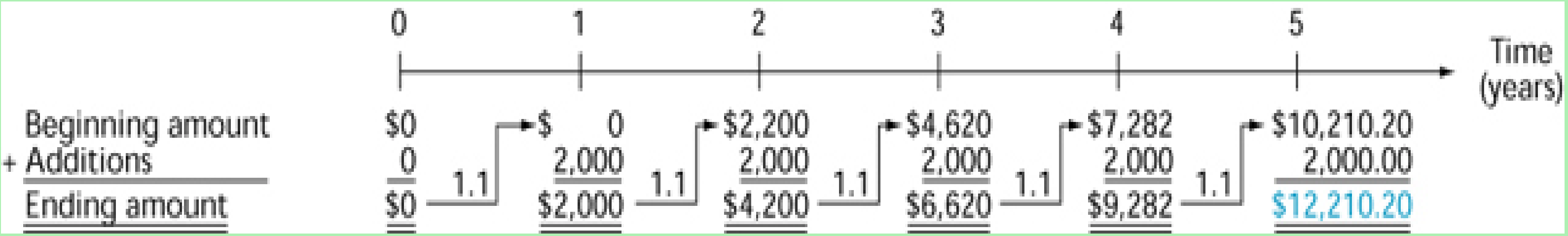
Chapter 6
Discounted Cash Flow Valuation

Chapter Organization

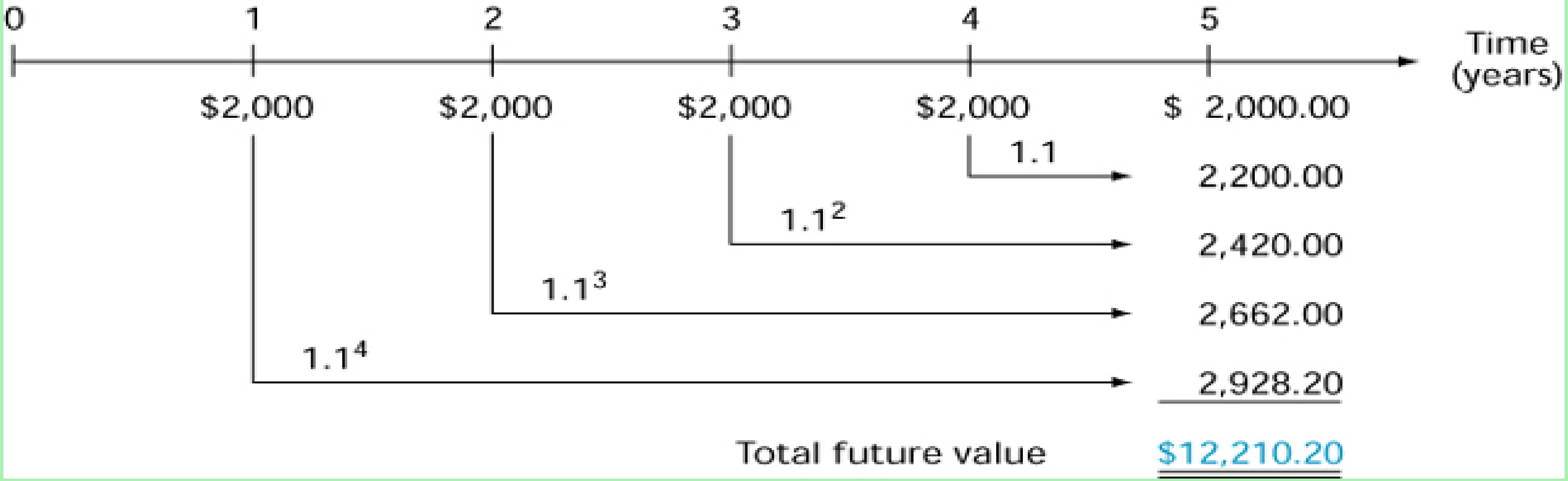
- 6.1 Future and Present Values of Multiple Cash Flows
- 6.2 Valuing Level Cash Flows: Annuities and Perpetuities
- 6.3 Comparing Rates: The Effect of Compounding
- 6.4 Loan Types and Loan Amortization
- 6.5 Summary and Conclusions

