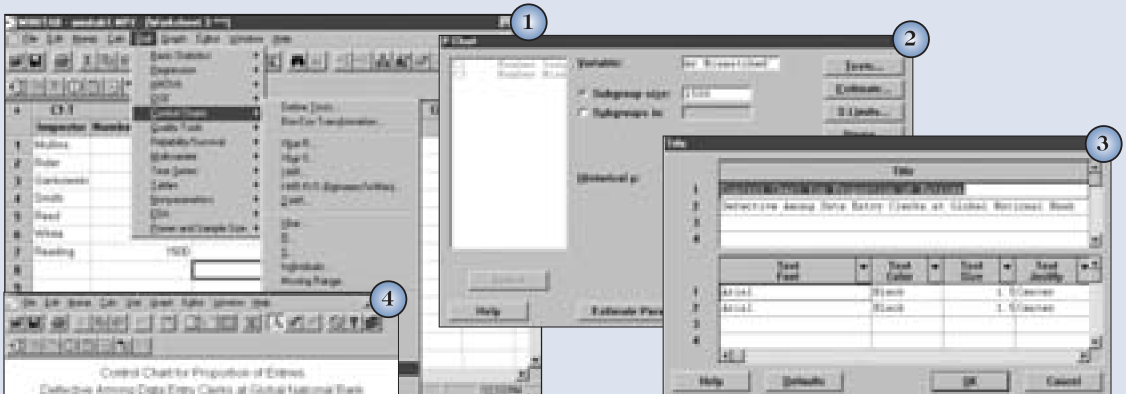
The upper and lower control limits are computed using Formula (20-8).

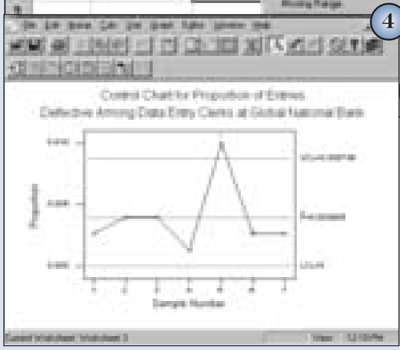
$$LCL, UCL = p \pm 3\sqrt{\frac{p(1-p)}{n}}$$

$$= \frac{41}{10\,500} \pm 3\sqrt{\frac{0.0039(1-0.0039)}{1500}} = 0.0039 \pm 0.0048$$

From the above calculations, the upper control limit is 0.0087, found by 0.0039 + 0.0048. The lower control limit is 0. Why? The lower limit by formula is determined by 0.0039 − 0.0048, which is equal to −0.0009. A negative proportion defective is not possible, so the smallest value is 0. We set the control limit at 0. Thus, any data entry clerk whose proportion defective is between 0 and 0.0087 is "in control." Clerk number 5, whose name is Reed, is out of control. Her proportion defective is 0.01, or 1.0 percent, which is outside the upper control limit. Perhaps she should receive additional training or be transferred to another position. This information is summarized in Chart 20-7, based on Minitab software.

MINITAB CHART 20-7: Control Chart for Proportion of Entries Defective Among Data Entry Clerks at Global National Bank





MINITAB INSTRUCTIONS

1. Enter the data on the number of mismatches from page 20.
2. Click Stat, Control Charts, P, and Enter.
3. Select the Variable indicating the number of mismatches, click on Subgroup size, and enter 1500.
4. In the lower right, click on Annotation, Title (type the title in the space provided), click OK, and OK again.

c-BAR CHART

The *c*-bar chart plots the number of defects or failures per unit. It is based on the Poisson distribution discussed in Chapter 6. The number of bags mishandled on a flight by Air Canada might be monitored by a *c*-bar chart. The "unit" under consideration is the flight. On most flights there are no bags mishandled. On others there may be only one, on others two, and so on. Canada Customs and Revenue Agency might count and develop a control chart for the number of errors in arithmetic per tax return. Most returns will not have any errors, some returns will have a single error, others will have two, and so on. We let $\bar{c}$ be the mean number of defects per unit. Thus, $\bar{c}$ is the mean number of bags mishandled by Air Canada per flight or the mean number of arithmetic errors per tax return. Recall from Chapter 6 that the standard deviation of a Poisson distribution is the square root of the mean. Thus, we can determine the 3-sigma, or 99.74 percent limits on a *c*-bar chart by:

Control Limits for the Number of Defects per Unit

$$LCL, UCL = \bar{c} \pm 3\sqrt{\bar{c}} \qquad \textbf{20-9}$$

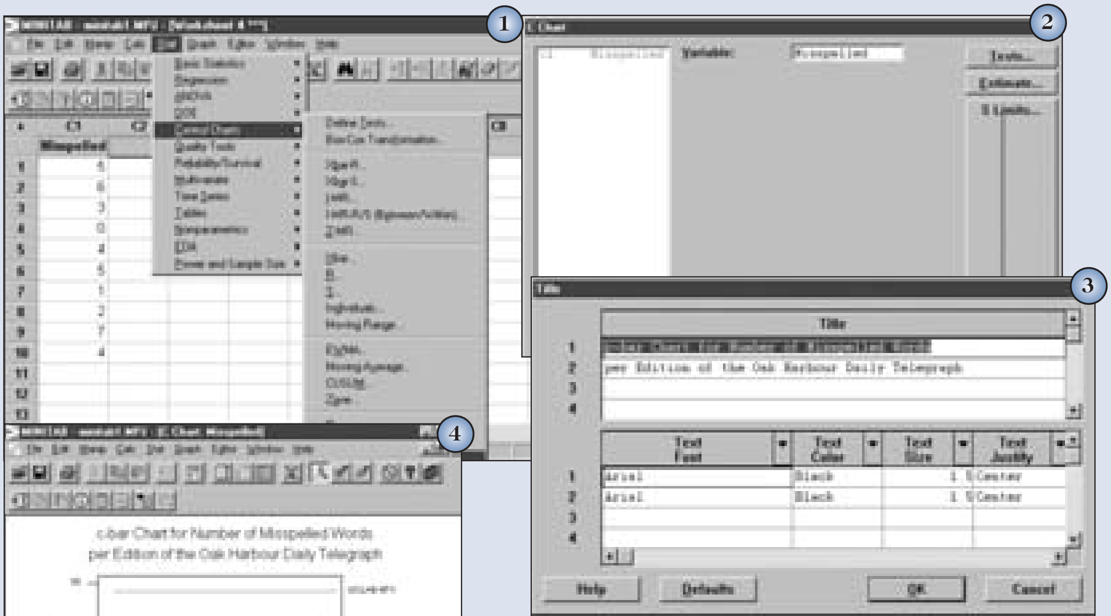
Example 20-5 The publisher of the *Oak Harbour Daily Telegraph* is concerned about the number of misspelled words in the daily newspaper. They do not print a paper on Saturday or Sunday. In an effort to control the problem and promote the need for correct spelling, a control chart is to be instituted. The number of misspelled words found in the final edition of the paper for the last 10 days is: 5, 6, 3, 0, 4, 5, 1, 2, 7, and 4. Determine the appropriate control limits and interpret the chart. Were there any days during the period that the number of misspelled words was out of control?

