Chapter 13
Return, Risk, and the Security Market Line

Chapter Organization

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T13.2 Expected Return and Variance: Basic Ideas

- The quantification of risk and return is a crucial aspect of modern finance. It is not possible to make "good" (i.e., value-maximizing) financial decisions unless one understands the relationship between risk and return.
- Rational investors like returns and dislike risk.
- Consider the following proxies for return and risk:

  Expected return - weighted average of the distribution of possible returns in the future.

  Variance of returns - a measure of the *dispersion* of the distribution of possible returns in the future.

  How do we calculate these measures? Stay tuned.

T13.3 Example: Calculating the Expected Return

State of Economy	p_i Probability of state i	R_i Return in state i
+1% change in GNP	.25	-5%
+2% change in GNP	.50	15%
+3% change in GNP	.25	35%

T13.3 Example: Calculating the Expected Return (concluded)

i	$(p_i \times R_i)$
i = 1	-1.25%
i = 2	7.50%
i = 3	8.75%

Expected return = (-1.25 + 7.50 + 8.75)

= 15%

T13.4 Calculation of Expected Return (Table 13.3)

		Stock L		*Stock U*	
(1) State of Economy	(2) Probability of State of Economy Occurs	(3) Rate of Return if State Occurs	(4) Product (2) × (3)	(5) Rate of Return if State Occurs	(6) Product (2) × (5)
Recession	.80	-.20	-.16	.30	.24
Boom	.20	.70	.14	.10	.02
			$E(R_L)$ = -2%		$E(R_U)$ = 26%

T13.5 Example: Calculating the Variance

State of Economy	p_i Probability of state i	r_i Return in state i
+1% change in GNP	.25	-5%
+2% change in GNP	.50	15%
+3% change in GNP	.25	35%

$$E(R) = \bar{R} = 15\% = .15$$

T13.5 Calculating the Variance (concluded)

i	$(R_i - \bar{R})^2$	$p_i \times (R_i - \bar{R})^2$
i=1	.04	.01
i=2	0	0
i=3	.04	.01
		Var(R) = .02

What is the standard deviation?

The standard deviation = $(.02)^{1/2}$ *= .1414 .*

T13.6 Example: Expected Returns and Variances

State of the economy	Probability of state	Return on asset A	Return on asset B
Boom	0.40	30%	-5%
Bust	0.60	-10%	25%
	1.00		

- A. Expected returns

$E(R_A) = 0.40 \times (.30) + 0.60 \times (-.10) = .06 = 6\%$

$E(R_B) = 0.40 \times (-.05) + 0.60 \times (.25) = .13 = 13\%$

T13.6 Example: Expected Returns and Variances (concluded)

- B. Variances

  $Var(R_A) = 0.40 \times (.30 - .06)^2 + 0.60 \times (-.10 - .06)^2 = .0384$

  $Var(R_B) = 0.40 \times (-.05 - .13)^2 + 0.60 \times (.25 - .13)^2 = .0216$

- C. Standard deviations

  $SD(R_A) = (.0384)^{1/2} = .196 = 19.6\%$

  $SD(R_B) = (.0216)^{1/2} = .147 = 14.7\%$

T13.7 Example: Portfolio Expected Returns and Variances

- Portfolio *weights*: put 50% in Asset A and 50% in Asset B:

State of the economy	Probability of state	Return on A	Return on B	Return on portfolio
Boom	0.40	30%	-5%	12.5%
Bust	0.60	-10%	25%	7.5%
	1.00			

T13.7 Example: Portfolio Expected Returns and Variances (continued)

- A. $E(R_P) = 0.40 \times (.125) + 0.60 \times (.075) = .095 = 9.5\%$
- B. $Var(R_P) = 0.40 \times (.125 - .095)^2 + 0.60 \times (.075 - .095)^2 = .0006$
- C. $SD(R_P) = (.0006)^{1/2} = .0245 = 2.45\%$
- NOTE: $E(R_P) = .50 \times E(R_A) + .50 \times E(R_B) = 9.5\%$
- BUT: $Var(R_P) \neq .50 \times Var(R_A) + .50 \times Var(R_B)$

In words: While the expected return of the portfolio is the weighted average of the asset returns, the variance is **not** just the weighted average of the asset variances.