T6.2 Future Value Calculated (Fig. 6.3-6.4)

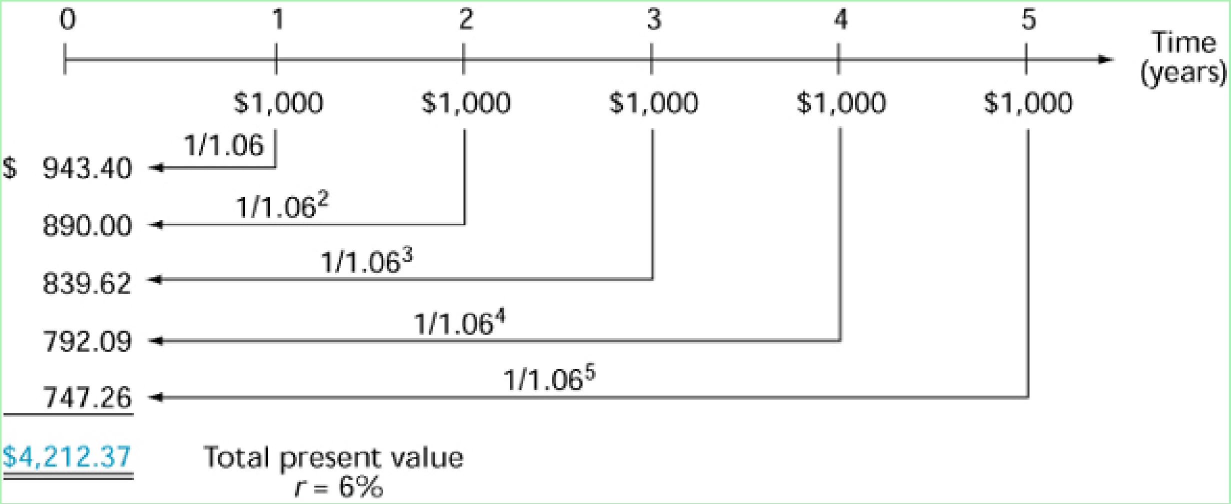
Future value calculated by compounding forward one period at a time



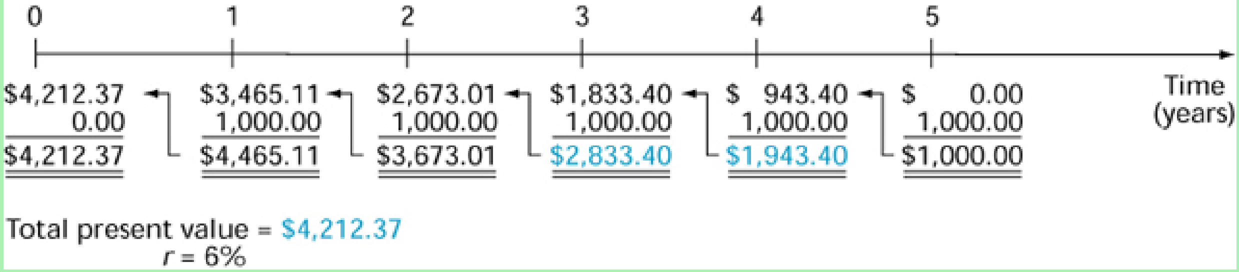
Future value calculated by compounding each cash flow separately



T6.3 Present Value Calculated (Fig 6.5-6.6)



Present value calculated by discounting each cash flow separately



Present value calculated by discounting back one period at a time

Example: Finding C

- **Q.** You want to buy a Mazda Miata to go cruising. It costs $25,000. With a 10% down payment, the bank will loan you the rest at 12% per year (1% per month) for 60 months. What will your monthly payment be?

