SIDELINE

Through the Ranks

Indifference Curves and Demand Theory

In *Spoilt for Choice*, we saw how William Stanley Jevons outlined a consumption theory based on the assumption that utility is measurable in imaginary units known as utils. Using a more contemporary tool, we can show how consumers maximize utility without the need to make this assumption. With this tool, known as an indifference curve, consumers merely state whether they prefer one option over another or whether they are indifferent to both options they are asked to choose from. In other words, they rank their options in order of preference, rather than being required to specify how much more utility one option provides compared with another.

An illustration will make this process clear. Suppose a consumer named Alice is asked to choose between bundles containing different amounts of milkshakes and hamburgers that she may consume during a particular week. For example, Alice may have to choose between bundle *a* (containing 4 milkshakes and 3 hamburgers) and bundle *b* (containing 3 milkshakes and 4 hamburgers). Given her preferences, she may select either *a* or *b*. Or, she may decide she is indifferent to both *a* and *b*, because each bundle gives her the same level of utility.

Graphing an Indifference Curve

If Alice is continually asked to choose between pairs of bundles containing milkshakes and hamburgers, there will be a range of bundles that give her the same level of utility, meaning, she is indifferent to any of them. A table summarizing this range of bundles is known as an indifference schedule, and its graph is an indifference curve. Figure A portrays a possible indifference schedule and its associated indifference curve for Alice.

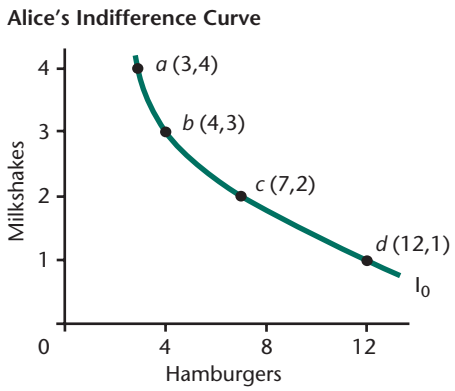
Bundles on the graph associated with points to the northeast of *a* give Alice a higher level of utility than at *a* because they contain more of one or both of the products. Likewise, points to the southwest of *a* are less desirable than *a*. As for the points on the indifference curve, I_0, they show an inverse relationship between the two products. To see why, let us visualize moving down I_0 from point *a* by taking away one of Alice's milkshakes. The only way Alice will have the same utility as before is if she receives more hamburgers (in this case, exactly one more hamburger) to make up for the milkshake she has lost. In other words, the new point *b* on the indifference curve must be to the southeast of the old point, which means I_0 is negatively sloped.

The slope of an indifference curve shows how the consumer substitutes one item for the other to maintain the same level of utility on the curve. The absolute value of the slope is known as the marginal rate of substitution (MRS). For

Figure A An Indifference Curve

Alice's Indifference Schedule

Milkshakes	Hamburgers	Point on Graph
4	3	*a*
3	4	*b*
2	7	*c*
1	12	*d*



When faced with bundles of milkshakes and hamburgers, Alice is indifferent to any of the four bundles shown. For example, the bundle containing 4 milkshakes and 3 hamburgers (point ***a***) gives her the same utility as 3 milkshakes and 4 hamburgers (point ***b***).

example, between points *a* and *b*, the marginal rate of substitution is [(3–4)/(4–3)] or 1, which is the ratio of the one lost milkshake (on the vertical axis) and the one extra hamburger (on the horizontal axis) when moving from *a* to *b*.

Note that the indifference curve becomes flatter and the marginal rate of substitution, therefore, decreases when travelling down I_0 to points where Alice consumes more hamburgers and fewer milkshakes. In other words, the indifference curve has a convex shape. This feature, known as the diminishing marginal rate of substitution, characterizes all indifference curves. In Alice's case, she will maintain the same level of utility when she keeps replacing hamburgers for milkshakes, only if she gains ever greater increases in hamburger consumption for each lost milkshake. For example, between points *b* and *c* on I_0, the marginal rate of substitution is [(2–3)/(7–4)] or 0.33, since 3 extra hamburgers are now needed to replace one lost milkshake, instead of 1 hamburger required to make up for the 1 lost milkshake when moving from points *a* to *b*.

The Map of Indifference Curves

Indifference curves can be drawn at various levels of utility, producing an indifference curve map (I_0, I_1, I_2) as shown in Figure B. Curves further to the northeast on this graph are linked to higher levels of utility for Alice, with points on I_1 preferable to those on I_0, and points on I_2 preferable to those on either I_1 or I_0.

