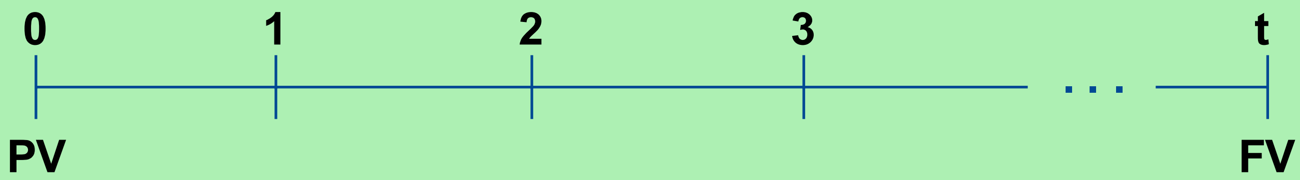
Chapter 5
Introduction to Valuation: The Time Value of Money

Chapter Organization

- 5.1 Future Value and Compounding
- 5.2 Present Value and Discounting
- 5.3 More on Present and Future Values
- 5.4 Summary and Conclusions

T5.2 Time Value Terminology

- Consider the time line below:



- PV is the Present Value, that is, the value <u>today</u>.
- FV is the Future Value, or the value at a future date.
- The number of time periods between the Present Value and the Future Value is represented by "t".
- The rate of interest is called "r".
- All time value questions involve the four values above: PV, FV, r, and t. Given three of them, it is always possible to calculate the fourth.

T5.3 Future Value for a Lump Sum

- **Notice that**
  - 1. $\$110 = \$100 \times (1 + .10)$
  - 2. $\$121 = \$110 \times (1 + .10) = \$100 \times 1.10 \times 1.10 = \$100 \times 1.10^2$
  - 3. $\$133.10 = \$121 \times (1 + .10) = \$100 \times 1.10 \times 1.10 \times 1.10$

    $= \$100 \times$ ________

T5.3 Future Value for a Lump Sum

- **Notice that**
  - **1. $110 = $100 $\times$ (1 + .10)**
  - **2. $121 = $110 $\times$ (1 + .10) = $100 $\times$ 1.10 $\times$ 1.10 = $100 $\times$ 1.10^2**
  - **3. $133.10 = $121 $\times$ (1 + .10) = $100 $\times$ 1.10 $\times$ 1.10 $\times$ 1.10**
    = $100 $\times$ $(1.10)^3$
- **In general, the future value, FV_t, of $1 invested today at r% for t periods is**

$$FV_t = \$1 \times (1 + r)^t$$

- **The expression $(1 + r)^t$ is the *future value interest factor.***

- **Q.** Deposit $5,000 today in an account paying 12%. How much will you have in 6 years? How much is simple interest? How much is compound interest?

- **A.** Multiply the $5000 by the future value interest factor:

$$\$5000 \times (1 + r)^t = \$5000 \times \_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \$5000 \times 1.9738227$$
$$= \$9869.11$$

At 12%, the simple interest is .12 × $5000 = $_____ per year. After 6 years, this is 6 × $600 = $_____ ; the difference between compound and simple interest is thus $_____ - $3600 = $_____

T5.4 Chapter 5 Quick Quiz - Part 1 of 4

- **Q.** **Deposit \$5,000 today in an account paying 12%. How much will you have in 6 years? How much is simple interest? How much is compound interest?**

- **A.** **Multiply the \$5000 by the future value interest factor:**

$$\$5000 \times (1 + r)^t = \$5000 \times (1.12)^6$$

$$= \$5000 \times 1.9738227$$

$$= \$9869.11$$

At 12%, the simple interest is .12 × \$5000 = \$600 per year. After 6 years, this is 6 × \$600 = \$3600; the difference between compound and simple interest is thus \$4869.11 - \$3600 = \$1269.11

T5.5 Interest on Interest Illustration

Q. You have just won a $1 million jackpot in the provincial lottery. You can buy a ten year certificate of deposit which pays 6% compounded annually. Alternatively, you can give the $1 million to your brother-in-law, who promises to pay you 6% *simple interest* annually over the ten year period. Which alternative will provide you with more money at the end of ten years?

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A. The future value of the CD is $1 million x $(1.06)^{10}$ = $1,790,847.70.
The future value of the investment with your brother-in-law, on the other hand, is $1 million + $1 million (.06)(10) = $1,600,000.

