CHAPTER 7

Using Discounted Cash-Flow Analysis to Make Investment Decisions

Think of the problems that General Motors faces when considering whether to introduce a new model. How much will we need to invest in new plant and equipment? What will it cost to market and promote the new car? How soon can we get the car into production? What is the projected production cost? What do we need in the way of inventories of raw materials and finished cars? How many cars can we expect to sell each year and at what price? What credit arrangements will we need to give our dealers? How long will the model stay in production? What happens at the end of that time? Can we use the plant and equipment elsewhere in the company? All of these issues affect the level and timing of project cash flows. In this chapter we continue our analysis of the capital budgeting decision by turning our focus to how the financial manager should prepare cash-flow estimates for use in net present value analysis.

In Chapter 6 we used the net present value rule to make a simple capital budgeting decision. You tackled the problem in four steps:

Step 1: Forecast the project cash flows.

Step 2: Estimate the opportunity cost of capital—that is, the rate of return that your shareholders could expect to earn if they invested their money in the capital market.

Step 3: Use the opportunity cost of capital to discount the future cash flows. The project's present value (PV) is equal to the sum of the discounted future cash flows.

Step 4: Net present value (NPV) measures whether the project is worth more than it costs. To calculate NPV you need to subtract the required investment from the present value of the future payoffs:

$$NPV = PV - \text{required investment}$$

You should go ahead with the project if it has a positive NPV.

We now need to consider how to apply the net present value rule to practical investment problems. The first step is to decide what to discount. We know the answer in principle: discount cash flows. This is why capital budgeting is often referred to as *discounted cash flow,* or *DCF,* analysis. But useful forecasts of cash flows do not arrive on a silver platter. Often the financial manager has to make do with raw data supplied by specialists in product design, production, marketing, and so on, and must adjust such data before they are useful. In addition, most financial forecasts are prepared in accordance with accounting principles that do not necessarily recognize cash flows when they occur. These data must also be adjusted.

We start this chapter with a discussion of the principles governing the cash flows that are relevant for discounting. We then present an example designed to show how standard accounting information can be used to compute those cash flows and why cash flows and accounting income usually differ. The example will lead us to various further points, including the links between depreciation and taxes and the importance of tracking investments in working capital.

After studying this chapter you should be able to

- Identify the cash flows properly attributable to a proposed new project.
- Calculate the cash flows of a project from standard financial statements.
- Understand how the company's tax bill is affected by depreciation and how this affects project value.
- Understand how changes in working capital affect project cash flows.

7.1 Discount Cash Flows, Not Profits

Up to this point we have been concerned mainly with the mechanics of discounting and with the various methods of project appraisal. We have had almost nothing to say about the problem of *what* you should discount. The first and most important point is this: to calculate net present value you need to discount cash flows, *not* accounting profits.

We stressed the difference between cash flows and profits in Chapter 2. Here we stress it again. Income statements are intended to show how well the firm has performed. They do not track cash flows.

If the firm lays out a large amount of money on a big capital project, you would not conclude that the firm performed poorly that year, even though a lot of cash is going out the door. Therefore, the accountant does not deduct capital expenditure when calculating the year's income but instead depreciates it over several years.

That is fine for computing year-by-year profits, but it could get you into trouble when working out net present value. For example, suppose that you are analyzing an investment proposal. It costs \$2,000 and is expected to bring in a cash flow of \$1,500 in the first year and \$500 in the second. You think that the opportunity cost of capital is 10 percent and so calculate the present value of the cash flows as follows:

$$PV = \frac{\$1{,}500}{1.10} + \frac{\$500}{(1.10)^2} = \$1{,}776.86$$

The project is worth less than it costs; it has a negative NPV:

$$NPV = \$1{,}776.86 - \$2{,}000 = -\$223.14$$

The project costs \$2,000 today, but accountants would not treat that outlay as an immediate expense. They would depreciate that \$2,000 over 2 years and deduct the depreciation from the cash flow to obtain accounting income:

	Year 1	Year 2
Cash inflow	+\$1,500	+\$ 500
Less depreciation	– 1,000	– 1,000
Accounting income	+ 500	– 500

Thus an accountant would forecast income of $500 in Year 1 and an accounting loss of $500 in Year 2.

Suppose you were given this forecast income and loss and naively discounted them. Now NPV *looks* positive:

$$\text{Apparent NPV} = \frac{\$500}{1.10} + \frac{-\$500}{(1.10)^2} = \$41.32$$

Of course we know that this is nonsense. The project is obviously a loser; we are spending money today ($2,000 cash outflow) and we are simply getting our money back ($1,500 in Year 1 and $500 in Year 2). We are earning a zero return when we could get a 10 percent return by investing our money in the capital market.

The message of the example is this:

When calculating NPV, recognize investment expenditures when they occur, not later when they show up as depreciation. Projects are financially attractive because of the cash they generate, either for distribution to shareholders or for reinvestment in the firm. Therefore, the focus of capital budgeting must be on cash flow, not profits.

Here is another example of the distinction between cash flow and accounting profits. Accountants try to show profit as it is earned, rather than when the company and the customer get around to paying their bills. For example, an income statement will recognize revenue when the sale is made, even if the bill is not paid for months. This practice also results in a difference between accounting profits and cash flow. The sale generates immediate profits, but the cash flow comes later.

EXAMPLE 7.1 *Sales before Cash*

Reggie Hotspur, ace computer salesperson, closed a $500,000 sale on December 15, just in time to count it toward his annual bonus. How did he do it? Well, for one thing he gave the customer 180 days to pay. The income statement will recognize Hotspur's sale in December, even though cash will not arrive until June. But a financial analyst tracking cash flows would concentrate on the latter event.

The accountant takes care of the timing difference by adding $500,000 to accounts receivable in December, then reducing accounts receivable when the money arrives in June. (The total of accounts receivable is just the sum of all cash due from customers.)

You can think of the increase in accounts receivable as an investment—it's effectively a 180-day loan to the customer—and therefore a cash outflow. That investment is recovered when the customer pays. Thus financial analysts often find it convenient to calculate cash flow as follows:

December		June	
Sales	$500,000	Sales	0
Less investment in accounts receivable	–$500,000	Plus recovery of accounts receivable	+$500,000
Cash flow	0	Cash flow	$500,000

Note that this procedure gives the correct cash flow of $500,000 in June.

It is not always easy to translate accounting data back into actual dollars. If you are in doubt about what is a cash flow, simply count the dollars coming in and take away the dollars going out.

Check Point 7.1

A regional supermarket chain is deciding whether to install a tewgit machine in each of its stores. Each machine costs $250,000. Projected income per machine is as follows:

Year:	1	2	3	4	5
Sales	$250,000	$300,000	$300,000	$250,000	$250,000
Operating expenses	200,000	200,000	200,000	200,000	200,000
Depreciation	50,000	50,000	50,000	50,000	50,000
Accounting income	0	50,000	50,000	0	0

Why would the stores continue to operate a machine in years 4 and 5 if it produces no profits? What are the cash flows from investing in a machine? Assume each tewgit machine is completely depreciated and has no salvage value at the end of its 5-year life.

7.2 Discount Incremental Cash Flows

A project's present value depends on the *extra* cash flows that it produces. Forecast the firm's cash flows first if you go ahead with the project. Then forecast the cash flows if you *don't* accept the project. Take the difference and you have the extra (or *incremental*) cash flows produced by the project:

$$\text{Incremental cash flow} = \text{cash flow with project} - \text{cash flow without project} \quad (7.1)$$

EXAMPLE 7.2 *Launching a New Product*

Consider the decision by Intel to launch its Pentium III microprocessor. A successful launch could mean sales of 50 million processors a year and several billion dollars in profits.

But are these profits all incremental cash flows? Certainly not. Our with-versus-without principle reminds us that we also need to think about what the cash flows would be *without* the new processor. Intel recognized that if it went ahead with the Pentium III, demand for its older Pentium II processors would be reduced. The incremental cash flows are therefore

Cash flow with Pentium III (including lower cash flow from Pentium II processors) – Cash flow without Pentium III (with higher cash flow from Pentium II processors)

The trick in capital budgeting is to trace all the incremental flows from a proposed project. Here are some things to look out for.

INCLUDE ALL INDIRECT EFFECTS

Intel's new processor illustrates a common indirect effect. New products often damage sales of an existing product. Of course, companies frequently introduce new products anyway, usually because they believe that their existing product line is under threat from competition.

Even if you don't go ahead with a new product, there is no guarantee that sales of the existing product line will continue at their present level. Sooner or later they will decline.

Sometimes a new project will *help* the firm's existing business. Suppose that you are the financial manager of an airline that is considering opening a new short-haul route from Prince George, B.C. to Vancouver's International Airport. When considered in isolation, the new route may have a negative NPV. But once you allow for the additional business that the new route brings to your other traffic out of Vancouver, it may be a very worthwhile investment.

To forecast incremental cash flow, you must trace out all indirect effects of accepting the project.

Some capital investments have very long lives once all indirect effects are recognized. Consider the introduction of a new jet engine. Engine manufacturers often offer attractive pricing to achieve early sales, because once an engine is installed, 15 years' sales of replacement parts are almost assured. Also, since airlines prefer to reduce the number of different engines in their fleet, selling jet engines today improves sales tomorrow as well. Later sales will generate further demands for replacement parts. Thus the string of incremental effects from the first sales of a new model engine can run out 20 years or more.

FORGET SUNK COSTS

Sunk costs are like spilled milk: they are past and irreversible outflows.

Sunk costs remain the same whether or not you accept the project. Therefore, they do not affect project NPV.

Unfortunately, often managers are influenced by sunk costs. For example, in 1971 Lockheed sought a federal guarantee for a bank loan to continue development of the Tristar airplane. Lockheed and its supporters argued that it would be foolish to abandon a project on which nearly $1 billion had already been spent. This was a poor argument, however, because the $1 billion was sunk. The relevant questions were how much more needed to be invested and whether the finished product warranted the *incremental* investment.

Lockheed's supporters were not the only ones to appeal to sunk costs. Some of its critics claimed that it would be foolish to continue with a project that offered no prospect of a satisfactory return on that $1 billion. This argument too was faulty. The $1 billion was gone, and the decision to continue with the project should have depended only on the return on the incremental investment.

INCLUDE OPPORTUNITY COSTS

opportunity cost Benefit or cash flow forgone as a result of an action.

Resources are almost never free, even when no cash changes hands. For example, suppose a new manufacturing operation uses land that could otherwise be sold for $100,000. This resource is costly; by using the land you pass up the opportunity to sell it. There is no out-of-pocket cost but there is an **opportunity cost,** that is, the value of the forgone alternative use of the land.

This example prompts us to warn you against judging projects "before versus after" rather than "with versus without." A manager comparing before versus after might not assign any value to the land because the firm owns it both before and after:

Before	Take Project	After	Cash Flow, Before versus After
Firm owns land	→	Firm still owns land	0

The proper comparison, *with versus without*, is as follows:

Before	Take Project	After	Cash Flow, with Project
Firm owns land	→	Firm still owns land	0

Before	Do Not Take Project	After	Cash Flow, without Project
Firm owns land	→	Firm sells land for $100,000	$100,000

Comparing the cash flows with and without the project, we see that $100,000 is given up by undertaking the project. The original cost of purchasing the land is irrelevant—that cost is sunk.

The opportunity cost equals the cash that could be realized from selling the land now, and therefore is a relevant cash flow for project evaluation.

When the resource can be freely traded, its opportunity cost is simply the market price.[1] However, sometimes opportunity costs are difficult to estimate. Suppose that you go ahead with a project to develop Computer Nouveau, pulling your software team off their work on a new operating system that some existing customers are not-so-patiently awaiting. The exact cost of infuriating those customers may be impossible to calculate, but you'll think twice about the opportunity cost of moving the software team to Computer Nouveau.

RECOGNIZE THE INVESTMENT IN WORKING CAPITAL

net working capital Current assets minus current liabilities.

Net working capital (often referred to simply as *working capital*) is the difference between a company's short-term assets and liabilities. The principal short-term assets are cash, accounts receivable (customers' unpaid bills), and inventories of raw materials and finished goods. The principal short-term liabilities are accounts payable (bills that *you* have not paid), notes payable, and accruals (liabilities for items such as wages or taxes that have recently been incurred but have not yet been paid).

Most projects entail an additional investment in working capital. For example, before you can start production, you need to invest in inventories of raw materials. Then, when you deliver the finished product, customers may be slow to pay and accounts receivable will increase. (Remember Reggie Hotspur's computer sale, described in Example 7.1. It required a $500,000, 6-month investment in accounts receivable.) Next year, as business builds up, you may need a larger stock of raw materials and you may have even more unpaid bills.

Investments in working capital, just like investments in plant and equipment, result in cash outflows.

[1] If the value of the land to the firm were less than the market price, the firm would sell it. On the other hand, the opportunity cost of using land in a particular project cannot exceed the cost of buying an equivalent parcel to replace it.