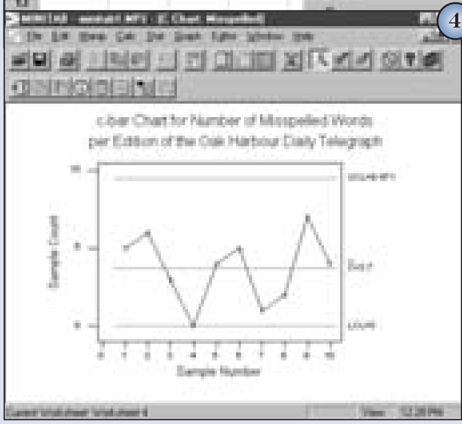
Solution The sum of the number of misspelled words over the 10-day period is 37. So the mean number of defects, $\bar{c}$, is 3.7. The square root of this number is 1.924. So the upper control limit is:

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 3.7 + 3\sqrt{3.7} = 3.7 + 5.77 = 9.47$$

The computed lower control limit would be $3.7 - 3(1.924) = -2.07$. However, the number of misspelled words cannot be less than 0, so we use 0 as the lower limit. The lower control limit is 0 and the upper limit is 9.47. When we compare each of the data points to the value of 9.47, we see they are all less than the upper control limit; the number of misspelled words is "in control." Of course, newspapers are going to strive to eliminate all misspelled words, but control charting techniques offer a means of tracking daily results and determining whether there has been a change. For example, if a new proofreader was hired, her work could be compared with others. These results are summarized in Chart 20-8, based on the Minitab software.

MINITAB CHART 20-8: ***c*-Bar Chart for Number of Misspelled Words Per Edition of the Oak Harbour Daily Telegraph**





MINITAB INSTRUCTIONS

1. Enter the data on the number of misspelled words from page 22.
2. Click on Stat, Control Chart, C Chart, and Enter.
3. Select the Variable indicating the number of misspelled words. In the lower right, click Annotation, and choose Title. Type the title in the space provided, click OK, and OK again.

SELF-REVIEW 20-3

The Auto-Lite Company manufactures car batteries. At the end of each shift the Quality Assurance Department selects a sample of batteries and tests them. The number of defective batteries found over the last 12 shifts is 2, 1, 0, 2, 1, 1, 7, 1, 1, 2, 6, and 1. Construct a control chart for the process and comment on whether the process is in control.

EXERCISES 20-9 TO 20-12

20-9. A bicycle manufacturer randomly selects 10 frames each day and tests for defects. The number of defective frames found over the last 14 days is 3, 2, 1, 3, 2, 2, 8, 2, 0, 3, 5, 2, 0, 4. Construct a control chart for this process and comment on whether the process is "in control."

20-10. Scott Paper tests its toilet paper by subjecting 15 rolls to a wet stress test to see whether and how often the paper tears during the test. Following are the number of defectives found over the last 15 days: 2, 3, 1, 2, 2, 1, 3, 2, 2, 1, 2, 2, 1, 0, and 0. Construct a control chart for the process and comment on whether the process is “in control.”

20-11. Sam’s Supermarkets tests its checkout clerks by randomly examining the printout receipts for scanning errors. The following numbers are the number of errors on each receipt for October 27: 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0. Construct a control chart for this process and comment on whether the process is “in control.”

20-12. Dave Christi runs a car wash chain scattered throughout Manitoba. He is concerned that some local managers are giving away free washes to their friends. He decides to collect data on the number of “voided” sales receipts. Of course, some of them are legitimate voids. Would the following data indicate a reasonable number of “voids” at his facilities: 3, 8, 3, 4, 6, 5, 0, 1, 2, 4? Construct a control chart for this process and comment on whether the process is “in control.”

20.7 ACCEPTANCE SAMPLING

STATISTICS IN ACTION

It was reported during the late 1980s that a Canadian firm ordered some parts from a Japanese company with instructions that there should be “no more than three defective parts per one thousand.” When the parts arrived, there was a note attached that said, “Your three defective parts are wrapped separately in the upper left compartment of the shipment.” This is a far cry from the days when “Made in Japan” meant cheap.

The previous section was concerned with maintaining the *quality of the product as it is being produced*. In many business situations we are also concerned with the *quality of the incoming finished* product. What do the following cases have in common?

- Sims Software, Inc., purchases CDs from CD International. The normal purchase order is for 100 000 CDs, packaged in lots of 1000. Todd Sims, president, does not expect each CD to be perfect. In fact, he has agreed to accept lots of 1000 with up to 10 percent defective. He would like to develop a plan to inspect incoming lots, to ensure that the quality standard is met. The purpose of the inspection procedure is to separate the acceptable from the unacceptable lots.
- Zenith Electric purchases magnetron tubes from Bono Electronics for use in their new microwave oven. The tubes are shipped to Zenith in lots of 10 000. Zenith allows the incoming lots to contain up to 5 percent defective tubes. They would like to develop a sampling plan to determine which lots meet the criterion and which do not.
- General Motors purchases windshields from many suppliers. GM insists that the windshields be in lots of 1000. They are willing to accept 50 or fewer defects in each lot, that is, 5 percent defective. They would like to develop a sampling procedure to verify that incoming shipments meet the criterion.

The common thread in these cases is a need to verify that an incoming product meets the stipulated requirements. The situation can be likened to a screen door, which allows the warm summer air to enter the room while keeping the bugs out. Acceptance sampling lets the lots of acceptable quality into the manufacturing area and screens out lots that are not acceptable.

Of course, the situation in modern business is more complex. The buyer wants protection against accepting lots that are below the quality standard. The best protection against inferior quality is 100 percent inspection. Unfortunately, the cost of 100 percent inspection is often prohibitive. Another problem with checking each item is that the test may be destructive. If all light bulbs were tested until burning out before they were shipped, there would be none left to sell. Also, 100 percent inspection may not lead to the identification of all defects, because boredom might cause a loss of perception on the part of the inspectors. Thus, complete inspection is rarely employed in practical situations.

Acceptance sampling

Acceptance number

The usual procedure is to screen the quality of incoming parts by using a statistical sampling plan. According to this plan, a sample of n units is randomly selected from the lots of N units (the population). This is called **acceptance sampling**. The inspection will determine the number of defects in the sample. This number is compared with a predetermined number called the **critical number** or the **acceptance number**. The acceptance number is usually designated c. If the number of defects in the sample of size n is less than or equal to c, the lot is accepted. If the number of defects exceeds c, the lot is rejected and returned to the supplier, or perhaps submitted to 100 percent inspection.

Consumer's risk

Producer's risk

Acceptance sampling is a decision-making process. There are two possible decisions: accept or reject the lot. In addition, there are two situations under which the decision is made: the lot is good or the lot is bad. These are the states of nature. If the lot is good and the sample inspection reveals the lot to be good, or if the lot is bad and the sample inspection indicates it is bad, then a correct decision is made. However, there are two other possibilities. The lot may actually contain more defects than it should, but it is accepted. This is called **consumer's risk**. Similarly, the lot may be within the agreed-upon limits, but it is rejected during the sample inspection. This is called the **producer's risk**. The following summary table for acceptance decisions shows these possibilities. Notice how this discussion is very similar to the ideas of Type I and Type II errors presented at the beginning of Chapter 10. (See text page 418.)

	States of Nature	
Decision	**Good Lot**	**Bad Lot**
Accept lot	Correct	Consumer's risk
Reject lot	Producer's risk	Correct

OC curve

To evaluate a sampling plan and determine that it is fair to both the producer and the consumer, the usual procedure is to develop an **operating characteristic curve**, or an **OC curve** as it is usually called. An OC curve reports the percent defective along the horizontal axis and the probability of accepting that percent defective along the vertical axis. A smooth curve is usually drawn connecting all the possible levels of quality. The binomial distribution is used to develop the probabilities for an OC curve.

