

# CHAPTER

# 5

## *Applications of Rational Choice and Demand Theories*

Inevitably, some claims for publicly provided social assistance or “welfare” payments involve misrepresentation or fraud. The proportion across Canada has been estimated to be as high as 2 percent of total social assistance payments. Consider a provincial government elected on a platform of “downsizing government” and “ending welfare fraud.” As part of their mandate to downsize government, they downsize the welfare fraud investigation unit. Not only does this step reduce government expenditures, but as a result of the increased zeal and productivity of the remaining investigators, the next year witnesses a decline in welfare fraud.

Or does it? Certainly the number of cases of fraud uncovered and investigated has declined. But an alternative explanation is that the total number of cases of potential fraud that can be investigated is roughly proportional to the number of investigators assigned to the detection and documentation of fraud. If a stable proportion of these potential-fraud investigations turn up evidence of *actual* fraud, then the reality is quite different from what the data suggest. With fewer investigators to do the legwork, a larger proportion of cases of actual welfare fraud goes undetected.

Several related misconceptions are at work here. The first is, “If we don’t know about it, it’s not happening.” The second is that “Information is a free good.” However, the gathering and production of information involves resources, particularly when people have a vested interest in concealment. The third related misconception involves treating welfare fraud investigators exclusively in terms of their *cost*, while neglecting the *benefits* that flow from their work. If an additional fraud investigator, costing \$80,000 per year, including overhead expenses, can produce annual welfare cost savings of \$300,000, then the marginal net “benefit” of laying off that investiga-

tor is *minus* \$220,000. And this loss is not the end of the story. If laying off fraud investigators increases the likelihood that attempts at welfare fraud will succeed, then we would expect the number of fraudulent claims to increase. Shortsighted downsizing in this case is thus false economy with a vengeance.

## Chapter Preview

The case of welfare fraud investigation shows the importance of rational choice theory. In this chapter we take up a variety of applications and examples involving the rational choice and demand theories developed in Chapters 3 and 4. We begin with two examples—a gasoline tax and school vouchers—that illustrate how the rational choice model can shed light on important economic policy questions. Next we consider the concept of consumer surplus, a measure of how much the consumer benefits from being able to buy a given product at a given price. And we will see how the rational choice model can be used to examine how price and income changes affect welfare.

Next on our agenda are some case studies that illustrate the role of price elasticity in policy analysis. Here we examine the effect of a transit fare increase and the effect of liquor taxes on alcohol consumption by heavy drinkers.

Finally, we consider how the rational choice model can be adapted to consider choices that have future consequences.

## Using the Rational Choice Model to Answer Policy Questions

Many government policies affect not only the incomes that people receive but also the prices they pay for specific goods and services. Sometimes these effects are the deliberate aim of government policy, and on other occasions they are the unintended consequence of policies directed toward other ends. In either case, both common sense and our analysis of the rational choice model tell us that changes in incomes and prices can normally be expected to alter the ways in which consumers spend their money. And as we will see, the rational choice model can yield crucial insights not always available to policy analysts armed with only common sense.

### APPLICATION: A GASOLINE TAX AND REBATE POLICY

Suppose it is determined that a reduction in overall gasoline consumption is desirable, for both environmental and energy-conservation reasons. One method of curtailing gas consumption would be to institute a “conservation surtax” on gasoline. To address concerns that the surtax imposes a hardship on lower-income households, the government decides to use the entire proceeds of the surtax to reduce other taxes. Each individual will therefore receive a lump-sum tax rebate. (Here the term “lump sum” means that the rebate does not vary with the amount of gasoline the individual consumes.) There can still be some distributional effects. A heavy gasoline user will receive a smaller rebate than he has paid in surtax, while pedestrians and cyclists will receive a larger rebate than they have paid in surtax. This bias actually reinforces the objectives of the surtax/rebate policy, by rewarding low gasoline consumption. For simplicity, we will assume reasonably, that the costs of administering the policy are negligible.

A critic might ask, "Why bother? If you give *all* the surtax revenues back to people, then they will just go out and spend their rebates on gasoline. Except for the slight redistributive effect you've mentioned, giving the tax proceeds back defeats the purpose of the policy." This argument is incorrect, as illustrated in the example below.

The current price of gasoline is \$.50/litre. A surtax of \$.25/litre is imposed that results in a \$.25 increase in the price of gasoline to consumers.<sup>1</sup> Consider a consumer whose income is \$75 per week, and whose lump-sum tax rebate happens to be *exactly equal* to the amount of gasoline surtax he pays.

This consumer's budget constraint before the imposition of the tax is shown as  $B_1$  in Figure 5-1.<sup>2</sup> On this budget constraint, he chooses bundle  $C$ , which contains 58 litres/wk of gasoline. His budget constraint with a \$.75/litre price of gasoline would be  $B_2$  if he received no rebate. On this budget constraint, he would consume bundle  $A$ , which contains only 30 litres/wk of gasoline. But how do we find the budget constraint that corresponds to a rebate equal to the amount collected from him in gasoline taxes?

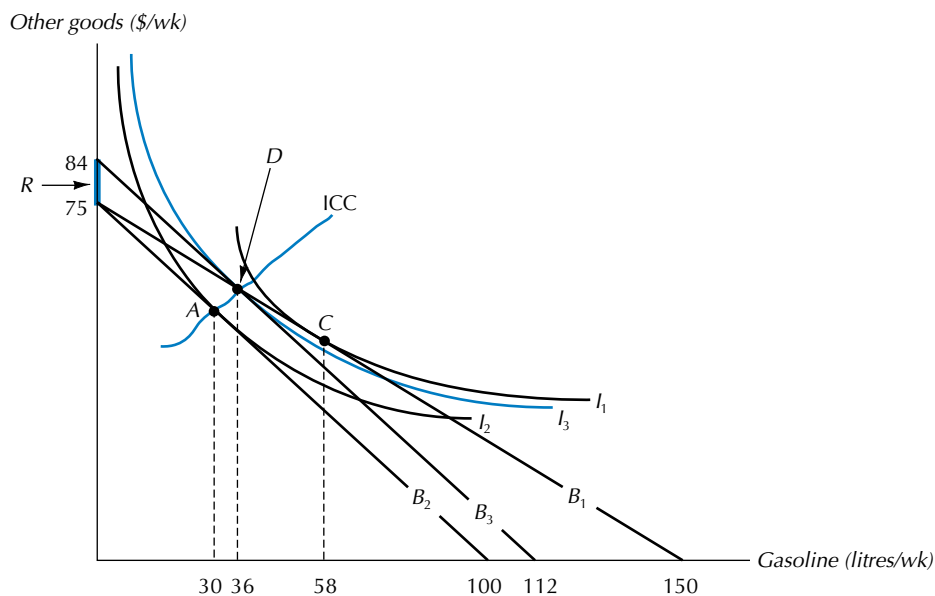
The first step is to note that for any given quantity of gasoline consumed, the vertical distance between budget constraints  $B_1$  and  $B_2$  corresponds to the total amount of tax paid on that amount of gasoline. Thus, at 1 litre/wk of gasoline, the vertical distance between  $B_1$  and  $B_2$  would be \$.25; at 2 litres/wk, it would be \$.50; and so on.

Our next step is to trace out how the consumer's consumption will vary as a function of the size of the rebate. To do this, note that a rebate is like income from any other source, so what we really want to do is to trace out how the consumer responds

**FIGURE 5-1**

The tax rotates the original budget constraint from  $B_1$  to  $B_2$ . The rebate shifts  $B_2$  out to  $B_3$ . The rebate does not alter the fact that the tax makes gasoline 50% more expensive relative to all other goods. The consumer shown in the diagram responds by consuming 22 litres per week less gasoline.

#### A GASOLINE TAX AND REBATE



<sup>1</sup>Recall from Chapter 2 that the rise in equilibrium price will be exactly the same as the tax when the supply curve for gasoline is perfectly horizontal.

<sup>2</sup>The equation for  $B_1$  is  $Y = 75 - .5G$ , for  $B_2$  is  $Y = 75 - .75G$ , and for  $B_3$  is  $Y = 84 - .75G$ , where  $G$  is gasoline (litres/wk) and  $Y$  is all other goods (\$/wk).

to changes in income. As we saw in Chapter 4, the appropriate tool for this task is the income-consumption curve, or ICC. Accordingly, we construct the ICC through bundle  $A$ , as shown in Figure 5-1.

Now look at bundle  $D$ , the point where the ICC through  $A$  intersects the original budget constraint  $B_1$ .  $D$  is the equilibrium bundle on the budget constraint labelled  $B_3$ , where the price of gasoline is \$.75/litre and the consumer has  $\$(75 + R)/\text{wk} = \$84/\text{wk}$  of income. This means that if we give the consumer a rebate of  $R = \$9/\text{wk}$ , he will consume bundle  $D$  and pay exactly  $\$9/\text{wk}$  in gasoline taxes. Note, however, that  $D$  lies well to the left of the original bundle  $C$ , which means that, despite the rebate, the consumer substantially curtails his gasoline consumption. If gasoline is a normal good, the effect of the rebate is to offset partially the income effect of the price increase. It does nothing to alter the substitution effect.

Note that as a result of the tax-and-rebate policy, the individual is on a *lower* indifference curve than he was initially ( $I_3$  is lower than  $I_1$ ). The reason is that his indifference curves as drawn give no weight to the benefits of gasoline conservation and reduced environmental damage. If the consumer attaches utility to these effects, then (in a *three-dimensional* diagram) he could view himself as better off. From a *social* perspective, however, by assumption overall welfare has increased even if some individuals disagree with the policy.

#### APPLICATION: SCHOOL VOUCHERS

A perennial policy concern in any country is how to ensure the best quality of primary and secondary education for students. One proposal for improving the quality of schooling assumes that a major problem with the present public school system is that it behaves too much like a complacent monopoly. Under this proposal, the present system would be replaced by a *voucher system*, under which each school-age child would be given a voucher which could be applied towards the costs of tuition at *any* school of the family's choosing. Proponents of vouchers argue that tuition-charging private schools cannot currently compete on an equal footing with public schools offering free tuition. According to them, the voucher system would introduce more competition into the market for educational services, and improve educational quality.

We will develop a simple model of the present system and the voucher alternative, and then consider some additional factors that need to be taken into account in appraising their relative merits. Under the present system, public school costs are paid for out of provincial and municipal tax revenues, which are collected from all households, those with and without school-age children. Children are entitled to free public education. If a family chooses to send its children to a private school, it does not receive a refund of an equivalent portion of its taxes, any more than those without children do. It can, however, deduct these private schooling costs—up to a specified maximum amount—in calculating its taxable income.

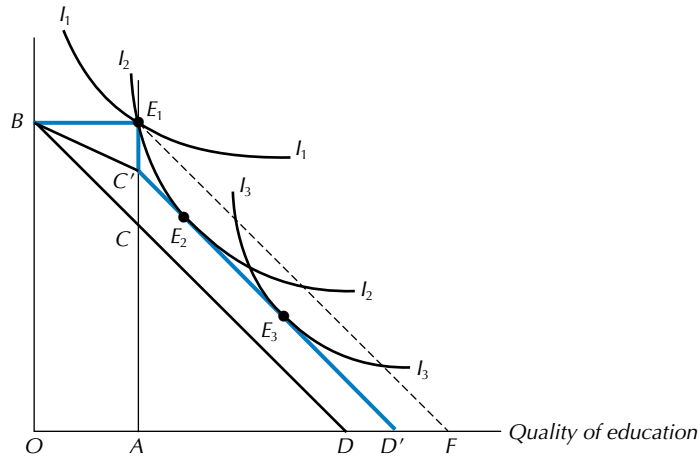
The present system is depicted in Figure 5-2. We will assume that educational “quality” can be measured, and that it is a linear function of the number of dollars spent on educating a child. Consider a family with after-tax income equal to  $OB$ , which it can spend on educational quality for its children or on all other goods. If we set the value of a unit of educational quality at \$1, and the family pays *all* of its children's educational expenses, then the family's budget constraint  $BCD$  has a slope of  $-1$ , and  $OB = OD$ . Childless families and individuals will consume at point  $B$ , because they have no such educational expenses. Suppose that families are legally required to provide a minimum quality of education (equal to  $OA$ ) for each child. Then a family

FIGURE 5-2

With  $OA$  in “free” education and marginal tax rate  $CC'/CE_1$ , a single-child family’s after-tax budget constraint is  $BE_1C'D'$ . Three families have the same after-tax income  $OB$ , before their schooling decision. Family 1 chooses  $E_1$ , in public school, Family 3 chooses  $E_3$  in private school, and Family 2 is indifferent between  $E_1$  (public) and  $E_2$  (private school).

## EDUCATIONAL CHOICE UNDER THE CURRENT SYSTEM

Other goods (\$)



with one child will consume along  $CD$ , spending at least  $OA$  on educational quality and at most  $AC$  on other goods. A family with three children will be required to spend at least  $3OA$  on education, with the balance for other goods. The burden on the family of meeting this socially imposed requirement out of its own resources could represent a genuine hardship. For this reason, and because an educated population is viewed as a social good, the public school system provides  $OA$  per child in tuition-free education. For a one-child family, the budget constraint with free public schooling now becomes  $BE_1F$ . Along  $E_1F$ , the family could augment the quality of education for their child by expenditures on computers, learning software, individual tutoring, extra-curricular programs, and the like. We have drawn  $E_1F$  as a dotted line, however, to highlight the difference between public schooling and tuition-charging private schooling in a way that is most favourable to voucher system proponents.