The Budget Line

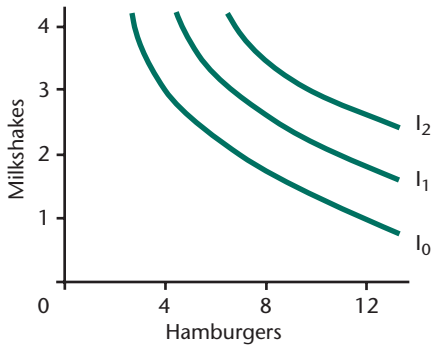
At the same time as Alice ranks various bundles of milkshakes and hamburgers on the basis of her individual preferences, she also faces a weekly budget ($15), all of which she spends on these two products, with the price of milkshakes at $3 and the price of hamburgers at $1.50. Her budget constraint can be summarized in an equation:

$$(\$3 \times M) + (\$1.50 \times H) = \$15$$

where M and H represent the quantities of milkshakes and hamburgers that Alice consumes during the week. This equation states that the sum of Alice's weekly expenditures on milkshakes ($3 × M) and hamburgers ($1.50 × H) equals her weekly budget of $15.

When portrayed on a graph, this equation is known as a budget line. As plotted in Figure C, this line divides the graph into an attainable region (the shaded area) and an unattainable region (the unshaded area) of possible purchases. The line's vertical intercept (5 milkshakes) shows how many milkshakes Alice can consume if all of her budget is used on milkshakes, and is found by dividing her budget by the price of milkshakes ($15 ÷ $3). The line's horizontal intercept (10 hamburgers) shows what happens if all of her budget is spent on hamburgers, and is therefore the ratio of her budget to the price of hamburgers ($15 ÷ $1.50). Because the line's vertical intercept is half as large as its horizontal intercept, the line has a slope of –.5. In general, this slope has an absolute value equal to the ratio of the two products' prices—the price of

Figure B A Map of Indifference Curves



Each of the indifference curves represents a different level of utility for Alice when she consumes any of the bundles on the curve. In moving northeast, she reaches indifference curves that give higher levels of utility, with I_1 preferable to I_0, and I_2 is preferable to either I_1 or I_0.

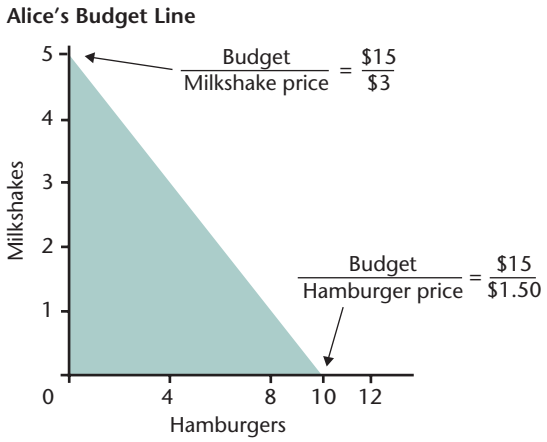
SIDELINE

SIDELINE

Figure C The Budget Line

Alice's Budget Schedule

Milkshakes	Hamburgers
5	0
4	2
3	4
2	6
1	8
0	10



Alice faces a region of attainable purchases (the shaded area) and unattainable purchases (the unshaded area), given her weekly budget ($15) and the prices of the two products she consumes ($3 for milkshakes and $1.50 for hamburgers).

hamburgers (the product on the horizontal axis) divided by the price of milkshakes (the product on the vertical axis).

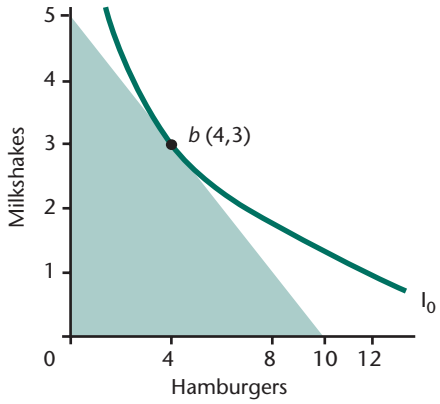
Figure C shows a variety of other possible bundles that Alice can purchase with her $15. For example, she can consume 3 milkshakes, at a cost of $9 (3 milkshakes times the $3 price), along with 4 hamburgers at a cost of $6 (4 hamburgers times the $1.50 price).

