Solving for the Unknown: Basic Linear Equations

A How-To Approach for Solving Equations

5

LEARNING UNIT OBJECTIVES

LU 5–1: Solving Equations for the Unknown: Basic Linear Equations

- Explain the basic procedures used to solve equations for the unknown (pp. 115–19).
- List the five rules and the mechanical steps used to solve for the unknown in seven situations; know how to check the answers (verification) (pp.).

LU 5–2: Solving Word Problems for the Unknown

- List the steps for solving word problems (p. 119).
- Complete Blueprint Aids to solve word problems; verify the solutions (pp. 120–22).

Not Too Deep

India rates low in Internet penetration, compared with the rest of its region (in millions)

	NET USERS	ADULT POP.	USERS AS PCT. OF ADULT POP.
Singapore	1.2	3.4	36.2%
Australia	4.1	15.4	26.3
Taiwan	4.0	17.7	22.8
Hong Kong	1.2	5.9	19.7
S. Korea	6.8	37.8	17.9
Japan	17.7	109.2	16.2
China	8.3	964.9	0.9
INDIA	**1.8**	**695.8**	**0.3**

If Singapore increases its Net users by three times, will Singapore's users be more than Taiwan's users?

The *MacLean's* clipping "Law: Patents and Rodents" states that due to the Supreme Court decision that higher life forms such as mice cannot be patented, Canadian biotech companies may consider heading for more favourable jurisdictions. According to the article, in a four-year period the number of companies leaving increased from 227 to 400. What was the increase in the number of Canadian biotech companies? This value is unknown, and we must find it. To solve this word problem we will apply the step approach.

Step 1. Let's read the problem once again and write what is given in the top left corner of our notebook. The number of firms has increased *from* 227. This means that the initial or old value is 227. The number of firms has increased *to* 400. This means that the new value is 400. We must now find the amount of change.

Step 2. Let's assign variables to known and unknown values and translate the English language into math. Initial/old value is V_o; new value is V_n; and the unknown value is X. Now, we can write down the problem in the following form:

$V_o = 227$

$V_n = 400$

$X = ?$

Step 3. Let's think of a formula that will help us find the solution. The amount of the change equals the new value minus the old value:

$$X = V_n - V_o$$

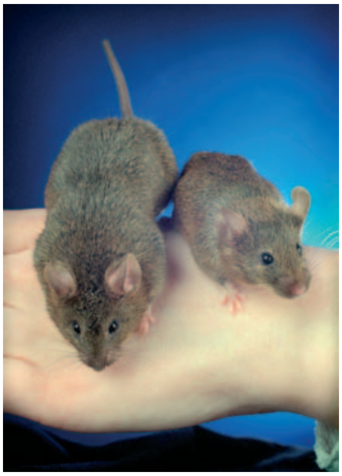
Step 4. Substitute the known values into the formula:

$$X = 400 - 227 = 173$$

Write the answer in full sentence form.

The number of Canadian biotech companies voting has increased *by* 173 companies.

Step 5. Verify the answer. Substitute the values for the variables. Always use the original formula.



Patents and Rodents

Does a genetically engineered mouse qualify for the same patent protection as, say, a hand-held computer? Not as far as the Supreme Court of Canada is concerned. In its long-anticipated, and ethically and morally charged ruling, the court decided that higher life forms—such as Harvard University's so-called OncoMouse which was at the heart of the case—cannot be patented.

Canada now stands almost alone—and biotechnologists aren't happy. The United States, Japan and much of Europe have long extended patent rights over the OncoMouse. Some critics now worry about brain drain as Canadian biotech companies may consider heading for more favourable jurisdictions. Between 1997 and 2001, the number of such firms in Canada increased from 227 to more than 400. Annual revenues reported by the sector's public companies now exceed $1.5 billion, a 160 per cent increase.

Learning Unit 5–1 explains how to solve for unknowns in equations. In Learning Unit 5–2 you learn how to solve for unknowns in word problems. When you complete these Units, you will not have to memorize as many formulas to solve business and personal math applications. Also, with the increasing use of computer software, a basic working knowledge of solving for the unknown has become necessary.

LEARNING UNIT 5–1

SOLVING EQUATIONS FOR THE UNKNOWN: BASIC EQUATIONS

The following heading appeared in a *Wall Street Journal* article:

Calculating Retirement? It's No Simple Equation

Many of you are familiar with the terms *variables* and *constants*. If you are planning to prepare for your retirement by saving only what you can afford each year, your saving is a *variable*. However, if you plan to save the same amount each year, your saving is a *constant*. This unit explains the importance of mathematical variables and constants when solving equations.