We will assume that educational quality is obtainable only during the school day, and that if the family wants more “quality” for its child, it must withdraw her from the public school system and enrol her in a private school. If the maximum tax-deductible education expenditure equals  $OA$ , and the family’s marginal tax rate over this range is given by  $CC'/CE_1$ , then the net *after-tax* cost of  $OA$  ( $=CE_1$ ) of private schooling is only  $CE_1 - CC' = C'E_1$ . If the family wants to spend more than  $OA$  on schooling, it can purchase additional educational quality along  $C'D'$ . It receives no tax deduction for school expenditures above the maximum deductible amount  $OA$ , however, and so the slope of  $C'D'$  is  $-1$ . The budget constraint is thus  $BE_1C'D'$ . Families with *higher* marginal tax rates, typically, higher-income families, pay less in after-tax dollars for private schooling than those with *lower* marginal tax rates, under the present system.

Consider three families, with identical marginal tax rates and after-tax income before making their schooling decision, and indifference curves  $I_1$ ,  $I_2$  and  $I_3$ , respectively. Family 1 reaches its highest indifference curve at  $E_1$ , in the public school system. Family 2 is indifferent between consuming at  $E_1$  or  $E_2$ , while Family 3 will opt for private school at  $E_3$ .

What effect will a voucher system have? Suppose that each family receives one voucher per child equal in value to  $OA$ , and there is no tax deduction for additional

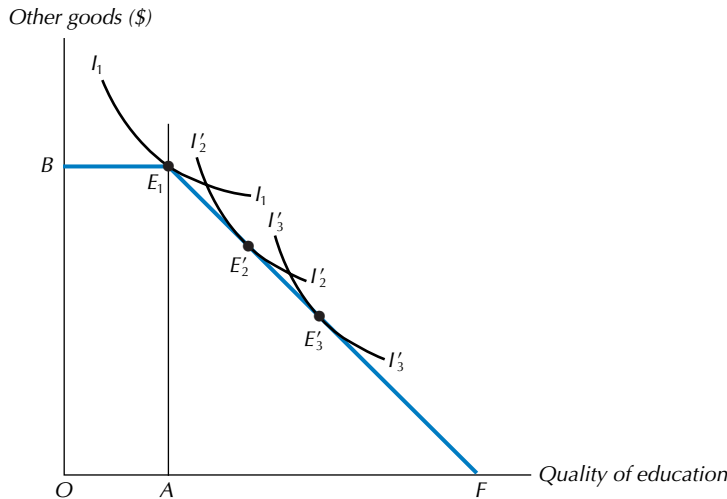
educational expenditures. As Figure 5-3 shows, the new budget constraint facing all three families will be  $BE_1F$ , and  $E_1F$  will have a slope of  $-1$ . Family 1 will remain at  $E_1$ . If educational quality is a normal good, however, then both Family 2 and Family 3 will spend *more* than before on educational quality, at  $E_2'$  and  $E_3'$  in the private school system. The total additional cost of the voucher system to taxpayers is the *sum* of the amounts  $C'E_1$  for *all* children who were in private schools under the *present* system. Establishment of the voucher system involves no additional cost to taxpayers if a child switches from a public to a private school, since that child was already consuming OA in resources under the present system.

The voucher system therefore increases the taxes paid by childless individuals and families with children in public schools under the present system, and lowers the taxes of those (typically higher-income) families who choose to send their children to private schools under the present system. The voucher system also increases the incentive for families to shift their children from public to private schools. Family 2, which was initially indifferent between public and private schooling, opts for private schooling under the voucher system, as will any single-child family whose MRS at  $E_1$  is greater than 1. Total expenditures on schooling should therefore increase under a voucher system.

The benefit of the voucher system, according to its proponents, is that increased competition will increase educational productivity. This is theoretically possible, and if it were to occur then it could result in cost savings and greater educational quality for the same expenditure of resources. However, we have no universally agreed upon, one-dimensional measure of what constitutes educational quality. What we do have is a wide range of diverse educational objectives, and a range of interrelated programs intended to achieve them. The *objectives* include the development of gross and fine motor skills; a balance of intellectual, physical, and emotional growth; mastery of the three Rs and a number of specific subject-areas; general cultural knowledge; familiarity with computers; the ability to work independently, and to cooperate with others; the capacity to benefit from cultural diversity; leadership; and the capacity for critical and creative thought. If we have no real measure of educational *quality*, then it is even less clear that a doubling of expenditures per student will double the quality of

**FIGURE 5-3**  
Under the voucher system, the budget constraint becomes  $BE_1F$ . Family 1's expenditure is unchanged at  $E_1$ , but Family 2 and Family 3 both spend *more* on education, at  $E_2'$  and  $E_3'$  respectively.

**EDUCATIONAL CHOICE UNDER A VOUCHER SYSTEM**



education. We can imagine one school with a stable of polo ponies, and another with ten used soccer balls. The former costs much more per student than the latter, but each program will contribute, in different ways, to the achievement of various educational objectives. If polo costs 500 times more per student than soccer, is its marginal contribution to educational quality 500 times greater than that of soccer? Merely posing the question in this way should suggest the difficulty with our initial assumptions that quality can be measured in a unidimensional way, and that a doubling of educational expenditure doubles quality. Yet the conclusions of our basic model require this assumption.

We can still explore whether the voucher system is likely to be more cost-effective, efficient, or productive. Let us note briefly some of the ways in which the shift to a voucher system could *reduce* the efficiency of the system. If private schools are “for-profit” institutions, then from each dollar spent on education a proportion will have to go to the school’s owners as profits. Interest costs will also be higher for the private schools, because the risk-premium on a loan to a private borrower is greater than on one to government. The public school system may also be able to achieve more economies in the purchase of textbooks, course materials, and school supplies through the use of its mass buying power. Costs of curriculum research and development can likewise be spread over a larger student population. Such costs are hence lower per student within the present system. If the costs of providing *equivalent* education are higher under the voucher system, for the above reasons, then vouchers also increase the risk of school bankruptcies, and the costs of disruption of service they entail.

These added costs of the voucher system relative to the present system are fairly clear. Other potential costs are less obvious. The rise of a whole host of diverse private schools increases what economists call problems of uncertainty and *asymmetric information* (which we will examine further in Chapter 6 and Appendix 6). The new schools know what sort of education they are providing. Parents, however, do not, and it is generally costly for them to discover the quality of education being provided by a particular school, except after the fact. Impressive brochures and websites are relatively cheap to produce; quality education, however measured, is generally not. One theoretical possibility would be to shift children from school to school until a good match was achieved, but this search process would involve major transactions costs for parents, children, and the educational system as a whole.

If there is greater diversity among schools under the voucher system, then the costs of teacher accreditation, core curriculum approval, and overall educational quality control are also likely to increase. Consider Family 1 in Figure 5-3. They would prefer to spend *less* than  $OA$  on their child’s education. Fortunately, under the voucher program, the Creative Home Educational Adventure Program school (CHEAP, for short) offers a curriculum where the school’s students work entirely at home, surfing the net, “to develop their independent study skills.” CHEAP receives  $OA$  per child, and then provides a cash rebate to the parents, so that they can purchase “necessary educational supplies,” and a diploma at the end of the year. The CHEAP campus is a post office box. Needless to say, CHEAP is not likely to be in business for long, but it will cost resources to put them out of business.

The voucher system, at least in the form outlined here, would also have consequences for children with special needs and their families. In the present system,  $OA$  is not the amount that is actually devoted to each public school student, but rather the average amount per student. Students with special needs or disabilities tend to require more than the average amount  $OA$ , while other students require less than  $OA$ . Under the voucher system, each student receives precisely  $OA$ , and the averaging-out

that is an integral part of the present system is no longer possible. Students without disabilities benefit from the change, since they received less than  $OA$  before, but students with disabilities and their families are further disadvantaged under the voucher system, since they receive fewer resources than they do under the present system.

This brief sketch of a few of the potential problems with school voucher systems is not intended to suggest that the voucher idea is not worth exploring further. We will be examining the question of the appropriate balance between public and private provision of goods and services in several subsequent chapters, including Chapter 12 and Chapter 18. Yet it should suggest that policy ideas which look good on paper may have unintended consequences when they are translated into the real world, unless they are implemented with forethought and care.

## Consumer Surplus

### Consumer surplus

A dollar measure of the extent to which a consumer benefits from participating in a transaction.

When exchange takes place voluntarily, economists generally assume that it makes all participants better off. Otherwise they would not have engaged in the exchange. It is often useful to have a dollar measure of the extent to which people benefit from a transaction. Such a measure, called *consumer surplus*, is particularly important for the purpose of evaluating potential government programs. It is relatively straightforward to measure the costs of, say, building a new road. But an intelligent decision about whether to build the road cannot be made without a reliable estimate of the extent to which consumers will benefit from it.

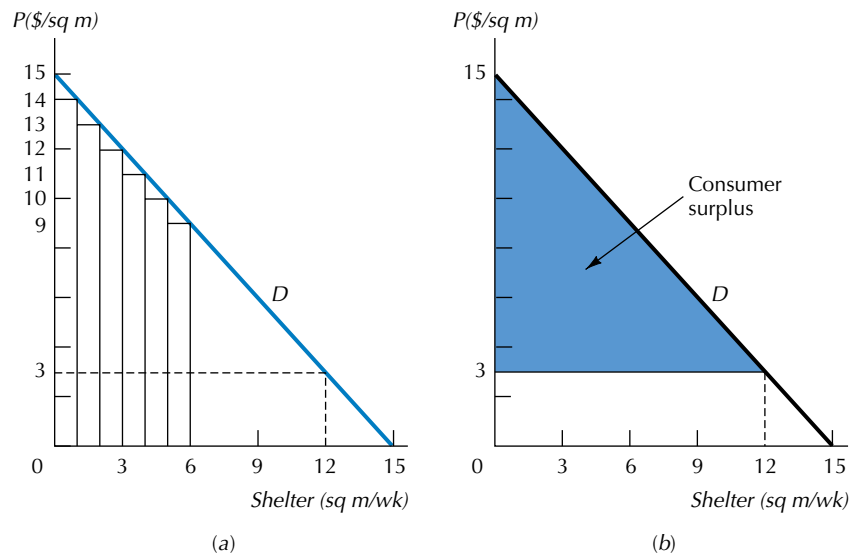
### USING DEMAND CURVES TO MEASURE CONSUMER SURPLUS

The easiest way to measure consumer surplus involves the consumer's demand curve for the product. In both panels in Figure 5-4, the line labelled  $D$  represents an individual's demand curve for shelter, which sells for a market price of  $\$3/\text{sq m}$ . In panel

**FIGURE 5-4**

(a) The height of the demand curve at any quantity measures the most the consumer would be willing to pay for an extra unit of shelter. That amount minus the market price is the surplus he gets from consuming the last unit. (b) The total consumer surplus is the shaded area between the demand curve and the market price.

### THE DEMAND CURVE MEASURE OF CONSUMER SURPLUS



(a), note that the most the consumer would have been willing to pay for the first square metre of shelter is \$14. Since shelter costs only \$3/sq m, this means that he obtains a surplus of \$11 from his purchase of the first square metre of shelter each week. The most he would be willing to pay for the second square metre of shelter is \$13, so his surplus from the purchase of that unit will be smaller, only \$10. His surplus from the third unit is smaller still, at \$9. For shelter or any other perfectly divisible good, the height of the individual's demand curve at any quantity represents the most the consumer would pay for an additional unit of it.<sup>3</sup> In this example, if we subtract the purchase price of \$3/sq m from that value and sum the resulting differences for every quantity out to 12 sq m/wk, we get roughly the shaded area shown in panel (b). (If we use infinitesimal increments along the horizontal axis, we get exactly the shaded area.) This shaded area represents the individual's consumer surplus from the purchase of 12 sq m/wk of shelter.

### EXAMPLE 5-1

An individual's demand curve for gasoline is given by  $P = 2.50 - \frac{1}{16}Q$ , where  $P$  is the price of gasoline (\$/litre), and  $Q$  is the quantity she consumes (litres/wk). If the individual's weekly income is \$1000 and the current price of gasoline is \$0.50/litre, by how much will her consumer surplus decline if an oil import restriction raises the price to \$0.75/litre?

At a price of \$0.50/litre, she consumes only 32 litres of gasoline per week, which amounts to less than 2 percent of her income. The income effect of the price increase is therefore likely to be insignificant, so we can use the demand curve approximation to measure her consumer surplus before and after the price increase. (See footnote 3.) Figure 5-5 displays her demand curve. Her consumer surplus at the price of \$.50/litre is given by the area of the triangle  $AEF$  in Figure 5-5,  $CS = \frac{1}{2}(2.5 - .5)(32) = \$32/\text{wk}$ . Following the price increase, her consumption falls from 32 to 28 litre/wk, and her surplus shrinks to the area of the triangle  $ACD$ ,  $CS = \frac{1}{2}(2.5 - .75)(28) = \$24.50/\text{wk}$ . Her loss in consumer surplus is the difference between these two areas, which is the area of the trapezoid  $DCEF$ , the shaded region in Figure 5-5. This area is equal to  $CS - CS' = 32 - 24.5 = \$7.50/\text{wk}$ .

### EXERCISE 5-1

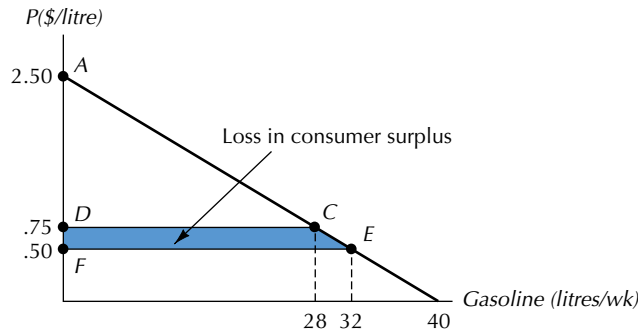
By how much would consumer surplus shrink in Example 5-1 if the price of gasoline rose from \$.75/litre to \$1/litre?