We find that working capital is one of the most common sources of confusion in forecasting project cash flows.[2] Here are the most common mistakes:

1. *Forgetting about working capital entirely.* We hope that you never fall into that trap.
2. *Forgetting that working capital may change during the life of the project.* Imagine that you sell $100,000 of goods per year and customers pay on average 6 months late. You will therefore have $50,000 of unpaid bills. Now you increase prices by 10 percent, so that revenues increase to $110,000. If customers continue to pay 6 months late, unpaid bills increase to $55,000, and therefore, you need to make an *additional* investment in working capital of $5,000.
3. *Forgetting that working capital is recovered at the end of the project.* When the project comes to an end, inventories are run down, any unpaid bills are (you hope) paid off, and you can recover your investment in working capital. This generates a cash *inflow.*

BEWARE OF ALLOCATED OVERHEAD COSTS

We have already mentioned that the accountant's objective in gathering data is not always the same as the investment analyst's. A case in point is the allocation of overhead costs such as rent, heat, or electricity. These overhead costs may not be related to a particular project, but they must be paid for nevertheless. Therefore, when the accountant assigns costs to the firm's projects, a charge for overhead is usually made. But our principle of incremental cash flows says that in investment appraisal we should include only the *extra* expenses that would result from the project.

A project may generate extra overhead costs, but then again, it may not. We should be cautious about assuming that the accountant's allocation of overhead costs represents the *incremental* cash flow that would be incurred by accepting the project.

Check Point 7.2

A firm is considering an investment in a new manufacturing plant. The site is already owned by the company, but existing buildings would need to be demolished. Which of the following should be treated as incremental cash flows?

a. The market value of the site.
b. The market value of the existing buildings.
c. Demolition costs and site clearance.
d. The cost of a new access road put in last year.
e. Lost cash flows on other projects due to executive time spent on the new facility.
f. Future depreciation of the new plant.

7.3 Discount Nominal Cash Flows by the Nominal Cost of Capital

The distinction between nominal and real cash flows and interest rates is crucial in capital budgeting. Interest rates are usually quoted in *nominal* terms. If you invest $100 in a bank deposit offering 6 percent interest, then the bank promises to pay you $106 at the end of

[2] If you are not clear why working capital affects cash flow, look back to Chapter 2, where we gave a primer on working capital and a couple of simple examples.

the year. It makes no promises about what that $106 will buy. The real rate of interest on the bank deposit depends on inflation. If inflation is 2 percent, that $106 will buy you only 4 percent more goods at the end of the year than your $100 could buy today. The *real* rate of interest is therefore about 4 percent.[3]

If the discount rate is nominal, consistency requires that cash flows be estimated in nominal terms as well, taking into account trends in selling price, labour and materials costs, and so on. This calls for more than simply applying a single assumed inflation rate to all components of cash flow. Some costs or prices increase faster than inflation, some slower. For example, perhaps you have entered into a 5-year fixed-price contract with a supplier. No matter what happens to inflation over this period, this part of your costs is fixed in nominal terms.

Of course, there is nothing wrong with discounting real cash flows at the real interest rate, although this is not commonly done. We saw in Chapter 3 that real cash flows discounted at the real discount rate give exactly the same present values as nominal cash flows discounted at the nominal rate.

It should go without saying that you cannot mix and match real and nominal quantities. Real cash flows must be discounted at a real discount rate, nominal cash flows at a nominal rate. Discounting real cash flows at a nominal rate is a *big* mistake.

While the need to maintain consistency may seem like an obvious point, analysts sometimes forget to account for the effects of inflation when forecasting future cash flows. As a result, they end up discounting real cash flows at a nominal interest rate. This can grossly understate project values.

EXAMPLE 7.3 *Cash Flows and Inflation*

City Consulting Services is considering moving into a new office building. The cost of a 1-year lease is $8,000, but this cost will increase in future years at the annual inflation rate of 3 percent. The firm believes that it will remain in the building for 4 years. What is the present value of its rental costs if the discount rate is 10 percent?

The present value can be obtained by discounting the nominal cash flows at the 10 percent discount rate as follows:

Year	Cash Flow	Present Value at 10% Discount Rate
1	8,000	8,000/1.10 = 7,272.73
2	$8,000 \times 1.03 = 8,240$	$8,240/1.10^2 = 6,809.92$
3	$8,000 \times 1.03^2 = 8,487.20$	$8,487.20/1.10^3 = 6,376.56$
4	$8,000 \times 1.03^3 = 8,741.82$	$8,741.82/1.10^4 = 5,970.78$
		$26,429.99

[3] Remember from Chapter 3,

$$\text{Real rate of interest} \approx \text{nominal rate of interest} - \text{inflation rate}$$

The exact formula is

$$1 + \text{real rate of interest} = \frac{1 + \text{nominal rate of interest}}{1 + \text{inflation rate}} = \frac{1.06}{1.02} = 1.0392$$

Therefore, the real interest rate is .0392, or 3.92 percent.

Alternatively, the real discount rate can be calculated as 1.10/1.03 – 1 = .067961 = 6.7961%. The present value of the cash flows can also be computed by discounting the real cash flows at the real discount rate as follows:

Year	Real Cash Flow	Present Value at 6.7961% Discount Rate
1	8,000/1.03 = 7,766.99	7,766.99/1.067961 = 7,272.73
2	$8,240/1.03^2$ = 7,766.99	$7,766.99/1.067961^2$ = 6,809.92
3	$8,487.20/1.03^3$ = 7,766.99	$7,766.99/1.067961^3$ = 6,376.56
4	$8,741.82/1.03^4$ = 7,766.99	$7,766.99/1.067961^4$ = 5,970.78
		$26,429.99

Notice the real cash flow is a constant, since the lease payment increases at the rate of inflation. The present value of *each* cash flow is the same regardless of the method used to discount. The sum of the present values is also identical, of course.

Check Point 7.3

Nasty Industries is closing down an outmoded factory and throwing all of its workers out on the street. Nasty's CEO, Cruella DeLuxe, is enraged to learn that it must continue to pay for workers' health insurance for 4 years. The cost per worker next year will be $2,400 per year, but the inflation rate is 4 percent, and health costs have been increasing at three percentage points faster than inflation. What is the present value of this obligation? The (nominal) discount rate is 10 percent.

7.4 Separate Investment and Financing Decisions

When we calculate the cash flows from a project, we ignore how that project is financed. The company may decide to finance partly by debt, but even if it did, we would *neither* subtract the debt proceeds from the required investment *nor* recognize the interest and principal payments as cash outflows. Regardless of the actual financing, we should view the project as if it were all-equity financed, treating all cash outflows required for the project as coming from shareholders and all cash inflows as going to them.

We do this to separate the analysis of the investment decision from the financing decision. We first measure whether the project has a positive net present value, assuming all-equity financing. Then we can undertake a separate analysis of the financing decision. We discuss financing decisions later in the book.

7.5 Calculating Cash Flow

A project cash flow is the sum of three components: investment in fixed assets such as plant and equipment, investment in working capital, and cash flow from operations:

Total cash flow = cash flow from investment in plant and equipment (7.2)
+ cash flow from investment in working capital
+ cash flow from operations

Let's examine each of these in turn.

CAPITAL INVESTMENT

To get a project off the ground, a company will typically need to make considerable up-front investments in plant, equipment, research, marketing, and so on. For example, Gillette spent about $750 million to develop and build the production line for its Mach3 razor cartridge and an additional $300 million in its initial marketing campaign, largely before a single razor was sold. These expenditures are negative cash flows—negative because they represent a cash outflow from the firm.

Conversely, if a piece of machinery can be sold when the project winds down, the sales price (net of any taxes on the sale) represents a positive cash flow to the firm.

EXAMPLE 7.4 *Cash Flow from Investments*

Gillette's competitor, Slick, invests $800 million to develop the Mock4 razor blade. The specialized blade factory will run for 7 years, until it is replaced by more advanced technology. At that point, the machinery will be sold for scrap metal, for a price of $50 million. Taxes of $10 million will be assessed on the sale.

Therefore, the initial cash flow from investment is –$800 million, and in 7 years, the cash flow from the disinvestment in the production line will be $50 million – $10 million = $40 million.

INVESTMENT IN WORKING CAPITAL

We pointed out earlier in the chapter that when a company builds up inventories of raw materials or finished product, the company's cash is reduced; the reduction in cash reflects the firm's investment in inventories. Similarly, cash is reduced when customers are slow to pay their bills—in this case, the firm makes an investment in accounts receivable. Investment in working capital, just like investment in plant and equipment, represents a negative cash flow. On the other hand, later in the life of a project, when inventories are sold off and accounts receivable are collected, the firm's investment in working capital is reduced as it converts these assets into cash.

EXAMPLE 7.5 *Cash Flow from Investments in Working Capital*

Slick makes an initial (Year 0) investment of $10 million in inventories of plastic and steel for its blade plant. Then in Year 1 it accumulates an additional $20 million of raw materials. The total level of inventories is now $10 million + $20 million = $30 million, but the cash expenditure in Year 1 is simply the $20 million addition to inventory. The $20 million investment in additional inventory results in a cash flow of –$20 million.

Later on, say in Year 5, the company begins planning for the next-generation blade. At this point, it decides to reduce its inventory of raw material from $20 million to $15 million. This reduction in inventory investment frees up $5 million of cash, which is a positive cash flow. Therefore, the cash flows from inventory investment are –$10 million in Year 0, –$20 million in Year 1, and +$5 million in Year 5.

In general,

An *increase* in working capital implies a *negative* cash flow; a decrease implies a positive cash flow. The cash flow is measured by the *change* in working capital, not the *level* of working capital.

CASH FLOW FROM OPERATIONS

The third component of project cash flow is cash flow from operations. There are several ways to work out this component.

Method 1. Take only the items from the income statement that represent cash flows. We start with cash revenues and subtract cash expenses and taxes paid. We do not, however, subtract a charge for depreciation because depreciation is an accounting entry not a cash expense. Thus

Cash flow from operations = revenues – cash expenses – taxes paid

Method 2. Alternatively, you can start with accounting profits and add back any deductions that were made for noncash expenses such as depreciation. (Remember from our earlier discussion that you want to discount cash flows, not profits.) By this reasoning,

Cash flow from operations = net profit + depreciation

Method 3. Although the depreciation deduction is *not* a cash expense, it does affect net profits and therefore taxes paid, which *is* a cash item.[4] For example, if the firm's tax bracket is 35 percent, each additional dollar of depreciation reduces taxable income by $1. Tax payments therefore fall by $.35, and cash flow increases by the same amount. The total **depreciation tax shield** equals the product of depreciation and the tax rate:

depreciation tax shield Reduction in taxes attributable to the depreciation allowance.

Depreciation tax shield = depreciation × tax rate (7.3)

This suggests a third way to calculate cash flow from operations. First, calculate net profit *assuming* zero depreciation. This item would be (revenues – cash expenses) × (1 – tax rate). Now add back the tax shield created by depreciation. We then calculate operating cash flow as follows:

Cash flow from operations = (revenues – cash expenses) × (1 – tax rate)
+ (depreciation × tax rate)

The following example confirms that the three methods for estimating cash flow from operations all give the same answer.

EXAMPLE 7.6 *Cash Flow from Operations*

A project generates revenues of $1,000, cash expenses of $600, and depreciation charges of $200 in a particular year. The firm's tax bracket is 35 percent. Net income is calculated as follows:

	Revenues	$1,000
–	Cash expenses	600
–	Depreciation expense	200
=	Profit before tax	200
–	Tax at 35%	70
=	Net income	130

Methods 1, 2, and 3 all show that cash flow from operations is $330:

Method 1: Cash flow from operations = revenues – cash expenses – taxes
= 1,000 – 600 – 70 = 330

[4] The discussion here is general, without reference to the tax laws of any country. In Section 7.6 we will examine the treatment of depreciation in Canadian tax law.

Method 2: Cash flow from operations = net profit + depreciation
= 130 + 200 = 330

Method 3: Cash flow from operations = (revenues – cash expenses) × (1 – tax rate)
+ (depreciation × tax rate)
= (1,000 – 600) × (1 – .35) + (200 × .35) = 330

CHECK POINT 7.4

A project generates revenues of $600, expenses of $300, and depreciation charges of $200 in a particular year. The firm's tax bracket is 35 percent. Find the operating cash flow of the project using all three approaches.

In many cases, a project will seek to improve efficiency or cut costs. A new computer system may provide labour savings. A new heating system may be more energy efficient than the one it replaces. These projects also contribute to the operating cash flow of the firm—not by increasing revenue, but by reducing costs. As the next example illustrates, we calculate the addition to operating cash flow on cost-cutting projects just as we would for projects that increase revenues.

EXAMPLE 7.7 *Operating Cash Flow on Cost-Cutting Projects*

Suppose the new heating system costs $100,000 but reduces heating expenditures by $30,000 a year. The system will be depreciated straight-line over a 5-year period, so the annual depreciation charge will be $20,000. The firm's tax rate is 35 percent. We calculate the *incremental* effects on revenues, expenses, and depreciation charges as follows. Notice that the reduction in expenses increases revenues minus cash expenses.