Example 20-6

Sims Software, as mentioned earlier, purchases CDs from CD International. The CDs are packaged in lots of 1000 each. Todd Sims, president of Sims Software, has agreed to accept lots with 10 percent or fewer defective CDs. Todd has directed his inspection department to select a random sample of 20 CDs and examine them carefully. He will accept the lot if it has two or fewer defectives in the sample. Develop an OC curve for this inspection plan. What is the probability of accepting a lot that is 10 percent defective?

Solution

Attribute sampling

This type of sampling is also called **attribute sampling** because the sampled item, a CD in this case, is classified as acceptable or unacceptable. No "reading" or "measurement" is obtained on the CD. Let's structure the problem in terms of the states of nature. Let π represent the actual proportion defective in the population.

The lot is good if $\pi \le 0.10$.
The lot is bad if $\pi > 0.10$.

Decision rule

Let X be the number of defects in the sample. The decision rule is:

Reject the lot if $X \ge 3$.
Accept the lot if $X \le 2$.

Here the acceptable lot is one with 10 percent or fewer defective CDs. If the lot is acceptable when it has exactly 10 percent defectives, it would be even more acceptable if it contained fewer than 10 percent defectives. Hence, it is the usual practice to work with the upper limit of the percent of defectives.

The binomial distribution is used to compute the various values on the OC curve. Recall that for us to use the binomial there are four requirements:

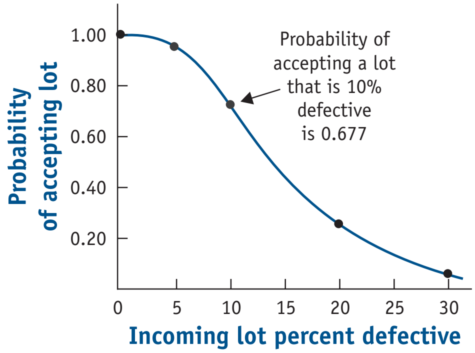
1. There are only two possible outcomes. Here the CD is either acceptable or unacceptable.
2. There is a fixed number of trials. In this instance the number of trials is the sample size of 20.
3. There is a constant probability of success. A success is the probability of finding a defective CD. It is assumed to be 0.10.
4. The trials are independent. The probability of obtaining a defective CD on the third one selected is not related to the likelihood of finding a defect on the fourth CD selected.

Appendix A in the text gives the various binomial probabilities. We need to convert the acceptance sampling vocabulary to that used in Chapter 6 for discrete probability distributions. Let $\pi = 0.10$, the probability of a success, and $n = 20$, the number of trials, c is the number of defects allowed—two in this case. We will now determine the probability of accepting an incoming lot that is 10 percent defective using a sample size of 20 and allowing zero, one, or two defects. First, locate within Appendix A the case where $n = 20$ and $\pi = 0.10$. Find the row where X, the number of defects, is 0. The probability is 0.122. Next find the probability of one defect, that is, where $X = 1$. It is 0.270. Similarly, the probability of $X = 2$ is 0.285. To find the probability of two or fewer defects, we need to add these three probabilities. The total is 0.677. Hence, the probability of accepting a lot that is 10 percent defective is 0.677. The probability of rejecting this lot is 0.323, found by 1 – 0.677. This result is usually written in shorthand notation as follows (the bar, |, means "given that"):

$$P(X \le 2 \mid \pi = 0.10 \text{ and } n = 20) = 0.677$$

The OC curve in Chart 20-9 shows various values of π and the corresponding probabilities of accepting a lot of that quality. Management of Sims Software will be able to quickly evaluate the probabilities of various quality levels.

CHART 20-9 OC Curve for Sampling Plan ($n = 20$, $c = 2$)



Lot percent defective	Probability of accepting lot
0	1.000
5	0.924
10	0.677
20	0.207
30	0.036

SELF-REVIEW 20-4

Compute the probability of accepting a lot of CDs that is actually 30 percent defective, using the sampling plan for Sims Software.

EXERCISES 20-13 TO 20-16

20-13. Determine the probability of accepting lots that are 10 percent, 20 percent, 30 percent, and 40 percent defective using a sample of size 12 and an acceptance number of 2.

20-14. Determine the probability of accepting lots that are 10 percent, 20 percent, 30 percent, and 40 percent defective using a sample of size 14 and an acceptance number of 3.

20-15. Warren Electric manufactures fuses for many customers. To ensure the quality of the outgoing product, they test 10 fuses each hour. If no more than one fuse is defective, they package the fuses and prepare them for shipment. Develop an OC curve for this sampling plan. Compute the probabilities of accepting lots that are 10 percent, 20 percent, 30 percent, and 40 percent defective. Draw the OC curve for this sampling plan using the four quality levels.

20-16. Grills Radio Products purchases transistors from Mira Electronics. According to his sampling plan, Art Grills, owner of Grills Radio, will accept a shipment of transistors if three or fewer are defective in a sample of 25. Develop an OC curve for these percents defective: 10 percent, 20 percent, 30 percent, and 40 percent.

CHAPTER OUTLINE

I. The objective of statistical quality control is to control the quality of the product or service as it is being developed.

II. A Pareto chart is a technique for tallying the number and type of defects that happen within a product or service.
   A. This chart was named after an Italian scientist, Vilfredo Pareto.
   B. The concept of the chart is that 80 percent of the activity is caused by 20 percent of the factors.

III. A fishbone diagram emphasizes the relationship between a possible problem cause that will produce the particular effect.
   A. It is also called a cause-and-effect diagram.
   B. The usual approach is to consider four problem areas: methods, materials, equipment, and personnel.

IV. The purpose of a control chart is to monitor graphically the quality of a product or service.
   A. There are two types of control charts.
      1. A variable control chart is the result of a measurement.
      2. An attribute chart shows whether the product or service is acceptable or not acceptable.
   B. There are two sources of variation in the quality of a product or service.
      1. Chance variation is random in nature and cannot be controlled or eliminated.
      2. Assignable variation is not due to random causes and can be eliminated.
   C. Four control charts were considered in this chapter.
      1. A mean chart shows the mean of a variable, and a range chart shows the range of the variable.
         (a) The upper and lower control limits are set at plus or minus 3 standard errors from the mean.
         (b) The formulas for the upper and lower control limits for the mean are:

$$UCL = \bar{\bar{X}} + A_2\bar{R} \qquad LCL = \bar{\bar{X}} - A_2\bar{R} \qquad \textbf{20-4}$$

         (c) The formulas for the upper and lower control limits for the range are:

$$UCL = D_4\bar{R} \qquad LCL = D_3\bar{R} \qquad \textbf{20-5}$$

      2. A percent defective chart is an attribute chart that shows the proportion of the product or service that does not conform to the standard.
         (a) The mean percent defective is found by

$$p = \frac{\text{Total number defective}}{\text{Total number of items sampled}} \qquad \textbf{20-6}$$

         (b) The control limits for the proportion defective are determined from the equation

$$LCL, UCL = p \pm 3\sqrt{\frac{p(1-p)}{n}} \qquad \textbf{20-8}$$

3. A c-bar chart refers to the number of defects per unit.
   (a) It is based on the Poisson distribution.
   (b) The mean number of defects per unit is $\bar{c}$.
   (c) The control limits are determined from the following equation.

$$LCL, UCL = \bar{c} \pm 3\sqrt{\bar{c}} \quad \textbf{20-9}$$

V. Acceptance sampling is a method to determine whether an incoming lot of a product meets specified standards.
   A. It is based on random sampling techniques.
   B. A random sample of n units is selected from a population of N units.
   C. c is the maximum number of defective units that may be found in the sample of n and the lot still considered acceptable.
   D. An OC (operating characteristic) curve is developed using the binomial probability distribution to determine the probability of accepting lots of various quality levels.