<sup>3</sup>These statements about willingness-to-pay are literally true only if the demand curve we are talking about is an income-compensated demand curve like the one discussed in the appendix to Chapter 4. If the demand curve shown were an ordinary demand curve of the sort we have been using, it would tell us that the consumer would be willing to buy 1 unit at a price of \$14, 2 units at a price of \$13, and so on. From this it would not be strictly correct to conclude that, having already paid \$14 for the first unit, the consumer would then be willing to spend an *additional* \$13 for the second unit. If the income effect of the demand for the good is positive, the fact that the consumer is now \$14 poorer than before means that he would be willing to pay somewhat less than \$13 for the second unit. But since income effects for most goods are small, it will generally be an acceptable approximation to measure consumer surplus using the ordinary demand curve. In a widely cited article, Robert Willig has argued that the demand curve method will almost always yield an acceptable approximation of the true value of consumer benefits. See R. Willig, "Consumer Surplus without Apology," *American Economic Review*, 66, 1976: 589–597.

FIGURE 5-5

At a price of \$.50/litre, consumer surplus is given by the area of triangle  $AEF$ . At a price of \$.75/litre, consumer surplus shrinks to the area of triangle  $ACD$ . The loss in consumer surplus is the difference in these two areas, which is the area of the shaded region.

THE LOSS IN CONSUMER SURPLUS FROM AN OIL PRICE INCREASE



## APPLICATION: TWO-PART PRICING

Economic reasoning suggests that a voluntary exchange will take place between a buyer and a seller if and only if that exchange makes both parties better off. On the buyer's side, we may say that willingness to exchange depends on the buyer's expectation of receiving consumer surplus from the transaction.

Economic theory does not tell us very much about how the gains from exchange will be divided between the buyer and the seller. Sometimes the buyer will be in an advantageous bargaining position, enabling her to capture most of the benefits. Other times the buyer's options will be more limited, and in these cases, her consumer surplus is likely to be smaller. Indeed, as the following example illustrates, the seller can sometimes design a pricing strategy that captures *all* the consumer surplus.

## EXAMPLE 5-2

*Why do some tennis clubs have an annual membership charge in addition to their hourly court fees?*

A suburban tennis club rents its courts for \$25 per person per hour. John's demand curve for court time,  $P = 50 - \frac{1}{4}Q$ , measured in hours per year, is given in Figure 5-6. Assuming there were no other tennis clubs in town, what is the maximum annual membership fee John would be willing to pay for the right to buy court time for \$25 per hour?

The answer to this question is the consumer surplus John receives from being able to buy as much court time as he wants at the \$25 per hour price. This is equal to the area of triangle  $ABC$  in Figure 5-6, which is  $CS = \frac{1}{2}(50 - 25)100 = \$1250/\text{yr}$ . If the club charged a fee higher than that, John would be better off not renting any court time at all.

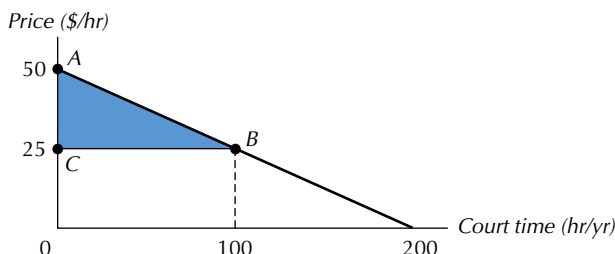
## EXERCISE 5-2

In Example 5-2, how much would the maximum annual membership fee be if the club charged only \$20 per hour for court time?

FIGURE 5-6

At a price of \$25 per hour, John receives \$1250 per year (the shaded area) of consumer surplus from renting court time. The maximum annual membership fee the club can charge is \$1250.

AN INDIVIDUAL DEMAND CURVE FOR TENNIS COURT TIME



Example 5-2 sheds light on many of the pricing practices we observe throughout the economy. Many amusement parks, for example, charge a fixed admission fee in addition to a charge for each ride. Many telephone companies charge a fixed monthly fee in addition to charges based on actual calls made. And some shopping clubs charge a fixed membership fee for the right to buy items carried in their stores or catalogs. Pricing schemes such as these are often called *two-part pricing*. Their effect is to transfer a portion of the consumer surplus from the buyer of the product to the seller.

### Two-part pricing

Pricing system with a fixed access fee and a charge per unit of a good purchased.

## Overall Welfare Comparisons

The concept of consumer surplus helps us identify the benefits (or costs) of changes that occur in particular markets. Often we will want to assess whether consumers are better or worse off as a result of changes not just in one market but in many. Here too our model of rational choice lets us draw a variety of useful inferences. Consider the following example.

### EXAMPLE 5-3

Jones spends all his income on two goods:  $X$  and  $Y$ . The prices he paid and the quantities he consumed last year are as follows:  $P_X = \$10/\text{unit}$ ,  $X = 50$  units,  $P_Y = \$20/\text{unit}$ , and  $Y = 25$  units. This year  $P_X$  and  $P_Y$  are both  $\$10/\text{unit}$ , and Jones's income is  $\$750$ . Assuming his tastes do not change, in which year was Jones better off, last year or this?

To answer this question, it is helpful to begin by comparing Jones's budget constraints for the two years. To do this, we note first that his income last year was equal to what he spent, namely,  $P_X X + P_Y Y = \$1000$ . For the prices given, we thus have the budget constraints shown in Figure 5-7a.

In Figure 5-7a, we see that Jones's budget constraint for this year contains the very same bundle he bought last year. Since his tastes have not changed, this tells us he cannot be worse off this year than last. After all, he can still afford to buy the same bundle as before. But our standard assumptions about preference orderings enable us to draw an even stronger inference. In particular, if his indifference curves have the usual convex shape, we know that an indifference curve—call it  $I_0$ —was tangent to last year's budget constraint at the point  $A$  in Figure 5-7b. We also know that this year's budget constraint is steeper than last year's, which tells us that part of  $I_0$  must lie inside this year's budget triangle. On  $I_0$ , bundle  $A$  is equally preferred to bundle  $C$ . And because more is better, we know that  $D$  is preferred to  $C$ . It thus follows that  $D$  is preferred to  $A$ , and so we know that Jones was able to purchase a bundle of goods this year that he

likes better than the one he bought last year. It follows that Jones was better off this year than last.

### EXERCISE 5-3

Jones spends all his income on two goods: X and Y. The prices he paid and the quantities he consumed last year are as follows:  $P_X = \$15/\text{unit}$ ,  $X = 20$  units,  $P_Y = \$25/\text{unit}$ , and  $Y = 30$  units. This year the prices have changed ( $P_X = \$15/\text{unit}$  and  $P_Y = \$20/\text{unit}$ ), and Jones's income is now \$900. Assuming his tastes have not changed, in which year was Jones better off, last year or this?

#### APPLICATION: THE WELFARE EFFECTS OF CHANGES IN HOUSING PRICES

Consider the following two situations:

1. You have just purchased a house for \$200,000. The very next day, the prices of all houses, including the one you just bought, unexpectedly double.
2. You have just purchased a house for \$200,000. The very next day, the prices of all houses, including the one you just bought, unexpectedly fall by half.

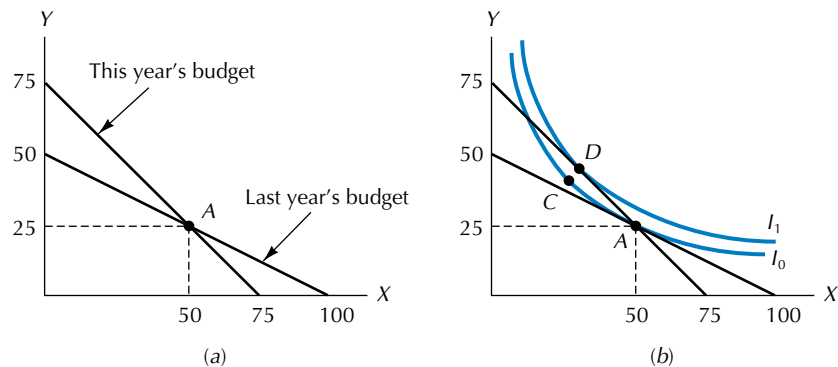
In each case, ignoring transactions costs and taxation effects, how does the price change affect your welfare? (Are you better off before the price change or after?)

A class of first-year graduate students in economics was asked these questions. The overwhelming majority responded that you are better off as a result of the price increase in situation 1, but worse off as a result of the price drop in situation 2. Although most students seemed very confident about these two responses, only one turns out to be correct.

To see why, let us first consider the case in which all housing prices double. Suppose your total wealth just before purchasing your house was \$400,000. Let the size of your current house correspond to one unit of housing and let the price of other

**FIGURE 5-7**  
 (a) If the consumer's budget constraint for this year contains the same bundle he bought last year (bundle A), he will be at least as well off this year as last. (b) If, in addition, relative prices are different in the two years, he will necessarily be able to buy a better bundle this year (bundle D).

**BUDGET CONSTRAINTS FOR 2 YEARS**



goods (the composite good) be \$ per unit. Your original budget constraint under situation 1 will then correspond to the line labelled  $B_1$  in Figure 5-8. Its vertical intercept, \$400,000, is the maximum amount you could have spent on other goods. Its horizontal intercept, 2 units of housing, corresponds to the maximum quantity of housing you could have bought (that is, a house twice as large as your current house). On  $B_1$ , the equilibrium at  $A$  represents your original purchase. At  $A$ , you have 1 unit of housing and \$200,000 left for other goods.

After the price of your house doubles, your budget constraint becomes the line labelled  $B_2$  in Figure 5-8. To calculate the vertical intercept of  $B_2$ , note that your current house can now be sold for \$400,000, which, when added to the \$200,000 you had left over after buying your house, yields a maximum of \$600,000 available for other goods. The horizontal intercept of  $B_2$  tells us that when the price of housing doubles to \$400,000/unit, your \$600,000 will buy a maximum of only 1.5 units of housing. Note finally that on  $B_2$  your optimal bundle is  $C$ , which contains  $H_2 < 1$  units of housing and  $O_2 > \$200,000$  worth of other goods. And since bundle  $C$  lies on a higher indifference curve than bundle  $A$ , you are better off than before the price increase.

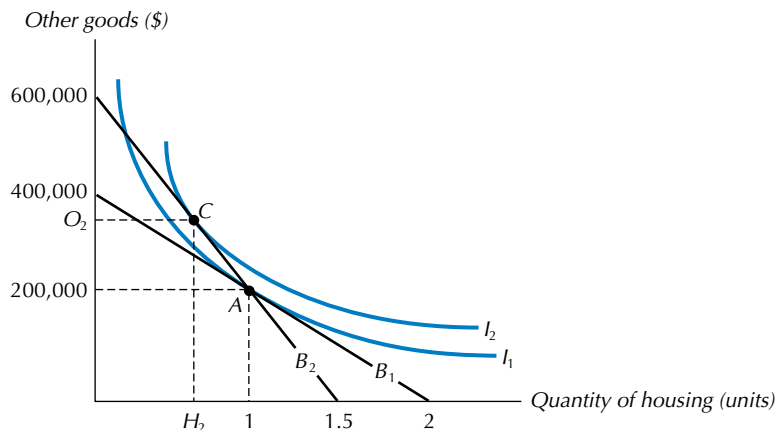
Not surprisingly, when the price of housing goes up, your best response is to buy fewer units of housing and more units of other goods. Note that you are insulated from the harm of the income effect of the price increase because the price increase makes the house you own more valuable.

So far, so good. Now let us consider what for many students was the more troubling case—namely, situation 2, in which housing prices fall by half. Again adopting the units of measurement used in situation 1, your budget constraint following the fall in housing prices is the line labelled  $B_3$  in Figure 5-9. To get its vertical intercept, note that sale of your current house will now yield only \$100,000, which, when added to the \$200,000 you already have, makes a maximum of \$300,000 available for the purchase of other goods. To calculate the horizontal intercept of  $B_3$ , note that when the price of housing falls to \$100,000, your \$300,000 will now buy a maximum of 3 units of housing. Given the budget constraint  $B_3$ , the best affordable bundle is the one labelled  $D$ , which contains  $H_3 > 1$  units of housing and  $O_3 < 200,000$  units of other goods. As in situation 1, the effect of the relative price change is again to move you to a higher indifference curve. This time, however, your direction of substitution is the opposite of the one in situation 1: because housing is now cheaper than before, you respond by purchasing more units of housing and fewer units of other goods.

**FIGURE 5-8**

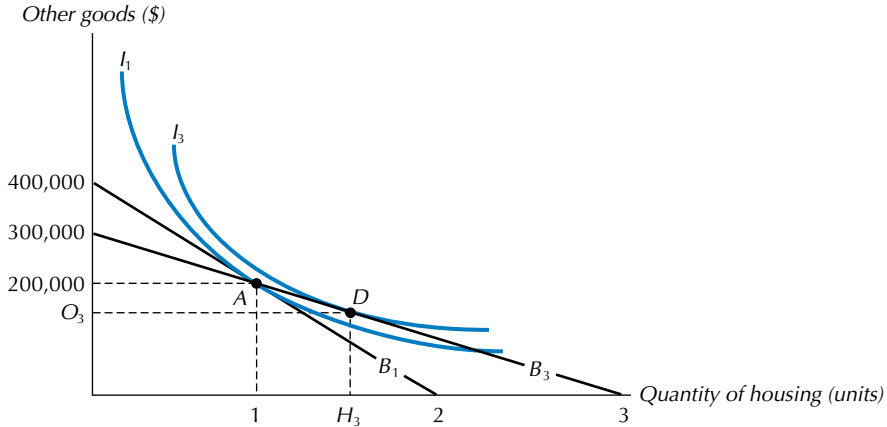
When the price of housing doubles, your budget constraint becomes  $B_2$ , which also contains your original bundle  $A$ . Because  $C$ , the optimal bundle on  $B_2$ , lies on a higher indifference curve than  $A$ , the effect of the housing price increase is to make you better off.