Basic Equation-Solving Procedures

Expression on the left side	=	Expression on the right side
$5b - 7$	=	$4b + 3$
$6x$	=	18
$4(a + 3)$	=	$-11(2a - 1)$
$3x + 2$	=	$5x - 2$

To solve an equation for an unknown variable is to present the equation in such a form that the unknown variable with coefficient positive one (1) will be on the left side of the equal sign and all the other members of the equation on the right.
$x = 3a + 4$
$y = 85$

Do you know the difference between a mathematical expression, an equation, and a formula? A mathematical **expression** is a meaningful combination of numbers and letters called *terms*. Operational signs (such as + or −) within the expression connect the terms to show a relationship between them. For example, $6 + 2$ or $6A - 4A$ are mathematical expressions. An **equation** is a mathematical statement with an equal sign showing that a mathematical expression on the left equals the mathematical expression on the right. Mathematical expressions on both sides of the equal sign are called **members of an equation.** An equation has an equal sign; an expression does not have an equal sign. A **formula** is an equation that expresses in symbols a general fact, rule, or principle. Formulas are shortcuts for expressing a word concept. For example, in Chapter 12 you will learn that the formula for simple interest is Interest (I) = Principal (P) × Rate (R) × Time (T). This means that when you see $I = P \times R \times T$, you recognize the simple interest formula. Now let's study basic equations.

As a mathematical statement of equality, equations show that two numbers or groups of numbers are equal. For example, $6 + 4 = 10$ shows the equality of an equation. Equations also use letters as symbols that represent one or more numbers. These symbols, usually a letter of the alphabet, are **variables** that stand for a number. We can use a variable even though we may not know what it represents. For example, $A + 2 = 6$. The variable A represents the number or **unknown** (4 in this example) for which we are solving. We distinguish variables from numbers, which have a fixed value. Numbers such as 3 or −7 are **constants** or **knowns,** whereas A and $3A$ (this means 3 times the variable A) are variables. So we can now say that variables and constants are *terms of mathematical expressions.*

Usually in solving for the unknown, we place variable(s) on the left side of the equation and constants on the right. The following rules for variables and constants are important.

VARIABLES AND CONSTANTS RULES

Step 1. If no number is in front of a letter, it is a 1: $B = 1B$; $C = 1C$.

Step 2. If no sign is in front of a letter or number, it is a +: $C = +C$; $4 = +4$.

You should be aware that in solving equations, the meaning of the symbols +, −, ×, and ÷ has not changed. However, some variations occur. For example, you can also write $A \times B$ (A times B) as $A \cdot B$, $A(B)$, or AB. Also, A divided by B is the same as A/B. If an equation contains only one variable, it is called **an equation in one unknown.** If this unknown is in the first power, the equation is called a **linear equation.** The four equations shown on the left are linear equations in one unknown. In each of these equations, the left member equals the right member. In the first equation, $5b - 7 = 4b + 3$. The expression $5b - 7$ is the left member and $4b + 3$ is the right member of the equation. They are equal.

To **solve an equation** means to find the value of unknown variable or to find the **root** or the **solution of the equation.** To solve an equation for an unknown variable means to present it in such a form that the unknown variable with positive coefficient 1 will be on the left side of the equal sign and all other members on the right side. In other words, to find the solution or root of an equation is to find a replacement value for the unknown, which when substituted for the unknown variable will make the equation true.

EXAMPLE

$3x + 2 = 5x - 2$

In this equation the solution is $x = 2$, because when we substitute 2 for x, the two members of equation will still be equal.

Left side: $3 \times 2 + 2 = 8$

Right side: $5 \times 2 - 2 = 8$

Left side equals right side: $8 = 8$

This process of checking an equation is called **verification.**

Remember that to solve an equation, you must find a number that can replace the unknown in the equation and make it a true statement. Now let's take a moment to look at how we can change verbal statements into variables.

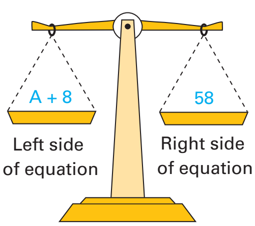
If you change both sides of an equation in the same way, the meaning of the equation will remain the same.

Verbal Statement	Variable *A* (age)
Dick's age 8 years ago	$A - 8$
Dick's age 8 years from today	$A + B$
Four times Dick's age	$4A$
One-fifth Dick's age	$A/5$

Assume Dick Hersh, an employee of Nike, is 50 years old. Let's assign Dick Hersh's changing age to the symbol *A*. The symbol *A* is a variable.