Increase in (revenues minus expenses)	$30,000
– Additional depreciation expense	– 20,000
= Incremental profit before tax	= 10,000
– Incremental tax at 35%	– 3,500
= Change in net income	= 6,500

Therefore, the increment to operating cash flow can be calculated by *method 1* as

Increase in (revenues – cash expenses) – additional taxes
= $30,000 – $3,500
= $26,500

or by *method 2:*

Increase in net profit + additional depreciation
= $6,500 + $20,000
= $26,500

or by *method 3:*

Increase in (revenues – cash expenses) × (1 – tax rate) + (additional depreciation × tax rate)
= $30,000 × (1 – .35) + ($20,000 × .35)
= $26,500

7.6

Business Taxes in Canada and the Capital Budgeting Decision

DEPRECIATION AND CAPITAL COST ALLOWANCE

capital cost allowance The amount of write-off on depreciable assets allowed by Canada Customs and Revenue Agency (CCRA) against taxable income.

undepreciated capital cost (UCC) The balance remaining in an asset class that has not yet been depreciated in that year.

CCA tax shield Tax savings arising from the capital cost allowance charge.

While calculating profit before tax, or taxable income, the business is allowed to deduct an amount for depreciation on its depreciable assets. This deduction, called the **capital cost allowance (CCA)** in Canada, enables the business to recover the original amount invested in the asset over a period of time, free of tax. In a general sense,

$$\text{Taxable income} = \text{revenues} - \text{expenses} - \text{CCA} \qquad (7.4)$$

The CCA for each year is calculated by multiplying the balance on the asset, called the **undepreciated capital cost (UCC)**, by the appropriate tax rate.[5] Although the CCA itself is a noncash charge, it does affect cash flow to the extent that it reduces the taxes paid. This tax saving, called the **CCA tax shield**, or sometimes, the *depreciation tax shield***,** is discussed in more detail later in the chapter.

It is important to remember that although the terms depreciation and CCA are often used interchangeably, they are not necessarily the same. In fact, the depreciation figure shown in a company's income statement is often calculated in a different manner than the CCA it reports to Canada Customs and Revenue Agency (CCRA). We should note that only the CCA amount has an effect on the company's cash flows since it determines its tax bill.

THE ASSET CLASS SYSTEM

asset class Eligible depreciable assets are grouped into specified asset classes by CCRA. Each asset class has a prescribed CCA rate.

straight-line depreciation Constant depreciation for each year of the asset's accounting life.

declining balance depreciation This is computed by applying the depreciation rate to the asset balance for each year.

half-year rule Only one-half of the purchase cost of the asset is added to the asset class and used to compute CCA in the year of purchase.

All eligible depreciable assets are grouped into one of over 30 CCA **asset classes**. Each asset class has been assigned a CCA rate by CCRA. Table 7.1 provides details regarding some of the asset classes. We note that buildings are generally included in asset classes 1 and 3 and allowed a lower CCA rate. For instance, most buildings acquired after 1987 would fall in Class 1, and a firm that owned such buildings would be entitled to a CCA amount equal to 4 percent of the value of this class. At the other extreme, chinaware, cutlery, most computer software, and videotape cassettes used for rental purposes fall into asset class 12, which is allowed a 100 percent CCA rate. For most assets, CCA is calculated by applying the appropriate asset class rate against the declining asset balance (UCC amount). Intangible assets such as leasehold improvements (a Class 13 asset) or patents (a Class 14 asset) follow the **straight-line depreciation** method for computing CCA. For such assets, CCA essentially represents an annuity series.

Under the asset class system, all assets within a particular CCA class are depreciated for tax purposes as if they were a single asset. To understand how the asset class system works, suppose that you have started a business as a tourbus operator and have invested $100,000 in a new bus. This would be a Class 10 asset for which the CCA is computed using a **declining balance depreciation** method and the applicable CCA rate is 30 percent. Suppose, in the second year, you decide to expand your business and buy another bus for $100,000. To keep things simple, let us ignore the **half-year rule** for now.[6] Your CCA claims at the end of the first and second years are provided in Table 7.2.

[5] We can define UCC as the total cost of all assets in an asset class minus the accumulated CCA in that class. It is similar to the concept of "net fixed assets" under GAAP.

[6] The half-year rule will allow your firm to include one-half of the purchase cost of the asset for calculating the year's CCA in that asset class in the year the asset is purchased. This is the case regardless of what part of the year the asset is purchased. The remaining half of the purchase cost is added to the asset class in the next year.

TABLE 7.1
CCA rates and classes

The following is a partial list describing the most common capital cost allowance (CCA) classes.

Class number	Description	CCA rate
1	Most buildings made of brick, stone, or cement acquired after 1987, including their component parts such as electric wiring, lighting fixtures, plumbing, heating and cooling equipment, elevators, and escalators	4%
3	Most buildings made of brick, stone, or cement acquired before 1988, including their component parts as listed in Class 1 above	5%
7	Canoes, boats, and most other vessels, including their furniture, fittings, or equipment	15%
9	Aircraft, including furniture, fittings, or equipment attached, and their spare parts	25%
10	Automobiles (except taxis and those used for lease or rent), vans, wagons, trucks, buses, tractors, trailers, drive-in theatres, general-purpose electronic data-processing equipment (e.g., personal computers) and systems software, and timber-cutting and -removing equipment	30%
12	Chinaware, cutlery, linen, uniforms, dies, jigs, moulds or lasts, computer software (except systems software), cutting or shaping parts of a machine, certain production costs associated with making a motion picture film, such as apparel or costumes, videotape cassettes, ...	100%
13	Property that is leasehold interest (the maximum CCA rate depends on the type of the leasehold and the terms of the lease)	N/A
38	Most power-operated movable equipment acquired after 1987 used for moving, excavating, placing, or compacting earth, rock, concrete, or asphalt	30%
39	Machinery and equipment acquired after 1987 that is used in Canada primarily to manufacture and process goods for sale or lease	25%
43	Manufacturing and processing machinery and equipment acquired after February 25, 1992, described in Class 39 above	30%

Source: "CCA Rates and Classes," Canada Customs and Revenue Agency website, www.ccra-adrc.gc.ca.

You would reduce your taxable income in the first year by $30,000. You begin the next year with an undepreciated balance (the *undepreciated capital cost*, *UCC*) of $70,000 to which you would add the purchase cost of the second bus of $100,000 for a total UCC amounting to $170,000. In the second year you would be entitled to a CCA deduction from your taxable income of $51,000. The UCC remaining at the end of the second year is $119,000.

Let us now introduce the half-year rule for the illustration above. As shown in Table 7.3, the CCA for asset Class 10 is $15,000 in Year 1 and $40,500 in Year 2. Taxable income is reduced by these amounts in the 2 years. In the first year, CCA is calculated on one-half of the purchase cost of the asset or .5 × $100,000 × .30. In the second year, the total CCA

TABLE 7.2
Undepreciated capital cost (UCC) and capital cost allowance (CCA) (without the half-year rule)

Year	Cost of Buses	Beginning of Year UCC	CCA	End of Year UCC
1	$100,000	$100,000	$30,000[1]	$ 70,000
2	$100,000	$170,000	$51,000	$119,000

[1] $100,000 × .30.

TABLE 7.3
Undepreciated capital cost (UCC) and capital cost allowance (CCA) (with the half-year rule)

Year	Cost of Buses	Beginning of Year UCC	CCA	End of Year UCC
1	$100,000	$100,000	$15,000	$85,000
2	$100,000	$185,000	$40,500	$144,500

of $40,500 for the asset class is calculated as follows: for the asset purchased in the first year, CCA computed on the UCC balance is $85,000 × .30, or $25,500, whereas the half-year rule applies to the second year's purchase, and the eligible CCA is .5 × $100,000 × .30, or $15,000. Notice that, for Year 3, if you do not make any further additions to this asset class, CCA will be calculated on the UCC balance of $144,500 at the applicable rate of 30 percent.

SALE OF ASSETS

A company is entitled to a CCA as long as it owns at least one asset in the asset class. When a depreciable asset is sold, the undepreciated capital cost of its asset class is reduced by either the asset's sale price or its initial cost, whichever is less. The result is called the *adjusted cost of disposal*. In the example in Table 7.3, if the bus purchased in Year 1 for $100,000 is sold in Year 3 for $80,000, the adjusted cost of disposal is $80,000 and gets deducted from the UCC of Class 10 in Year 3.

In any given year, the company may buy new assets and sell old assets from within the same asset class. In this case, we would apply the *net acquisitions rule*. That is, we would determine the total cost of all additions to an asset class and then subtract the adjusted cost of disposal of all assets in that class. If the net acquisition is positive, we would apply the half-year rule and calculate CCA as shown earlier. Continuing with our tourbus operator example, suppose in Year 3, in addition to selling the bus bought in Year 1 for $80,000, we buy another luxury coach for $150,000. Our net acquisition in Year 3 will be $150,000 – $80,000, or $70,000. To calculate CCA for Year 3, we will apply the half-year rule on $70,000. This result will be added to the CCA computed on the UCC balance of $144,500 to get the overall CCA for Year 3. If, on the other hand, the net acquisition is negative, we do not adjust for the half-year rule. Instead, we will subtract the negative net acquisition amount from the beginning UCC balance of the asset class. CCA for the year will be calculated by applying the CCA rate on this net amount.

☑ CHECK POINT 7.5

In Year 1 of your new business as a boat rental company, you have bought a motorboat for $100,000 and a sailboat for $80,000. In Year 2, you sell the motorboat for $110,000 and the sailboat for $75,000. You also buy a speedboat in Year 2 for $150,000. For CCA purposes, the boats are grouped in Class 7, which carries a 15 percent CCA rate. Calculate the CCA for years 1 and 2. What is the UCC balance at the end of Year 2?

TERMINATION OF ASSET POOL

What happens if the company disposes of its entire pool of assets in an asset class? Once again, we determine the adjusted cost of disposal (as the lower of the sale proceeds or the initial cost of this pool of assets) and subtract this amount from the undepreciated capital cost of the asset class. If this leaves a positive balance in the asset class and there are *no*

terminal loss When an asset class has a positive balance following the disposal of all assets in the class, this balance is called terminal loss. The UCC of the asset class is set to zero after a terminal loss is recognized.

other assets remaining in the class, this remaining balance is called a **terminal loss** and is deducted from taxable income. Also, the UCC then becomes zero, and the asset class ceases to generate CCA tax shields.[7]

If, on the other hand, we arrive at a negative balance after deducting the adjusted cost of disposal from the UCC of the asset class, this amount is called **recaptured depreciation** and is added back to taxable income. Once again, the undepreciated capital cost of the asset class becomes zero.

When an asset is sold for more than its initial cost, the difference between the sale price and initial cost is called a capital gain. Presently capital gains, net of any capital losses, are taxed at 50 percent of the firm's applicable marginal tax rate.

EXAMPLE 7.8 *Recaptured Depreciation*

recaptured depreciation If the sale of an asset causes a negative balance in an asset class, the amount of the negative balance is known as recaptured depreciation and is added to taxable income.

Remember our example where you bought two buses in years 1 and 2, respectively, each costing $100,000. To calculate CCA, the buses fall into Class 10 and are eligible for a 30 percent CCA rate. From Table 7.3, we saw that after taking 2 years of CCA, the undepreciated capital cost in the asset class is $144,500. Suppose you now decide to end your career as a tourbus operator and terminate the pool of assets in this class by selling both buses. You sell each bus for $120,000 for a total sale value of $240,000. Notice that because the sale price of the buses exceeds their purchase cost, they have actually appreciated in value instead of depreciating. The adjusted cost of disposal is their total purchase cost of $200,000. In such a situation, CCRA will determine that you have taken $200,000 – $144,500 = $55,500 depreciation that did not reflect the economic depreciation on the assets. Therefore, this amount ($55,500) will be "recaptured" and added to your income to calculate tax. In addition, you have a capital gain of $240,000 – $200,000 = $40,000. The capital gain will be taxed at 50 percent of your firm's applicable marginal tax rate.

Suppose that instead of selling the two buses, you sell only one bus for $120,000 and continue running your business using the other bus. Notice that because one bus still remains in the asset class, the asset pool is not terminated. The UCC of $144,500 gets reduced by the adjusted cost of disposal of $100,000[8] but is still a positive amount of $44,500 and so there is no recaptured depreciation. You have also made a capital gain on the sale of $120,000 – $100,000 = $20,000.

☑ CHECK POINT 7.6

Think again about Example 7.8. Suppose that instead of selling the two buses for $120,000 each, you sell the bus bought in Year 1 for $65,000 and the one bought in Year 2 for $70,000. What are the tax consequences of this transaction, assuming that the firm has no other Class 10 assets?

PRESENT VALUES OF CCA TAX SHIELDS

Suppose we start a new asset class by buying an asset. We'll use the following notation for our subsequent discussion:

[7] We should note that if the asset pool is not completely terminated—that is, there are other assets remaining in the asset class—then a positive balance would simply become the UCC of the class and continue to generate CCA tax shields.

[8] This amount, representing the initial purchase cost, is lower than the sale price of $120,000.