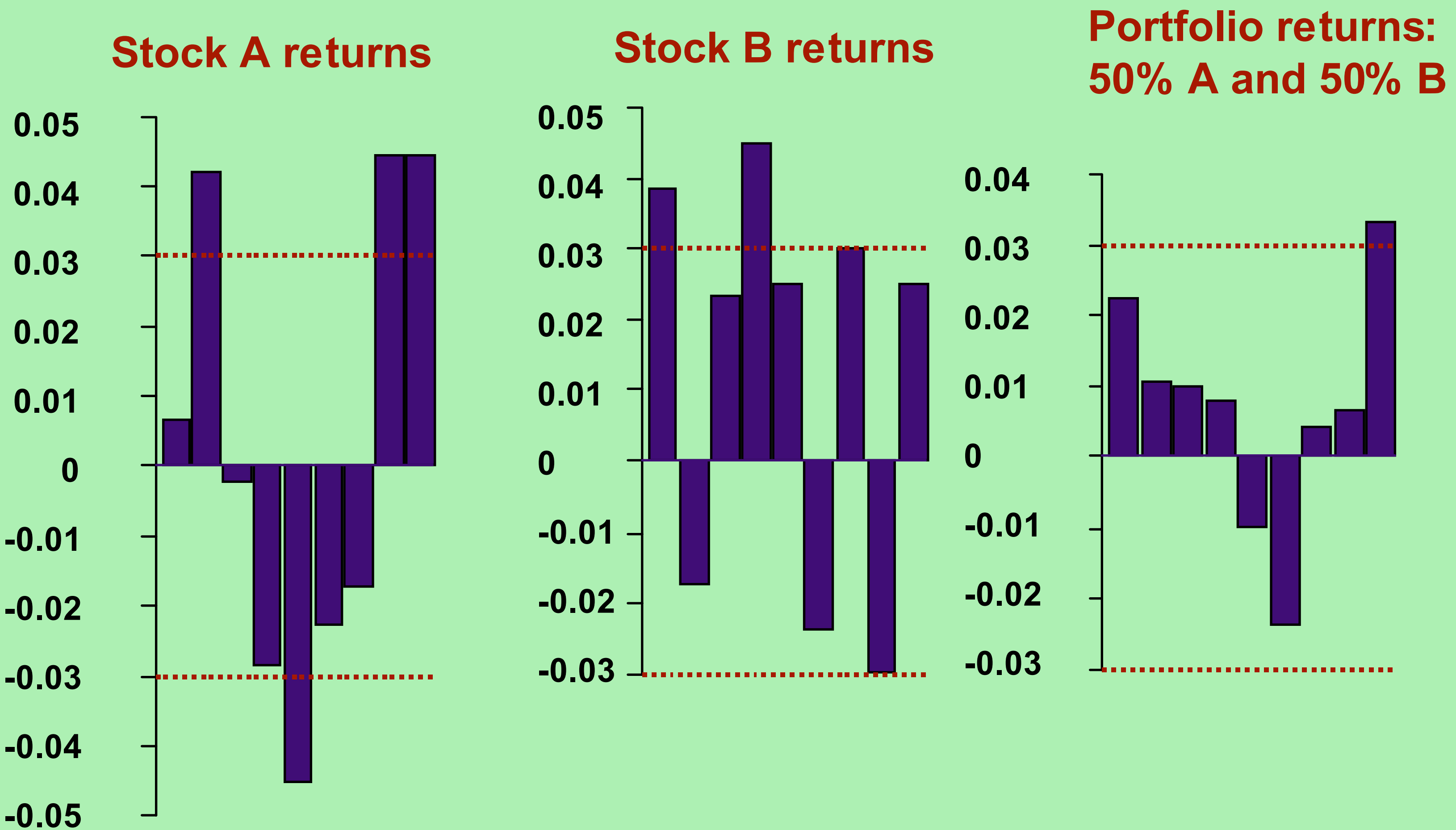
T13.7 Example: Portfolio Expected Returns and Variances (concluded)

- New portfolio weights: put 3/7 in A and 4/7 in B:

State of the economy	Probability of state	Return on A	Return on B	Return on portfolio
Boom	0.40	30%	-5%	10%
Bust	0.60	-10%	25%	10%
	1.00			

- **A. $E(R_P)$ = 10%**
- **B. $SD(R_P)$ = 0% (Why is this zero?)**

T13.8 The Effect of Diversification on Portfolio Variance



T13.9 Announcements, Surprises, and Expected Returns

- Key issues:
  - What are the components of the total return?
  - What are the different types of risk?