FIGURE 5–1

Equality in equations



Dick's age in 8 years will equal 58.

To visualize how equations work, think of the old-fashioned balancing scale shown in Figure 5.1. The pole of the scale is the equal sign. The two sides of the equation are the two pans of the scale. In the left pan or left side of the equation, we have $A + 8$; in the right pan or right side of the equation, we have 58. To solve for the unknown (Dick's present age), we isolate or place the unknown (variable) on the left side and the numbers on the right. We will do this soon. For now, remember that to keep an equation (or scale) in balance, we must perform mathematical operations (addition, subtraction, multiplication, and division) to *both* sides of the equation.

SOLVING FOR THE UNKNOWN RULE

Whatever you do to one side of an equation, you must do to the other side.

If you change both sides of an equation in the same way, you will get **equivalent equations,** equations that have the same root.

EXAMPLE

$5b - 7 = 4b + 3$ and $b - 7 = 3$ are equivalent equations, because they have the same solution or root: $b = 10$. The equations $6x = 18$ and $2x = 6$ are also equivalent.

HOW TO SOLVE FOR UNKNOWNS IN EQUATIONS

This section presents seven drill situations and the rules that will guide you in solving for unknowns in these situations. We begin with two basic rules—the opposite process rule and the equation equality rule.

OPPOSITE PROCESS RULE

If an equation indicates a process such as addition, subtraction, multiplication, or division, solve for the unknown or variable by using the opposite process. For example, if the equation process is addition, solve for the unknown by using subtraction.

EQUATION EQUALITY RULE

You can add the same quantity or number to both sides of the equation and subtract the same quantity or number from both sides of the equation without affecting the equality of the equation. You can also divide or multiply both sides of the equation by the same quantity or number (*except zero*) without affecting the equality of the equation.

To check (verify) your answer(s), substitute your answer(s) for the letter(s) in the equation. The sum of the left side should equal the sum of the right side.

Drill Situation 1: Subtracting Same Number from Both Sides of Equation

EXAMPLE

$A + 8 = 58$
Dick's age A plus 8 equals 58.

Mathematical steps

$$\begin{aligned} A + 8 &= 58 \\ -8 &\quad -8 \\ \hline A &= 50 \end{aligned}$$

Explanation

8 is subtracted from *both* sides of equation to isolate variable A on the left.

Check

$50 + 8 = 58$
$58 = 58$

These two equations are called "equivalent" equations.

Note: Since the equation process used *addition,* we use the opposite process rule and solve for variable A with *subtraction.* We also use the equation equality rule when we subtract the same quantity from both sides of the equation.

Drill Situation 2: Adding Same Number to Both Sides of Equation

EXAMPLE

$B - 50 = 80$
Some number B less 50 equals 80.

Mathematical steps

$$\begin{aligned} B - 50 &= 80 \\ +50 &\quad +50 \\ \hline B &= 130 \end{aligned}$$

Explanation

50 is added to *both* sides to isolate variable B on the left.

Check

$130 - 50 = 80$
$80 = 80$

Equivalent equations

Note: Since the equation process used *subtraction,* we use the opposite process rule and solve for variable B with *addition.* We also use the equation equality rule when we add the same quantity to both sides of the equation.

Drill Situation 3: Dividing Both Sides of Equation by Same Number

EXAMPLE

$7G = 35$
Some number G times 7 equals 35.

Mathematical steps

$$7G = 35$$

$$\frac{\cancel{7}G}{\cancel{7}} = \frac{35}{7}$$

$$G = 5$$

Explanation

By dividing both sides by 7, G equals 5.

Check

$7(5) = 35$
$35 = 35$

Equivalent equations

Note: Since the equation process used *multiplication*, we use the opposite process rule and solve for variable G with *division*. We also use the equation equality rule when we divide both sides of the equation by the same quantity.

Drill Situation 4: Multiplying Both Sides of Equation by Same Number

EXAMPLE

$\frac{V}{5} = 70$

Some number V divided by 5 equals 70.

Mathematical steps

$$\frac{V}{5} = 70$$

$$5\left(\frac{V}{5}\right) = 70(5)$$

$$V = \boxed{350}$$

Explanation

By multiplying both sides by 5, V is equal to 350

Check

$$\frac{350}{5} = 70$$

$$70 = 70$$

Equivalent equations

Note: Since the equation process used *division*, we use the opposite process rule and solve for variable V with *multiplication*. We also use the equation equality rule when we multiply both sides of the equation by the same quantity.