C	=	capital cost of an asset acquired at the beginning of Year 1
d	=	CCA rate for the asset class to which the asset belongs
UCC_t	=	undepreciated capital cost in Year t after deducting CCA for the year
T_c	=	the firm's tax rate
r	=	discount rate
S	=	salvage amount from the sale of the asset at the end of Year t

To properly evaluate a project and calculate its present value, we need to compute the present value of the CCA tax shields accruing from capital investments in the project. As we show later in our discussion of Example 7.9, an asset can continue to generate CCA tax shields for the firm even after it is sold, provided there are other assets remaining in its class and the total UCC balance for the asset class is positive. This suggests that the CCA tax shield from investing in an asset can continue in perpetuity, since we are essentially deducting a fraction of the remaining UCC balance over an infinite period. Equation 1 below can be used to compute the present value of a perpetual tax shield.[9]

$$\frac{CdT_c}{r+d}\left[\frac{1+.5r}{1+r}\right] \tag{1}$$

Let us now introduce a residual or salvage value arising from the sale of an asset into the discussion. We would deduct this salvage value from the UCC of the asset class, and thereby reduce the CCA deductions and CCA tax shields for later years. Let us assume that other assets remain in the asset class and the total UCC exceeds the salvage value.[10] If the asset is sold at the end of Year t for a salvage amount, S, then the total tax shield lost is a perpetuity with a present value at the end of Year t of

$$\frac{SdT_c}{d+r}$$

When we discount this back to the present, we get the present value of lost CCA tax shields due to salvage value:

$$\left[\frac{SdT_c}{d+r}\right]\left[\frac{1}{(1+r)^t}\right] \tag{2}$$

Combining equations 1 and 2 enables us to provide a general formula for the present value of the CCA tax shield:

Present value of CCA tax shield =

$$\left[\frac{CdT_c}{r+d}\right]\left[\frac{1+.5r}{1+r}\right]-\left[\frac{SdT_c}{d+r}\right]\left[\frac{1}{(1+r)^t}\right] \tag{3}$$

We can now look at an example on how to get the present value of the tax shields. Keep in mind that we are looking for incremental changes in CCA and UCC that arise because of the purchase (or sale) of assets for the project. From Chapter 2, we know that the combined federal and provincial tax rate varies from province to province. Also, the tax rate could be different depending on whether the firm is a large corporation or a small business, and also, whether it is in manufacturing and processing or some other industry. To illustrate the calculation of taxes, we will assume that the company pays a total of 35 percent of its taxable income to the federal and provincial government.

[9] A detailed discussion of how this equation is obtained follows in Appendix 7A.

[10] Notice that, otherwise, we may have to consider recapture of CCA and capital gains or terminal losses.

EXAMPLE 7.9 *PV of CCA Tax Shields*

Suppose that in Year 1 you buy heavy equipment for your factory for an amount C of \$250,000. The equipment belongs to asset class 39 with a CCA rate $d = .25$. You intend to sell the equipment in Year 8 for a salvage value, *S*, of \$8,000. At the time of sale, you still anticipate having other assets in the class and a UCC that exceeds the salvage value of the asset, so you will not have to deal with recaptured CCA or a terminal loss. Your tax rate is 35 percent and your discount rate is 12 percent. You want to know the present value of the incremental tax shields generated from owning and eventually selling the asset.

To calculate the present value of such incremental tax shields, we would have to reduce the present value of CCA tax shields by the present value of the tax shields lost due to the sale of the asset in Year 8.

$$\text{PV of CCA tax shields} = \begin{array}{l}\text{PV of perpetual tax}\\\text{shield on asset acquired}\\\text{in Year 1}\end{array} - \begin{array}{l}\text{PV of perpetual tax}\\\text{shield on salvage}\\\text{value in Year 8}\end{array}$$

$$= \left[\frac{CdT_c}{r+d}\right]\left[\frac{1+.5r}{1+r}\right] - \left[\frac{SdT_c}{d+r}\right]\left[\frac{1}{(1+r)^t}\right]$$

$$= \left[\frac{250{,}000 \times .25 \times .35}{.12+.25}\right]\left[\frac{1+(.5 \times .12)}{1+.12}\right]$$

$$- \left[\frac{8{,}000 \times .25 \times .35}{.25+.12}\right]\left[\frac{1}{(1+.12)^8}\right]$$

$$= \left[\frac{21{,}875}{.37}\right]\left[\frac{1.06}{1.12}\right] - \left[\frac{700}{.37}\right]\left[\frac{1}{2.48}\right]$$

$$= 55{,}954 - 763 = 55{,}191$$

From Table 7.4, note that the UCC generated by the machinery after 8 years of CCA tax shields is \$29,200. Even *after* selling the equipment in Year 8, you will continue to depreciate \$29,200 – \$8,000 = \$21,200 over future years. This example shows us a unique feature of Canadian tax law, that it is possible for an asset to generate CCA tax shields for the firm even after it is sold. Notice that two basic conditions have to be met for this to happen: (1) there are other assets remaining in its class and (2) the proceeds from disposing of any such assets are less than the total UCC for the asset class.

CHECK POINT 7.7

You are evaluating a project that requires an investment of \$5,000 and generates revenues of \$3,000 and expenses of \$1,500. CCA on the project will be based on a declining balance system with an applicable rate of 15 percent. The project will last for 5 years, at which time the machinery will be worthless and the firm will no longer produce cash flows. The firm's tax bracket is 35 percent.

a. Find the relevant cash flows for the first 5 years.
b. Assume that depreciation is on a straight-line basis over 5 years. What is the yearly depreciation charge? How would this change the cash flows for the first 5 years?

TABLE 7.4
CCA and UCC generated by the heavy equipment till Year 8

Year	1	2	3	4	5	6	7	8
C	250,000							
CCA	31,250	54,688	41,016	30,762	23,071	17,303	12,978	9,732
UCC	218,750	164,062	123,046	92,284	69,213	51,910	38,932	29,200

7.7 Example: Blooper Industries

Now that we have examined many of the pieces of a cash-flow analysis, let's try to put them together into a coherent whole. As the newly appointed financial manager of Blooper Industries, you are about to analyze a proposal for mining and selling a small deposit of high-grade magnoosium ore.[11] You are given the forecasts shown in Table 7.5. We will walk through the lines in the table.

Capital Investment (line 1). The project requires an investment of $10 million in mining machinery. At the end of 5 years the machinery has no further value. The machinery falls into asset class 38, which has a CCA rate of 30 percent. The company owns other assets that also fall into this asset class. These other assets will remain in the asset class even after the magnoosium project ceases to exist after 5 years.

Working Capital (lines 2 and 3). Line 2 shows the level of working capital. As the project gears up in the early years, working capital increases, but later in the project's life, the investment in working capital is recovered.

Line 3 shows the *change* in working capital from year to year. Notice that in years 1 to 4 the change is positive; in these years the project requires a continuing investment in

TABLE 7.5
Financial projections for Blooper's magnoosium mine (figures in thousands of dollars)

Year:	0	1	2	3	4	5	6
1. Capital investment	10,000						
2. Working capital	1,500	4,075	4,279	4,493	4,717	3,039	0
3. Change in working capital	1,500	2,575	204	214	225	–1,678	–3,039
4. Revenues		15,000	15,750	16,538	17,364	18,233	
5. Expenses		10,000	10,500	11,025	11,576	12,155	
6. CCA of mining equipment (asset class 38, d = 30%)		1,500[1]	2,550	1,785	1,250	875	612 …
7. Pretax profit		3,500	2,700	3,728	4,538	5,203	
8. Tax (35%)		1,225	945	1,305	1,588	1,821	
9. Profit after tax		2,275	1,755	2,423	2,950	3,382	

[1]In the first year, CCA is computed using the half-year rule.
Note: Some entries are subject to rounding error.

[11] Readers have inquired whether magnoosium is a real substance. Here, now, are the facts. Magnoosium was created in the early days of TV, when a splendid-sounding announcer closed a variety show by saying, "This program has been brought to you by Blooper Industries, proud producer of aleemium, magnoosium, and stool." We forget the company, but the blooper really happened.

working capital. Starting in Year 5 the change is negative; there is a disinvestment as working capital is recovered.

Revenues (line 4). The company expects to be able to sell 750,000 kilograms of magnoosium a year at a price of $20 a kilogram in Year 1. That points to initial revenues of 750,000 × 20 = $15,000,000. But be careful: inflation is running at about 5 percent a year. If magnoosium prices keep pace with inflation, you should up your forecast of the second-year revenues by 5 percent. Third-year revenues should increase by a further 5 percent, and so on. Line 4 in Table 7.5 shows revenues rising in line with inflation.

The sales forecasts in Table 7.5 are cut off after 5 years. That makes sense if the ore deposit will run out at that time. But if Blooper could make sales for Year 6, you should include them in your forecasts. We have sometimes encountered financial managers who assume a project life of (say) 5 years, even when they confidently expect revenues for 10 years or more. When asked the reason, they explain that forecasting beyond 5 years is too hazardous. We sympathize, but you just have to do your best. Do not arbitrarily truncate a project's life.

Expenses (line 5). We assume that the expenses of mining and refining also increase in line with inflation at 5 percent per year.

CCA (line 6). Mining equipment falls in asset class 38, which has a CCA rate of 30 percent. We compute CCA using the declining balance method which is prescribed by CCRA for this asset class. Notice from Table 7.6, that even though the magnoosium mine stops producing in Year 5, an undepreciated capital cost (UCC) balance of $2.04 million remains at the end of Year 5. This suggests that the initial investment has not been completely depreciated. In fact, computing with the declining balance system will continue to provide smaller CCA values each year over an infinite period. To get around this problem, we will use a method of computing the present value of the CCA tax shields under the declining balance system we described with equation 3, page 219. Even after the magnoosium mine shuts its operations in Year 5, we assume that the company has other assets in Class 38 and that the asset pool will not be terminated.

Pretax Profits (line 7). Profit after depreciation equals (revenues – expenses – CCA).

Tax (line 8). Company taxes are 35 percent of pretax profits. For example, in Year 1,

$$\text{Tax} = .35 \times 3{,}500 = 1{,}225, \text{ or } \$1{,}225{,}000$$

Profit after Tax (line 9). Profit after tax is simply equal to pretax profit less taxes.

TABLE 7.6
Computation of CCA and UCC balances for Blooper's magnoosium mine (figures in thousands of dollars)

Year	Capital Investment UCC	CCA	End of Year (UCC)
1	10,000	1,500	8,500
2	8,500	2,550	5,950
3	5,950	1,785	4,165
4	4,165	1,250	2,915
5	2,915	875	2,040
6	2,040	612	1,428

CALCULATING BLOOPER'S PROJECT CASH FLOWS

Table 7.5 provides most of the information you need to figure out the cash flows on the magnoosium project. These cash flows are the sum of three broad components: investment in plant and equipment, investment in working capital, and cash flows from operations. In turn, cash flows from operations comprise (1) operating cash flows excluding depreciation (CCA) and (2) the CCA tax shield.

Cash flow from investment in plant and equipment
+ Cash flow from investment in working capital
+ Cash flow from operations, including
  - Operating cash flows
  - CCA tax shield
= Total project cash flows

Table 7.7 provides calculations for yearly operating cash flows excluding CCA tax shields, Table 7.8 sets out the project cash flows excluding the CCA tax shield, and Table 7.9 provides details by year regarding the CCA tax shield. First, let's see where these figures come from.

Capital Investment. Investment in plant and equipment is taken from line 1 of Table 7.5. Blooper's initial investment is a negative cash flow of –$10 million shown in line 1 of Table 7.8.

Investment in Working Capital. We've seen that investment in working capital, just like investment in plant and equipment, produces a negative cash flow. For instance, when the company builds up inventories of refined magnoosium, the company's cash is reduced, or when customers are slow to pay their bills, cash is reduced. An increase in working capital implies a negative cash flow; a decrease implies a positive cash flow.

The numbers required for these calculations come from lines 2 and 3 of Table 7.5. Line 2 shows the amount or level of working capital whereas line 3 shows the change in working capital. Notice the cash flow is measured by the *change* in working capital not the level of working capital. For instance, from Table 7.5, line 2, we see that Blooper makes an initial (Year 0) investment of $1,500,000 in working capital, which goes up to $4,075,000 in Year 1. This *total* level of working capital in Year 1 is arrived at by an additional investment in working capital of $2,575,000 in Year 1 to Year 0's investment, that is, $1,500,000 + $2,575,000 = $4,075,000. The additional investment in Year 1 of $2,575,000 is shown in line 3, Table 7.5, and results in a negative cash flow by this amount (line 2, Table 7.8).

TABLE 7.7
Operating cash flows excluding CCA tax shields for Blooper's magnoosium mine (figures in thousands of dollars)

Year:	0	1	2	3	4	5	6
Revenues		15,000	15,750	16,538	17,364	18,233	
– Expenses		10,000	10,500	11,025	11,576	12,155	
= Profit before tax		5,000	5,250	5,513	5,788	6,078	
– Tax at 35%		1,750	1,838	1,930	2,026	2,127	
= Operating cash flows (excluding CCA tax shield)		3,250	3,412	3,583	3,762	3,951	

TABLE 7.8
Cash flows for Blooper's magnoosium mine (figures in thousands of dollars)

Year:	0	1	2	3	4	5	6
1. Capital investment	–10,000						
2. Change in working capital	–1,500	–2,575	–204	–214	–225	1,678	3,039
3. Cash flows from operations (excluding CCA tax shield)		3,250	3,412	3,583	3,762	3,951	
4. Total cash flows (excluding CCA tax shield)	–11,500	675	3,208	3,369	3,537	5,629	3,039

Cash Flows from Operations. The third component of project cash flows is cash flow from operations. As we have discussed earlier, we can segregate this component into two parts: (1) operating cash flows, excluding CCA, and (2) the CCA tax shield. Let us discuss these one by one.