CHAPTER EXERCISES 20-17 TO 20-36

20-17. The production supervisor at Westburg Electric, Inc., noted an increase in the number of electric motors rejected at the time of final inspection. Of the last 200 motors rejected, 80 of the defects were due to poor wiring, 60 contained a short in the coil, 50 involved a defective plug, and 10 involved other defects. Develop a Pareto chart to show the major problem areas.

20-18. The manufacturer of athletic shoes conducted a study on their newly developed jogging shoe. Listed below are the type and frequency of the nonconformities and failures found. Develop a Pareto chart to show the major problem areas.

Type of Nonconformity	Frequency	Type of Nonconformity	Frequency
Sole separation	34	Lace breakage	14
Heel separation	98	Eyelet failure	10
Sole penetration	62	Other	16

20-19. Wendy's fills their soft drinks with an automatic machine that operates based on the weight of the soft drink. When the process is in control, the machine fills each cup so that the grand mean is 10.0 ounces and the mean range is 0.25 for samples of 5.
   (a) Determine the upper and lower control limits for the process for both the mean and the range.
   (b) The manager of a store tested five soft drinks served last hour and found that the mean was 10.16 ounces and the range was 0.35 ounces. Is the process in control? Should other action be taken?

20-20. A new machine has just been installed to cut and rough-shape large slugs. The slugs are then transferred to a precision grinder. One of the critical measurements is the outside diameter. The quality control inspector randomly selected five slugs each hour, measured the outside diameter, and recorded the results. The measurements (in millimetres) for the period 8:00 A.M. to 10:30 A.M. follow.

	Outside Diameter (millimeters)				
Time	**1**	**2**	**3**	**4**	**5**
8:00	87.1	87.3	87.9	87.0	87.0
8:30	86.9	88.5	87.6	87.5	87.4
9:00	87.5	88.4	86.9	87.6	88.2
9:30	86.0	88.0	87.2	87.6	87.1
10:00	87.1	87.1	87.1	87.1	87.1
10:30	88.0	86.2	87.4	87.3	87.8

(a) Determine the control limits for the mean and the range.
(b) Plot the control limits for the mean outside diameter and the range.
(c) Are there any points on the mean or the range chart that are out of control? Comment on the chart.

20-21. The Long Last Tire Company, as part of its inspection process, tests its tires for tread wear under simulated road conditions. Twenty samples of three tires each were selected from different shifts over the last month of operation. The tread wear is reported below in hundredths of an inch.

Sample	**Tread Wear**			**Sample**	**Tread Wear**		
1	44	41	19	11	11	33	34
2	39	31	21	12	51	34	39
3	38	16	25	13	30	16	30
4	20	33	26	14	22	21	35
5	34	33	36	15	11	28	38
6	28	23	39	16	49	25	36
7	40	15	34	17	20	31	33
8	36	36	34	18	26	18	36
9	32	29	30	19	26	47	26
10	29	38	34	20	34	29	32

(a) Determine the control limits for the mean and the range.
(b) Plot the control limits for the mean outside diameter and the range.
(c) Are there any points on the mean or the range chart that are "out of control"? Comment on the chart.

20-22. The National Bank has a staff of loan officers located in its branch offices throughout Quebec. The vice president in charge of the loan officers would like some information on the typical amount of loans and the range in the amount of the loans. A staff analyst of the vice president selected a sample of 10 loan officers and from each officer selected a sample of five loans he or she made last month. The data are reported below. Develop a control chart for the mean and the range. Do any of the officers appear to be "out of control"? Comment on your findings.

	Loan Amount ($000)						Loan Amount ($000)				
Officer	**1**	**2**	**3**	**4**	**5**	**Officer**	**1**	**2**	**3**	**4**	**5**
Weinraub	59	74	53	48	65	Bowyer	66	80	54	68	52
Visser	42	51	70	47	67	Kuhlman	74	43	45	65	49
Moore	52	42	53	87	85	Ludwig	75	53	68	50	31
Brunner	36	70	62	44	79	Longnecker	42	65	70	41	52
Wolf	34	59	39	78	61	Simonetti	43	38	10	19	47

20-23. The producer of a candy bar, called the "A Rod" Bar, reports on the package that the calorie content is 420 per 50-gram bar. A sample of 5 bars from each of the last 10 days is sent for a chemical analysis of the calorie content. The results are shown below. Does it appear that there are any days where the calorie count is out of control? Develop an appropriate control chart and analyze your findings.

	Calorie Count						Calorie Count				
Sample	**1**	**2**	**3**	**4**	**5**	**Sample**	**1**	**2**	**3**	**4**	**5**
1	426	406	418	431	432	6	427	417	408	418	422
2	421	422	415	412	411	7	422	417	426	435	426
3	425	420	406	409	414	8	419	417	412	415	417
4	424	419	402	400	417	9	417	432	417	416	422
5	421	408	423	410	421	10	420	422	421	415	422

20-24. The Early Morning Delivery Service guarantees delivery of small packages by 10:30 A.M. Of course, some of the packages are not delivered by 10:30 A.M. For a sample of 200 packages delivered each of the last 15 working days, the following number of packages were delivered after the deadline: 9, 14, 2, 13, 9, 5, 9, 3, 4, 3, 4, 3, 3, 8, and 4.

(a) Determine the mean proportion of packages delivered after 10:30 A.M.

(b) Determine the control limits for the proportion of packages delivered after 10:30 A.M. Were any of the sampled days out of control?

(c) If 10 packages out of 200 in the sample were delivered after 10:30 A.M. today, is this sample within the control limits?

20-25. An automatic machine produces 5.0 mm bolts at a high rate of speed. A quality control program has been started to control the number of defectives. The quality control inspector selects 50 bolts at random and determines how many are defective. The number of defectives in the first 10 samples is 3, 5, 0, 4, 1, 2, 6, 5, 7, and 7.

(a) Design a percent defective chart. Insert the mean percent defective, *UCL*, and *LCL*.

(b) Plot the percent defective for the first 10 samples on the chart.

(c) Interpret the chart.

20-26. Steele Breakfast Foods, Inc. produces a popular brand of raisin bran cereal. The package indicates it contains 25.0 ounces of cereal. To ensure the product quality, the Steele Inspection Department makes hourly checks on the production process. As a part of the hourly check, 4 boxes are selected and their contents weighed. The results are reported below.

Sample	Weights			
1	26.1	24.4	25.6	25.2
2	25.2	25.9	25.1	24.8
3	25.6	24.5	25.7	25.1
4	25.5	26.8	25.1	25.0
5	25.2	25.2	26.3	25.7
6	26.6	24.1	25.5	24.0
7	27.6	26.0	24.9	25.3
8	24.5	23.1	23.9	24.7
9	24.1	25.0	23.5	24.9
10	25.8	25.7	24.3	27.3
11	22.5	23.0	23.7	24.0
12	24.5	24.8	23.2	24.2
13	24.4	24.5	25.9	25.5
14	23.1	23.3	24.4	24.7
15	24.6	25.1	24.0	25.3
16	24.4	24.4	22.8	23.4
17	25.1	24.1	23.9	26.2
18	24.5	24.5	26.0	26.2
19	25.3	27.5	24.3	25.5
20	24.6	25.3	25.5	24.3
21	24.9	24.4	25.4	24.8
22	25.7	24.6	26.8	26.9
23	24.8	24.3	25.0	27.2
24	25.4	25.9	26.6	24.8
25	26.2	23.5	23.7	25.0

Develop an appropriate control chart. What are the limits? Is the process out of control at any time?