#### RISING HOUSING PRICES AND THE WELFARE OF HOMEOWNERS



**FIGURE 5-9**  
When the price of housing falls by half, your budget constraint becomes  $B_3$ , which also contains your original bundle  $A$ . Because  $D$ , the optimal bundle on  $B_3$ , lies on a higher indifference curve than  $A$ , the effect of the housing price drop is to make you better off.

**FALLING HOUSING PRICES AND THE WELFARE OF HOMEOWNERS**



In each situation, note that your new budget constraint contains your original bundle, which means that you have to be at least as well off after the price change as before. Note also that in each case the change in relative prices means that your new budget constraint contains bundles which lie beyond your original indifference curve, making it possible to achieve a better outcome in each situation.

**APPLICATION: A BIAS IN THE CONSUMER PRICE INDEX**

The consumer price index (CPI) measures changes in the “cost of living,” the amount a consumer must spend to maintain a given standard of living. Published each month by Statistics Canada, the CPI is calculated by first computing the cost of a representative bundle of goods and services during a reference period and then dividing that cost into the current cost of the same bundle. Thus, if it cost \$100 to buy the representative bundle in the reference period and \$150 to buy the same bundle today, the CPI would be 1.5. Announcing this figure, government spokespersons would explain that it meant the cost of living had increased by 50 percent compared to the reference period.

What the CPI fails to take into account, however, is that when the prices of different goods rise by different proportions, consumers do not generally buy the same bundle of goods they used to buy. Instead, the typical pattern is to substitute away from those goods whose prices have risen the most. By reallocating their budgets in this fashion, consumers are able to escape at least part of the harmful effects of price increases. Because the CPI fails to take substitution into account, it tends to overstate increases in the cost of living.

A simple example using the rational choice model makes this point unmistakably clear. Suppose the only goods in the economy were rice and wheat and that the representative consumer consumed 20 kg/mo of each in the reference period. If rice and wheat each cost \$1/kg in the reference period, what will be the CPI in the current period if rice now costs \$2/kg and wheat costs \$3/kg? The cost of the reference period bundle at reference period prices was \$40, while at current prices the same bundle now costs \$100. The CPI thus takes the value of  $\$100/\$40 = 2.5$ . But is it really correct to say that the cost of living is now 2.5 times what it was?

To consider an extreme case, suppose our representative consumer regarded rice

and wheat as perfect one-for-one substitutes, meaning that her indifference curves are negatively sloped 45° lines. In Figure 5-10, her original bundle is denoted as *A* and her original indifference curve (which coincides exactly with her original budget constraint) is labelled  $I_0$ . Now suppose we ask, how much income would she need in the current period to achieve the same level of satisfaction she achieved in the reference period? At the new prices, the slope of her budget constraint is no longer  $-1$ , but  $-3/2$ . With a budget constraint with this new slope, she could reach her original indifference curve most cheaply by buying the bundle labelled *C* in Figure 5-10. And since the cost of *C* at current prices is only \$80, we can say that the cost of maintaining the original level of satisfaction is only 2.0, not 2.5 times as great.

In general, we can say that the extent to which the CPI overstates the cost of living will go up as substitution possibilities increase. The bias will also be larger when there are greater differences in the rates of increase of different prices.

### QUALITY CHANGE: ANOTHER BIAS IN THE CPI?

Gathering data on the prices of goods and services might seem like a straightforward task. In practice, however, it is complicated by the existence of discounts, rebates, and other promotional offers in which the actual transaction price may be substantially different from the official list price.

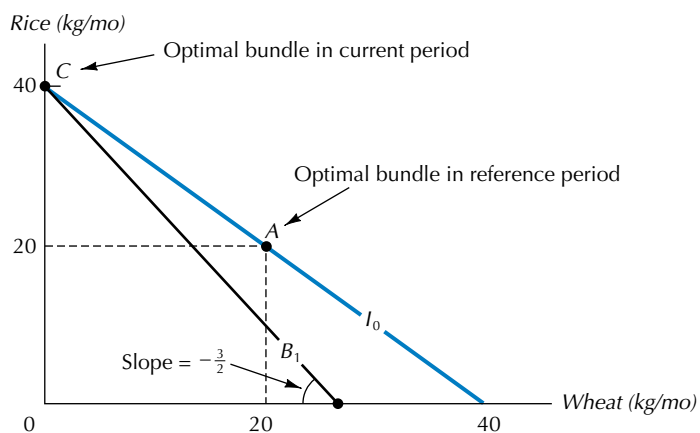
Yet important as they are, accurate price data are not sufficient for estimating changes in the cost of living. We must also account for changes in the quality of what we buy. And this, unfortunately, turns out to be a far more complicated task than measuring changes in prices.

Twenty-five years ago, a scientific hand-held calculator cost \$99.95. Now, you can buy a respectable one for \$9.95. The price has fallen to a tenth of its former level. But

**FIGURE 5-10**

For this consumer, rice and wheat are perfect substitutes. When the price of each was \$1/kg, she bought 20 kg/mo of each in the reference period, for a total expenditure of \$40/mo. If the current prices of rice and wheat are \$2/kg and \$3/kg, respectively, the expenditure required to buy the original bundle is \$100/mo. The CPI is the ratio of these two expenditures,  $\$100/\$40 = 2.5$ . But the consumer can attain her original indifference curve,  $I_0$ , by buying bundle *C*, which costs only \$80 at current prices. The cost of maintaining the original level of satisfaction is thus only 2.0 times the original level.

**THE BIAS INHERENT IN THE CONSUMER PRICE INDEX**



the earlier calculator could perform only 18 functions, while the current one can perform 136 functions. And for less than \$99.95, you can now have a programmable handheld calculator which performs a range of operations that would test the knowledge of a good undergraduate math major.

In the case of automobiles, average automobile prices since the early 1980s have increased at a somewhat higher rate than the Consumer Price Index. Yet at the same time, a number of features that were non-existent, optional, or luxury items then are common or standard now. Airbags, antilock brakes, significantly reduced hydrocarbon and nitrogen oxide emission levels, rear window defrosters, power windows, increased interior room, improved engine performance, increased average fuel economy, greater reliability, crashworthiness, and rust resistance: this is only a partial list of the quality improvements that we now take for granted. If we factor in reasonable estimates of the value of all of these quality changes, then automobile prices have actually risen *less* rapidly than consumer prices generally.

With other goods, there has likely been some *lowering* of quality over time. The replacement of wood flooring by particleboard in housing construction has lowered the rate of increase of housing prices, but has also reduced the strength and durability of the housing stock. A digital wristwatch that keeps accurate time can be obtained for a few dollars, but such wristwatches have become almost disposable items.

One method of dealing with such quality changes is to impute a value to the various product characteristics, and make an adjustment downwards (if there has been an improvement in quality,) or upwards (in the case of quality deterioration,) from the market price actually observed to correct for the quality change. This adjustment will never be exact, because goods come as a *bundle* of characteristics which cannot be sold separately. And there is an arbitrary element in the weights we attach to specific characteristics, separately or in combination. Carefully done, however, such adjustments can provide an approximation of the change in value due to quality change, and thus improve the accuracy of our price indices.<sup>4</sup>

The problem of quality change becomes even more acute, however, with the appearance of completely new products and the disappearance of old ones. The CPI tracks the weighted average prices of a representative bundle of consumer goods, where the weights are given by the quantities purchased by consumers in the base year. If the base year is 1950, then the quantity weight assigned to colour televisions, home microwave ovens, Walkmans, personal computers, e-mail services, CD burners, and precooked bacon is *zero*: these products did not exist in 1950. In contrast, if this year is the base year, the quantity weight attached to eight-track tapes is zero, and the weight attached to buggy whips would be almost zero, except for the specialty trade in the back of some adult stores, next to the leather goods section.

Obviously, if a good that appears in the base year bundle is not produced at all in the comparison year, it will have *no* price in the comparison year. Conversely, if it is produced in the comparison year but not in the base year, it *will* have a price, but the quantity weight attached to that price is *zero*. Over time, with the appearance of qualitatively new products and the disappearance of old ones, the CPI starts to develop holes or gaps. In response to this problem, Statistics Canada has adopted the expedient of periodically revising its standard consumption bundle and changing its base year. The price index for the new base year is calculated in two ways: using the quantity weights from the *old* base year *and* using quantity weights derived from the stan-

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<sup>4</sup>A good introduction to some of these issues is Statistics Canada, *Your Guide to the Consumer Price Index*, available free online at <<http://www.statscan.ca>>.

standard commodity bundle of the new base year. This overlap permits the construction of spliced-together *chain indexes*. Economists, business, labour, and government shamelessly use such chain indexes for economic analysis and policy purposes, even when they are fully aware of their defects, simply because there is no generally accepted alternative.

The CPI, warts and all, is of considerable practical importance, since (among its other uses) it is often the basis for cost-of-living adjustment (COLA) provisions in wage settlements and pension plans. We may believe that the CPI is biased, but we will continue to use it as long as it is “the best game in town.” In the appendix to the chapter we explore price indexes in greater detail.

## Using Price Elasticity of Demand

In the sphere of applied economic analysis, few tools are more important than the concept of price elasticity of demand. In this section we examine applications of this concept in two different settings.

### APPLICATION: TTC FARE INCREASES

The Toronto Transit Commission (TTC) prides itself on being “the most productive, least subsidized transit authority in North America.”<sup>5</sup> From a social standpoint, for reasons we will examine further in Chapter 12, it may even be *under*-subsidized! Traffic congestion on the major road routes in the Greater Toronto Area (GTA) is estimated to cost several billion dollars annually in lost time and wasted fuel. If more motorists could be enticed to use public transit through lower TTC fares and increased service, reduced road congestion would result in major time and cost savings. Here we are concerned with a narrower but related and important issue, namely the price elasticity of demand for rides on the TTC.

Between 1995 and 2000 there were three fare increases on the TTC, one of 14.6 percent in 1995, one of 9.3 percent in 1996 and one of 5.2 percent in 1999. We will use the data on fare increases and ridership levels to calculate approximate price elasticity of demand estimates. In the process, we will discover some pitfalls to avoid in making such calculations. In the months after the 1995 fare increase, ridership fell from an annualized figure of 392 million to 380 million riders, a 3.06 percent drop. The price elasticity of demand  $\eta$  is simply  $(\Delta Q/Q) / (\Delta P/P)$ , the ratio of the percentage change in ridership to the percentage change in fares, or  $-0306 / .146 = -0.21$ . This rough-hewn price-elasticity estimate is *negative* and *inelastic* (significantly less than 1 in absolute value): quantity demanded dropped by about 0.21 percent for each 1 percent increase in fares. The 1995 fare increase resulted in an *increase* in revenues. The 9.6 percent fare increase in 1996 similarly resulted in a decline in annualized ridership from 380 million to 372 million rides, a decline of  $8/380 = 2.1$  percent, which yields a rough price-elasticity estimate of  $-.021 / .096 = -0.22$ . If we apply the same method to the 5.2 percent fare increase of 1999, however, we encounter a problem: ridership actually *increased* from 387 million to 396 million rides, an *increase* of  $9/387 = 2.33$  percent, and so the estimated price elasticity of demand is  $.0233 / .052 = +0.45$ ! This suggests that the demand curve in 1999 was *upward* sloping, like that of a Giffen good.

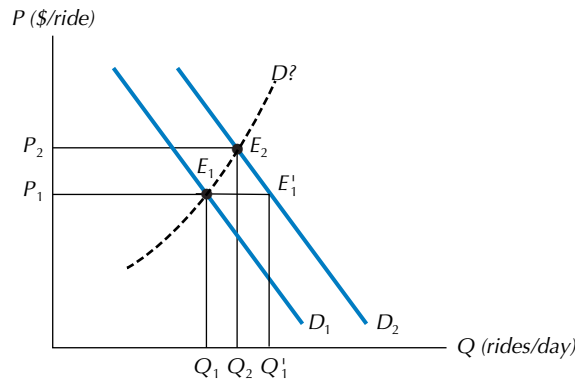
What went wrong? After all, we don't really believe that public transit has an

<sup>5</sup>Toronto Transit Commission, “TTC Year 2000 Operating Budget,” December 14, 1999, p. 1.

FIGURE 5-11

After the fare increases from  $P_1$  to  $P_2$ , ridership increases from  $Q_1$  to  $Q_2$  because of two separate shifts: the shift of the demand curve from  $D_1$  to  $D_2$ , and the shift along  $D_2$  from  $E'_1$  to  $E_2$ . Neglecting the shift from  $D_1$  to  $D_2$  gives the imaginary upward-sloping demand curve,  $D?$

THE 1999 TTC FARE INCREASE



upward-sloping demand curve. Figure 5-11 shows what likely occurred. Recall that price elasticity of demand estimates require that *other* determinants of quantity demanded besides price remain *unchanged*. In at least two respects, however, this was not a reasonable assumption. First, the TTC had been shut down for two days by a strike shortly before the fare increase, so that annualized ridership was artificially lower than it would have been under normal circumstances. Second, and equally important, since early 1997 there had been a steady increase in ridership with fares constant in nominal terms (and hence declining slightly in real terms), as a result of increased service and of demographic and other factors. As Figure 5-11 suggests, the shift from  $E_1$  to  $E_2$  was likely the result of a combination of *two* shifts. The shift of *demand* from  $D_1$  to  $D_2$  would have resulted, at the original price  $P_1$ , in an increase in ridership from  $Q_1$  to  $Q'_1$ . Along  $D_2$ , the increase in price from  $P_1$  to  $P_2$  would have resulted in a decrease in quantity demanded from  $Q'_1$  to  $Q_2$ . The *price* elasticity of demand relates only to this *second* shift *along*  $D_2$ . Ignoring the first shift gives us the illusion that we are measuring the elasticity of the upward-sloping dotted demand curve  $D?$ , which does not in fact exist.