Operating cash flows, excluding CCA: Table 7.5 has the necessary data to calculate such operating cash flows. The details, in thousands of dollars, are provided in Table 7.7.

The cash flow amounts for Blooper's magnoosium mine are in line 3 of Table 7.8. Line 4 provides the yearly total cash flows excluding the CCA tax shield. Notice that the table does not fully capture the entire amount of cash flows for the project since the CCA tax shield is not included. Let us now examine the CCA tax shield.

CCA Tax Shield: In Section 7.6, we saw that CCA has an important effect on cash flows because it reduces taxable income and the firm's tax bill. For any year, this *tax shield* is calculated as the product of the CCA and tax rate.

$$\textbf{CCA tax shield} = \textbf{CCA} \times \textbf{tax rate} \tag{7.5}$$

For Blooper, which pays tax at a rate of 35 percent, this means that each additional dollar of CCA reduces taxable income by $1 and taxes owed by 35 cents. Table 7.9 provides details regarding the CCA tax shields for Blooper. Notice that both the CCA and the CCA tax shield increase between years 1 and 2, but thereafter, are represented by a series of declining balances. This is because in Year 1 when Blooper purchases the machine, the half-year rule applies, but all subsequent CCA tax shield calculations use a declining balance system. Also, CCA tax shields from the project will continue to be generated beyond Year 5, assuming that Blooper will have other assets in Class 38 after the magnoosium mine is shut down.

TABLE 7.9
Computation by year of CCA tax shields for Blooper's magnoosium mine (figures in thousands of dollars)

Year:	0	1	2	3	4	5	6
CCA		1,500	2,550	1,785	1,250	875	612
× Tax Rate		.35	.35	.35	.35	.35	.35
= CCA Tax Shield		525	893	625	438	306	214

CALCULATING THE NPV OF BLOOPER'S PROJECT

Table 7.10 sets out the calculations for the total present value of cash flows excluding the CCA tax shield. Assume that investors expect a return of 12 percent from investments in the capital market with the same risk as the magnoosium project. This is the opportunity cost of the shareholders' money that Blooper is proposing to invest in the project. Therefore, to calculate NPV you need to discount the cash flows at 12 percent. Remember that to calculate the present value of a cash flow in Year t you can divide the cash flow by $(1 + r)^t$ or you can multiply by a discount factor which is equal to $1/(1 + r)^t$.

The total present value in Table 7.10 is not the net present value of the project because we still have to deal with the CCA tax shield. We deal with the tax shields separately because even though the magnoosium project terminates in Year 5 and we have computed cash flows till Year 6, from Table 7.6 we have noted that there is still a UCC balance at the end of Year 6 on which CCA can be computed for future years.[12] To compute the present value of the CCA tax shield, we use equation 3 (page 219). Notice that in the example we have assumed a zero salvage value, S, at the end of the project's life.

$$\text{PV of CCA tax shield} = \left[\frac{CdT_c}{r + d}\right]\left[\frac{1 + .5r}{1 + r}\right] - \left[\frac{SdT_c}{d + r}\right] \times \left[\frac{1}{(1 + r)^t}\right]$$

$$= \left[\frac{10{,}000 \times .3 \times .35}{.12 + .3}\right]\left[\frac{1 + (.5 \times .12}{1 + .12}\right]$$

$$- \left[\frac{0 \times .3 \times .35}{.3 + .12}\right]\left[\frac{1}{(1 + .12)^6}\right]$$

$$= \$2{,}366$$

We now have all the necessary information to determine the net present value of Blooper's magnoosium project. It is the sum of the total present value excluding CCA tax shields in Table 7.10 and the present value of the CCA tax shield, that is

$$\begin{aligned} \text{NPV} &= \text{total PV excluding CCA tax shields + PV of CCA tax shield} \\ &= \$1{,}040 + \$2{,}366 = \$3{,}406 \end{aligned}$$

We see that when all cash flows are discounted and added up, the magnoosium project offers a positive net present value of about $3.4 million.

TABLE 7.10
Cash flows and total present value of Blooper's project excluding the CCA tax shield (figures in thousands of dollars)

Year:	0	1	2	3	4	5	6
Total cash flow excluding tax shields	–11,500	675	3,208	3,369	3,537	5,629	3,039
Discount factor	1.0000	.8929	.7972	.7118	.6355	.5674	.5066
Present value (excluding CCA tax shields	–11,500	603	2,557	2,398	2,248	3,194	1,540
Total present value (excluding CCA tax shields	1,040						

[12] Of course, this assumes that the firm has other assets in the asset class.

A SPREADSHEET MODEL FOR BLOOPER (FORMULA INSERTS)

	A	B	C	D	E	F	G	H
1	Year:	0	1	2	3	4	5	6
2	Capital investment	10,000						
3	Working capital	=0.15*C6+2/12*B5	=0.15*D6+2/12*C5	=0.15*E6+2/12*D5	=0.15*F6+2/12*E5	=0.15*G6+2/12*F5	=0.15*H6+2/12*G5	=0.15*I6+2/12*H5
4	Change in working capital	=B3	=C3-B3	=D3-C3	=E3-D3	=F3-E3	=G3-F3	=H3-G3
5	Revenues		15000	=C5*1.05	=D5*1.05	=E5*1.05	=F5*1.05	
6	Expenses		10000	=C6*1.05	=D6*1.05	=E6*1.05	=F6*1.05	
7	Profit before tax (excluding CCA tax shield)		=C5-C6	=D5-D6	=E5-E6	=F5-F6	=G5-G6	
8	Tax (35%)		=C7*0.35	=D7*0.35	=E7*0.35	=F7*0.35	=G7*0.35	
9	Operating cash flows (excluding CCA tax shield)		=C7-C8	=D7-D8	=E7-E8	=F7-F8	=G7-G8	
10	Salvage value							0
11	Total cash flow (excluding CCA tax shield)	=-B2-B4	=C2-C4+C9	=D2-D4+D9	=E2-E4+E9	=F2-F4+F9	=G2-G4+G9	=H2-H4+H9+H10
12	PV of cash flow (excluding CCA tax shield)	=B11/(1.12)^B1	=C11/(1.12)^C1	=D11/(1.12)^D1	=E11/(1.12)^E1	=F11/(1.12)^F1	=G11/(1.12)^G1	=H11/(1.12)^H1
13	Total present value (excluding CCA tax shield) (A)	=SUM(B12:H12)						
14	CCA		=0.5*B2*0.3	=(B2-C14)*0.3	=D14*0.7	=E14*0.7	=F14*0.7	=G14*0.7
15	CCA tax shield		=10000*0.3/2*0.35	=8500*0.3*0.35	=5950*0.3*0.35	=4165*0.3*0.35	=2916*0.3*0.35	=2041*0.3*0.35
16	PV of CCA tax shield (B)	=(B2*0.3*0.35/(0.12+0.3))*((1+(0.5*0.12))/(1+0.12))-(H10*0.3*0.35/(0.3+0.12)*(1/(1+0.12)^H1))						
17	Total net present value (A) + (B)	=(B13+B16)						

You might have guessed that discounted cash-flow analysis such as that of the Blooper case is tailor-made for spreadsheets. The worksheet directly above shows the formulas from the Excel spreadsheet that we used to generate the Blooper example. The first spreadsheet on the facing page shows the resulting values, which appear in the text in tables 7.5 through 7.10. This model assume that there is no salvage value on Blooper's equipment. We have also included a spreadsheet model in which we assume a salvage value of $1.5 million in Year 6.

The assumed values are the capital investment (cell B2), the initial level of revenues (cell C5), and expenses (cell C6). Rows 5 and 6 show that each entry for revenues and expenses equals the previous value times (1 + inflation rate), or 1.05. Row 3, which is the amount of working capital, is the sum of inventories and accounts receivable. To capture the fact that inventories tend to rise with production, we set working capital equal to .15 times the following year's expenses. Similarly, accounts receivable rise with sales, so we assumed that accounts receivable would be 1/6 times the current year's revenues. Each entry in row 3 is the sum of these two quantities.[13] Net investment in working capital (row 4) is the increase in working capital from one year to the next. Total cash flow excluding the CCA tax shield (row 11) is capital investment plus change in working capital plus profit after tax, which we have called operating cash flows. Cell H10 includes a provision for salvage value in Year 6 which is set at zero in the first spreadsheet model and $1.5 million in the second model. In row 12, we discount the cash flow amounts at a 12 percent discount rate and in cell B13 we add the present value of each cash flow to find the total present value excluding the CCA tax shield. Row 14 includes CCA values. Notice that the first year's CCA in cell C14 is computed using the half-year rule, whereas the subsequent CCA amounts follow the declining balance system. Row 15 includes the yearly CCA tax shields and row 16 incorporates the present value of the CCA tax shield. The project's net present value is provided in cell B17 as the sum of the total present value excluding the CCA tax shield (cell B13) and the present value of the CCA tax shield (cell B16).

Once the spreadsheet is up and running it is easy to do various sorts of "what if" analysis. Here are a few questions to try your hand.

Questions (Answers on page 234)

1. Suppose the firm can economize on working capital by managing inventories more efficiently. If the firm can reduce inventories from 15 percent to 10 percent of next year's cost of goods sold, what will be the effect on project NPV?
2. What happens to NPV if the inflation rate falls from 5 percent to zero and the discount rate falls from 12 percent to 7 percent? Given that the real discount rate is almost unchanged, why does project NPV increase?
3. Suppose that Blooper's mining equipment could be depreciated on a 5-year, straight-line basis. What is the present value of the depreciation tax shield? What happens to cash flow in each year and to the project NPV?

[13] For convenience we assume that Blooper pays all its bills immediately and therefore accounts payable equals zero. If it didn't, working capital would be reduced by the amount of the payables.

A SPREADSHEET MODEL FOR BLOOPER (WITHOUT SALVAGE VALUE)[1]

	A	B	C	D	E	F	G	H	I
1	Year:	0	1	2	3	4	5	6	…∞
2	Capital investment	10,000							
3	Working capital	1,500	4,075	4,279	4,493	4,717	3,039	0	
4	Change in working capital	1,500	2,575	204	214	225	–1,679	–3,039	
5	Revenues		15,000	15,750	16,538	17,364	18,233		
6	Expenses		10,000	10,500	11,025	11,576	12,155		
7	Profit before tax (excluding CCA tax shield)		5,000	5,250	5,513	5,788	6,078		
8	Tax (35%)		1,750	1,838	1,930	2,026	2,127		
9	Operating cash flows (excluding CCA tax shield)		3,250	3,412	3,583	3,762	3,950		
10	Salvage value							0	
11	Total cash flow (excluding CCA tax shield)	–11,500	675	3,208	3,369	3,538	5,629	3,039	
12	PV of cash flow (excluding CCA tax shield)	–11,500	603	2,558	2,398	2,248	3,194	1,540	
13	Total present value (excluding CCA tax shield) (A)	1,041							
14	CCA[2]		1,500	2,550	1,785	1,250	875	612	→ 0
15	CCA tax shield[2]		525	893	625	437	306	214	→ 0
16	PV of CCA tax shield[2] (B)	2,366							
17	Total net present value (A) + (B)	3,407							

Notes: [1] Some entries in this table may differ from those in tables 7.5 to 7.10 due to rounding error.
[2] The CCA and the CCA tax shield (lines 14 and 15) will continue even after Year 6. The PV of the CCA tax shield (line 16) has been calculated assuming that the CCA and the CCA tax shield will continue in perpetuity.

A SPREADSHEET MODEL FOR BLOOPER (WITH SALVAGE VALUE)[1]

	A	B	C	D	E	F	G	H	I
1	Year:	0	1	2	3	4	5	6	…∞
2	Capital investment	10,000							
3	Working capital	1,500	4,075	4,279	4,493	4,717	3,039	0	
4	Change in working capital	1,500	2,575	204	214	225	–1,679	–3,039	
5	Revenues		15,000	15,750	16,538	17,364	18,233		
6	Expenses		10,000	10,500	11,025	11,576	12,155		
7	Profit before tax (excluding CCA tax shield)		5,000	5,250	5,513	5,788	6,078		
8	Tax (35%)		1,750	1,838	1,930	2,026	2,127		
9	Operating cash flows (excluding CCA tax shield)		3,250	3,412	3,583	3,762	3,950		
10	Salvage value							1,500	
11	Total cash flow (excluding CCA tax shield)	–11,500	675	3,208	3,369	3,538	5,629	4,539	
12	PV of cash flow (excluding CCA tax shield)	–11,500	603	2,558	2,398	2,248	3,194	2,299	
13	Total present value (excluding CCA tax shield) (A)	1,801							
14	CCA[2]		1,500	2,550	1,785	1,250	875	612	→ 0
15	CCA tax shield[2]		525	893	625	437	306	214	→ 0
16	PV of CCA tax shield[2] (B)	2,176							
17	Total net present value (A) + (B)	3,977							

Notes: [1] Some entries in this table may differ from those in tables 7.5 to 7.10 due to rounding error.
[2] The CCA and the CCA tax shield (lines 14 and 15) will continue even after Year 6. The PV of the CCA tax shield (line 16) has been calculated assuming that the CCA and the CCA tax shield will continue in perpetuity.