20-27. An investor believes there is a 50-50 chance that a stock will increase in price on a particular day. To investigate this idea, for 30 consecutive trading days the investor selects a random sample of 50 stocks and counts the number whose prices increased. The number of stocks in the sample whose prices increased is reported below.

14	12	13	17	10	18	10	13	13	14
13	10	12	11	9	13	14	11	12	11
15	13	10	16	10	11	12	15	13	10

Develop a percent defective chart and write a brief report summarizing your findings. Based on these sample results, is it reasonable that the odds are 50-50 that a stock's price will increase? What percent of the stocks would need to increase in a day for the process to be "out of control"?

20-28. Lahey Motors specializes in selling cars to buyers with a poor credit history. Listed below is the number of cars that were repossessed from Lahey customers because they did not meet the payment obligations over the last 36 months.

6	5	8	20	11	10	9	3	9	9
15	12	4	11	9	9	6	18	6	8
9	7	13	7	11	8	11	13	6	14
13	5	5	8	10	11				

Develop a *c*-bar chart for the number repossessed. Were there any months when the number was out of control? Write a brief report summarizing your findings.

20-29. A process engineer is considering two sampling plans. In the first a sample of 10 will be selected and the lot accepted if 3 or fewer are found defective. In the second the sample size is 20 and the acceptance number is 5. Develop an OC curve for each. Compare the probability of acceptance for lots that are 5, 10, 20, and 30 percent defective. Which of the plans would you recommend if you were the supplier?

20-30. The Inter Global Moving and Storage Company is setting up a control chart to monitor the proportion of residential moves that result in written complaints due to late delivery, lost items, or damaged items. A sample of 50 moves is selected for each of the last 12 months. The number of written complaints in each sample is 8, 7, 4, 8, 2, 7, 11, 6, 7, 6, 8, and 12.

(a) Design a percent defective chart. Insert the mean percent defective, *UCL*, and *LCL*.

(b) Plot the proportion of written complaints in the last 12 months.

(c) Interpret the chart. Does it appear that the number of complaints is out of control for any of the months?

20-31. Eric's Cookie House sells chocolate chip cookies in shopping malls. Of concern is the number of chocolate chips in each cookie. Eric, the owner and president, would like to establish a control chart for the number of chocolate chips per cookie. He selects a sample of 15 cookies from today's production and counts the number of chocolate chips in each. The results are as follows: 6, 8, 20, 12, 20, 19, 11, 23, 12, 14, 15, 16, 12, 13, and 12.

(a) Determine the centreline and the control limits.

(b) Develop a control chart and plot the number of chocolate chips per cookie.

(c) Interpret the chart. Does it appear that the number of chocolate chips is out of control in any of the cookies sampled?

20-32. The number of "near misses" recorded for the last 20 months at the Lima International Airport is 3, 2, 3, 2, 2, 3, 5, 1, 2, 2, 4, 4, 2, 6, 3, 5, 2, 5, 1, and 3. Develop an appropriate control chart. Determine the mean number of misses per month and the limits on the number of misses per month. Are there any months where the number of near misses is out of control?

20-33. The following number of robberies were reported during the last 10 days to the Robbery Division of the Metro City Police: 10, 8, 8, 7, 8, 5, 8, 5, 4, and 7. Develop an appropriate control chart. Determine the mean number of robberies reported per day and determine the control limits. Are there any days when the number of robberies reported is out of control?

20-34. Seiko purchases watch stems for their watches in lots of 10 000. Seiko's sampling plan calls for checking 20 stems, and if 3 or fewer stems are defective, the lot is accepted.

(a) Based on their sampling plan, what is the probability that a lot of 40 percent defective will be accepted?

(b) Design an OC curve for incoming lots that have zero, 10 percent, 20 percent, 30 percent, and 40 percent defective stems.

20-35. Automatic Screen Door Manufacturing Company purchases door latches from a number of vendors. The purchasing department is responsible for inspecting the incoming latches. Automatic purchases 10 000 door latches per month and inspects 20 latches selected at random. Develop an OC curve for the sampling plan if three latches can be defective and the incoming lot is still accepted.

20-36. At the beginning of each football season Team Sports, the local sporting goods store, purchases 5000 footballs. A sample of 25 balls is selected, and they are inflated, tested, and then deflated. If more than two balls are found defective, the lot of 5000 is returned to the manufacturer. Develop an OC curve for this sampling plan.

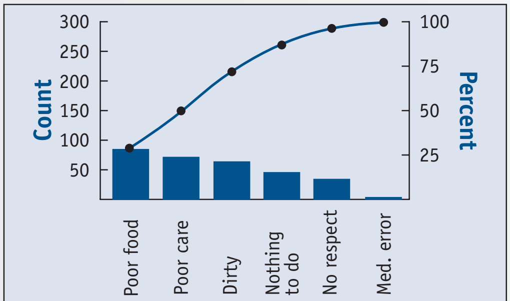
(a) What are the probabilities of accepting lots that are 10 percent, 20 percent, and 30 percent defective?

(b) Estimate the probability of accepting a lot that is 15 percent defective.

(c) John Brennen, owner of Team Sports, would like the probability of accepting a lot that is 5 percent defective to be more than 90 percent. Does this appear to be the case with this sampling plan?

CHAPTER 20 ANSWERS TO SELF-REVIEW

20-1.



Count	84	71	83	45	35	2
Percent	28	24	21	15	12	0
Cum. %	28	52	73	88	100	100

Seventy-three percent of the complaints involve poor food, poor care, or dirty conditions. These are the factors the administrator should address.

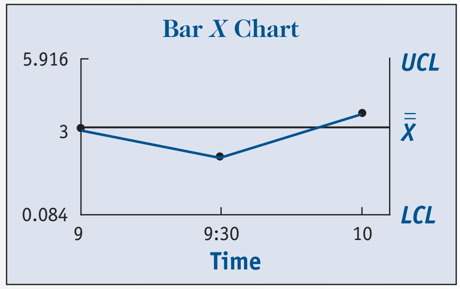
20-2. (a)

Sample Piece 1	2	3	4	Total	Average	Range
1	4	5	2	12	3	4
2	3	2	1	8	2	2
1	7	3	5	16	4	6
					9	12

$\bar{\bar{X}} = \frac{9}{3} = 3 \qquad \bar{R} = \frac{12}{3} = 4$

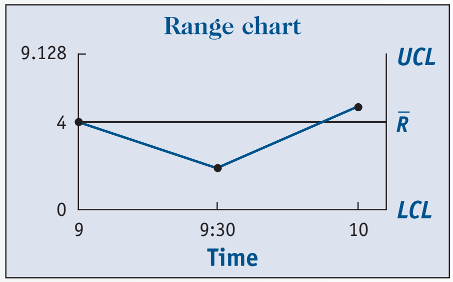
$$UCL \text{ and } LCL = \bar{\bar{X}} \pm A_2\bar{R}$$
$$= 3 \pm 0.729(4)$$

$UCL = 5.916 \qquad LCL = 0.084$



$LCL = D_3\bar{R} = 0(4) = 0$

$UCL = D_4\bar{R} = 2.282(4) = 9.128$



(b) Yes. Both the mean chart and the range chart indicate that the process is in control.