Drill Situation 5: Equation That Uses Subtraction and Multiplication to Solve Unknown

MULTIPLE PROCESSES RULE

When solving for an unknown that involves more than one process, do the addition and subtraction before the multiplication and division.

EXAMPLE

$\frac{H}{4} + 2 = 5$

When we divide unknown H by 4 and add the result to 2, the answer is 5.

Mathematical steps

$$\frac{H}{4} + 2 = 5$$

$$\frac{H}{4} + 2 - 2 = 5 - 2$$

$$\frac{H}{4} = 3$$

$$4\left(\frac{H}{4}\right) = 4(3)$$

$$H = \boxed{12}$$

Explanation

1. Move constant to right side by subtracting 2 from both sides.
2. To isolate H, which is divided by 4, we do the opposite process and multiply 4 times *both* sides of the equation.

Check

$$\frac{12}{4} + 2 = 5$$

$$3 + 2 = 5$$

$$5 = 5$$

Equivalent equations

Drill Situation 6: Using Parentheses in Solving for Unknown
BEDMAS
PEMDAS

PARENTHESES RULE

When equations contain parentheses (which indicate grouping together), you solve for the unknown by first multiplying each item inside the parentheses by the number or letter just outside the parentheses. Then you continue to solve for the unknown with the opposite process used in the equation. Do the additions and subtractions first; then the multiplications and divisions.

EXAMPLE

$5(P - 4) = 20$
The unknown P less 4, multiplied by 5 equals 20.

Mathematical steps

$$5(P - 4) = 20$$
$$5P - 20 = 20$$
$$\underline{+20} \quad \underline{+20}$$
$$\frac{\cancel{5}P}{\cancel{5}} = \frac{40}{5}$$
$$P = 8$$

Explanation

1. Parentheses tell us that everything inside parentheses is multiplied by 5. Multiply 5 by P and 5 by -4.
2. Add 20 to both sides to isolate $5P$ on left.
3. To remove 5 in front of P, divide both sides by 5 to result in P equals 8.

Check

$5(8 - 4) = 20$
$5(4) = 20$
$20 = 20$

Drill Situation 7: Combining Like Unknowns

LIKE UNKNOWNS RULE

To solve equations with like unknowns, you first combine the unknowns and then solve with the opposite process used in the equation.

EXAMPLE

$4A + A = 20$

Mathematical steps

$$4A + A = 20$$
$$\frac{\cancel{5}A}{\cancel{5}} = \frac{20}{5}$$
$$A = 4$$

Explanation

To solve this equation: $4A + 1A = 5A$. Thus, $5A = 20$. To solve for A, divide both sides by 5, leaving A equals 4.

Before you go to Learning Unit 5–2, let's check your understanding of this Unit.

LU 5–1 PRACTICE QUIZ

1. Write equations for the following (use the letter Q as the variable). Do not solve for the unknown.
   a. Nine less than one-half a number is fourteen.
   b. Eight times the sum of a number and thirty-one is fifty.
   c. Ten decreased by twice a number is two.
   d. Eight times a number less two equals twenty-one.
   e. The sum of four times a number and two is fifteen.
   f. If twice a number is decreased by eight, the difference is four.
2. Solve the following:
   a. $B + 24 = 60$ **b.** $D + 3D = 240$ **c.** $12B = 144$
   d. $\frac{B}{6} = 50$ **e.** $\frac{B}{4} + 4 = 16$ **f.** $3(B - 8) = 18$

LEARNING UNIT 5–2
SOLVING WORD PROBLEMS FOR THE UNKNOWN

On the first day of your business math class, you count 29 students in the class. A week later, 11 additional students had joined the class. The semester ended with 35 students in attendance. How many students dropped out of class? You can solve this unknown as follows:

29 students started the class + 11 students joined the class = 40 total students
40 total students − 35 students completed the class = 5 students dropped the class

Whether you are in or out of class, you are continually solving word problems. In this Unit we give you a road map showing you how to solve word problems with unknowns by using a Blueprint Aid. We have already used the step approach to solve word problems in the earlier chapters and in Learning Unit 5–1. In Chapters 1 through 3, we also presented Blueprint Aids for dissecting and solving word problems. Now the Blueprint Aid focuses on solving for the unknown.

We look at six different situations in this Unit. Be patient and *persistent*. The more problems you work, the easier the process becomes. Note how we dissect and solve each problem in the Blueprint Aids. Do not panic! Repetition is the key. Now let's study the five steps.

SOLVING WORD PROBLEMS FOR UNKNOWNS

Step 1. Carefully read the entire problem. You may have to read it several times. In the left corner of your notebook write what is given.