Compounding (or interest on interest), results in incremental wealth of nearly *$191,000*. (Of course we haven't even begun to address the risk of handing your brother-in-law $1 million!)

T5.6 Future Value of $100 at 10 Percent (Table 5.1)

Year	Beginning Amount	Simple Interest	Compound Interest	Total Interest Earned	Ending Amount
1	$100.00	$10.00	$ 0.00	$10.00	$110.00
2	110.00	10.00	1.00	11.00	121.00
3	121.00	10.00	2.10	12.10	133.10
4	133.10	10.00	3.31	13.31	146.41
5	146.41	10.00	4.64	14.64	161.05
	Totals	$50.00	$ 11.05	$ 61.05	

- **Want to be a millionaire? No problem! Suppose you are currently 21 years old, and can earn 10 percent on your money. How much must you invest *today* in order to accumulate $1 million by the time you reach age 65?**

T5.7 Chapter 5 Quick Quiz - Part 2 of 4 (concluded)

- **First define the variables:**

  FV = \$1 million r = 10 percent

  t = 65 - 21 = 44 years   PV = ?

- **Set this up as a future value equation and solve for the present value:**

  $\$1 \text{ million} = PV \times (1.10)^{44}$

  $PV = \$1 \text{ million}/(1.10)^{44} = \$15,091.$

- **Of course, we've ignored taxes and other complications, but stay tuned - right now you need to figure out where to get \$15,000!**

T5.8 Present Value for a Lump Sum

- **Q. Suppose you need $20,000 in three years to pay your college tuition. If you can earn 8% on your money, how much do you need today?**
- **A. Here we know the future value is $20,000, the rate (8%), and the number of periods (3). What is the unknown present amount (i.e., the *present value*)? From before:**

$FV_t = PV \times (1 + r)^t$

$\$20{,}000 = PV \times$ __________

Rearranging:

$PV = \$20{,}000/(1.08)^3$

$= \$$_________

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$$FV_t = PV \times (1 + r)^t$$

$$\$20{,}000 = PV \times (1.08)^3$$

Rearranging:

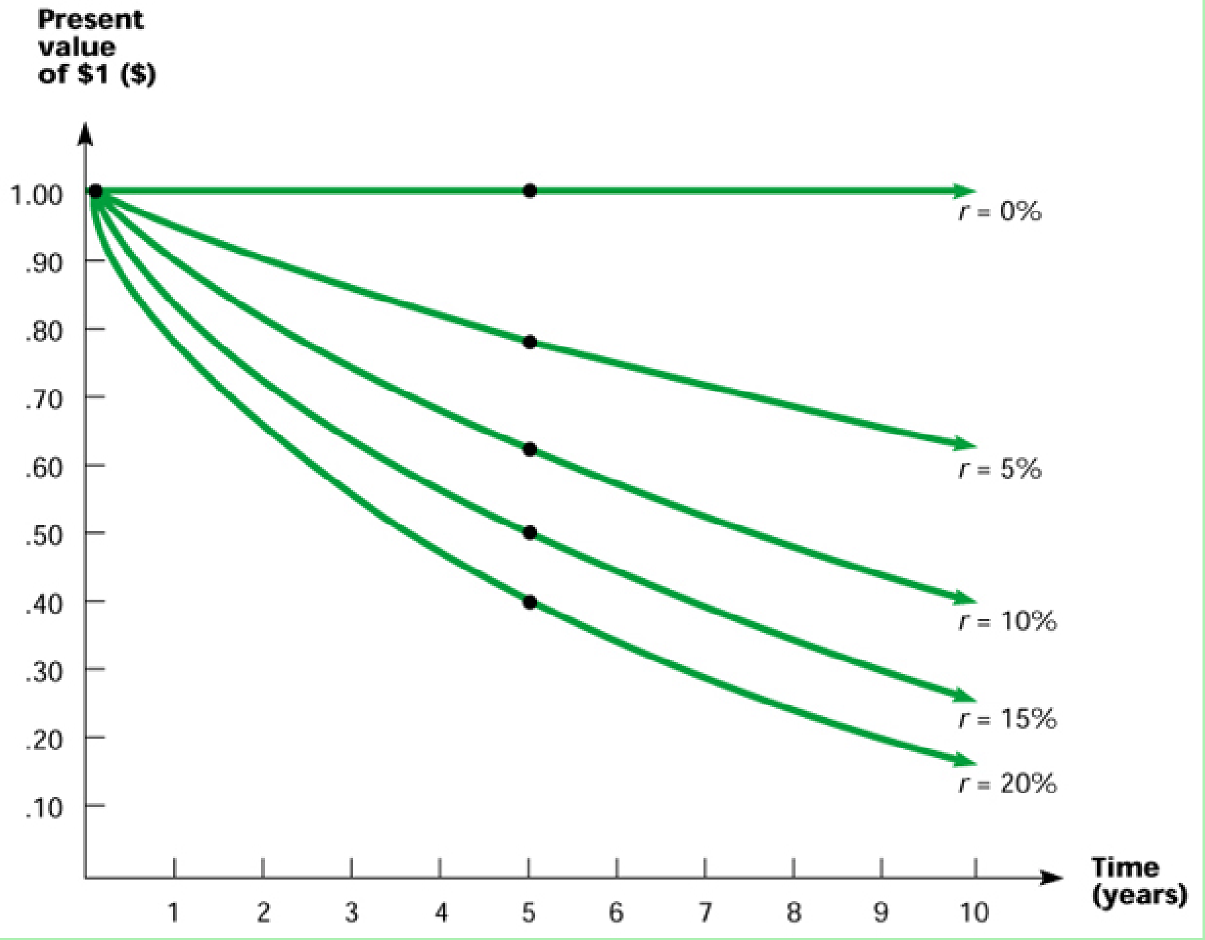
$$PV = \$20{,}000/(1.08)^3$$

$$= \$15{,}876.64$$

The PV of a $1 to be received in *t* periods when the rate is *r* is

$$PV = \$1/(1 + r)^t$$

T5.9 Present Value of $1 for Different Periods and Rates (Figure 5.3)



T5.10 Example: Finding the Rate

- Benjamin Franklin died on April 17, 1790. In his will, he gave 1,000 pounds sterling to Massachusetts and the city of Boston. He gave a like amount to Pennsylvania and the city of Philadelphia. The money was paid to Franklin when he held political office, but he believed that politicians should not be paid for their service(!).

  Franklin originally specified that the money should be paid out 100 years after his death and used to train young people. Later, however, after some legal wrangling, it was agreed that the money would be paid out 200 years after Franklin's death in 1990. By that time, the Pennsylvania bequest had grown to about $2 million; the Massachusetts bequest had grown to $4.5 million. The money was used to fund the Franklin Institutes in Boston and Philadelphia.

- Assuming that 1,000 pounds sterling was equivalent to 1,000 dollars, what rate did the two states earn? (Note: the dollar didn't become the official U.S. currency until 1792.)

T5.10 Example: Finding the Rate (continued)

- Q. Assuming that 1,000 pounds sterling was equivalent to 1,000 dollars, what rate did the two states earn?

- A. For Pennsylvania, the future value is $________ and the present value is $______ . There are 200 years involved, so we need to solve for *r* in the following:

  ________ = _____________/$(1 + r)^{200}$

  $(1 + r)^{200}$ = ________

  Solving for *r*, the Pennsylvania money grew at about 3.87% per year. The Massachusetts money did better; check that the rate of return in this case was 4.3%. Small differences can add up!

T5.10 Example: Finding the Rate (concluded)

- **Q.** Assuming that 1,000 pounds sterling was equivalent to 1,000 dollars, what rate did the two states earn?

- **A.** For Pennsylvania, the future value is \$ 2 million and the present value is \$ 1,000. There are 200 years involved, so we need to solve for *r* in the following:

  $\$ 1,000 = \$ 2 \text{ million}/(1 + r)^{200}$

  $(1 + r)^{200} = 2,000.00$

  Solving for *r*, the Pennsylvania money grew at about 3.87% per year. The Massachusetts money did better; check that the rate of return in this case was 4.3%. Small differences can add up!

T5.11 The Rule of 72

- **The "Rule of 72" is a handy rule of thumb that states the following:**

  If you earn *r* % per year, your money will double in about 72/*r* % years.

  So, for example, if you invest at 6%, your money will double in 12 years.

- **Why do we say "about?" Because at higher-than-normal rates, the rule breaks down.**

  What if *r* = 72%? ⇒ FVIF(72,1) = 1.72, not 2.00

  And if *r* = 36%? ⇒ FVIF(36,2) = 1.8496

  The lesson? The Rule of 72 is a useful rule of thumb, but it is *only* a rule of thumb!