20-3. $\bar{c} = \frac{25}{12} = 2.083$

$UCL = 2.083 + 3\sqrt{2.083} = 6.413$

$LCL = 2.083 - 3\sqrt{2.083} = 0$

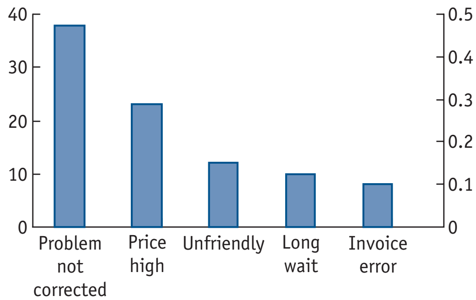
20-4. The shift with 7 defects is out of control.

$P(X \leq 2 \mid \pi = 0.30 \text{ and } n = 20) = 0.036$

ANSWERS

to Chapter 20 Exercises

20-1.

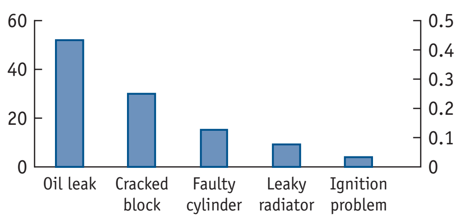


Count	38	23	12	10	8
Percent	42	25	13	11	9
Cum %	42	67	80	91	100

About 67% of the complaints concern the problem of not being corrected and the price being too high.

20-2.

Defect	Frequency	Percent
Oil leak	52	47
Cracked block	30	27
Faulty cylinder	15	14
Leaky radiator	9	8
Ignition problem	4	4
	110	



Count	52	30	15	9	4
Percent	47	27	14	8	4
Cum %	47	74	88	96	100

Almost 75% of the complaints relate to oil leaks and cracked blocks.

20-3. Chance variation is random in nature and because the cause is a variety of factors it cannot be entirely eliminated. Assignable variation is not random, it is usually due to a specific cause and can be eliminated.

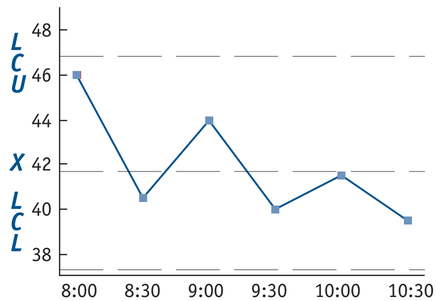
20-4. Variable control charts are concerned with actual measurements (weight, diameter, etc.). Attribute control charts are based on a classification of acceptable or not acceptable (door lock either works or it does not).

20-5. a. The A_2 factor is 0.729.

b. The value for D_3 is 0, and for D_4 is 2.282.

20-6. 0.215, found by $[(0.577)(0.50)(\sqrt{5})]/3$.

20-7. a.



Time	$\bar{X}$ Arithmetic means	R range
8:00 A.M.	46.0	16
8:30 A.M.	40.5	6
9:00 A.M.	44.0	6
9:30 A.M.	40.0	2
10:00 A.M.	41.5	9
10:30 A.M.	39.5	1
	251.5	40

$$\bar{\bar{X}} = \frac{251.5}{6} = 41.92 \qquad \bar{R} = \frac{40}{6} = 6.67$$

$$UCL = 41.92 + 0.729(6.67) = 46.78$$

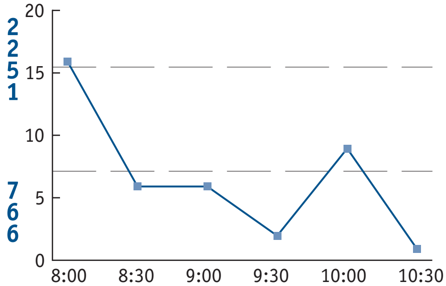
$$LCL = 41.92 - 0.729(6.67) = 37.06$$

b. Interpreting, the mean reading was 341.92 degrees Fahrenheit. If the oven continues operating as evidenced by the first six hourly readings, about 99.7% of the mean readings will lie between 337.06 degrees and 346.78 degrees Fahrenheit.

20-8. a. $UCL = D_4\bar{R} = 2.282(6.67) = 15.22$

$LCL = D_3\bar{R} = 0$

b. Oven is operating within bounds except for the first range (8 A.M.), which is outside the control limits.



20-9. $\bar{p} = \frac{37}{140} = 0.26$

$$0.26 \pm 3\sqrt{\frac{0.26(0.74)}{10}} = 0.26 \pm 0.42$$

The control limits are from 0 to 0.68. The process is out of control on the seventh day.

20-10. $\bar{c} = \frac{24}{15} = 1.6$

$$\bar{c} \pm 3\sqrt{\bar{c}} = 1.6 \pm 3\sqrt{1.6} = 1.6 \pm 3.79$$

The control limits are from 0 to 5.39 tears. Each sample roll is within these limits.

20-11. $\bar{c} = \frac{6}{11} = 0.545$

$$0.545 \pm 3\sqrt{0.545} = 0.545 \pm 2.215$$

The control limits are from 0 to 2.760, so there are no receipts out of control.

20-12. $\bar{c} = \frac{36}{10} = 3.6$

$$3.6 \pm 3\sqrt{3.6} = 3.6 \pm 5.69$$

The control limits are from 0 to 9.29. There is no case where the number of voids is greater than 9, so the process is in control.

20-13.

Percent defective	Probability of accepting lot
10	0.889
20	0.558
30	0.253
40	0.083

20-14.

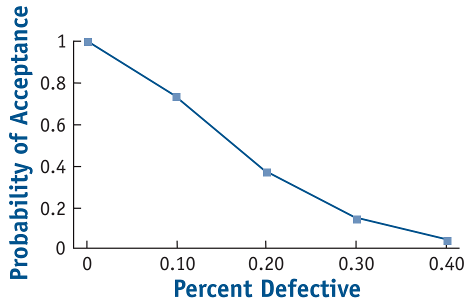
Percent defective	Probability of accepting lot
10	0.956
20	0.698
30	0.355
40	0.125

20-15. $P(X \le 1 \mid n = 10, \pi = 0.10) = 0.736$

$P(X \le 1 \mid n = 10, \pi = 0.20) = 0.375$

$P(X \le 1 \mid n = 10, \pi = 0.30) = 0.149$

$P(X \le 1 \mid n = 10, \pi = 0.40) = 0.046$

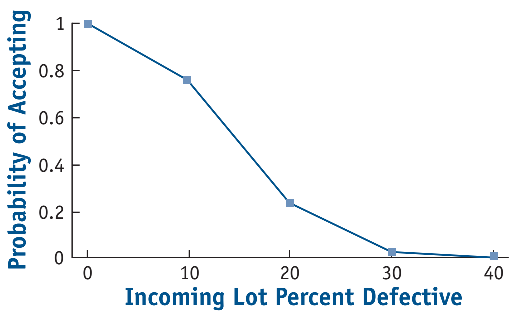


20-16. $P(X \le 3 \mid n = 25, \pi = 0.10) = 0.763$

$P(X \le 3 \mid n = 25, \pi = 0.20) = 0.235$

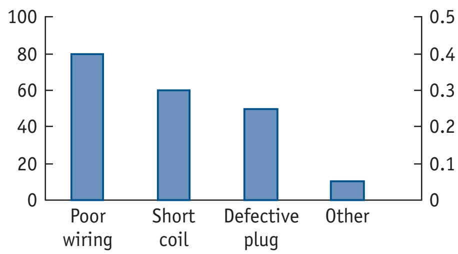
$P(X \le 3 \mid n = 25, \pi = 0.30) = 0.032$

$P(X \le 3 \mid n = 25, \pi = 0.40) = 0.002$



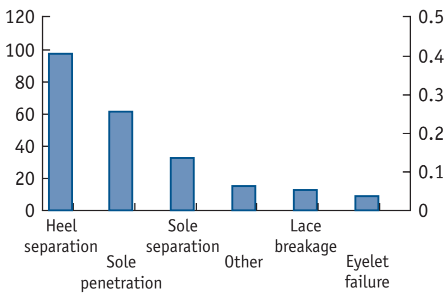
20-17.

