*APPENDIX A Logarithms

A strictly algebraic approach to the solution of certain compound interest problems involves the use of logarithms. However, if the financial functions of a calculator are employed, logarithms are not needed.

Financial calculators all have a function labelled **ln** or **ln *x***. This denotes the "natural logarithm" function.

Example 9AA CALCULATING NATURAL LOGARITHMS

a. $\ln 1 = 0$

b. $\ln 10 = 2.30258$

c. $\ln 1.1 = 0.09531$

d. $\ln 100 = 4.60517$

e. $\ln (1+ 0.01) = \ln 1.01 = 0.009950$

f. $\ln (1 + 0.06) = \ln 1.06 = 0.05827$

g. $\ln\left(\frac{\$5000}{\$2000}\right) = \ln 2.5 = 0.91629$

h. $\ln\left(\frac{\$3491.67}{\$1820.19}\right) = \ln 1.91830 = 0.65144$

Rules of Logarithms The Rules of Logarithms show how the logarithm of a product, quotient, or power may be expanded in terms of the logarithms of the individual factors. Their derivation is straightforward and can be found in any algebra text that introduces logarithms.

Rules of Logarithms:

1. The Product Rule: $\ln (ab) = \ln a + \ln b$
2. The Quotient Rule: $\ln (a/b) = \ln a - \ln b$
3. The Power Rule: $\ln (a^k) = k(\ln a)$

TRAP

It's Tempting But . . .

The following expansions are illegal.

$\ln (a + b) \neq \ln a + \ln b$

$\ln (ab) \neq (\ln a)(\ln b)$

$\ln (a - b) \neq \ln a - \ln b$

$\ln (a/b) \neq \ln a\,/\ln b$

Example 9AB DERIVATION OF FORMULA (9-2)

Solve $FV = PV(1 + i)^n$ for n.

Solution

First divide both sides of the equation by PV and then interchange the two sides, giving

$$(1 + i)^n = \frac{FV}{PV}$$

Next take logarithms of both sides. Use the Power Rule to expand $\ln(1+i)^n$. We obtain

$$n \times \ln(1 + i) = \ln\left(\frac{FV}{PV}\right)$$

Finally, division of both sides by $\ln(1+i)$ yields

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1 + i)}$$