- Expected and Unexpected Returns

  Total return = Expected return + Unexpected return

  $$R = E(R) + U$$

- Announcements and News

  Announcement = Expected part + Surprise

T13.10 Risk: Systematic and Unsystematic

- Systematic and Unsystematic Risk
  - Types of surprises

    1. Systematic or “market” risks

    2. Unsystematic/unique/asset-specific risks

  - Systematic and unsystematic components of return

    Total return = Expected return + Unexpected return

    R = E(R) + U

    = E(R) + systematic portion + unsystematic portion

T13.11 Peter Bernstein on Risk and Diversification

"Big risks are scary when you cannot diversify them, especially when they are expensive to unload; even the wealthiest families hesitate before deciding which house to buy. Big risks are not scary to investors who can diversify them; big risks are interesting. No single loss will make anyone go broke . . . by making diversification easy and inexpensive, financial markets enhance the level of risk-taking in society."

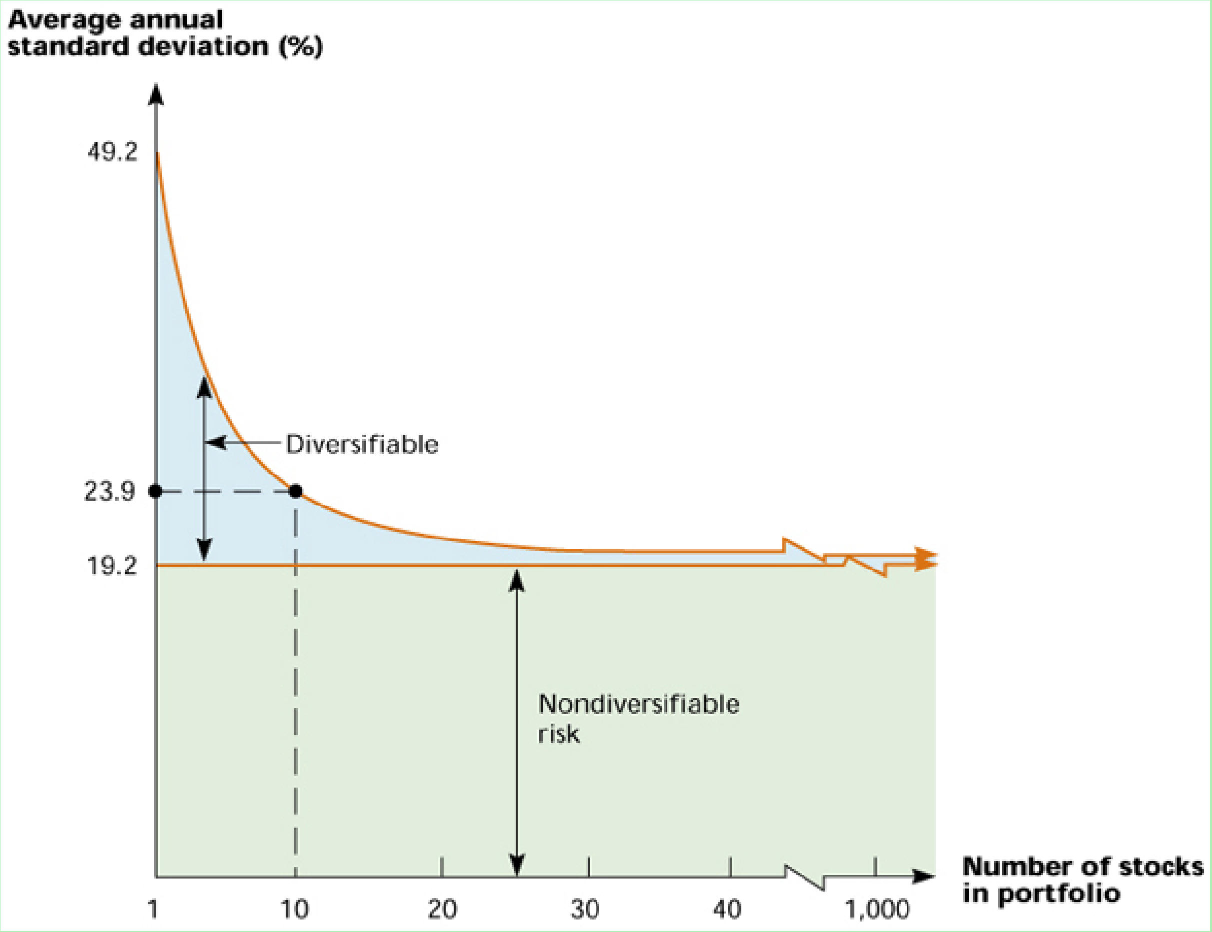
Peter Bernstein, in his book, Capital Ideas