Defect	Percent	Number of defects
Poor wiring	40.0	80
Short in coil	30.0	60
Defective plug	25.0	50
Other	5.0	10
	100.0	200



Count	80	60	50	10
Percent	40	30	25	5
Cum %	40	70	95	100

20-18.



Count	98	62	34	16	14	10
Percent	41.9	26.5	14.5	6.8	6.0	4.3
Cum %	41.9	68.4	82.9	89.7	95.7	100.0

A total of 68.4% of the defects relate to heel and sole separation.

20-19. a. $UCL = 10.0 + 0.577(0.25)$

$= 10.0 + 0.14425 = 10.14425$

$LCL = 10.0 - 0.577(0.25)$

$= 10.0 - 0.14425 = 9.85575$

$UCL = 2.115(0.25) = 0.52875$

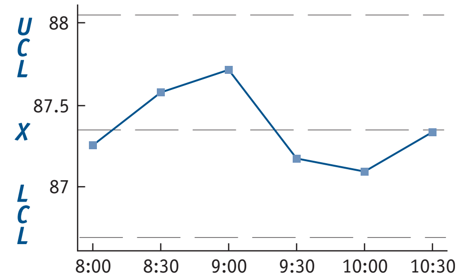
$LCL = 0(0.25) = 0$

b. The mean is 10.16, which is above the upper control limit and is out of control. There is too much cola in the soft drinks, an adjustment is needed. The process is in control for variation.

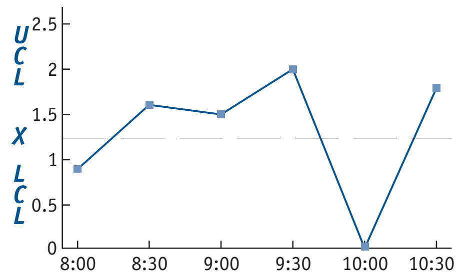
20-20. a. $\bar{\bar{X}} = 87.36$ $UCL = 88.11$ $LCL = 86.61$

$\bar{R} = 1.3$ $UCL = 2.75$ $LCL = 0$

b.



c.



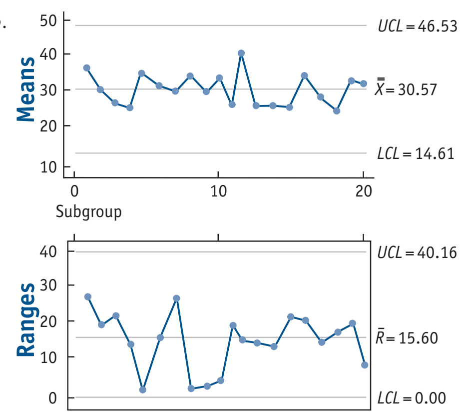
20-21. a. $\bar{\bar{X}} = \dfrac{611.23}{20} = 30.5665 \quad \bar{R} = \dfrac{312}{20} = 15.6$

$UCL = 30.5665 + (1.023)(15.6) = 46.53$

$LCL = 30.5665 - (1.023)(15.6) = 14.61$

$UCL = 2.575(15.6) = 40.17$

b.



c. The points all seem to be within the control limits. No adjustments are necessary.

20-22. $\bar{\bar{X}} = \dfrac{551.4}{10} = 55.14$

$\bar{R} = \dfrac{355}{10} = 35.5$

$UCL = 55.14 + (0.577)(35.5) = 75.62$

$LCL = 55.14 - (0.577)(35.5) = 34.66$

$UCL = 2.115(35.5) = 75.08$

The last bank officer, Simonetti, is below the lower control limit for the group.

20-23. $\bar{\bar{X}} = \dfrac{4183}{10} = 418.3$

$\bar{R} = \dfrac{162}{10} = 16.2$

$UCL = 418.3 + (0.577)(16.2) = 427.65$

$LCL = 418.3 - (0.577)(16.2) = 408.95$

$UCL = 2.115(16.2) = 34.26$

All points are in control for both the mean and the range.

20-24. a. $\bar{p} = \dfrac{93}{15(200)} = 0.031$

b. $3\sqrt{\dfrac{0.031(1-0.031)}{200}} = 0.037$

$UCL = 0.031 + 0.037 = 0.068$

$LCL = 0.031 - 0.037 = 0$

c. Ten packages mean $\bar{p} = 0.05$, which is well within the limits.

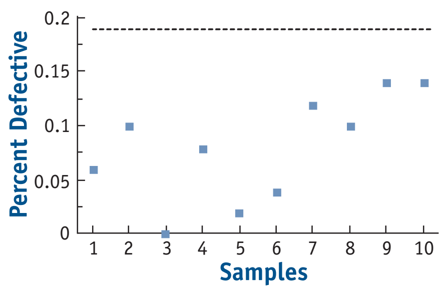
20-25. a. $\bar{p} = \dfrac{40}{10(50)} = 0.08$

$3\sqrt{\dfrac{0.08(1-0.08)}{50}} = 0.115$

$UCL = 0.08 + 0.115 = 0.195$

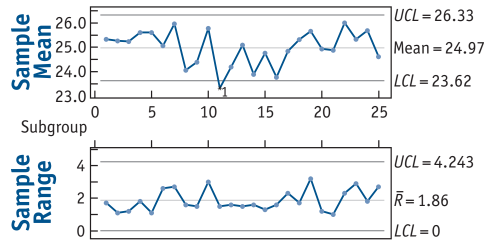
$LCL = 0.08 - 0.115 = 0$

b.



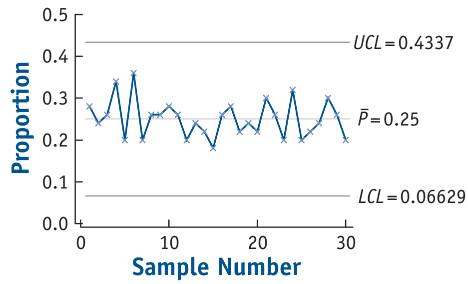
c. There are no points that exceed the limits.

20-26. Bar X/R Chart Weights



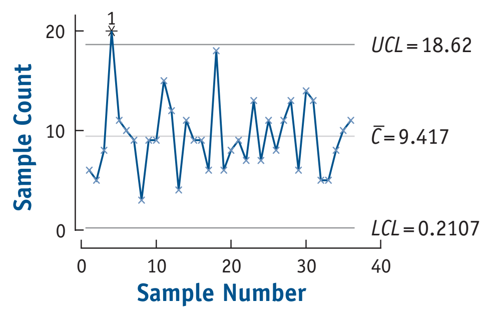
The sample 11 mean is 23.3, which is below the lower control limit of 23.62. The process is "out of control" at that point.