Step 2. Ask yourself: "What is the problem looking for?" Assign variables to known and unknown values. If the problem has more than one unknown, try to represent the second unknown in terms of the first. For example, if the problem has two unknowns, and you decide to call one of them Y, the second unknown might be called $4Y$ if you are told in the problem that it is four times the first unknown. You have translated English into mathematical terms.

Step 3. Think of a formula that will help you find a solution. Visualize the relationships between unknowns and variables. Set up an equation that expresses these relationships.

Step 4. Substitute the known values into the formula and find the unknown.

Step 5. Check (verify) the answer to see if it is accurate.

Bill Aron/PhotoEdit

The Flagging Division ...

A snapshot of the Disney Stores

- **NUMBER OF STORES:** 740
- **STORE VISITORS:** 250 million annually
- **LOCATIONS:** 11 countries including Britain, Australia and Japan
- **PRODUCTS:** Toys, costumes, apparel, jewelry, accessories, videos and games, among others
- **PRODUCT PLANS:** Each store will have 1,800 product offerings, down from 3,400 in the past. Focus on adults will be narrowed to sleepwear and parenting products.

Word Problem Situation 1: Number Problems From the *Wall Street Journal* clip "The Flagging Division," you can determine that Disney Stores reduced its product offerings by 1,600. Disney now has 1,800 product offerings. What was the original number of product offerings?

BLUEPRINT AID

Unknown(s)	Variable(s)	Relationship*
Original number of product offerings	P	$P - 1{,}600$ = New offerings New offerings = 1,800

*This column will help you visualize the equation before setting up the actual equation.

MATHEMATICAL STEPS

$$P - 1{,}600 = 1{,}800$$
$$\underline{+1{,}600 \quad +1{,}600}$$
$$P = 3{,}400$$

EXPLANATION

The original offerings less 1,600 = 1,800. Note that we added 1,600 to both sides to isolate P on the left. Remember, $1P = P$.

CHECK

$3{,}400 - 1{,}600 = 1{,}800$

$1{,}800 = 1{,}800$

STEP APPROACH

STEP 1. Read the problem, and in the top left corner of your notebook write in brief form what is given.

New offerings after reduction = 1,800
Amount of reduction = 1,600
Original number of product offerings = ?

STEP 2. Assign variables to known and unknown values.

New offerings after reduction: B
Amount of reduction: A
Original number of product offerings: P
Rewrite the given information, using variables.

$B = 1{,}800$
$A = 1{,}600$
$P = ?$

STEP 3. Think of a formula that will help find the solution.

Original number − Amount of reduction = New number
$P - A = B$

To find the original number P, we must add the amount of reduction A and the new number B.
$P = B + A$

STEP 4. Substitute the values into the formula.
$P = 1{,}800 + 1{,}600$
$P = 3{,}400$

STEP 5. Verify the answer. Always substitute the values into the original formula.
$P - A = B$

The left side must equal the right side.

Left Side	=	Right Side
$3{,}400 - 1{,}600$	=	1,800
1,800	=	1,800

THE ANSWER IS CORRECT

Word Problem Situation 2: Finding the Whole When Part Is Known A local Burger King budgets $\frac{1}{8}$ of its monthly profits on salaries. Salaries for the month were $12,000. What were Burger King's monthly profits?

BLUEPRINT AID

Unknown(s)	Variable(s)	Relationship
Monthly profits	P	$\frac{1}{8}P$ Salaries = $12,000

MATHEMATICAL STEPS

$$\frac{1}{8}P = \$12{,}000$$
$$8\left(\frac{P}{8}\right) = \$12{,}000(8)$$
$$P = \$96{,}000$$

EXPLANATION

$\frac{1}{8}P$ represents Burger King's monthly salaries. Since the equation used division, we solve for P by multiplying both sides by 8.

CHECK

$\frac{1}{8}(\$96{,}000) = \$12{,}000$

$\$12{,}000 = \$12{,}000$

STEP APPROACH

STEP 1. Read the problem, and in the top left corner of your notebook write in brief form what is given.

Salaries for the month = $12,000
Salaries = $\frac{1}{8}$ of monthly profits
Monthly profits = ?

STEP 2. Assign variables to known and unknown values.

Salaries for the month: D
Monthly profits: P
$D = \frac{1}{8}$ of P

Rewrite the given information, using variables.