- **A.** You will borrow ___ × $25,000 = $______ . This is the amount today, so it's the ___________ . The rate is ___ , and there are __ periods:

$$\$\ \_\_\_\_\_\_ = C \times \{\_\_\_\_\_\_\_\_\_\_\_\_\}/.01$$
$$= C \times \{1 - .55045\}/.01$$
$$= C \times 44.955$$
$$C = \$22{,}500/44.955$$
$$C = \$\_\_\_\_\_\_\_\_$$

Example: Finding C

- Q. You want to buy a Mazda Miata to go cruising. It costs $25,000. With a 10% down payment, the bank will loan you the rest at 12% per year (1% per month) for 60 months. What will your monthly payment be?

- A. You will borrow .90 × $25,000 = $22,500 . This is the amount today, so it's the present value. The rate is 1%, and there are 60 periods:

$$\$\,22{,}500 = C \times \{1 - (1/(1.01)^{60}\}/.01$$
$$= C \times \{1 - .55045\}/.01$$
$$= C \times 44.955$$

$$C = \$22{,}500/44.955$$
$$C = \$500.50 \text{ per month}$$

T6.5 Annuities and Perpetuities -- Basic Formulas

- **Annuity Present Value**

$$PV = C \times \{1 - [1/(1 + r)^t]\}/r$$

$$= \frac{\left\{1-\left[\frac{1}{(1+\quad)}\right]\right\}}{\quad}$$

- **Annuity Future Value**

$$FV_t = C \times \{[(1 + r)^t - 1]/r\}$$

$$= \frac{\left\{(1+\quad)\ -1\right\}}{\quad}$$

- **Perpetuity Present Value**

$$PV = C/r$$

- **The formulas above are the basis of many of the calculations in Corporate Finance. It will be worthwhile to keep them in mind!**

Annuity Present Value

- **Suppose you need $20,000 each year for the next three years to make your tuition payments.**

  Assume you need the first $20,000 in exactly one year. Suppose you can place your money in a savings account yielding 8% compounded annually. How much do you need to have in the account *today*?

  (Note: Ignore taxes, and keep in mind that you don't want any funds to be left in the account after the third withdrawal, nor do you want to run short of money.)

Annuity Present Value - Solution

Here we know the periodic cash flows are $20,000 each. Using the most basic approach:

$$PV = \$20{,}000/1.08 + \$20{,}000/1.08^2 + \$20{,}000/1.08^3$$
$$= \$18{,}518.52 + \$\_\_\_\_\_\_\_\_ + \$15{,}876.65$$
$$= \$51{,}541.94$$

Here's a shortcut method for solving the problem using the *annuity present value factor*:

$$PV = \$20{,}000 \times [\_\_\_\_\_\_\_\_\_\_\_\_\_\_]/\_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \$20{,}000 \times 2.577097$$
$$= \$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Annuity Present Value - Solution

Here we know the periodic cash flows are $20,000 each. Using the most basic approach:

$$PV = \$20{,}000/1.08 + \$20{,}000/1.08^2 + \$20{,}000/1.08^3$$
$$= \$18{,}518.52 + \$17{,}146.77 + \$15{,}876.65$$
$$= \$51{,}541.94$$

Here's a shortcut method for solving the problem using the *annuity present value factor*:

$$PV = \$20{,}000 \times [1 - 1/(1.08)^3]/.08$$
$$= \$20{,}000 \times 2.577097$$
$$= \$51{,}541.94$$

Annuity Present Value

- **Let's continue our tuition problem.**

  Assume the same facts apply, but that you can only earn 4% compounded annually. Now how much do you need to have in the account today?