T13.12 Standard Deviations of Annual Portfolio Returns (Table 13.8)

(1) Number of Stocks in Portfolio	(2) Average Standard Deviation of Annual Portfolio Returns	(3) Ratio of Portfolio Standard Deviation to Standard Deviation of a Single Stock
1	49.24%	1.00
10	23.93	0.49
50	20.20	0.41
100	19.69	0.40
300	19.34	0.39
500	19.27	0.39
1,000	19.21	0.39

These figures are from Table 1 in Meir Statman, “How Many Stocks Make a Diversified Portfolio?” *Journal of Financial and Quantitative Analysis* 22 (September 1987), pp. 353–64. They were derived from E. J. Elton and M. J. Gruber, “Risk

T13.13 Portfolio Diversification (Figure 13.6)



T13.14 Beta Coefficients for Selected Companies (Table 13.10)

Canadian Company	Beta Coefficient	U.S. Company	Beta Coefficient
Bank of Nova Scotia	0.65	American Electric Power	.65
Bombardier	0.71	Exxon	.80
Canadian Utilities	0.50	IBM	.95
C-MAC Industries	1.85	Wal-Mart	1.15
Investors Group	1.22	General Motors	1.05
Maple Leaf Foods	0.83	Harley-Davidson	1.20
Nortel Networks	1.61	Papa Johns	1.45
Rogers Communication	1.26	America Online	1.65

Source: (Canadian) Scotia Capital markets and (US) Value Line *Investment Survey*, May 8, 1998.

T13.15 Example: Portfolio Beta Calculations

Stock	Amount Invested	Portfolio Weights	Beta	
(1)	(2)	(3)	(4)	(3) × (4)
Haskell Mfg.	$ 6,000	50%	0.90	0.450
Cleaver, Inc.	4,000	33%	1.10	0.367
Rutherford Co.	2,000	17%	1.30	0.217
Portfolio	$12,000	100%		1.034

T13.16 Example: Portfolio Expected Returns and Betas

- Assume you wish to hold a portfolio consisting of asset A and a riskless asset. Given the following information, calculate portfolio expected returns and portfolio betas, letting the proportion of funds invested in asset A range from 0 to 125%.

  Asset A has a beta of 1.2 and an expected return of 18%.

  The risk-free rate is 7%.

  Asset A weights: 0%, 25%, 50%, 75%, 100%, and 125%.

T13.16 Example: Portfolio Expected Returns and Betas (concluded)

Proportion Invested in Asset A (%)	Proportion Invested in Risk-free Asset (%)	Portfolio Expected Return	Portfolio Beta
0	100	7.00	0.00
25	75	9.75	0.30
50	50	12.50	0.60
75	25	15.25	0.90
100	0	18.00	1.20
125	-25	20.75	1.50

T13.17 Return, Risk, and Equilibrium

- Key issues:
  - What is the relationship between risk and return?
  - What does security market equilibrium look like?

  The fundamental conclusion is that the ratio of the risk premium to beta is the same for every asset. In other words, the reward-to-risk ratio is *constant* and equal to

$$\textbf{Reward/risk ratio} = \frac{E(R_i) - R_f}{\beta_i}$$

T13.17 Return, Risk, and Equilibrium (concluded)

- Example:

  Asset A has an expected return of 12% and a beta of 1.40. Asset B has an expected return of 8% and a beta of 0.80. Are these assets valued correctly relative to each other if the risk-free rate is 5%?

  a. For A, (.12 - .05)/1.40 = ________

  b. For B, (.08 - .05)/0.80 = ________

- What would the risk-free rate have to be for these assets to be correctly valued?

  $(.12 - R_f)/1.40 = (.08 - R_f)/0.80$

  R_f = ________

T13.17 Return, Risk, and Equilibrium (concluded)

- Example:

  Asset A has an expected return of 12% and a beta of 1.40. Asset B has an expected return of 8% and a beta of 0.80. Are these assets valued correctly relative to each other if the risk-free rate is 5%?

  a. For A, (.12 - .05)/1.40 = .05

  b. For B, (.08 - .05)/0.80 = .0375

- What would the risk-free rate have to be for these assets to be correctly valued?