T5.12 Chapter 5 Quick Quiz - Part 3 of 4

- **Suppose you deposit $5000 today in an account paying *r* percent per year. If you will get $10,000 in 10 years, what *rate of return* are you being offered?**

- **Set this up as present value equation:**

  FV = $10,000 PV = $ 5,000 *t* = 10 years

  $PV = FV_t/(1 + r)^t$

  $\$5000 = \$10,000/(1 + r)^{10}$

- **Now solve for *r*:**

  $(1 + r)^{10} = \$10,000/\$5,000 = 2.00$

  $r = (2.00)^{1/10} - 1 = .0718$ = **7.18 percent**

T5.13 Example: The (Really) Long-Run Return on Common Stocks

According to *Stocks for the Long Run*, by Jeremy Siegel, the average annual compound rate of return on common stocks was 8.4% over the period from 1802-1997. Suppose a distant ancestor of yours had invested $1000 in a diversified common stock portfolio in 1802. Assuming the portfolio remained untouched, how large would that portfolio be at the end of 1997? (Hint: if you owned this portfolio, you would never have to work for the rest of your life!)

Common stock values increased by 28.59% in 1998 (as proxied by the growth of the S&P 500). How much would the above portfolio be worth at the end of 1998?

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t = 195 years, r = 8.4%, and FVIF(8.4,195) = 6,771,892.09695

So the value of the portfolio would be: *$6,771,892,096.95!*

Common stock values increased by 28.59% in 1998 (as proxied by the growth of the S&P 500). How much would the above portfolio be worth at the end of 1998?

The 1998 value would be $6,771,892,096.95 × (1 + .2859) = *$8,707,976,047.47*!

T5.14 Summary of Time Value Calculations (Table 5.4)

I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r **= Interest rate, rate of return, or discount rate per period**

t **= number of periods**

C = cash amount

II. Future value of *C* invested at *r* percent per period for *t* periods:

$FV_t = C \times (1 + r)^t$

The term $(1 + r)^t$ is called the *future value factor*.

T5.14 Summary of Time Value Calculations (Table 5.4) (concluded)

III. Present value of *C* to be received in *t* periods at *r* percent per period:

$PV = C/(1 + r)^t$

The term $1/(1 + r)^t$ is called the *present value factor*.

IV. The basic present value equation giving the relationship between present and future value is:

$PV = FV_t/(1 + r)^t$

- **Now let's see what you remember!**

1. Which of the following statements is/are true?

- **Given *r* and *t* greater than zero, future value interest factors FVIF(*r*,*t*) are always greater than 1.00.**
- **Given r and t greater than zero, present value interest factors PVIF(*r*,*t*) are always less than 1.00.**

2. True or False: For given levels of *r* and *t*, PVIF(*r*,*t*) is the reciprocal of FVIF(*r*,*t*).

3. All else equal, the higher the discount rate, the (lower/higher) the present value of a set of cash flows.

T5.15 Chapter 5 Quick Quiz - Part 4 of 4 (concluded)

1. Both statements are true. If you use time value tables, use this information to be sure that you are looking at the correct table.

2. This statement is also true. PVIF(*r*,*t*) = 1/FVIF(*r*,*t*).

3. The answer is *lower* - discounting cash flows at higher rates results in lower present values. And compounding cash flows at higher rates results in higher future values.

T5.16 Solution to Problem 5.6

- **Assume the total cost of a university education will be $200,000 when your child enters college in 18 years. You presently have $15,000 to invest. What annual rate of interest must you earn on your investment to cover the cost of your child's university education?**

  Present value = $15,000

  Future value = $200,000

  ***t* = 18 *r* = ?**

- **Solution: Set this up as a future value problem.**

  $200,000 = $15,000 × FVIF(*r*,18)

  FVIF(*r*,18) = $200,000 / $15,000 = 13.333 . . .

  Solving for *r* gives 15.48%.

T5.17 Solution to Problem 5.10

- **Imprudential, Inc. has an unfunded pension liability of $425 million that must be paid in 20 years. To assess the value of the firm's stock, financial analysts want to discount this liability back to the present. If the relevant discount rate is 8 percent, what is the present value of this liability?**

  Future value = FV = $425 million

  ***t* = 20   *r* = 8 percent   Present value = ?**

- **Solution: Set this up as a present value problem.**

  PV = $425 million × PVIF(8,20)

  PV = $91,182,988.15 or about $91.18 million