Annuity Present Value - Solution

Again we know the periodic cash flows are $20,000 each. Using the basic approach:

$$PV = \$20,000/1.04 + \$20,000/1.04^2 + \$20,000/1.04^3$$
$$= \$19,230.77 + \$18,491.12 + \$17,779.93$$
$$= \$55,501.82$$

Here's a shortcut method for solving the problem using the *annuity present value factor*:

$$PV = \$20,000 \times [1 - 1/(1.04)^3]/.04$$
$$= \$20,000 \times 2.775091$$
$$= \$55,501.82$$

Example 1: Finding t

- Q. Suppose you owe $2000 on a Visa card, and the interest rate is 2% per month. If you make the minimum monthly payments of $50, how long will it take you to pay off the debt? (Assume you quit charging stuff immediately!)

Example 1: Finding t

- **Q.** Suppose you owe $2000 on a Visa card, and the interest rate is 2% per month. If you make the minimum monthly payments of $50, how long will it take you to pay off the debt? (Assume you quit charging stuff immediately!)

A. A time:

$2000 = $50 × {1 - 1/(1.02) }/.02

.80 = 1 - 1/1.02

1.02 = 5.0

= 81.3 months, or about 6.78 !

Example 2: Finding C

- Previously we determined that a 21-year old could accumulate $1 million by age 65 by investing $15,091 today and letting it earn interest (at 10%compounded annually) for 44 years.

  Now, rather than plunking down $15,091 in one chunk, suppose she would rather invest smaller amounts annually to accumulate the million. If the first deposit is made in one year, and deposits will continue through age 65, how large must they be?

- Set this up as a FV problem:

$$\$1,000,000 = C \times [(1.10)^{44} - 1]/.10$$

$$C = \$1,000,000/652.6408 = \$1,532.24$$

Becoming a millionaire just got easier!

T6.8 Example: Annuity Future Value

- **Previously we found that, if one begins saving at age 21, accumulating $1 million by age 65 requires saving only $1,532.24 per year.**

  Unfortunately, most people don't start saving for retirement that early in life. (Many don't start at all!)

  Suppose Bill just turned 40 and has decided it's time to get serious about saving. Assuming that he wishes to accumulate $1 million by age 65, he can earn 10% compounded annually, and will begin making equal annual deposits in one year and make the last one at age 65, how much must each deposit be?

- **Setup:** **$1 million = $C \times$ [$(1.10)^{25}$ - 1]/.10**

  Solve for *C*: *C* = $1 million/98.34706 = $10,168.07

  By waiting, Bill has to set aside over *six times* as much money each year!

T6.9 Chapter 6 Quick Quiz -- Part 3 of 5

- **Consider Bill's retirement plans one more time.**

  Again assume he just turned 40, but, recognizing that he has a lot of time to make up for, he decides to invest in some high-risk ventures that may yield 20% annually. (Or he may lose his money completely!) Anyway, assuming that Bill still wishes to accumulate $1 million by age 65, and will begin making equal annual deposits in one year and make the last one at age 65, *now* how much must each deposit be?

- **Setup:** **$1 million = $C \times$ [$(1.20)^{25}$ - 1]/.20**

  Solve for *C*: *C* = $1 million/471.98108 = $2,118.73

  So Bill can catch up, but only if he can earn a *much higher return* (which will probably require taking a lot more risk!).

T6.10 Summary of Annuity and Perpetuity Calculations (Table 6.2)

I. Symbols

PV	=	Present value, what future cash flows bring today
FV_t	=	Future value, what cash flows are worth in the future
r	=	Interest rate, rate of return, or discount rate per period
t	=	Number of time periods
C	=	Cash amount

II. FV of *C* per period for *t* periods at *r* percent per period:

$$FV_t = C \times \{[(1 + r)^t - 1]/r\}$$

$$= \frac{\{(1+\ )\ -1\}}{}$$

III. PV of *C* per period for *t* periods *at r* percent per period:

$$PV = C \times \{1 - [1/(1 + r)^t]\}/r$$

$$= \frac{\left\{1-\left[\frac{1}{(1+\ )}\right]\right\}}{}$$

IV. PV of a perpetuity of C per period:

$$PV = C/r$$

T6.11 Example: Perpetuity Calculations

- Suppose we expect to receive $1000 at the end of each of the next 5 years. Our opportunity rate is 6%. What is the value today of this set of cash flows?