Although the price elasticities we calculated using the 1995 and 1996 fare increase data conformed to our expectations, we have no more reason to trust their accuracy than we do the 1999 estimate, unless we are certain that *all factors affecting demand except price did remain constant*. The same warning applies to all empirical elasticity measures, including those included in the tables of Chapter 4. As circumstances change, elasticities typically change as well, and before we rely on such estimates we need to satisfy ourselves that proper estimating procedures were used initially and that circumstances have not altered excessively.

Given the uncertainties about our rough elasticity estimates, which we could refine using more sophisticated econometric methods, can we say anything about the policy issue with which we began this chapter? Strictly speaking, the answer is “No.” But a reasonable hypothesis might be that if demand was shifting outward over the entire 1995–2000 period, then demand is *more* price-elastic than our estimates indicate. Lowering the price would be more effective in attracting new riders than we might have thought. Yet if demand is still fairly price-inelastic, then increasing service, which shifts the demand curve to the right, might be an equally effective way to entice motorists to leave their cars at home, utilize public transit, and relieve traffic congestion.

## APPLICATION: THE PRICE ELASTICITY OF DEMAND FOR ALCOHOL

How does the consumption of alcoholic beverages respond to changes in their price? For many decades, the conventional wisdom on this subject responded, “not very much.” Unfortunately, however, estimates of the price elasticity of demand for alcohol tend to be highly unreliable. The problem is that liquor prices usually don’t vary sufficiently to permit an accurate estimate of their effects.

In a careful study,<sup>6</sup> Philip Cook made use of some previously unexploited United States data on significant changes in alcohol prices. He suggested that the price elasticity of demand for alcohol may be much higher than had been thought.

Cook’s method was to examine changes in alcohol consumption that occurred in response to changes in U.S. state liquor taxes. Of the 48 contiguous states, 30 license and tax the private sale of liquor. Periodically, most of these states increase their nominal liquor taxes to compensate for the effects of inflation. The pattern is for the real value of a state’s liquor tax to be highest right after one of these tax increases, then to erode steadily as the cost of living rises over the next several years. The fact that taxes are not adjusted continuously to keep their real value constant provides the real price variability we need to estimate the responsiveness of alcohol purchases to price changes.

There were 39 liquor tax increases in Cook’s 30-state sample during the period 1960–1975. In 30 of these 39 cases, he found that liquor consumption declined relative to the national trend in the year following the tax increase. His estimate of the price elasticity of demand was  $-1.8$ , a substantially higher value than had been found in previous studies.

Cook’s interpretation of his findings provides an interesting case study in the factors that govern price elasticity. One salient fact about the alcohol market, he noted, is that heavy drinkers, though a small fraction of the total population, account for a large fraction of the total alcohol consumed. This fact has led many people to expect that alcohol consumption would be unresponsive to variations in price. The common view of heavy drinkers, after all, is that they drink primarily out of habit, not because of rational deliberations about price. Stated another way, analysts always expected the substitution effect to be small for these people. But even if the substitution effect were zero for heavy drinkers, there would remain the income effect to consider. The budget share devoted to alcohol tends to be large among heavy drinkers for two reasons. The obvious one is that heavy drinkers buy a lot of liquor. Less obvious, perhaps, is that their incomes tend to be significantly smaller than average. Many heavy drinkers have difficulty holding steady jobs and often cannot work productively in the jobs they do hold. The result is that the income effect of a substantial increase in the price of liquor forces many heavy drinkers to consume less. In support of this interpretation, Cook observed that mortality from cirrhosis of the liver declines sharply in the years following significant liquor tax increases. This is a disease that for the most part afflicts only people with protracted histories of alcohol abuse, and clinical experience reveals that curtailed drinking can delay or prevent its onset in long-term heavy drinkers.

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<sup>6</sup>Philip J. Cook, “The Effect of Liquor Taxes on Drinking, Cirrhosis, and Auto Accidents,” in *Alcohol and Public Policy*, Mark Moore and Dean Gerstein (eds.), Washington, DC: National Academy Press, 1982.

## The Intertemporal Choice Model

The choices we have considered thus far have involved trade-offs between alternatives in the present—the choice between food now and clothing now, between travel now and stereo equipment now, and so on. There was no hint in any of these choices that the alternative chosen today might affect the menu of alternatives available in the future.

Yet such effects are a prominent feature of many of our most important decisions. Our task in this section is to enlarge the basic consumer choice model in Chapter 3 to accommodate them.

### INTERTEMPORAL CONSUMPTION BUNDLES

When deciding what to do with their incomes, people may either consume them all now or save part for the future. The question we want to be able to answer is, “How would rational consumers distribute their consumption over time?” To keep the analysis manageable, it is helpful to begin by supposing that there are only two time periods, namely, *current* and *future*. In the standard, or *atemporal*, choice model in Chapter 3, the alternatives were different goods that could be consumed in the current period—apples now versus oranges now, etc. In our simple *intertemporal choice model*, the alternatives instead will be *current consumption* (denoted  $C_1$ ) versus *future consumption* (denoted  $C_2$ ). Each of these is an amalgam—the functional equivalent of the composite good (see Chapter 3). For the sake of simplicity, we set aside the question of how to apportion current and future consumption among the various specific consumption goods.

In the atemporal choice model, any bundle of goods can be represented as a point in a simple two-dimensional diagram. We use an analogous procedure in the intertemporal choice model. In Figure 5-12, for example, current consumption of \$6000 combined with future consumption of \$6000 is represented by the bundle *E*. Bundle *D* represents current consumption of \$3000 and future consumption of \$9000.

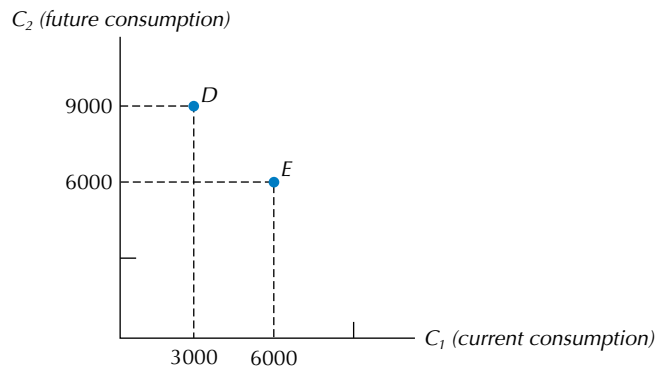
### THE INTERTEMPORAL BUDGET CONSTRAINT

Suppose you receive \$50,000 income in the current period and \$60,000 income in the future period. Suppose also that if you deposit some of your income from the current

**FIGURE 5-12**

Alternative combinations of current and future consumption are represented as points in the  $C_1, C_2$  plane. By convention, the horizontal axis measures current consumption; the vertical axis, future consumption.

### INTERTEMPORAL CONSUMPTION BUNDLES



period in a bank, you can receive your principal plus 20 percent in the future period. Similarly, if you wish to borrow against your future income, you may receive \$1 in the current period for every \$1.20 you must repay in the future period. (See Figure 5-13.) To construct your intertemporal budget constraint, first note that you can always merely consume your income in each period, so  $C_1 = \$50,000$  and  $C_2 = \$60,000$  must be a point on your intertemporal budget constraint. Another option is to deposit all \$50,000 (maximum lending) and thus receive  $1.2(50,000) = \$60,000$  in addition to your \$60,000 future income for  $C_2 = \$120,000$  future consumption with no current consumption ( $C_1 = 0$ ). Yet another option is to borrow  $60,000/1.2 = \$50,000$  (maximum borrowing) in addition to your \$50,000 current income for  $C_1 = \$100,000$  current consumption with no future consumption ( $C_2 = 0$ ). The equation for your intertemporal budget constraint is  $C_2 = 120,000 - 1.2C_1$ , or, equivalently,  $1.2C_1 + C_2 = \$120,000$ .

In general, suppose you receive  $M_1$  of your income in the first period and  $M_2$  in the second, and can either borrow or lend at the interest rate  $r$ . Under these circumstances, what is the most you can consume in the future period? Maximum future consumption occurs when you set all your current income aside for future use. Setting aside  $M_1$  in the current period at the interest rate  $r$  means your deposit will grow to  $M_1(1 + r)$  by the future period. So the most you can possibly consume in the future is that amount plus your future income, or  $M_1(1 + r) + M_2$ .

What is the most you could consume in the current period? The answer is your current income plus the maximum amount you can borrow against your future income. The most you can borrow against a future income of  $M_2$  is called the *present value* of  $M_2$ , denoted  $PV(M_2)$ . It is the amount that, if deposited today at the interest rate  $r$ , would be worth exactly  $M_2$  in the future period. Accordingly, we can find the present value of  $M_2$  by solving  $PV(M_2)(1 + r) = M_2$  for  $PV(M_2)$ :

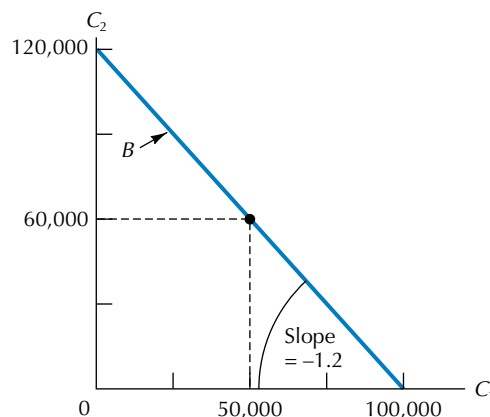
$$PV(M_2) = \frac{M_2}{1 + r}. \quad (5.1)$$

For example, if  $M_2$  were \$110,000 and the interest rate were 10 percent per period (that is,  $r = 0.10$ ), the present value of  $M_2$  would be  $\$110,000/1.1 = \$100,000$ . Present value is a simple equivalence relationship between sums of money that are payable

**Present value** The present value of a payment of  $X$  dollars  $T$  years from now is  $X/(1 + r)^T$ , where  $r$  is the annual rate of interest.

**FIGURE 5-13**  
For every dollar by which current consumption is reduced, it is possible to increase future consumption by \$1.2.

#### THE INTERTEMPORAL BUDGET CONSTRAINT



at different points in time. If  $r = 10$  percent ( $=0.10$ ) per period, then \$100,000 today will be worth \$110,000 in the future, and \$110,000 in the future is worth \$100,000 today.

It is not necessary, of course, to borrow or save the maximum amounts possible. The consumer who wishes to shift some of her future income into the current period can borrow any amount up to the maximum at the rate of  $1/(1+r)$  dollars today for every dollar given up in the future. Or, she can save any amount of her current income and get back  $(1+r)$  dollars in the future for every dollar not consumed today. The intertemporal budget constraint, shown as the locus  $B$  in Figure 5-14, is thus the straight line that joins the maximum current consumption and maximum future consumption points. Its slope will be  $-(1+r)$ . As in the atemporal model, here too the slope of the budget constraint may be interpreted as a relative price ratio. This time it is the ratio of the prices of current and future consumption. With  $r > 0$ , current consumption has a higher price than future consumption because of the opportunity cost of the interest forgone when money is spent rather than saved. It is conventional to refer to the horizontal intercept of the intertemporal budget constraint as the *present value of lifetime income*.

### EXERCISE 5-4

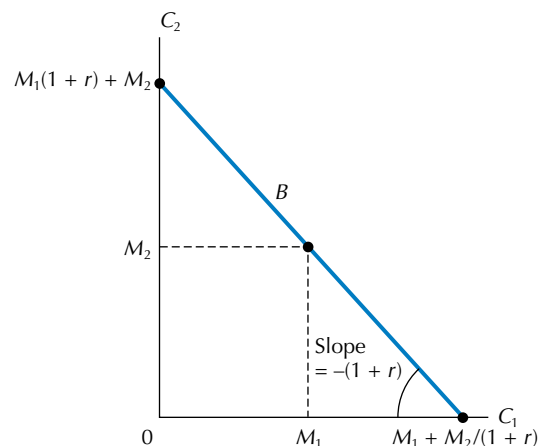
You have \$50,000 of current income and \$42,000 of future income. If the interest rate between the current and future period is 5 percent, what is the present value of your lifetime income? What is the maximum amount you could consume in the future? What is the equation describing your intertemporal budget constraint?

As in the atemporal case considered in Chapter 3, the intertemporal budget constraint is a convenient way of summarizing the consumption bundles that someone is *able* to buy. And again as before, it tells us nothing about which particular combination a person will *choose* to buy.

**FIGURE 5-14**

The opportunity cost of \$1 of present consumption is  $(1+r)$  dollars of future consumption. The horizontal intercept of the intertemporal budget constraint is the present value of lifetime income,  $M_1 + M_2/(1+r)$ .