Utility Maximization

To find Alice's utility-maximizing point, her map of indifference curves can be combined with her budget line. Given her weekly budget and the prevailing prices, she reaches the highest possible level of utility within her attainable region of possible purchases when she is on the indifference curve that just touches the budget line at a single point. In Figure D, this is point *b* on the indifference curve I_0, with a utility-maximizing bundle of three milkshakes and four hamburgers.

At this point, the marginal rate of substitution along Alice's highest possible indifference curve equals the ratio of the price of hamburgers ($1.50) to the price of milkshakes ($3). In more general terms, the utility-maximizing condition for products 1 and 2 (with product 1 on the vertical axis and

Figure D The Utility-Maximizing Point



Alice's greatest achievable utility is on the indifference curve I_0, which touches her budget line at the single point *b*, since this is the highest possible indifference curve within the attainable region defined by her budget line.

product 2 on the horizontal axis), is met when the ratio of the two products' prices P_1 and P_2 equals the marginal rate of substitution between the two products:

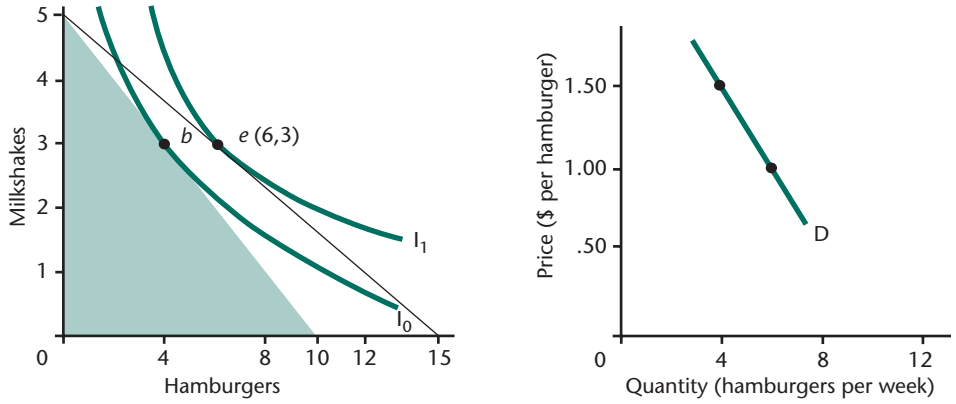
$$P_2/P_1 = MRS$$

Deriving a Demand Curve

This model of utility maximization can be applied to derive Alice's demand curve for hamburgers. Figure E treats the case where the price of hamburgers falls from $1.50 to $1. This price change leaves the budget line's vertical intercept (5 milkshakes) unaffected, but its horizontal intercept increases to 15 hamburgers (Alice's $15 budget divided by the new $1 hamburger price). Meanwhile, the line's slope has changed from –.5 (–$1.50/$3) to –.33 (–$1/$3).

With her new budget line, Alice finds that her attainable region of possible purchases has expanded, allowing her to increase her level of utility by switching from point *b* on indifference curve I_0 to point e on indifference curve I_1. As a result of the changes on the first graph in Figure E, the decline in the price of hamburgers from $1.50 to $1 has caused Alice's consumption of hamburgers to increase from 4 (at point *b*) to 6 (at point *e*). Therefore, we have derived two points on Alice's demand curve for hamburgers, as shown on the second graph in Figure E.

Figure E Deriving a Demand Curve



When the price of hamburgers declines from $1.50 to $1, Alice's budget line rotates outwards, and her attainable region of purchases expands, as shown on the left graph. Her utility-maximizing point then moves from *b* on indifference curve I_0 to *e* on I_1, and her consumption of hamburgers rises from 4 to 6. The connection between the price and Alice's quantity demanded of hamburgers is shown on the right graph, which highlights two points on her hamburger demand curve.

1. Draw a graph to show what happens to Alice's budget line if:
   a the price of hamburgers remains at $1.50, and her budget continues to be $15, but the price of milkshakes falls to $1.50.
   b the prices of hamburgers and milkshakes remain at $1.50 and $3, respectively, but her budget rises to $30.

2 A consumer named Adam has a weekly budget of $32 to spend on sodas and nachos, and the prices of sodas and nachos are $2 and $4, respectively. Adam has an indifference schedule as shown in the table below. Graph Adam's budget line and indifference curve (placing sodas on the vertical axis) and show his utility-maximizing point. Explain why Adam chooses this point over all others in his attainable region of purchases.

Sodas	Nachos
18	1
13	2
10	3
9	4

SIDELINE