$$\begin{aligned} PV &= \$1000 \times \{1 - 1/(1.06)^5\}/.06 \\ &= \$1000 \times \{1 - .74726\}/.06 \\ &= \$1000 \times 4.212364 \\ &= \$4212.36 \end{aligned}$$

- Now suppose the cash flow is $1000 per year *forever*. This is called a *perpetuity*. And the PV is easy to calculate:

  PV = C/r = $1000/.06 = $16,666.66…

- So, payments in years 6 thru ∞ have a total PV of $12,454.30!

T6.12 Chapter 6 Quick Quiz -- Part 4 of 5

Consider the following questions.

- **The present value of a perpetual cash flow stream has a finite value (as long as the discount rate, *r*, is greater than 0). Here's a question for you: How can an infinite number of cash payments have a *finite* value?**
- **Here's an example related to the question above. Suppose you are considering the purchase of a perpetual bond. The issuer of the bond promises to pay the holder $100 per year forever. If your opportunity rate is 10%, what is the <u>most</u> you would pay for the bond today?**
- **One more question: Assume you are offered a bond identical to the one described above, but with a life of 50 years. What is the difference in value between the 50-year bond and the perpetual bond?**

- An infinite number of cash payments has a finite present value is because the present values of the cash flows in the distant future become infinitesimally small.
- The value today of the perpetual bond = $100/.10 = $1,000.
- Using Table A.3, the value of the 50-year bond equals

  $100 × 9.9148 = $991.48

  So what is the present value of payments 51 through infinity (also an infinite stream)?

  Since the perpetual bond has a PV of $1,000 and the otherwise identical 50-year bond has a PV of $991.48, the value today of payments 51 through infinity must be

  $1,000 - 991.48 = $8.52 (!)

T6.13 Compounding Periods, EARs, and APRs

Compounding period	Number of times compounded	Effective annual rate
Year	1	10.00000%
Quarter	4	10.38129
Month	12	10.47131
Week	52	10.50648
Day	365	10.51558
Hour	8,760	10.51703
Minute	525,600	10.51709

T6.13 Compounding Periods, EARs, and APRs (continued)

- **EARs and APRs**
- Q. If a rate is quoted at 16%, compounded semiannually, then the actual rate is 8% per six months. Is 8% per six months the same as 16% per year?
- A. If you invest $1000 for one year at 16%, then you'll have $1160 at the end of the year. If you invest at 8% per period for two periods, you'll have
- $$\begin{aligned} FV &= \$1000 \times (1.08)^2 \\ &= \$1000 \times 1.1664 \\ &= \$1166.40, \end{aligned}$$

or $6.40 more. Why? What rate per year is the same as 8% per six months?

- **The *Effective Annual Rate* (EAR) is _____%. The "16% compounded semiannually" is the quoted or stated rate, <u>not</u> the effective rate.**

- **By law, in consumer lending, the rate that must be quoted on a loan agreement is equal to the rate per period multiplied by the number of periods. This rate is called the __________________ (____).**

- **Q. A bank charges 1% per month on car loans. What is the APR? What is the EAR?**

- **A. The APR is __ × __ = ___%. The EAR is:**

  EAR = _________ - 1 = 1.126825 - 1 = 12.6825%

T6.13 Compounding Periods, EARs, and APRs (concluded)

- The *Effective Annual Rate* (EAR) is 16.64%. The "16% compounded semiannually" is the quoted or stated rate, not the effective rate.
- By law, in consumer lending, the rate that must be quoted on a loan agreement is equal to the rate per period multiplied by the number of periods. This rate is called the Annual Percentage Rate *(APR)*.
- Q. A bank charges 1% per month on car loans. What is the APR? What is the EAR?
- A. The APR is $1\% \times 12 = 12\%$. The EAR is:

  $$EAR = (1.01)^{12} - 1 = 1.126825 - 1 = 12.6825\%$$

  The APR is thus a quoted rate, not an effective rate!