20-27. P Chart for $C1$



These sample results indicate that the odds are much less than 50-50 for an increase. The percent of stocks that increase is "in control" around 0.25 or 25%. The control limits are 0.06629 and 0.4337.

20-28. C Chart for $C1$



The fourth month (20) was "out of control" above the upper control limit of 18.62. Otherwise, the rest of the months appear "in control" around a mean of 9.417.

20-29. $P(X \le 3 \mid n = 10, \pi = 0.05) = 0.999$

$P(X \le 3 \mid n = 10, \pi = 0.10) = 0.987$

$P(X \le 3 \mid n = 10, \pi = 0.20) = 0.878$

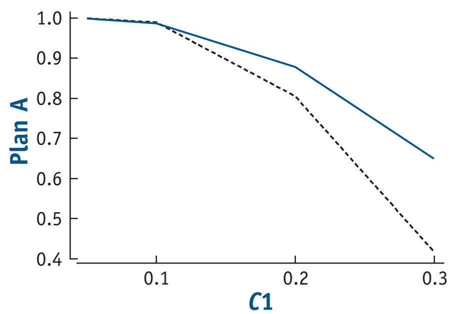
$P(X \le 3 \mid n = 10, \pi = 0.30) = 0.649$

$P(X \le 5 \mid n = 20, \pi = 0.05) = 0.999$

$P(X \le 5 \mid n = 20, \pi = 0.10) = 0.989$

$P(X \le 5 \mid n = 20, \pi = 0.20) = 0.805$

$P(X \le 5 \mid n = 20, \pi = 0.30) = 0.417$



The solid line represents the operating characteristic curve for the first plan; the dashed line represents the second plan. The supplier would prefer the first because the probability of acceptance is higher (above). However, if he is really sure of his quality, the second plan seems slightly higher at the very low range of defect percentages and might be preferred.

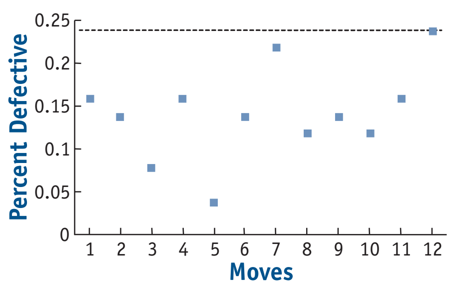
20-30. a. $\bar{p} = \dfrac{86}{12(50)} = 0.1433$

$$3\sqrt{\frac{0.1433(1-0.1433)}{50}} = 0.15$$

$UCL = 0.14 + 0.15 = 0.29$

$LCL = 0.14 - 0.15 = 0$

b.



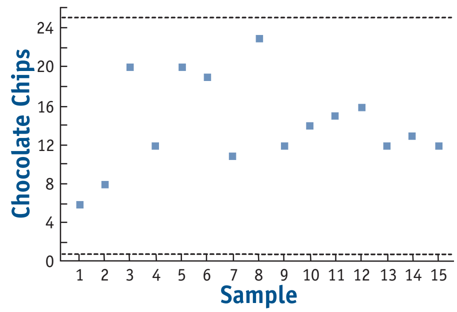
c. None of the moves are out of control in terms of the number of complaints.

20-31. a. $\bar{c} = \dfrac{213}{15} = 14.2 \qquad 3\sqrt{14.2} = 11.30$

$UCL = 14.2 + 11.3 = 25.5$

$LCL = 14.2 - 11.3 = 2.9$

b.

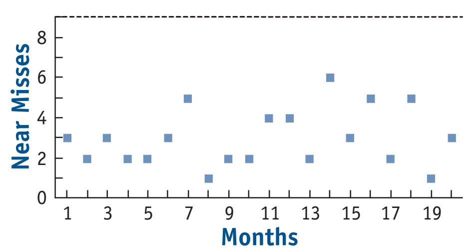


c. All the points are in control.

20-32. $\bar{c} = \dfrac{60}{20} = 3.0$

$UCL = 3.0 + 3.0\sqrt{3} = 3.00 + 5.20 = 8.20$

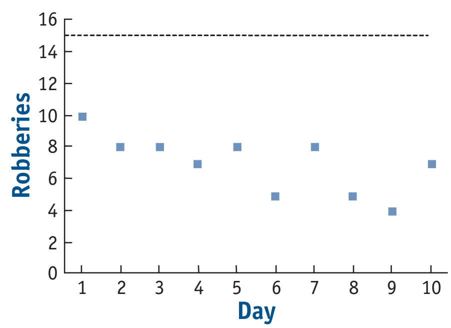
$LCL = 3.0 - 3.0\sqrt{3} = 3.00 - 5.20 = 0$



20-33. $\bar{c} = \dfrac{70}{10} = 7.0$

$UCL = 7.0 + 3.0\sqrt{7} = 7.00 + 7.9 = 14.9$

$LCL = 7.0 - 3.0\sqrt{7} = 7.00 - 7.9 = 0$



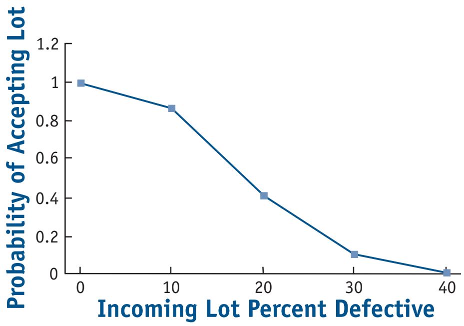
20-34. a. 0.015, found by $0.012 + 0.003$

b. $P(c \le 3/\pi = 0.10, n = 20)$
$= 0.122 + 0.270 + 0.285 + 0.190 = 0.867$

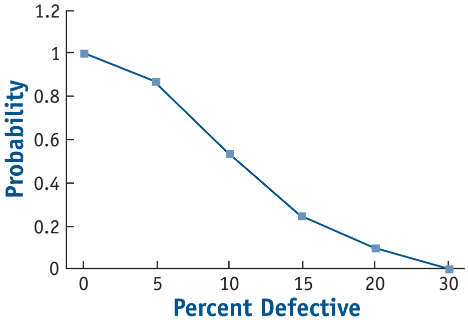
$P(c \le 3/\pi = 0.20, n = 20)$
$= 0.012 + 0.058 + 0.137 + 0.205 = 0.412$

$P(c \leq 3/\pi = 0.30, n = 20)$
$= 0.001 + 0.007 + 0.028 + 0.072 = 0.108$

$P(c \leq 3/\pi = 0.40, n = 20)$
$= 0.003 + 0.12 + 0.015$



20-35. $P(X \leq 3 \mid n = 20, \pi = 0.10) = 0.867$
$P(X \leq 3 \mid n = 20, \pi = 0.20) = 0.412$
$P(X \leq 3 \mid n = 20, \pi = 0.30) = 0.108$



20-36. a. $P(X \leq 2 \mid n = 25, \pi = 0.10) = 0.537$
$P(X \leq 2 \mid n = 25, \pi = 0.20) = 0.099$
$P(X \leq 2 \mid n = 25, \pi = 0.30) = 0.008$

b. $P(X \leq 2 \mid n = 25, \pi = 0.15)$
$= 0.0172 + 0.0759 + 0.1607 = 0.2538$

c. $P(X \leq 2 \mid n = 25, \pi = 0.05)$
$= 0.2774 + 0.3650 + 0.2305 = 0.8729$

The probability (0.8729) is somewhat less than desired (0.90). (Note: A more extensive table, or some computer system such as Minitab, is required for part b.)