Now let's consider a small point that often causes confusion. To calculate the present value of the first year's cash flow, we divide by $(1 + r) = 1.12$. Strictly speaking, this makes sense only if all the sales and all the costs occur exactly 365 days, 0 hours, and 0 minutes from now. But, of course, the year's sales don't all take place on the stroke of midnight December 31. However, when making capital budgeting decisions, companies are usually happy to pretend that all cash flows occur at 1-year intervals—for one reason only—simplicity. When sales forecasts are sometimes little more than intelligent guesses, it may be pointless to inquire how the sales are likely to be spread out during the year.[14]

FURTHER NOTES AND WRINKLES ARISING FROM BLOOPER'S PROJECT

Before we leave Blooper and its magnoosium project, we should cover a few extra wrinkles.

How to Deal with Salvage Value. So far, we have assumed that Blooper will not receive any salvage value from the mining equipment when the magnoosium mine is closed. But suppose Blooper forecasts that the equipment can be sold for \$1.5 million in Year 6.

You recorded the initial \$10 million investment as a negative cash flow. Now, in Year 6, you have a forecast return of \$1.5 million of that investment. That is a positive cash flow estimate, which has a present value of

$$\frac{S}{(1 + r)^6} = \frac{1.5}{(1.12)^6} = .76$$

The salvage value will also reduce future tax shields, and therefore, the present value of the CCA tax shield as follows:[15]

$$\frac{1}{(1 + r)^6} \times \frac{SdT_c}{(r + d)} = \frac{1}{(1.12)^6} \times \frac{1.5 \times .3 \times .35}{(.12 + .3)} = .19$$

So we see that while the sale of the mining equipment increases the net present value of the project by \$760,000, the present value of the lost tax shield from the salvage will reduce net present value by \$190,000. Overall, net present value of the project will increase by \$760,000 – \$190,000 = \$570,000.

A Further Note on CCA. We warned you earlier not to assume that all cash flows are likely to increase with inflation. The CCA tax shield is a case in point because CCRA lets companies depreciate only the amount of the original investment. For example, if you go back to CCRA to explain that inflation mushroomed since you made the investment and you should be allowed to depreciate more, CCRA won't listen. The *nominal* amount of CCA is fixed, and therefore, the higher the rate of inflation, the lower the *real* value of the CCA that you can claim.

[14] Financial managers sometimes assume cash flows arrive in the middle of the calendar year, that is, the end of June. This also makes NPV a midyear number. If you are standing at the start of the year, the NPV must be discounted for a further half-year. To do this, divide the midyear NPV by the square root of (1 + r).

This midyear convention is roughly equivalent to assuming cash flows are distributed evenly throughout the year. This is a bad assumption for some industries. In retailing, for example, most of the cash flow comes late in the year as the holiday season approaches.

[15] Remember that we have assumed that Blooper has other machines in this asset class so the sale will not close out the asset class.

7.8 Summary

1. **How should the cash flows properly attributable to a proposed new project be calculated?**

   Here is a checklist to bear in mind when forecasting a project's cash flows:

   - Discount cash flows, not profits.
   - Estimate the project's incremental cash flows—that is, the difference between the cash flows *with* the project and those *without* the project.
   - Include all indirect effects of the project, such as its impact on the sales of the firm's other products.
   - Forget sunk costs.
   - Include **opportunity costs**, such as the value of land which you could otherwise sell.
   - Beware of allocated overhead charges for heat, light, and so on. These may not reflect the incremental effects of the project on these costs.
   - Remember the investment in working capital. As sales increase, the firm may need to make additional investments in working capital and, as the project finally comes to an end, it will recover these investments.
   - Do not include debt interest or the cost of repaying a loan. When calculating NPV, assume that the project is financed entirely by the shareholders and that they receive all the cash flows. This isolates the investment decision from the financing decision.

2. **How can the cash flows of a project be computed from standard financial statements?**

   Project cash flow does not equal profit. You must allow for changes in working capital as well as noncash expenses such as depreciation. Also, if you use a nominal cost of capital, consistency requires that you forecast *nominal* cash flows—that is, cash flows that recognize the effect of inflation.

3. **How is the company's tax bill affected by capital cost allowance (CCA) and how does this affect project value?**

   CCA is not a cash flow. However, because CCA reduces taxable income, it reduces taxes. This tax reduction is called the **CCA tax shield**. For computing tax depreciation in Canada, assets are assigned into different **asset classes**, which have specified CCA rates. Most asset classes follow a declining balance system for computing CCA, and therefore, most assets continue to generate CCA tax shields over an infinite time frame. Because of this, we find the present value of operating cash flows separately from the present value of the CCA tax shields to determine the net present value of a project.

4. **How do changes in working capital affect project cash flows?**

   Increases in **net working capital**, such as accounts receivable or inventory, are investments, and therefore, use cash. That is, they reduce the net cash flow provided by the project in that period. When working capital is run down, cash is freed up, so cash flow increases.

Related Web Links

www-ec.njit.edu/~mathis/interactive/FCCalcBase4.html A net present value calculator from Professor Roswell Mathis

www.4pm.com/articles/palette.html Try the online demonstration here to see how good business judgment is used to formulate cash-flow projections

www.irs.ustreas.gov/prod/bus_info/index.html Tax rules affecting project cash flows can be found here

Key Terms

opportunity cost	207
net working capital	208
depreciation tax shield	213
capital cost allowance (CCA)	215
undepreciated capital cost (UCC)	215
CCA tax shield	215
asset class	215
straight-line depreciation	215
declining balance depreciation	215
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terminal loss	218
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Questions and Problems

*Answers in Appendix B

Basic

1. **Cash Flows.** A new project will generate sales of $74 million, costs of $42 million, and depreciation expense of $10 million in the coming year. The firm's tax rate is 35 percent. Calculate cash flows for the year using all three methods discussed in the chapter and confirm that they are equal.

2. **Cash Flows.** Canyon Tours showed the following components of working capital last year:

	Beginning	End of Year
Accounts receivable	$24,000	$22,500
Inventory	12,000	13,000
Accounts payable	14,500	16,500

   a. What was the change in net working capital during the year?
   b. If sales were $36,000 and costs were $24,000, what was cash flow for the year? Ignore taxes.

*3. **Cash Flows.** Tubby Toys estimates that its new line of rubber ducks will generate sales of $7 million, operating costs of $4 million, and a depreciation expense of $1 million. If the tax rate is 40 percent, what is the firm's operating cash flow? Show that you get the same answer using all three methods to calculate operating cash flow.

4. **Cash Flows.** We've emphasized that the firm should pay attention only to cash flows when assessing the net present value of proposed projects. Depreciation is a noncash expense. Why then does it matter whether we assume straight-line depreciation or declining balance CCA depreciation when we assess project NPV?

*5. **Proper Cash Flows.** Quick Computing currently sells 10 million computer chips each year at a price of $20 per chip. It is about to introduce a new chip, and it forecasts annual sales of 12 million of these improved chips at a price of $25 each. However, demand for the old chip will decrease, and sales of the old chip are expected to fall to 3 million per year. The old chip costs $6 each to manufacture, and the new ones will cost $8 each. What is the proper cash flow to use to evaluate the present value of the introduction of the new chip?

*6. **Calculating Net Income.** The owner of a bicycle repair shop forecasts revenues of $160,000 a year. Variable costs will be $45,000, and rental costs for the shop are $35,000 a year. Depreciation on the repair tools will be $10,000. Prepare an income statement for the shop based on these estimates. The tax rate is 35 percent.

7. **Cash Flows.** Calculate the operating cash flow for the repair shop in the previous problem using all three methods suggested in the chapter: (a) net income plus depreciation; (b) cash inflow/cash outflow analysis; and (c) the depreciation tax shield approach. Confirm that all three approaches result in the same value for cash flow.

*8. **Cash Flows and Working Capital.** A house painting business had revenues of $16,000 and expenses of $9,000. There were no depreciation expenses. However, the business reported the following changes in various components of working capital:

	Beginning	End
Accounts receivable	$1,200	$4,500
Accounts payable	600	200

   Calculate net cash flow for the business for this period.

9. Incremental Cash Flows. A corporation donates a valuable painting from its private collection to an art museum. Which of the following are incremental cash flows associated with the donation?

   a. The price the firm paid for the painting.
   b. The current market value of the painting.
   c. The deduction from income that it declares for its charitable gift.
   d. The reduction in taxes due to its declared tax deduction.

*10. **Operating Cash Flows.** Laurel's Lawn Care, Ltd., has a new mower line that can generate revenues of $120,000 per year. Direct production costs are $40,000 and the fixed costs of maintaining the lawn mower factory are $15,000 a year. The factory originally cost $1 million and is included in an asset class with a CCA rate of 5 percent. Calculate the operating cash flows of the project for the next 6 years if the firm's tax bracket is 35 percent.

Practice

*11. **Operating Cash Flows.** Talia's Tutus bought a new sewing machine for $40,000 that will be depreciated in asset class 39 which has a CCA rate of 30 percent. This firm's tax bracket is 35 percent.

a. Find the CCA amount each year for the next 3 years.

b. If the sewing machine is sold after 3 years for $20,000, what will be the after-tax proceeds on the sale if the firm's tax bracket is 35 percent? Assume that Talia's Tutus has other assets in class 39.

c. Now rework your calculations assuming that Talia's Tutus has no other assets in Class 39 and the asset class will be terminated upon the sale of the sewing machine in Year 3.

12. **Proper Cash Flows.** Conference Services Inc. has leased a large office building for $4 million per year. The building is larger than the company needs: two of the building's eight storeys are almost empty. A manager wants to expand one of her projects, but this will require using one of the empty floors. In calculating the net present value of the proposed expansion, upper management allocates one-eighth of $4 million of building rental costs (i.e., $.5 million) to the project expansion, reasoning that the project will use one-eighth of the building's capacity.

a. Is this a reasonable procedure for the purposes of calculating NPV?

b. Can you suggest a better way to assess the cost of the office space used by the project?

13. **Cash Flows and Working Capital.** A firm had net income last year of $1.2 million. Its depreciation expenses were $.5 million, and its total cash flow was $1.2 million. What happened to net working capital during the year?

14. **Cash Flows and Working Capital.** The only capital investment required for a small project is investment in inventory. Profits this year were $10,000, and inventory increased from $4,000 to $5,000. What was the cash flow from the project?

15. **Cash Flows and Working Capital.** A firm's balance sheets for year-ends 2000 and 2001 contain the following data. What happened to investment in net working capital during 2001? All items are in millions of dollars.

	Dec. 31, 2000	**Dec. 31, 2001**
Accounts receivable	32	35
Inventories	25	30
Accounts payable	12	25

*16. **Salvage Value.** Quick Computing (from problem 5) installed its previous generation of computer chip manufacturing equipment 3 years ago. Some of that older equipment will become unnecessary when the company goes into production of its new product. The obsolete equipment, which originally cost $40 million, has been depreciated straight-line over an assumed tax life of 5 years, but it can now be sold for $18 million. The firm's tax rate is 35 percent. What is the after-tax cash flow from the sale of the equipment?

17. **Salvage Value.** Your firm purchased machinery for $10 million. The machinery falls into asset class 38, which has a CCA rate of 25 percent. The project will end after 5 years. If the equipment can be sold for $4 million at the completion of the project and your firm's tax rate is 35 percent, what is the after-tax cash flow from the sale of the machinery? Assume that the firm has no other assets in Class 38 and the asset class will be terminated upon the sale of the machinery.

*18. **CCA, Depreciation, and Project Value.** Bottoms Up Diaper Service is considering the purchase of a new industrial washer. It can purchase the washer for $6,000 and sell its old washer for $2,000. The new washer will last for 6 years and save $1,500 a year in expenses. If the old washer is retained, it will also last for 6 more years after which it will have to be junked. The washers fall into an asset class with a CCA rate of 30 percent. Bottoms Up owns other washing machines that also fall into this asset class. The opportunity cost of capital is 15 percent and the firm's tax rate is 40 percent.

a. If the salvage value of the washer is expected to be zero at the end of its 6-year life, what are the cash flows of the project in years 0 to 6?
b. What is the project NPV?
c. What will the NPV and IRR be if the firm uses straight-line depreciation with a 6-year tax life?

19. **Equivalent Annual Cost.** What is the equivalent annual cost of the washer in the previous problem if the firm uses straight-line depreciation?

20. **Cash Flows and NPV.** Johnny's Lunches is considering purchasing a new, energy-efficient grill. The grill will cost $20,000 and will be depreciated in an asset class which carries a CCA rate of 30 percent. It will be sold for scrap metal after 3 years for $5,000. The grill will have no effect on revenues but will save Johnny's $10,000 in energy expenses. The firm has other assets in this asset class. The tax rate is 35 percent.

a. What are the operating cash flows in years 1 to 3?
b. What are total cash flows in years 1 to 3?
c. If the discount rate is 12 percent, should the grill be purchased?