**INTERTEMPORAL BUDGET CONSTRAINT WITH INCOME IN BOTH PERIODS, AND BORROWING OR LENDING AT THE RATE  $r$**



## INTERTEMPORAL INDIFFERENCE CURVES

To discover which bundle the consumer will select from those that are feasible, we need some convenient way of representing the consumer's preferences over current and future consumption. Here again the analytical device is completely analogous to one we used in the atemporal case. Just as a consumer's preferences over two current consumption goods may be captured by an indifference map, so too may his preferences over current and future goods be represented in this fashion. In Figure 5-15, the consumer is indifferent between the bundles that lie on the locus  $I_1$ , each of which is less desirable than the bundles on  $I_2$ , and so on.

The absolute value of the slope of the intertemporal indifference curve at any point is the marginal rate of substitution between future and current consumption. At point  $A$  in Figure 5-15, it is given by  $|\Delta C_2/\Delta C_1|$ , and this ratio is also referred to as the *marginal rate of time preference (MRTP)* at  $A$  (see footnote 7). If  $|\Delta C_2/\Delta C_1| > 1$  at  $A$ , the consumer is said to exhibit *positive time preference* at that point. This means that he requires more than 1 unit of future consumption to compensate him for the loss of a unit of current consumption. If  $|\Delta C_2/\Delta C_1| < 1$  at a point, he is said to exhibit *negative time preference* at that point. Such a person is willing to forgo 1 unit of current consumption in return for less than 1 unit of future consumption. Finally, if  $|\Delta C_2/\Delta C_1| = 1$  at a point, the consumer is said to have *neutral time preference* at that point. With neutral time preference, present and future consumption trade off against one another at the rate of 1 to 1.

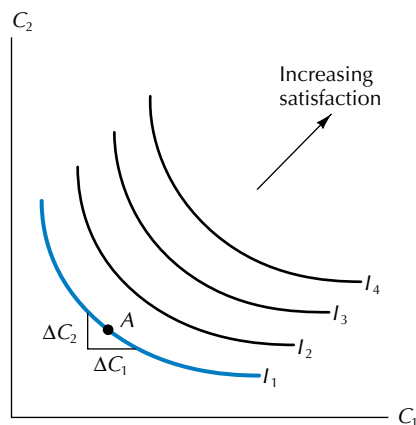
As in the atemporal case, it appears justified to assume that the marginal rate of time preference declines as one moves downward along an indifference curve. The more current consumption a person already has, the more she will be willing to give up in order to obtain an additional unit of future consumption. For most of us then, the question of whether time preference is positive, negative, or neutral will be a matter of where we happen to be on our indifference maps. The scion of a wealthy family who is unable to borrow against the \$5 billion he is due to inherit in 2 years very

**Marginal rate of time preference** The number of units of consumption in the future a consumer would exchange for 1 unit of consumption in the present.

**FIGURE 5-15**

As in the atemporal model, movements to the northeast represent increasing satisfaction. The absolute value of the slope of an indifference curve at a point is called the marginal rate of time preference (MRTP) at that point. The MRTP at  $A$  is  $|\Delta C_2/\Delta C_1|$ .

**AN INTERTEMPORAL INDIFFERENCE MAP**

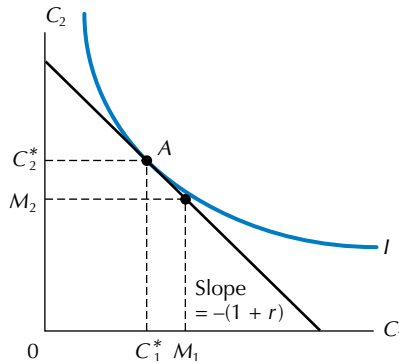


<sup>7</sup>In calculus terms, the marginal rate of time preference is given by  $|dC_2/dC_1|$ .

**FIGURE 5-16**

As in the atemporal model, the optimal intertemporal consumption bundle (bundle *A*) lies on the highest attainable indifference curve. Here, that occurs at a point of tangency.

**THE OPTIMAL INTERTEMPORAL ALLOCATION**



likely has strongly positive time preference. In contrast, the subsistence farmer whose food stocks are perishable is likely to have negative time preference in the wake of having harvested a bumper crop.

The optimal allocation between current and future consumption is determined exactly as in the atemporal model. The consumer selects the point along his budget constraint that corresponds to the highest attainable indifference curve. If the intertemporal indifference curves have the conventional convex shape, we ordinarily get a tangency solution like the one shown in Figure 5-16. If the MRTP is everywhere larger than (or everywhere smaller than) the absolute value of the slope of the budget constraint, corner solutions result, just as in the atemporal case.

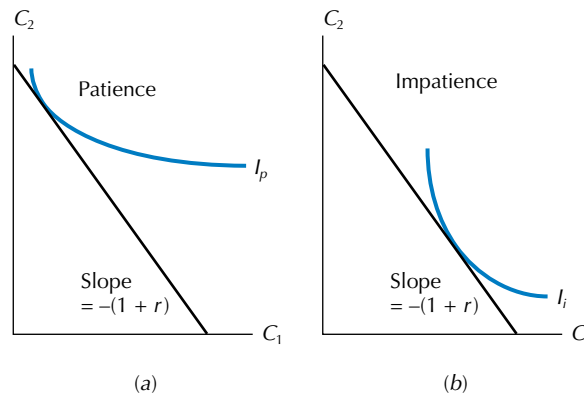
In Figure 5-16, suppose that the consumer has the same income in each time period ( $M_1 = M_2$ ) but consumes more in period 2 than in period 1. Does this pattern imply that she has positive time preference at the optimum point *A*? Not necessarily. If the rate of interest is negative ( $r < 0$ ), then  $1 + r < 1$ , and at *A* she will have negative time preference. What is true is that (with an interior solution) her time preference at the optimum point will be positive, neutral, or negative as  $r > 0$ ,  $r = 0$  or  $r < 0$ , respectively.

The optimal allocation will of course be different for different consumers. The optimum shown in Figure 5-17*a*, for example, is for a consumer whose preferences are much more heavily tilted in favour of future consumption. The one shown in Figure 5-17*b*, in

**FIGURE 5-17**

(*a*) The patient consumer postpones the bulk of consumption until the future period.  
 (*b*) The impatient consumer consumes much more heavily in the current period. But in equilibrium, the marginal rate of time preference ( $1 + r$ ) is the same for both types of consumers.

**PATIENCE AND IMPATIENCE**



contrast, is for a consumer who cares much more about present consumption. But in each case, note that the slope of the indifference curve at the optimal point is the same. As long as consumers can borrow and lend at the interest rate  $r$ , the marginal rate of time preference at the optimal bundle will be  $(1 + r)$  (except, of course, in the case of corner solutions). Hence with  $r > 0$ , for interior solutions, positive time preference is the rule, regardless of the consumer's preferences.

If (as we usually assume) both current and future consumption are normal goods, then an increase in the present value of lifetime income, all other factors constant, will cause both current and future consumption to rise.

### EXAMPLE 5-4

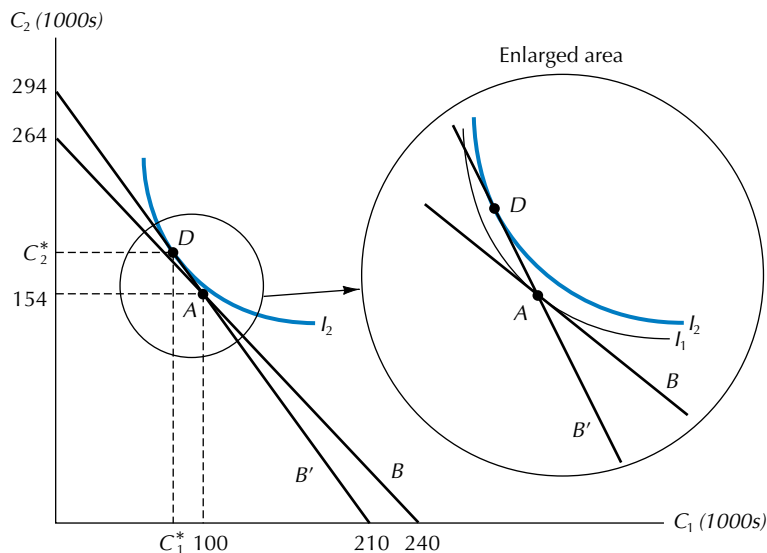
*In a two-period world, you have current income of \$100,000 and future income of \$154,000, and can borrow and lend at the rate  $r = 10$  percent/period. Under these conditions, you consume exactly your income in each period. True or false: An increase in  $r$  to  $r = 40$  percent/period will cause you to save some of your current income.*

Line  $B$  in Figure 5-18 is the original budget constraint. Its horizontal intercept is the present value of lifetime income when  $r = 0.1$ :  $\$100,000 + \$154,000/1.1 = \$240,000$ . Its vertical intercept is future income plus  $(1 + r)$  times current income:  $\$154,000 + (1.1)(\$100,000) = \$264,000$ . The optimal bundle occurs at  $A$ , by assumption, which implies that the MRTP at  $A$  is 1.1. When the interest rate rises to 0.4, the intertemporal budget constraint becomes  $B'$ . Its horizontal intercept is  $\$100,000 + \$154,000/1.4 = \$210,000$ . Its vertical intercept is  $\$154,000 + (1.4)(\$100,000) = \$294,000$ . Because the MRTP at  $A$  is less than the absolute value of the slope of the budget constraint  $B$ , it follows that the consumer will be better off by consuming less in the present and more in the future than he did at  $A$ . The new bundle is shown at  $D$  in Figure 5-18.

**FIGURE 5-18**

When the interest rate goes up, the intertemporal budget constraint rotates about the current endowment point. If the current endowment point ( $A$ ) was optimal at the lower interest rate, the new optimal bundle ( $D$ ) will have less current consumption and more future consumption.

**THE EFFECT OF A RISE IN THE INTEREST RATE**



## APPLICATION: THE PERMANENT INCOME AND LIFE-CYCLE HYPOTHESES

Economists once assumed that a person's current consumption depends primarily on her current income. Thus if a consumer received a windfall roughly equal to her current income, the prediction was that her consumption would roughly double.

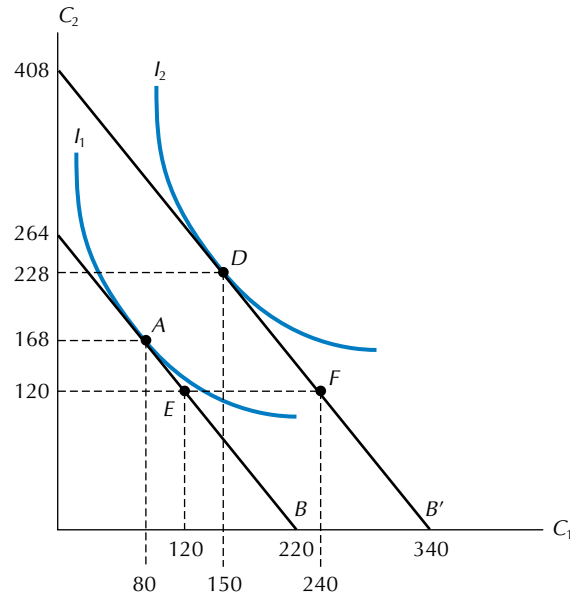
In the 1950s, however, Milton Friedman, Franco Modigliani, Richard Brumberg, and others argued that the intertemporal choice model suggests otherwise.<sup>8</sup> To illustrate, consider a consumer with current and future incomes both equal to 120, who can borrow and lend at the rate  $r = 20$  percent/period. The locus labelled  $B$  in Figure 5-19 is the consumer's intertemporal budget constraint, and the optimal bundle along it is denoted by  $A$ . Note that the horizontal intercept of  $B$  is the present value of lifetime income, namely,  $120 + (120/1.2) = 220$ .

Notice what happens when this consumer's current income rises from 120 to 240. His budget constraint is now the locus labelled  $B'$ , and the optimal bundle is  $D$ . The effect of increasing current income is thus to increase not only current consumption (from 80 to 150) but future consumption as well (from 168 to 228). If intertemporal indifference curves exhibit diminishing marginal rates of time preference,<sup>9</sup> the consumer generally does best not to concentrate too much of his consumption in any one period. By spreading his windfall over both periods, he is able to achieve a better outcome.

**FIGURE 5-19**

The effect of a rise in current income (from 120 to 240) will be felt as an increase not only in current consumption (from 80 to 150), but also in future consumption (from 168 to 228).

**PERMANENT INCOME, NOT CURRENT INCOME, IS THE PRIMARY DETERMINANT OF CURRENT CONSUMPTION**



<sup>8</sup>See Franco Modigliani and R. Brumberg, "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," in K. Kurihara (ed.), *Post-Keynesian Economics*, London: Allen & Unwin, 1955; and Milton Friedman, *A Theory of the Consumption Function*, Princeton, NJ: Princeton University Press, 1957.

<sup>9</sup>Recall that diminishing marginal rate of time preference is the intertemporal analogue of diminishing marginal rate of substitution in the atemporal model.

**Permanent income**  
The present value of  
lifetime income.

Friedman's *permanent income hypothesis* says that the primary determinant of current consumption is not current income but what he called *permanent income*. In terms of our simple intertemporal choice model, permanent income is simply the present value of lifetime income. (Following the increase in current income in Figure 5-19, permanent income is  $240 + 120/1.2 = 340$ .) When we consider that in reality the future consists of not just one but many additional periods, it becomes clear that current income constitutes only a small fraction of permanent income. (If there were 10 such future periods we were concerned about, for example, then a 10 percent increase in current income would cause permanent income to increase by just over 2 percent.)<sup>10</sup> Accordingly, Friedman argued, a given proportional change in current income should give rise to a much smaller proportional change in current consumption, just as we saw in Figure 5-19. (The *life-cycle hypothesis* of Modigliani and Brumberg tells essentially the same story.)