$D = \$12{,}000$
$D = \frac{1}{8} \times P$
$P = ?$

STEP 3. Think of a formula that will help find the solution.
$D = \frac{1}{8} \times P$

$P = D \div \left(\frac{1}{8}\right)$ (Use property of multiplications of fractions)
$P = D \times 8 = 8D$

STEP 4. Substitute the values into the formula.
$P = 8 \times 12{,}000$
$P = 96{,}000$

STEP 5. Verify the answer. Always substitute the values into the original formula.
$D = \frac{1}{8} \times P$

The left side must equal the right side.

Left Side	=	Right Side
12,000	=	$\frac{1}{8} \times 96{,}000$
12,000	=	12,000

THE ANSWER IS CORRECT

Word Problem Situation 3: Difference Problems ICM Company sold 4 times as many computers as Ring Company. The difference in their sales is 27. How many computers of each company were sold?

BLUEPRINT AID

Unknown(s)	Variable(s)	Relationship
ICM Ring	$4C$ C	$4C$ $-\ C$ 27

Note: If problem has two unknowns, assign the variable to smaller item or one who sells less. Then assign the other unknown using the same variable. *Use the same letter.*

MATHEMATICAL STEPS

$$4C - C = 27$$

$$\frac{3C}{3} = \frac{27}{3}$$

$$C = 9$$

Ring = 9 computers
ICM = 4(9)
= 36 computers

EXPLANATION

The variables replace the names ICM and Ring. We assigned Ring the variable C, since it sold fewer computers. We assigned ICM $4C$, since it sold 4 times as many computers.

CHECK

36 computers
− 9
27 computers

STEP APPROACH

STEP 1. Read the problem, and in the top left corner of your notebook write in brief form what is given. In this problem we are talking about two companies, ICM and Ring. ICM has sold four times more than Ring. In other words, the sales of ICM are expressed in terms of those of Ring.
Difference in their sales = 27
Sales of each company = ?

STEP 2. Assign variables to known and unknown values.
Sales of Ring = X
Sales of ICM = Y (Sales of ICM is $4 \times X$)
Difference in sales = $(Y - X)$
$Y = ?$
$X = ?$
Rewrite the given information, using variables.
$Y - X = 27$

STEP 3. Think of a formula, and visualize the relationships between the variables and unknowns.
Express Y in terms of X: $Y = 4X$
Substitute this expression in the formula: $4X - X = 27$

STEP 4. To find X (sales of the Ring company) we need to combine the like terms:
$3X = 27$
and then divide both parts of the equation by 3:
$X = 9$
To find Y (sales of ICM company) substitute the value of X into the formula $Y = 4X$.
$Y = 4 \times 9$
$Y = 36$

STEP 5. Verify the answer. Always substitute the values into the original formula.
$Y - X = 27$
The left side must equal the right side.

Left Side	=	Right Side
39 − 9	=	27
27	=	27

THE ANSWER IS CORRECT

Word Problem Situation 4: Calculating Unit Sales Together Barry Sullivan and Mitch Ryan sold a total of 300 homes for Regis Realty. Barry sold 9 times as many homes as Mitch. How many did each sell?

BLUEPRINT AID

Unknown(s)	Variable(s)	Relationship
Homes sold: B. Sullivan M. Ryan	$9M$ M	$9M$ $+\ M$ 300 homes

MATHEMATICAL STEPS

$$9M + M = 300$$

$$\frac{10M}{10} = \frac{300}{10}$$

$$M = 30$$

Ryan: 30 homes
Sullivan: 9(30) = 270 homes

EXPLANATION

We assigned Mitch M, since he sold fewer homes.
We assigned Barry $9M$, since he sold 9 times as many homes.
Together Barry and Mitch sold 300 homes.

CHECK

30 + 270 = 300

STEP APPROACH

STEP 1. Read the problem, and in the top left corner of your notebook write in brief form what is given. In this problem we are talking about two sales agents, Barry and Mitch. Together they have sold 300 homes. Barry has sold nine times as many homes as Mitch has. In other words, Barry's sales are expressed in terms of Mitch's sales.

STEP 2. *M:* Number of homes Mitch has sold
B: Number of homes Barry has sold
$M = ?$
$B = ?$
$B + M = 300$

STEP 3. Think of a formula, and visualize the relationship between the variables and unknowns.
Express B in terms of M: $B = 9 \times M$
Substitute this expression into the formula: $9M + M = 300$

STEP 4. To find M (Mitch's sales) we need to combine the like terms.
$10M = 300$ and then divide both parts of the equation by 10
$M = 30$
To find B, substitute the value of M into the formula $B = 9M$.
$B = 9 \times 30$
$Y = 270$

STEP 5. Verify the answer. Always substitute the values into the original formula.
$B + M = 300$
The left side must equal the right side.