*21. **Project Evaluation.** Revenues generated by a new fad product are forecast as follows:

Year	Revenues
1	$40,000
2	30,000
3	20,000
4	10,000
Thereafter	0

Expenses are expected to be 40 percent of revenues, and working capital required in each year is expected to be 20 percent of revenues in the following year. The product requires an immediate investment of $50,000 in plant and equipment.

a. What is the initial investment in the product? Remember working capital.
b. If the plant and equipment are in an asset class which has a CCA rate of 25 percent, and the firm's tax rate is 40 percent, what are the project cash flows in each year?
c. If the opportunity cost of capital is 10 percent, what is the project NPV?

22. **Buy versus Lease.** You can buy a car for $25,000 and sell it in 5 years for $5,000. Or you can lease the car for 5 years for $5,000 a year. The discount rate is 10 percent per year.

a. Which option do you prefer?
b. What is the maximum amount you should be willing to pay to lease rather than buy the car?

*23. **Project Evaluation.** Kinky Copies may buy a high-volume copier. The machine costs $100,000 and will be depreciated straight-line over 5 years to a salvage value of $20,000. Kinky anticipates that the machine can be sold in 5 years for $30,000. The machine will save $20,000 a year in labour costs but will require an increase in working capital, mainly paper supplies, of $10,000. The firm's marginal tax rate is 35 percent. Ignore the CCA system and assume that the straight-line depreciation method adopted by Kinky Copies will suffice for tax purposes. Should Kinky buy the machine? The discount rate is 8 percent.

24. **Project Evaluation.** Fireplaces Etc. is about to launch a new range of wood stoves, priced at $110 per unit. The unit cost of the wood stoves is $65. The firm expects to sell the wood stoves over the next

5 years. The venture will require an initial investment in plant and equipment of $25,000. Assume that the investment will be in an asset class with a CCA rate of 15 percent. At the end of 5 years, the plant and equipment will have a zero salvage value but Fireplaces Etc. will continue to have other assets in this asset class. Sales projections for the wood stoves are as follows:

Year	Unit Sales
1	300
2	350
3	400
4	500
5	500

The net working capital requirement (including the initial working capital needed in Year 0) is expected to be 20 percent of the following year's sales. The firm's tax rate is 35 percent. Using a discount rate of 15 percent, calculate the net present value of the project.

*25. **Project Evaluation.** Blooper Industries must replace its magnoosium purification system. Quick & Dirty Systems sells a relatively cheap purification system for $10 million. The system will last 5 years. Do-It-Right sells a sturdier but more expensive system for $12 million; it will last for 8 years. Both systems entail $1 million in operating costs; both will be depreciated in an asset class which has a CCA rate of 30 percent; neither will have any salvage value at the end of its life. The firm's tax rate is 35 percent, and the discount rate is 12 percent. Which system should Blooper install?

26. **Project Evaluation.** The following table presents sales forecasts for Golden Gelt Giftware. The unit price is $40. The unit cost of the giftware is $25.

Year	Unit Sales
1	22,000
2	30,000
3	14,000
4	5,000
Thereafter	0

It is expected that net working capital will amount to 25 percent of sales in the following year. For example, the store will need an initial (Year 0) investment in working capital of $.25 \times 22{,}000 \times \$40 = \$220{,}000$. Plant and equipment necessary to establish the giftware business will require an additional investment of $200,000. This investment will be depreciated in an asset class with a CCA rate of 25 percent. We will assume that the firm has other assets in this asset class. After 4 years, the equipment will have an economic and book value of zero. The firm's tax rate is 35 percent. The discount rate is 20 percent. What is the net present value of the project?

*27. **Project Evaluation.** Ilana Industries, Inc., needs a new lathe. It can buy a new high-speed lathe for $1 million. The lathe will cost $35,000 to run, will save the firm $125,000 in labour costs, and will be useful for 10 years. Suppose that for tax purposes, the lathe will be in an asset class with a CCA rate of 25 percent. Ilana has many other assets in this asset class. The lathe is expected to have a 10-year life with a salvage value of $100,000. The actual market value of the lathe at that time will also be $100,000. The discount rate is 10 percent and the corporate tax rate is 35 percent. What is the NPV of buying the new lathe?

CHALLENGE

28. **Project Evaluation.** The efficiency gains resulting from a just-in-time inventory management system will allow a firm to reduce its level of inventories permanently by $250,000. What is the most the firm should be willing to pay for installing the system?

*29. **Project Evaluation.** Better Mousetraps has developed a new trap. It can go into production for an initial investment in equipment of $6 million. The equipment will be depreciated straight-line over 5 years to a value of zero, but in fact it can be sold after 5 years for $500,000. The firm believes that working capital at each date must be maintained at a level of 10 percent of next year's forecast sales. The firm estimates production costs equal to $1.50 per trap and believes that the traps can be sold for $4 each. Sales forecasts are given in the following table. The project will come to an end in 5 years, when the trap becomes technologically obsolete. The firm's tax bracket is 35 percent, and the required rate of return on the project is 12 percent. What is project NPV?

Year:	**0**	**1**	**2**	**3**	**4**	**5**	**Thereafter**
Sales (millions of traps)	0	.5	.6	1.0	1.0	.6	0

30. **Working Capital Management.** Return to the previous problem. Suppose the firm can cut its requirements for working capital in half by using better inventory control systems. By how much will this increase project NPV?

*31. **Project Evaluation.** PC Shopping Network may upgrade its modem pool. It last upgraded 2 years ago, when it spent $115 million on equipment with an assumed life of 5 years and an assumed salvage value of $15 million for tax purposes. The firm uses straight-line depreciation. The old equipment can be sold today for $80 million. A new modem pool can be installed today for $150 million. This will have a 3-year life, and will be depreciated to zero using straight-line depreciation. The new equipment will enable the firm to increase sales by $25 million per year and decrease operating costs by $10 million per year. At the end of 3 years, the new equipment will be worthless. Assume the firm's tax rate is 35 percent and the discount rate for projects of this sort is 12 percent.

a. What is the net cash flow at time 0 if the old equipment is replaced?
b. What are the incremental cash flows in Years 1, 2, and 3?
c. What are the NPV and IRR of the replacement project?
d. Now ignore straight-line depreciation and assume that both new and old equipment are in an asset class with a CCA rate of 30 percent. PC Shopping Network has other assets in this asset class. What is the NPV of the replacement project? For this part, assume that the new equipment will have a salvage value of $30 million at the end of 3 years.

Solutions to Excel Spreadsheet Model Questions

1.

Year:	0	1	2	3	4	5	6	…∞
Capital investment	10,000							
Working capital	1,000	3,550	3,728	3,914	4,110	3,039	0	
Change in working capital	1,000	2,550	178	186	196	–1,071	–3,039	
Revenues		15,000	15,750	16,538	17,364	18,233		
Expenses		10,000	10,500	11,025	11,576	12,155		
Profit before tax (excluding CCA tax shield)		5,000	5,250	5,513	5,788	6,078		
Tax (35%)		1,750	1,838	1,929	2,026	2,127		
Operating cash flows (excluding CCA tax shield)		3,250	3,413	3,583	3,762	3,950		
Salvage value							0	
Total cash flow (excluding CCA tax shield)	–11,000	700	3,235	3,397	3,567	5,021	3,039	
PV of cash flow (excluding CCA tax shield)	–11,000	625	2,579	2,418	2,267	2,849	1,540	
Present value (excluding CCA tax shield) (A)	1,277							
CCA[1]		1,500	2,550	1,785	1,250	875	612	→ 0
CCA tax shield[1]		525	893	625	437	306	214	→ 0
PV of CCA tax shield[1] (B)	2,366							
Net present value (A) + (B)	3,643							

Notes: [1] The CCA and the CCA tax shield will continue even after Year 6. The PV of the CCA tax shield has been calculated assuming that the CCA and the CCA tax shield will continue in perpetuity.

2.

Year:	0	1	2	3	4	5	6	…∞
Capital investment	10,000							
Working capital	1,500	4,000	4,000	4,000	4,000	2,500	0	
Change in working capital	1,500	2,500	0	0	0	–1,500	–2,500	
Revenues		15,000	15,000	15,000	15,000	15,000		
Expenses		10,000	10,000	10,000	10,000	10,000		
Profit before tax (excluding CCA tax shield)		5,000	5,000	5,000	5,000	5,000		
Tax (35%)		1,750	1,750	1,750	1,750	1,750		
Operating cash flows (excluding CCA tax shield)		3,250	3,250	3,250	3,250	3,250		
Salvage value							0	
Total cash flow (excluding CCA tax shield)	–11,500	750	3,250	3,250	3,250	4,750	2,500	
PV of cash flow (excluding CCA tax shield)	–11,500	701	2,839	2,653	2,479	3,387	1,666	
Present value (excluding CCA tax shield) (A)	2,225							
CCA[1]		1,500	2,550	1,785	1,250	875	612	→ 0
CCA tax shield[1]		525	893	625	437	306	214	→ 0
PV of CCA tax shield[1] (B)	2,745							
Net present value (A) + (B)	4,970							

Notes: [1] The CCA and the CCA tax shield will continue even after Year 6. The PV of the CCA tax shield has been calculated assuming that the CCA and the CCA tax shield will continue in perpetuity.

Although the real discount rate is barely affected by the change in inflation, the real value of CCA and the present value of the CCA tax shield increase, which increases project NPV.

3.

Year:	0	1	2	3	4	5	6
Capital investment	10,000						
Working capital	1,500	4,075	4,279	4,493	4,717	3,039	0
Change in working capital	1,500	2,575	204	214	225	–1,679	–3,039
Revenues		15,000	15,750	16,538	17,364	18,233	
Expenses		10,000	10,500	11,015	11,576	12,155	
Profit before tax (excluding depreciation tax shield)		5,000	5,250	5,513	5,788	6,078	
Tax (35%)		1,750	1,838	1,930	2,026	2,127	
Operating cash flows (excluding depreciation tax shield)		3,250	3,412	3,583	3,762	3,950	
Salvage value							0
Total cash flow (excluding depreciation tax shield)	–11,500	675	3,209	3,369	3,538	5,629	3,039
PV of cash flow (excluding depreciation tax shield)	–11,500	603	2,558	2,398	2,248	3,194	1,540
Present value (excluding depreciation tax shield) (A)	1,041						
Depreciation		2,000	2,000	2,000	2,000	2,000	
Depreciation tax shield	0	700	700	700	700	700	0
PV of depreciation tax shield	0	625	558	498	445	396	0
Total present value of depreciation tax shield (B)	2,523						
Net present value (A) + (B)	3,564						

It is worthwhile noting that often large corporations keep two sets of books, one for shareholders and one for Canada Customs and Revenue Agency (CCRA). It is common to use straight-line depreciation on the shareholder books and the CCA system on the tax books. Only the CCA recorded in the tax books is relevant in capital budgeting.

Solutions to Check Points

7.1 Remember, discount cash flows, not profits. Each tewgit machine costs $250,000 right away; recognize that outlay, but forget accounting depreciation. Cash flows per machine are

Year:	0	1	2	3	4	5
Investment (outflow)	–250,000					
Sales		250,000	300,000	300,000	250,000	250,000
Operating expenses		–200,000	–200,000	–200,000	–200,000	–200,000
Cash flow	–250,000	+ 50,000	+100,000	+100,000	+ 50,000	+ 50,000

Each machine is forecast to generate $50,000 of cash flow in years 4 and 5. Thus it makes sense to keep operating for 5 years.

7.2 a., b. The site and buildings could have been sold or put to another use. Their values are opportunity costs, which should be treated as incremental cash outflows.

c. Demolition costs are incremental cash outflows.

d. The cost of the access road is sunk and not incremental.

e. Lost cash flows from other projects are incremental cash outflows.

f. Depreciation is not a cash expense and should not be included, except as it affects taxes. (Taxes are discussed later in this chapter.)

7.3 Actual health costs will be increasing at about 7 percent a year.

Year:	1	2	3	4
Cost per worker	$2,400	$2,568	$2,748	$2,940

The present value at 10 percent is $9,214 if the first payment is made immediately. If it is delayed a year, present value falls to $8,377.

7.4 The tax rate is T = 35 percent. Taxes paid will be

$$T \times (\text{revenue} - \text{expenses} - \text{depreciation}) = .35 \times (600 - 300 - 200) = \$35$$

Operating cash flow can be calculated as follows.

a. Revenue – expenses – taxes = 600 – 300 – 35 = $265

b. Net profit + depreciation = (600 – 300 – 200 – 35) + 200
= 65 + 200 = 265

c. (Revenues – cash expenses) × (1 – tax rate) + (depreciation × tax rate)
= (600 – 300) × (1 – .35) + (200 × .35) = 265

7.5

	Year 1	Year 2
Beginning UCC	—	$166,500
Net Acquisition	$180,000	(25,000)
CCA	13,500	21,225
Ending UCC	166,500	120,275

Calculations:

Year 1: CCA = $180, 000 × .5 × .15 = $13,500

Year 2: Adjusted cost of disposal = $100,000 + 75,000 = $175,000

Net acquisition = Total cost of additions – Adjusted cost of disposal
= $150,000 – $175,000 = –$25,000

CCA = $141,500 × .15 = $21,225

Ending UCC = $141,500 – $21,225 = $120,275

7.6 Adjusted cost of disposal = $65,000 + $70,000 = $135,000
UCC is reduced by this amount to $144,500 – $135,000 = $9,500.