### FACTORS ACCOUNTING FOR DIFFERENCES IN TIME PREFERENCE

Uncertainty regarding the future is one reason to prefer current to future consumption. In countries at war, for example, people often live as though there were no tomorrow, as indeed for some of them there will not be. By contrast, a peaceful international climate, secure employment, stable social networks, good health, and a variety of similar factors tend to reduce uncertainty about the future, in the process justifying greater weight on future as opposed to current consumption.

Intertemporal indifference maps, like the atemporal variety, will also vary according to the disposition of the individual. As Figure 5-17 suggests, for some individuals, like the grasshopper in Aesop's fable, the only problem with immediate gratification is that it takes too long. Others, like Aesop's ant, are reluctant to consume any significant amount until adequate provision is made for the future. Grasshoppers will have very steep intertemporal indifference curves, while those of ants will be flatter. Such differences in individual time preference result from a number of factors, which vary in importance among individuals. There are differences in the intensity of desire for immediate want-satisfaction, in the capacity to gain pleasure from anticipating future consumption, in the degree of risk-aversion (given uncertainty about the future), and in the capacity for prevision and planning.

Time preference depends also on the specific circumstances of the choice at hand. Experimental studies have isolated certain situations in which most people have strongly positive time preference, others in which they show strongly negative time preference. Carnegie-Mellon University economist George Loewenstein, for example, told experimental subjects to imagine they had won a kiss from their favourite movie star and then asked them when they would most like to receive it. Even though getting it right away was one of the options, most subjects elected to wait an average of several days. These choices imply negative time preference, and Loewenstein theorized that most subjects simply wanted a little while to savour the anticipation of the kiss.<sup>11</sup> Loewenstein also told a group of subjects to imagine that they were going to receive a painful electric shock and then asked them when they would like to receive it. This time most subjects chose to get it right away. They apparently wanted to spend as little time as possible dreading the shock. But since an electric shock is a "bad"

<sup>10</sup>Again, we assume an interest rate of  $r = 20$  percent/period.

<sup>11</sup>See George Loewenstein, "Anticipation and the Valuation of Delayed Consumption," *Economic Journal*, 97, September 1987: 666–684.

rather than a “good,” these choices too imply negative time preference.

These situations where people display negative time preference can be set against instances where people show a preference for present over future consumption. Put a large bowl of hot popcorn down in front of those same subjects shortly before dinner time. Even among those who would prefer the dinner to the popcorn, there are likely some who will “spoil their dinner,” because they lack the self-control to wait. Eugen von Böhm-Bawerk (1851–1914) suggested that one reason for such behaviour is that current consumption opportunities confront our senses directly, whereas future ones can only be imagined. Böhm-Bawerk believed that our “faulty telescopic faculty” was no good reason to assign greater weight to current than to future pleasures. Uncertainty aside, he felt that people would reap greater satisfaction from their lives if they weighed the present and the future equally.

## OTHER FACTORS THAT AFFECT INTERTEMPORAL CHOICE

**Preference for a Rising Consumption Standard.** Some evidence suggests that people prefer a consumption pattern that gives even more weight to future than to current consumption, when satisfaction depends not only on the level of consumption but also on its rate of change. An improving standard of living can often bring greater satisfaction than a static one that is initially at a higher level.

At the most fundamental level, support for this idea comes from our knowledge of how the human nervous system perceives and processes information. As psychologists and biologists explain it, we are much less sensitive to the absolute level of any sensory stimulus than to deviations from norms or reference standards we adopt from experience. The pedestrian in Montreal, for example, often fails to notice the horns that blare at him, whereas small town residents are often startled by much fainter sounds. Like the din of the metropolis, consumption at any constant level becomes a norm. As such, it is at least partly taken for granted, serving as the standard against which future consumption levels are measured.

To test your own intuition about the importance of a rising consumption profile, imagine yourself living on a deserted island faced with a once-and-for-all choice between the two consumption profiles shown in Figure 5-20. Which one would you pick?

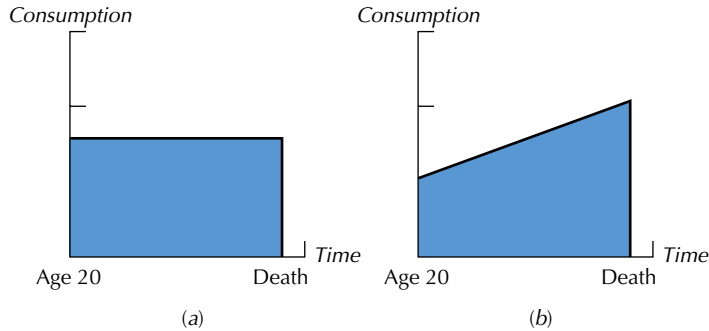
In a sample of 112 undergraduates who were asked to make a choice similar to the one portrayed in Figure 5-20, 87 (78 percent) chose the rising profile (panel *b*).<sup>12</sup> This pattern seems strongly at odds with Böhm-Bawerk’s notion that people generally put too much weight on current consumption. One reason for the apparent discrepancy is that the savings incentives we face individually differ from the ones we face as whole societies. Indeed, as we see in the next section, individuals face strong incentives to concentrate their consumption in the present, even though they might wish that everyone would save more for the future.

**Positional Concerns and Intertemporal Choice.** Skating excellence is inescapably relative. In 1962, Canadian skater Donald Jackson won the World Championship, largely because he was the first to land a triple lutz jump in competition. Now, triple lutz and triple axels are part of the standard repertoire of every world-class skater. Kurt

<sup>12</sup>See R. Frank and R. Hutchens, “Wages, Seniority, and the Demand for Rising Consumption Profiles,” *Journal of Economic Behaviour and Organization*, 21, 1993: 251–276.

**FIGURE 5-20**

Many people declare a strong preference for a growing consumption profile (panel *b*) over a static profile (panel *a*).

**LEVEL VERSUS GROWING CONSUMPTION PROFILES**

**Positional good** A good whose value depends strongly on how it compares with similar goods consumed by others; also called a *status good*.

Browning was the first to land a quadruple toe loop in competition, at the 1988 Worlds. Now, winning the World Championships typically requires not just a single quad, but rather multiple quads or quads in combination.

The late British economist Fred Hirsch coined the term *positional goods* to refer to goods and services whose value is strongly influenced by their *relative quality*.<sup>13</sup> Luxury goods like diamonds are a quintessential example. As with top skaters, a “good” diamond is one that compares favourably with other available diamonds. A good job, similarly, is one that is better than most other jobs. The standards that define a good job in the twentieth century are very different from those that applied in the fifteenth century. Education is likewise a good with a strong positional component. Whether society considers you well educated, and thus qualified for a good job, depends to a large extent on how your education compares with the education received by your peers.



Drawing by William Walker Hafeli. Copyright © 1979 Saturday Review, Inc.

***“I admit it does look very impressive. But you see, nowadays everyone graduates in the top ten per cent of his class.”***

<sup>13</sup>See Fred Hirsch, *Social Limits to Growth*, Cambridge, MA: Harvard University Press, 1976.

The central feature of a positional good is its *inherent* scarcity. Although it is possible to imagine an environment in which people had more food than they could possibly consume, the same cannot be said of a positional good.

This feature of positional goods shapes a variety of important decisions, including the one of how much to save. A family can save more of its income for retirement, or spend more now for, say, a house in a better school district. For most parents, the lure of providing relative educational advantages for their children is powerful. Yet simple arithmetic tells us that no matter how much each family spends on housing, only 10 percent of the children can occupy seats in the top decile of the educational quality distribution. In the aggregate, saving less and spending more for houses in better school districts in itself serves only to bid up the prices of those houses. It does nothing to alter the relative distribution of educational opportunities.<sup>14</sup>

Spending on positional goods is thus very much like a military arms race. Each side spends more in an effort not to fall behind, but in the end, their efforts merely offset one another. Positional concerns cause the individual payoff from spending to appear spuriously large, the payoff from saving spuriously small. These concerns are thus in direct competition with the desire to have a rising consumption profile. To provide the best possible education for your children, you must spend heavily during the early years of the life cycle. In contrast, to achieve a rising consumption profile, you must save a lot during the early years. If positional concerns are strong, then even people who strongly value a rising consumption profile might nonetheless save very little. Indeed, many families would have grossly inadequate incomes for retirement were it not for the Canada Pension Plan and private forced-savings programs. These programs may be interpreted as collective actions to shelter part of our incomes from spending on positional goods, in the process helping achieve a more preferred lifetime consumption profile.<sup>15</sup>

The importance of positional concerns suggests an important qualification to the life-cycle and permanent income models of savings. If positional concerns are important, people in the lower part of the income distribution will naturally have more difficulty than others in keeping up with community consumption standards. The clear prediction is that the savings rate will rise with position in the income distribution. In every country for which the relevant data are available, savings rates rise sharply with income.

**The Self-Control Pitfall.** Another reason for low savings rates coexisting with a preference for a rising consumption profile is that people often seem to have difficulty carrying out plans they believe to be in their own interests. Thomas Schelling notes, for example, that most cigarette smokers say they want to quit.<sup>16</sup> Many of them, with great effort, have done so. Many more, however, have tried to quit and failed.

One way of solving the self-control problem is captured by the example of Homer's Ulysses, who was faced with having to sail past dangerous reefs where the sirens lay. Ulysses realized that once he was within earshot of the sirens' cries, he would be drawn irresistibly toward them and sail to his doom on the reefs. Able to foresee this temporary change in his preferences, he came up with an effective *commitment device*: he instructed his crewmen to strap him tightly to the mast and not to release him, even though he might beg them to, until they had sailed safely past.

#### **Commitment device**

A device that commits a person to behave in a certain way in the future, even though he may wish to behave otherwise when the time comes.

<sup>14</sup>Nor, for that matter, does it alter the absolute value of educational services consumed by each student.

<sup>15</sup>For a detailed argument in support of this interpretation, see R. Frank, *Choosing the Right Pond: Human Behavior and the Quest for Status*, New York: Oxford University Press, 1985. Formally, the problem is like the "prisoners' dilemmas" we discuss further in Chapters 7 and 13.

<sup>16</sup>See Thomas Schelling, *Choice and Consequence*, Cambridge, MA: Harvard University Press, 1984.

Similar sorts of commitment devices are familiar in modern life. Fearing they will be tempted to spend their savings, people join “Christmas clubs,” special accounts that prohibit withdrawals until late autumn; they buy whole-life insurance policies, which impose substantial penalties on withdrawals before retirement. Fearing they will spoil their dinners, they put the salted nuts out of easy reach. Fearing they will gamble too much, they limit the amount of cash they take to Casino Niagara. Fearing they will stay up too late watching TV, they move the television out of the bedroom.

The moral of the burgeoning self-control literature is that *devising* a rational intertemporal consumption plan is only part of the problem. There is also the task of *implementing* it. But here too, rational deliberation can help us avoid some of the most important pitfalls. The consumer who has just given up smoking, for example, can predict that he will desperately want a cigarette if he goes out drinking with his friends on Friday nights. And he can also insulate himself from that temptation by committing himself to alternative weekend activities for the next month or so. By the same token, the person who wants to shield herself from the temptation to spend too much may have part of her pay diverted automatically into a savings program, and this is precisely what millions of people do.

These issues once again highlight the distinction between the positive and normative roles of the rational choice model discussed in Chapter 1. Because the rational choice model takes no account of self-control problems and the like, it will sometimes fail to predict how people actually behave. But this does not mean that the model, even in its narrowest form, is wrong or useless. For here, as in other instances, it can play the important normative role of guiding people toward better decisions, ones that accord more fully with their real objectives.

## Summary

In this chapter our primary focus was on applications of the rational choice and demand theories developed in Chapters 3 and 4. We also considered the concept of consumer surplus, which measures the amount by which a consumer benefits by being able to buy a given product at a given price. We saw that consumer surplus is well approximated by the area bounded above by the individual demand curve and below by the market price. Two-part pricing structures are a device by which a portion of consumer surplus is transferred from the buyer to the seller.

The rational choice model is also useful for evaluating the welfare effects of price and income changes. It suggests why the consumer price index, the government’s measure of changes in the cost of living, may often overstate the true cost of achieving a given level of satisfaction.

The intertemporal choice model is, in every essential respect, analogous to the atemporal choice model in Chapter 3. In the two-dimensional case, it begins with a commodity graph that depicts current and future consumption levels of a composite good. The consumer’s initial endowment is the point,  $(M_1, M_2)$ , that corresponds to current and future income. If the consumer can borrow and lend at the rate  $r$ , his intertemporal budget constraint is then the line passing through the endowment point with a slope of  $-(1 + r)$ . The opportunity cost of a unit of current consumption is  $1 + r$  units of future consumption. The horizontal intercept of the intertemporal budget constraint is the present value of all current and future income, which is also called the present value of lifetime wealth.

The consumer’s intertemporal preferences may be represented by an indifference map with essentially the same properties as in the atemporal case. A consumer is said

to exhibit positive, neutral, or negative time preference at a point if his marginal rate of time preference (the absolute value of the slope of his indifference curve) at that point is greater than 1, equal to 1, or less than 1, respectively. In the case of interior solutions, equilibrium occurs at a tangency between the intertemporal budget constraint and an indifference curve. Hence with an interior solution, when  $r > 0$ , consumers will exhibit positive time preference in equilibrium, irrespective of the shape of their indifference curves.

An important application of the intertemporal choice model is to the study of decisions about how much to save. The permanent income and life-cycle hypotheses employ the model to demonstrate that it is the present value of lifetime wealth, not current income alone, that governs current consumption (and hence current savings).