  $(.12 - R_f)/1.40 = (.08 - R_f)/0.80$

  $R_f = .02666$

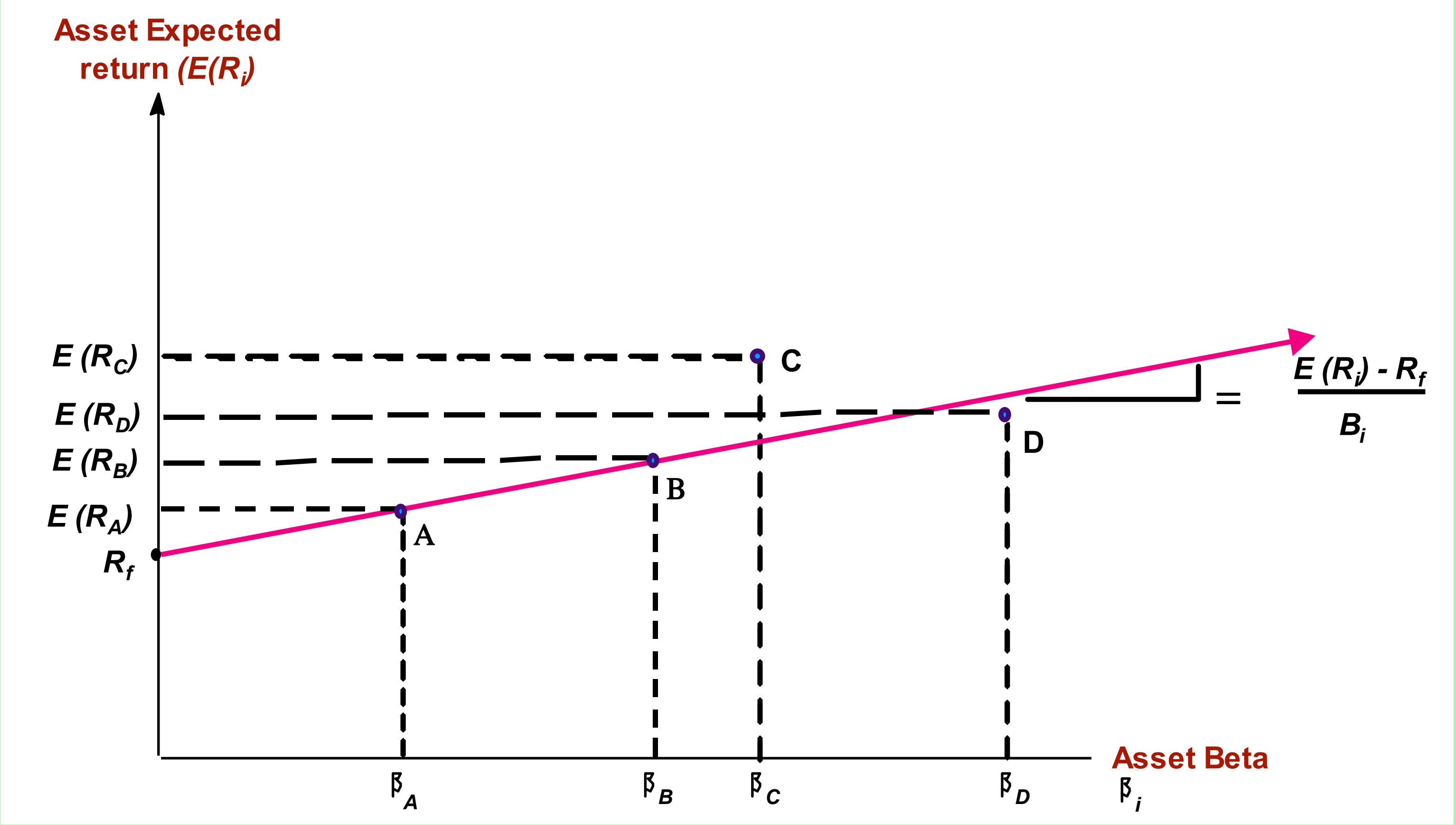
- The Capital Asset Pricing Model (CAPM) - an equilibrium model of the relationship between risk and return.