If the firm has no other assets in Class 10, then the asset pool will be terminated, and the amount of $9,500 will be treated as a terminal loss.

7.7 a.

All figures in dollars

Year:	0	1	2	3	4	5
1. Capital investment	–5,000					
2. Revenues		3,000	3,000	3,000	3,000	3,000
3. Expenses		1,500	1,500	1,500	1,500	1,500
4. Profit before tax (2 – 3)		1,500	1,500	1,500	1,500	1,500
5. Tax at 35%		525	525	525	525	525
6. Operating cash flow excluding CCA tax shield (4 – 5)		975	975	975	975	975
7. UCC		4,625	3,931	3,341	2,840	2,414
8. CCA		375	694	590	501	426
9. CCA tax shield (.35 × 8)		131	243	207	175	149
10. Total yearly cash flows including CCA tax shield (1 + 6 + 9)	5,000	1,106	1,218	1,182	1,150	1,124

Notice that the amounts in row 10 do not reflect *all* cash flows, since there remains a UCC balance of $2,414 in Year 5, which should continue to provide CCA tax shields beyond Year 5 if there are other assets in the asset class.

b. If depreciation is on a staight-line basis over 5 years, we would change items 8, 9, and 10 of the table in part (a) above as follows:

All figures in dollars

Year:	0	1	2	3	4	5
8. Depreciation		1,000	1,000	1,000	1,000	1,000
9. Depreciation tax shield (.35 × 8)		350	350	350	350	350
10. Total yearly cash flows including depreciation tax shield (1 + 6 + 9)	–5,000	1,325	1,325	1,325	1,325	1,325

MINICASE

Jack Tar, CFO of Sheetbend & Halyard, Inc., opened the company confidential envelope. It contained a draft of a competitive bid for a contract to supply duffel canvas to the Canadian Armed Forces. The cover memo from Sheetbend's CEO asked Mr. Tar to review the bid before it was submitted.

The bid and its supporting documents had been prepared by Sheetbend's sales staff. It called for Sheetbend to supply 100,000 yards of duffel canvas per year for 5 years. The proposed selling price was fixed at $30 per yard.

Mr. Tar was not usually involved in sales, but this bid was unusual in at least two respects. First, if accepted by the navy, it would commit Sheetbend to a fixed price, long-term contract. Second, producing the duffel canvas would require an investment of $1.5 million to purchase machinery and to refurbish Sheetbend's plant in Saint John, New Brunswick.

Mr. Tar set to work and by the end of the week had collected the following facts and assumptions:

- The plant in Saint John was built in the early 1900s and is now idle. The plant was fully depreciated on Sheetbend's books, except for the purchase cost of the land (in 1947) of $10,000.
- Now that the land was valuable shorefront property, Mr. Tar thought the land and the idle plant could be sold, immediately or in the future, for $600,000.
- Refurbishing the plant would cost $500,000. This investment would be depreciated for tax purposes in an asset class that has a CCA rate of 5 percent.
- The new machinery would cost $1 million. This investment could be depreciated in an asset class that has a CCA rate of 30 percent.
- The refurbished plant and new machinery would last for many years. However, the remaining market for duffel canvas was small, and it was not clear that additional orders could be obtained once the Forces contract was finished. The machinery was custom built and could be used only for duffel canvas. Its second-hand value at the end of 5 years was probably zero.
- Table 7.11 shows the sales staff's forecasts of income from the navy contract. Mr. Tar reviewed this forecast and decided that its assumptions were reasonable, except that the forecast used book, not tax, depreciation.
- But the forecast income statement contained no mention of working capital. Mr. Tar thought that working capital would average about 10 percent of sales.

Armed with this information, Mr. Tar constructed a spreadsheet to calculate the NPV of the duffel canvas project, assuming that Sheetbend's bid would be accepted by the Forces.

He had just finished debugging the spreadsheet when another confidential envelope arrived from Sheetbend's CEO. It contained a firm offer from a New Brunswick real estate developer to purchase Sheetbend's Saint John land and plant for $1.5 million in cash.

Should Mr. Tar recommend submitting the bid to the Forces at the proposed price of $30 per yard? The discount rate for this project is 12 percent.

TABLE 7.11

Forecast income statement for the navy duffel canvas project (dollar figures in thousands, except price per yard)

Year	1	2	3	4	5
1. Yards sold	100.00	100.00	100.00	100.00	100.00
2. Price per yard	30.00	30.00	30.00	30.00	30.00
3. Revenue (1 × 2)	3,000.00	3,000.00	3,000.00	3,000.00	3,000.00
4. Cost of goods sold	2,100.00	2,184.00	2,271.36	2,362.21	2,456.70
5. Operating cash flow (3 – 4)	900.00	816.00	728.64	637.79	543.30
6. Depreciation	250.00	250.00	250.00	250.00	250.00
7. Income (5 – 6)	650.00	566.00	478.64	387.79	293.30
8. Tax at 35%	227.50	198.10	167.52	135.72	102.65
9. Net income (7 – 8)	$422.50	$367.90	$311.12	$252.06	$190.64

Notes:

1. Yards sold and price per yard would be fixed by contract.
2. Cost of goods includes fixed cost of $300,000 per year plus variable costs of $18 per yard. Costs are expected to increase at the inflation rate of 4 percent per year.
3. Depreciation: A $1 million investment in machinery is depreciated straight-line over 5 years ($200,000 per year). The $500,000 cost of refurbishing the Saint John plant is depreciated straight-line over 10 years ($50,000 per year).

Appendix 7A: Deriving the CCA Tax Shield

Earlier, in Section 7.6, we saw that under Canada's CCA system, the tax shield from investing in an asset can continue in perpetuity. Equation 1 (page 219) provided a formula to compute the present value of this perpetual tax shield. In the following discussion, we will explain how equation 1 was arrived at. We will continue with the notation used earlier; that is,

C	=	capital cost of an asset acquired at the beginning of Year 1
d	=	CCA rate for the asset class to which the asset belongs
UCC_t	=	undepreciated capital cost in Year t after deducting CCA for the year
T_c	=	the firm's tax rate
r	=	discount rate
S	=	salvage amount from the sale of the asset at the end of Year t

We begin by making two simplifying assumptions, which will be relaxed later in the discussion. The assumptions are to ignore the half-year rule and the rules governing the disposal of assets. Suppose you start a new asset class by buying an asset. Our assumptions will enable you to get your first CCA tax shield from the asset in Year 1. The UCC of the class starts out at the beginning of Year 1 and is denoted $UCC_0 = C$.

Table 7.12 provides details regarding yearly beginning and ending UCC balances and the CCA, which is obtained as the product of the beginning UCC and the CCA rate, d. So, for instance, the CCA in Year 1 (CCA_1) is $d \times UCC_0 = Cd$.

After deducting CCA_1, the UCC at the end of year Year 1 becomes

$$\begin{aligned} UCC_1 &= UCC_0 - CCA \\ &= C - Cd \\ &= C(1 - d) \end{aligned}$$

Suppose C is \$200,000 and d is 10 percent, then CCA_1 is \$200,000 × .1 = \$20,000 and UCC_1 is \$200,000 × (1 – .1) = \$180,000. Similarly, the CCA for Year 2 is

$$CCA_2 = d \times UCC_1 = Cd(1 - d),$$

which is

$$\$20{,}000 \times (1 - .1) = \$200{,}000 \times .1 \times (1 - .1) = \$18{,}000$$

for our example. In Year t, you will have taken $t - 1$ previous CCA tax shields, so the UCC at the start of the year is $UCC_{t-1} = C(1 - d)^{t-1}$. The CCA for Year t will be

$$d \times UCC_{t-1} = Cd(1 - d)^{t-1}$$

For instance, if you take t to be 15 years, then, not surprisingly, the UCC balance in Year 15 will be quite small at

$$\$200{,}000 \times .1 \times (1 - .1)^{14} = \$4{,}575.36$$

TABLE 7.12
UCC and CCA

Year	Beginning of Year UCC	CCA	End of Year UCC
1	UCC_0	Cd	$UCC_1 = C(1 - d)$
2	UCC_1	$Cd(1 - d)$	$UCC_2 = C(1 - d)^2$
3	UCC_2	$Cd(1 - d)^2$	$UCC_3 = C(1 - d)^3$
–			
t	UCC_t	$Cd(1 - d)^{t-1}$	$UCC_t = C(1 - d)^t$

Until now, given the simplifying assumptions, we show that in a declining balance system, each asset will generate a stream of CCA amounts that will continue in perpetuity. Our algebraic expression also enables us to calculate the CCA and UCC amounts for any given year. Earlier we established that the CCA adds value to a capital investment project by reducing the firm's tax bill. A CCA stream in perpetuity will generate an infinite stream of tax savings. We now need to calculate the present value of this infinite stream of tax savings—that is, the PV of the CCA tax shield. For a given year, the CCA tax shield is simply the product of the year's CCA and the firm's tax rate, or CCA × T_c. A project will therefore generate the following CCA tax shields for years 1 through t:

Year:	**1**	**2**	**3**	**—**	***t***	**—**
CCA tax shield = CCA × T_c	CdT_c	$CdT_c(1-d)$	$CdT_c(1-d)^2$	—	$CdT_c(1-d)^{t-1}$	—

The CCA tax shield forms a perpetuity that is growing at the rate $-d$; which means, in effect, it is declining at this rate. Conceptually, this is like the constant growth dividend we looked at in Section 5.4. Remember, when the dividend grows at a constant rate g, we arrive at the stock price as the present value of expected future dividends as follows:

$$\text{Stock price} = \frac{DIV_1}{(r-g)}$$

Similar to the stock value, the CCA tax shield has the following present value:

$$\text{PV of CCA tax shield} = \frac{CdT_c}{(r+d)}$$

To determine the value added to an investment project by the tax savings generated by CCA, we can add the present value of the perpetual tax shield to the present value of the "after-tax" revenues and expenses to get the overall present value of the project.

Let us now relax our two simplifying assumptions. Until now, we had ignored the half-year rule feature of Canadian tax law requiring that, in the year of purchase of a depreciable asset, only half of the asset's value is added to the asset class balance. The remaining half is added in the following year. When we consider the half-year rule, the present value of the CCA tax shield has to be computed for the two halves separately and then added up. To see how this is done, let us find the CCA stream on the two halves of the asset value, as follows:

Year:	**1**	**2**	**3**	**...**	***t***	**...**
CCA on first half of asset value	$Cd/2$	$Cd(1-d)/2$	$Cd(1-d)^2/2$	...	$Cd(1-d)^t/2$	...
CCA on second half of asset value	0	$Cd/2$	$Cd(1-d)/2$	...	$Cd(1-d)^{t-1}/2$	...

If we multiply by T_c then the PV of perpetual CCA tax shields for the first half becomes

$$\frac{1}{2}\left[\frac{CdT_c}{1+r} + \frac{CdT_c(1-d)}{(1+r)^2} + \frac{CdT_c(1-d)^2}{(1+r)^3} + \frac{CdT_c(1-d)^{t-1}}{(1+r)^t}\right] = \frac{1}{2}\left[\frac{CdT_c}{r+d}\right]$$

Notice that in the second half below, the same CCA stream (and CCA tax shields) are deferred 1 year.

$$\frac{1}{2}\left[\frac{CdT_c}{(1+r)^2} + \frac{CdT_c(1-d)}{(1+r)^3} + \frac{CdT_c(1-d)^2}{(1+r)^4} \ldots \frac{CdT_c(1-d)^{t-2}}{(1+r)^t}\right]$$

To find the present value of the second half, we have to discount the CCA tax shields back to time 0:

$$\frac{1}{2}\left[\frac{CdT_c}{r+d}\right]\left[\frac{1}{1+r}\right]$$

When we add up the two present values we get the total present value of the CCA tax shield under the half-year rule:

$$\frac{1}{2}\left[\frac{CdT_c}{r+d}\right]+\frac{1}{2}\left[\frac{CdT_c}{r+d}\right]\left[\frac{1}{1+r}\right]$$

Using a bit of algebra, this formula can be simplified to provide equation 1, which we discussed earlier:

$$=\frac{1}{2}\left[\frac{CdT_c}{r+d}\right]\left[1+\frac{1}{1+r}\right]$$

$$=\frac{1}{2}\left[\frac{CdT_c}{r+d}\right]\left[\frac{1+r+1}{1+r}\right]$$

$$=\frac{CdT_c}{r+d}\left[\frac{1+.5r}{1+r}\right]$$

This general formula enables us to compute the present value of the tax shield on CCA under the half-year rule. Relaxing our second assumption, when a residual or salvage value arising from the sale of an asset is introduced into the discussion, we get the present value of lost CCA tax shields due to salvage value, as discussed in Section 7.6 and shown as equation 2 (page 219).

When we combine equations 1 and 2, we get a general formula for the present value of the CCA tax shield, which we presented earlier as equation 3:

Present value of CCA tax shield =

$$\left[\frac{CdT_c}{r+d}\right]\left[\frac{1+.5r}{1+r}\right]-\left[\frac{SdT_c}{(d+r)}\right]\left[\frac{1}{(1+r)^t}\right]$$