The appendix to this chapter discusses additional applications of rational choice and demand theories, including cost-of-living indices and the use of indifference curves to measure consumer surplus.

### Questions for Review

1. Explain in your own words why a gasoline tax whose proceeds are refunded to the consumer in a lump-sum amount will nonetheless reduce the consumption of gasoline.
2. Explain in your own words what a two-part pricing scheme is and why sellers might use one.
3. Do you think a college education has a high- or low-price (tuition) elasticity of demand?
4. Explain in your own words why even long-term heavy drinkers might be highly responsive to increases in the price of alcohol.
5. Explain why  $1$  plus the interest rate in the intertemporal choice model is analogous to the relative price ratio in the consumer choice model discussed in Chapter 3.
6. Public transit services are generally more energy-efficient than individuals using cars to commute to work. However, the trend over the past 30 years has been a decline in the proportion of commuters using public transit, despite an increase in real energy prices. Why?
7. Jennifer, who earns an annual salary of \$20,000, wins \$25,000 in the lottery. Explain why she most likely will not spend all her winnings during the next year.

### Problems

1. Using a diagram like Figure 5-2, explain why, under our current method of educational finance, a rich family is much more likely than a poor family to send its children to a private school.
2. When the price of gasoline is \$.50/litre, you consume 2000 litres/yr. Then two things happen: (1) The price of gasoline rises to \$1/litre and (2) a distant uncle dies, with the instruction to his executor to send you a cheque for \$1000/yr. If no other changes in prices or income occur, do these two changes leave you better off than before?
3. Larry demands strawberries according to the schedule  $P = 4 - (Q/2)$ , where  $P$  is the price of strawberries (\$/pack) and  $Q$  is the quantity (packs/wk). Assuming that the income effect is negligible, how much will he be hurt if the price of strawberries goes from \$1/pack to \$2/pack?

4. The only video rental club available to you charges \$4 per movie per day. If your demand curve for movie rentals is given by  $P = 20 - 2Q$ , where  $P$  is the rental price (\$/day) and  $Q$  is the quantity demanded (movies per year), what is the maximum annual membership fee you would be willing to pay to join this club? What is the average amount you would pay per video movie, counting membership and rental costs?
5. Jane spends all her income on hot dogs and caviar. Her demand curve for caviar is inelastic over the relevant price range. Unfortunately, overfishing has caused the supply of caviar to fall and the price to rise. If Jane's preferences and income and the price of hot dogs are unchanged, what has happened to Jane's consumption of hot dogs? Explain.
6. Jones spends all his income on two goods,  $X$  and  $Y$ . The prices he paid and the quantities he consumed last year are as follows:  $P_X = \$15/\text{unit}$ ,  $X = 20$  units,  $P_Y = \$25/\text{unit}$ , and  $Y = 30$  units. If the prices next year are  $P_X = \$6/\text{unit}$  and  $P_Y = \$30/\text{unit}$ , and Jones's income is \$1020, will he be better or worse off than he was in the previous year? (Assume that his tastes do not change.)
7. Smith lives in a world with two time periods, this period and the next period. His income in each period, which he receives at the beginning of each period, is \$210. If the interest rate is 5 percent/period, what is the present value of his lifetime income? Draw his intertemporal budget constraint. On the same axes, draw Smith's intertemporal budget constraint when  $r = 20$  percent/period.
8. Suppose Smith from Problem 7 views current and future consumption as perfect, one-for-one substitutes for one another. Find his optimal consumption bundle, in both cases.
9. Suppose Smith from Problem 7 views current and future consumption as one-to-one complements. Find his optimal consumption bundle, in both cases.
10. Karen earns \$75,000 in the current period and will earn \$75,000 in the future.
  - a. Assuming that these are the only two periods, and that banks in her country borrow and lend but at an interest rate  $r = 0$ , draw her intertemporal budget constraint.
  - b. Now suppose banks offer 10 percent interest on funds deposited during the current period, and offer loans at this same rate. Draw her new intertemporal budget constraint.
11. Find the present value of \$50,000 to be received after 1 year if the rate of interest for 1 year is
  - a. 8 percent.
  - b. 10 percent.
  - c. 12 percent.
12. Crusoe will live this period and the next period as the lone inhabitant of his island. His only income is a crop of 100 coconuts that he harvests at the beginning of each period. Coconuts not consumed in the current period spoil at the rate of 10 percent per period.
  - a. Draw Crusoe's intertemporal budget constraint. What will be his consumption in each period if he regards future consumption as a perfect, one-for-one substitute for current consumption?
  - b. What will he consume each period if he regards 0.8 unit of future consumption as being worth 1 unit of current consumption?
13. This fall, Crusoe puts 50 coconuts from his harvest into a cave just before a family of shipwrecked circus bears goes in to hibernate. As a result, he is unable to get the coconuts out before the bears emerge the following period. Coconuts spoil at the same rate no matter where he stores them. Why might he do this?
14. Kathy earns \$55,000 in the current period and will earn \$60,000 in the future period. What is the maximum interest rate that would allow her to spend \$105,000 in the current period?

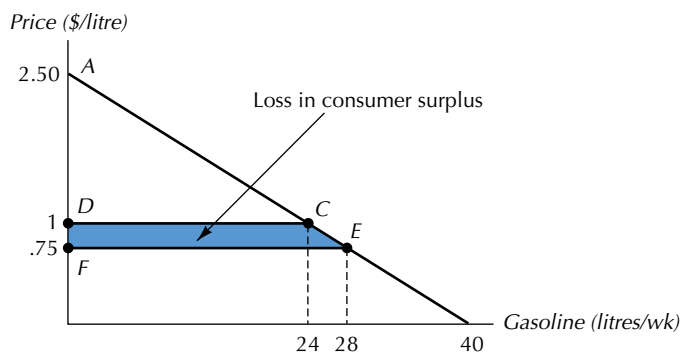
What is the minimum interest rate that would allow her to spend \$120,500 in the future period?

15. Smith receives \$100 of income this period and \$100 next period. At an interest rate of 10 percent per period, he consumes all his current income in each period. He has a diminishing marginal rate of time preference between consumption next period and consumption this period. *True or false:* If the interest rate rises to 20 percent, Smith will save some of his income this period. Explain.
16. At current prices, housing costs \$50 per unit and the composite good has a price of \$1 per unit. A wealthy benefactor has given Joe, a penniless person, 1 unit of housing and 50 units of the composite good. Now the price of housing falls by half. *True or false:* Joe is necessarily better off as a result of the price change. Explain.
- \*17. Tom and Karen are economists. In an attempt to limit their son Harry's use of the family car, they charge him a user fee of \$0.20/kilometre. At that price he still uses the car more than they would like, but they do not want simply to raise the price further. So Tom and Karen ask him the following question: "What is the minimum increase in your weekly allowance you would accept in return for having the fee raised to \$0.40/kilometre?" Harry, who is a known truth-teller and has conventional preferences, answers, "\$10/wk."
  - a. If Tom and Karen increase Harry's allowance by \$10/wk and charge him 40 cents/kilometre, will he drive less than before? Explain.
  - b. Will the additional revenue from the higher charges per kilometre be more than, less than, or equal to \$10/wk? Explain.
- \*18. All book buyers have the same preferences, and under current arrangements, those who buy used books at \$22 receive the same utility as those who buy new books at \$50. The annual interest rate is 10 percent, and there are no transaction costs involved in the buying and selling of used books. Each new textbook costs  $m$  to produce and lasts for two years.
  - a. What is the buyer's reservation price for the use of a new book for one year?
  - b. How low would  $m$  have to be before a publisher would find it worthwhile to print books with disappearing ink—ink that vanishes one year from the point of sale of a new book, thus eliminating the used-book market? (Assume that eliminating the used-book market will exactly double the publisher's sales.)
- \*\*19. Herb wants to work exactly 12 hr/wk to supplement his graduate fellowship. He can either work as a clerk in the library at \$6/hr or individually tutor first-year graduate students in economics. Pay differences aside, he is indifferent between these two jobs. Each of three first-year students has a demand curve for tutoring given by  $P = 10 - Q$ , where  $P$  is the price in dollars per hour, and  $Q$  is the number of hours per week. If Herb has the option of setting a two-part tariff for his tutoring services, how many hours per week should he tutor and how many hours should he work in the library? If he does any tutoring, what should his rate structure be?
- \*\*20. How would your answer to the preceding question change if Herb could earn \$7 per hour in the library?
- \*\*21. Grace Hopper and her Aunt Prudence live in a two-period universe. Both have an income of \$60,000 in period 1, and no income in period 2. The interest rate is zero. Grace's utility function has the form  $U = (C_1)^2 C_2$ , while Aunt Prudence chooses to consume exactly the same amount in each period. How much will each spend in each period? [*Hint:* If you do not have calculus, see Appendix 3.]
- \*\*22. How will your answers to the preceding question change if the interest rate increases to 100 percent per period?
- \*23. Harry runs a small movie theatre, whose customers all have identical tastes. Each customer's reservation price just to see a movie is \$5, and lower prices will not induce them to attend

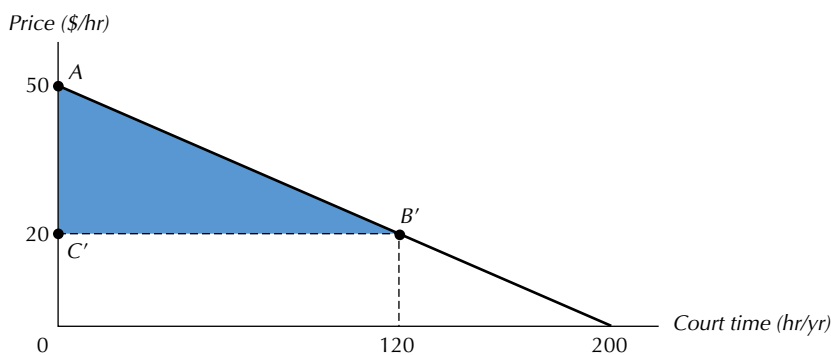
more frequently. Each customer's demand curve for popcorn at Harry's concession stand is given by  $P_c = 4 - Q_c$ , where  $P_c$  is the price of popcorn in dollars per container and  $Q_c$  is the number of containers purchased. Customers treat seeing a movie and eating popcorn at it as a unified "theatre-going experience." If the marginal cost of allowing another patron to watch the movie is zero, and the marginal cost of popcorn is \$1, at what price should Harry sell tickets and popcorn if his goal is to maximize his profits? (Assume that Harry is able to advertise his cost structure costlessly to potential patrons.)

### Answers to In-Chapter Exercises

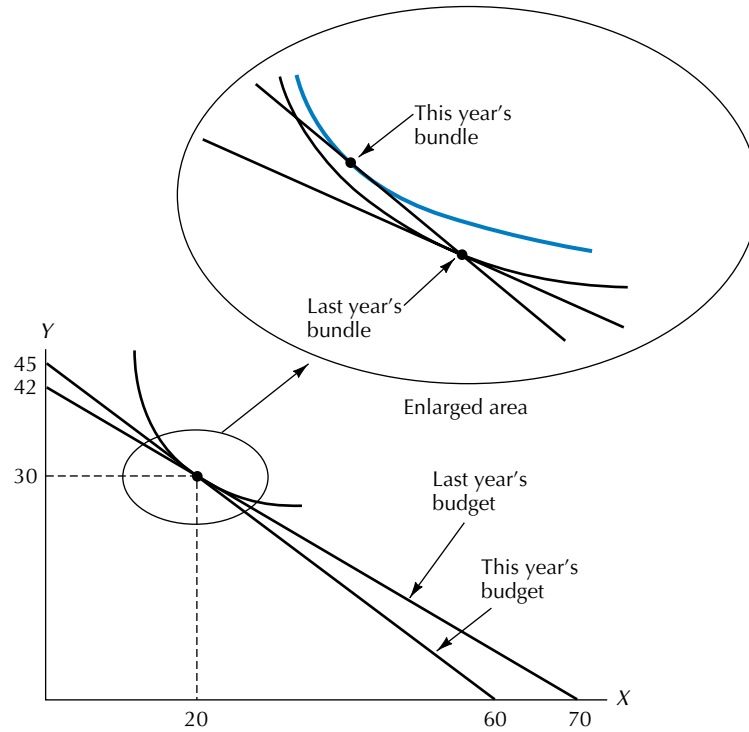
- 5-1. Initial consumer surplus at  $P = \$0.75/\text{litre}$  (and  $Q = 28$  litres/wk) is  $CS = \frac{1}{2}(2.5 - 0.75)(28) = \$24.50/\text{wk}$ . Consumer surplus at the higher price  $P' = \$1/\text{litre}$  (and  $Q' = 24$  litres/wk) is  $CS' = \frac{1}{2}(2.5 - 1)(24) = \$18/\text{wk}$ . The loss in consumer surplus is given by the area of  $DCEF$ , which equals  $24.5 - 18 = \$6.50/\text{wk}$ .



- 5-2. The maximum membership fee is now given by the area of triangle  $AB'C'$  which is  $CS = \frac{1}{2}(50 - 20)120 = \$1800/\text{yr}$ .



- 5-3. The two budget lines and last year's optimal bundle are shown in the diagram. A closer look at the tangency point (enlarged area) shows that this year Jones can now afford to purchase a bundle he prefers to the one he bought last year.



- 5-4.  $PV = \$50,000 + \$42,000/1.05 = \$90,000$ . Maximum future consumption =  $\$50,000(1.05) + \$42,000 = \$94,500$ . The equation for your intertemporal budget constraint is  $C_2 = 94,500 - 1.05C_1$ .