  What determines an asset's expected return?

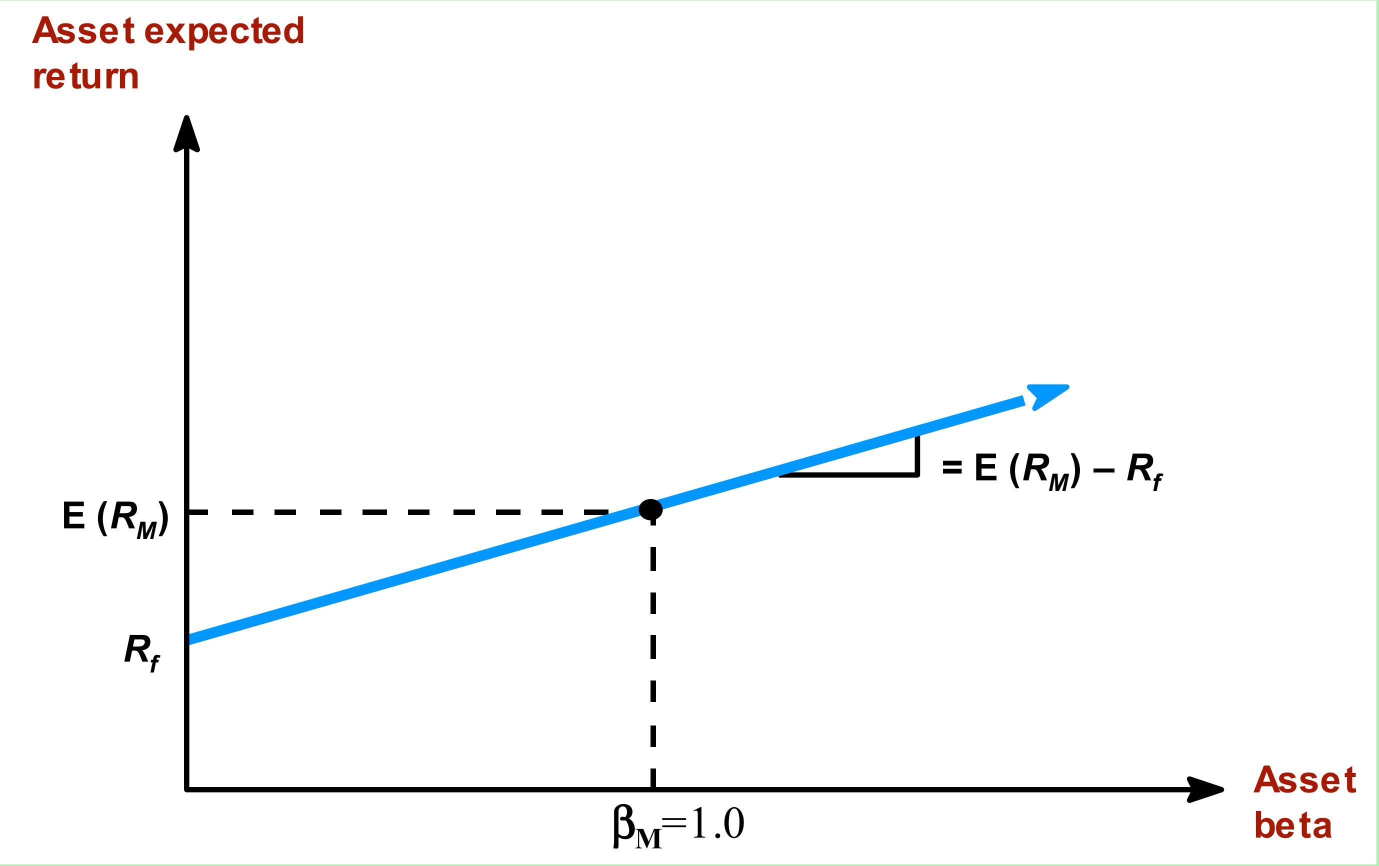
  - The risk-free rate - the pure time value of money
  - The market risk premium - the reward for bearing systematic risk
  - The beta coefficient - a measure of the amount of systematic risk present in a particular asset

  The CAPM: $E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$

T13.19 The Security Market Line (SML) (Figure 13.9)



T13.19 The Security Market Line (SML) (Figure 13.11)



T13.20 Summary of Risk and Return (Table 13.9)

I. Total risk - the variance (or the standard deviation) of an asset's return.

II. Total return - the expected return + the unexpected return.

III. Systematic and unsystematic risks

Systematic risks are unanticipated events that affect almost all assets to some degree because the effects are economywide.