Left Side	=	Right Side
270 + 30	=	300
300	=	300

THE ANSWER IS CORRECT

Word Problem Situation 5: Calculating Unit and Dollar Sales (Cost per Unit) When Total Units Are Not Given Andy sold watches ($9) and alarm clocks ($5) at a flea market. Total sales were $287. People bought 4 times as many watches as alarm clocks. How many of each did Andy sell? What were the total dollar sales of each?

BLUEPRINT AID

Unknown(s)	Variable(s)	Price	Relationship
Unit sales:			
Watches	$4C$	$9	$36C$
Clocks	C	5	$+ 5C$
			$287 total sales

MATHEMATICAL STEPS

$36C + 5C = 287$

$$\frac{41C}{41} = \frac{287}{41}$$

$C = 7$

7 clocks

$4(7) = 28$ watches

EXPLANATION

Number of watches times $9 sales price plus number of alarm clocks times $5 equals $287 total sales.

CHECK

7($5) + 28($9) = $287
$35 + $252 = $287
$287 = $287

STEP APPROACH

STEP 1. In this problem we are talking about sales of two items: clocks and watches. Sales of watches exceeded the sales of clocks by four times. So sales of watches are expressed in terms of sales of clocks.

STEP 2. C: Number of clocks sold
$4C$: Number of watches sold
Revenue from sales of watches was: $4C \times 9$
Revenue from sales of clocks was: $C \times 5$
Total revenue was: $287
$C = ?$
$4C = ?$

STEP 3. $36C + 5C = 287$

STEP 4. $41C = 287$
$C = 7$
$4C = 28$

STEP 5. Verify the answer. Always substitute the values into the original formula. The left side must equal the right side.

Left Side	=	Right Side
$9 \times 28 + 7 \times 5$	=	287
$252 + 35$	=	287
287	=	287

THE ANSWER IS CORRECT

Word Problem Situation 6: Calculating Unit and Dollar Sales (Cost per Unit) When Total Units Are Given Andy sold watches ($9) and alarm clocks ($5) at a flea market. Total sales for 35 watches and alarm clocks were $287. How many of each did Andy sell? What were the total dollar sales of each?

BLUEPRINT AID

Unknown(s)	Variable(s)	Price	Relationship
Unit sales:			
Watches	W*	$9	$9W$
Clocks	$35 - W$	5	$+ 5(35 - W)$
			$287 total sales

*The more expensive item is assigned to the variable first only for this situation to make the mechanical steps easier to complete.

MATHEMATICAL STEPS

$9W + 5(35 - W) = 287$

$9W + 175 - 5W = 287$

$4W + 175 = 287$

$\quad - 175 \quad - 175$

$$\frac{4W}{4} = \frac{112}{4}$$

$W = 28$

Watches = 28

Clocks = 35 − 28 = 7

EXPLANATION

Number of watches (W) times price per watch plus number of alarm clocks times price per alarm clock equals $287. Total units given was 35.

CHECK

28($9) + 7($5) = $287
$252 + $35 = $287
$287 = $287

STEP APPROACH

STEP 1. In this problem, once again we are speaking about sales of two items: clocks and watches.

STEP 2. W: number of watches sold
$35 - W$: number of clocks sold
Revenue from sales of watches was: $9 \times W$
Revenue from sales of clocks was: $5 \times (35 - W)$
Total revenue was: $287
$W = ?$
$(35 - W) = ?$

STEP 3. $9W + 5(35 - W) = 287$

STEP 4. $9W + 175 - 5W = 287$
$4W + 175 = 287$
$4W = 287 - 175$
$4W = 112$
$W = 28$
$35 - W = 35 - 28 = 7$

STEP 5. Verify the answer. The left side must equal the right side.

Left Side	=	Right Side
$9 \times 28 + 7 \times 5$	=	287
$252 + 35$	=	287
287	=	287

THE ANSWER IS CORRECT

Why did we use $35 - W$? Assume we had 35 pizzas (some cheese, others meatball). If I said that I ate all the meatball pizzas (5), how many cheese pizzas are left? Thirty? Right, you subtract 5 from 35. Think of $35 - W$ as meaning one number.

Note in Word Problem Situations 5 and 6 that the situation is the same. In Word Problem Situation 5, we were not given total units sold (but we were told which sold better). In Word Problem Situation 6, we were given total units sold, but we did not know which sold better.

Now try these six types of word problems in the Practice Quiz. Do it in either way: complete Blueprint Aids or follow steps for solving word problems with the unknown(s).