T6.14 Example: Amortization Schedule - Fixed Principal

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	$5,000	$1,450	$450	$1,000	$4,000
2	4,000	1,360	360	1,000	3,000
3	3,000	1,270	270	1,000	2,000
4	2,000	1,180	180	1,000	1,000
5	1,000	1,090	90	1,000	0
Totals		$6,350	$1,350	$5,000	

T6.15 Example: Amortization Schedule - Fixed Payments

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	$5,000.00	$1,285.46	$ 450.00	$ 835.46	$4,164.54
2	4,164.54	1,285.46	374.81	910.65	3,253.88
3	3,253.88	1,285.46	292.85	992.61	2,261.27
4	2,261.27	1,285.46	203.51	1,081.95	1,179.32
5	1,179.32	1,285.46	106.14	1,179.32	0.00
Totals		$6,427.30	$1,427.31	$5,000.00	

How to lie, cheat, and steal with interest rates:

RIPOV RETAILING

Going out *for* business sale!

$1,000 instant credit!

12% simple interest!

Three years to pay!

Low, low monthly payments!

Assume you buy $1,000 worth of furniture from this store and agree to the above credit terms. What is the APR of this loan? The EAR?

T6.16 Solution to Chapter 6 Quick Quiz -- Part 5 of 5 (concluded)

- **Your payment is calculated as:**
  - **1. Borrow $1,000 today at 12% per year for three years, you will owe $1,000 + $1000(.12)(3) = $1,360.**
  - **2. To make it easy on you, make 36 low, low payments of $1,360/36 = $37.78.**
  - **3. Is this a 12% loan?**

$1,000 = $37.78 x (1 - 1/(1 + *r*)^{36})/*r*

r* = 1.767% *per month

APR = 12(1.767%) = 21.204%

EAR = 1.01767^{12} - 1 = 23.39% (!)

T6.17 Solution to Problem 6.10

- **Seinfeld's Life Insurance Co. is trying to sell you an investment policy that will pay you and your heirs $1,000 per year forever. If the required return on this investment is 12 percent, how much will you pay for the policy?**

- **The present value of a perpetuity equals *C/r*. So, the *most* a rational buyer would pay for the promised cash flows is**

  C/r = $1,000/.12 = $8,333.33

- **Notice: $8,333.33 is the amount which, invested at 12%, would throw off cash flows of $1,000 per year forever. (That is, $8,333.33 × .12 = $1,000.)**

T6.18 Solution to Problem 6.11

- **In the previous problem, Seinfeld's Life Insurance Co. is trying to sell you an investment policy that will pay you and your heirs $1,000 per year forever. Seinfeld told you the policy costs $10,000. At what interest rate would this be a fair deal?**

- **Again, the present value of a perpetuity equals *C*/*r*. Now solve the following equation:**

$$\$10{,}000 = C/r = \$1{,}000/r$$

$$r = .10 = 10.00\%$$

- **Notice: If your opportunity rate is less than 10.00%, this is a good deal for you; but if you can earn more than 10.00%, you can do better by investing the $10,000 yourself!**

T6.18 Solution to Problem 6.11

- **Congratulations! You've just won the $20 million first prize in the Subscriptions R Us Sweepstakes. Unfortunately, the sweepstakes will actually give you the $20 million in $500,000 annual installments over the next 40 years, beginning next year. If your appropriate discount rate is 12 percent per year, how much money did you really win?**

- **"How much money did you really win?" translates to, "What is the value today of your winnings?" So, this is a present value problem.**

$$PV = \$500{,}000 \times [1 - 1/(1.12)^{40}]/.12$$

$$= \$500{,}000 \times [1 - .0107468]/.12$$

$$= \$500{,}000 \times 8.243776$$

$$= \$4{,}121{,}888.34$$ (Not quite $20 million, eh?)