Unsystematic risks are unanticipated events that affect single assets or small groups of assets. Also called *unique* or *asset-specific* risks.

IV. The effect of diversification - the elimination of unsystematic risk via the combination of assets into a portfolio.

V. The systematic risk principle and beta - the reward for bearing risk depends *only* on its level of systematic risk.

VI. The reward-to-risk ratio - the ratio of an asset's risk premium to its beta.

VII. The capital asset pricing model - $E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$.

T13.21 Chapter 13 Quick Quiz

1. Assume: the historic market risk premium has been about 8.5%. The risk-free rate is currently 5%. GTX Corp. has a beta of .85. What return should you expect from an investment in GTX?

   $E(R_{GTX})$ = 5% + _______ × .85% = 12.225%

2. What is the effect of diversification?

3. The ______ is the *equation* for the SML; the *slope* of the SML = ______ .

T13.21 Chapter 13 Quick Quiz

1. Assume: the historic market risk premium has been about 8.5%. The risk-free rate is currently 5%. GTX Corp. has a beta of .85. What return should you expect from an investment in GTX?

   $E(R_{GTX}) = 5\% + 8.5 \times .85 = 12.225\%$

2. What is the effect of diversification?

   Diversification reduces *unsystematic* risk.

3. The CAPM is the *equation* for the SML; the *slope* of the SML = $E(R_M) - R_f$.

T13.22 Solution to Problem 13.9

- Consider the following information:

State of Economy	Prob. of State of Economy	Stock A Return	Stock B Return	Stock C Return
Boom	0.35	0.14	0.15	0.33
Bust	0.65	0.12	0.03	-0.06

- What is the expected return on an equally weighted portfolio of these three stocks?
- What is the variance of a portfolio invested 15 percent each in A and B, and 70 percent in C?

T13.22 Solution to Problem 13.9 (continued)

- Expected returns on an equal-weighted portfolio
- a. Boom $E[R_p] = (.14 + .15 + .33)/3 = .2067$

  Bust: $E[R_p] = (.12 + .03 - .06)/3 = .0300$

  so the overall portfolio expected return must be

  $E[R_p] = .35(.2067) + .65(.0300) = .0918$

T13.22 Solution to Problem 13.9 (concluded)

- b. Boom: $E[R_p] = \_\_ (.14) + .15(.15) + .70(.33) = \_\_\_\_$

  Bust: $E[R_p] = .15(.12) + .15(.03) + .70(-.06) = \_\_\_\_$

  $E[R_p] = .35(\_\_\_\_) + .65(\_\_\_\_) = \_\_\_\_$

  so

  $\sigma^2_p = .35(\_\_\_\_ - \_\_\_\_)^2 + .65(\_\_\_\_ - \_\_\_\_)^2$

  $= \_\_\_\_\_$

T13.22 Solution to Problem 13.9 (concluded)

- b. Boom: $E[R_p] = .15(.14) + .15(.15) + .70(.33) = .2745$

  Bust: $E[R_p] = .15(.12) + .15(.03) + .70(-.06) = -.0195$

  $E[R_p] = .35(.2745) + .65(-.0195) = .0834$

  so

  $\sigma^2_p = .35(.2745 - .0834)^2 + .65(-.0195 - .0834)^2$

  $= .01278 + .00688 = .01966$

T13.23 Solution to Problem 13.21

- Using information from the previous chapter on capital market history, determine the return on a portfolio that is equally invested in Canadian stocks and long-term bonds.

- What is the return on a portfolio that is equally invested in small-company stocks and Treasury bills?

T13.23 Solution to Problem 13.21 (concluded)

Solution

- The average annual return on common stocks over the period 1948-1999 was 13.2 percent, and the average annual return on long-term bonds was 7.6 percent. So, the return on a portfolio with half invested in common stocks and half in long-term bonds would have been:

$$E[R_{p1}] = .50(13.2) + .50(7.6) = 10.4\%$$

If on the other hand, one would have invested in the common stocks of *small* firms and in Treasury bills in equal amounts over the same period, one's portfolio return would have been:

$$E[R_{p2}] = .50(14.8) + .50(3.8) = 9.3\%.$$