LU 5–2 PRACTICE QUIZ

Situations

1. An L. L. Bean sweater was reduced \$30. The sale price was \$90. What was the original price?
2. Kelly Doyle budgets $\frac{1}{8}$ of her yearly salary for entertainment. Kelly's total entertainment bill for the year is \$6,500. What is Kelly's yearly salary?
3. Micro Knowledge sells 5 times as many computers as Morse Electronics. The difference in sales between the two stores is 20 computers. How many computers did each store sell?
4. Susie and Cara sell stoves at Elliott's Appliances. Together they sold 180 stoves in January. Susie sold 5 times as many stoves as Cara. How many stoves did each sell?
5. Pasquale's Pizza sells meatball pizzas (\$6) and cheese pizzas (\$5). In March, Pasquale's total sales were \$1,600. People bought 2 times as many cheese pizzas as meatball pizzas. How many of each did Pasquale sell? What were the total dollar sales of each?
6. Pasquale's Pizza sells meatball pizzas (\$6) and cheese pizzas (\$5). In March, Pasquale's sold 300 pizzas for \$1,600. How many of each did Pasquale's sell? What was the dollar sales price of each?

CHAPTER ORGANIZER AND REFERENCE GUIDE

Properties of equations	Key points, procedures, formulas	Examples
If you change both sides of the equation in the same way, you will get equivalent equations, p.	Add the same number to both members of an equation. Subtract the same value from both members of an equation. Multiply or divide both members by the same number/value.	$4X = 8$ $4X + 2 = 10$ $(8 + 2 = 10)$ $15Y + 3 = 8$ $15Y + 3 - 3 = 8 - 3 = 5$ $15Y = 5$ $15Y = 5$ (÷ by 5) $3Y = 1$ $15Y = 5$ (× by 4) $60Y = 20$
Solving for unknowns from basic equations	**Mathematical steps to solve unknowns**	**Key point(s)**
Situation 1: Subtracting same number from both sides of equation, p. 116	$D + 10 = 12$ $\quad - 10 = -10$ $D = 2$	Subtract 10 from both sides of equation to isolate variable D on the left. Since equation used addition, we solve by using opposite process—subtraction.
Situation 2: Adding same number to both sides of equation, p. 117	$L - 24 = 40$ $\quad + 24 = +24$ $L = 64$	Add 24 to both sides to isolate unknown L on left. We solve by using opposite process of subtraction—addition.

(continues)

CHAPTER ORGANIZER AND REFERENCE GUIDE (CONTINUED)

SOLVING FOR UNKNOWNS FROM BASIC EQUATIONS	MATHEMATICAL STEPS TO SOLVE UNKNOWNS	KEY POINT(S)
Situation 3: Dividing both sides of equation by same number, p. 117	$6B = 24$ $\frac{6B}{6} = \frac{24}{6}$ $B = 4$	To isolate *B* by itself on the left, divide both sides of the equation by 6. Thus, the 6 on the left cancels—leaving *B* equal to *4*. Since equation used multiplication, we solve unknown by using opposite process—division.
Situation 4: Multiplying both sides of equation by same number, p. 117	$\frac{R}{3} = 15$ $3\left(\frac{R}{3}\right) = 15(3)$ $R = 45$	To remove denominator, multiply both sides of the equation by 3—the 3 on the left side cancels, leaving *R* equal to 45. Since equation used division, we solve unknown by using opposite process—multiplication.
Situation 5: Equation that uses subtraction and multiplication to solve for unknown, p. 117	$\frac{B}{3} + 6 = 13$ $-6 \quad -6$ $\frac{B}{3} = 7$ $3\left(\frac{B}{3}\right) = 7(3)$ $B = 21$	1. Move constant 6 to right side by subtracting 6 from both sides. 2. Isolate *B* by itself on left by multiplying both sides by 3.
Situation 6: Using parentheses in solving for unknown, p. 118	$6(A - 5) = 12$ $6A - 30 = 12$ $+30 = +30$ $\frac{6A}{6} = \frac{42}{6}$ $A = 7$	Parentheses indicate multiplication. Multiply 6 times *A* and 6 times −5. Result is 6*A* − 30 on left side of the equation. Now add 30 to both sides to isolate 6*A* on left. To remove 6 in front of *A*, divide both sides by 6, to result in *A* equal to 7. Note that when deleting parentheses, we did not have to multiply the right side.
Situation 7: Combining like unknowns, p. 118	$6A + 2A = 64$ $\frac{8A}{8} = \frac{64}{8}$ $A = 8$	6*A* + 2*A* combine to 8*A*. To solve for *A*, we divide both sides by 8.

SOLVING FOR UNKNOWNS FROM WORD PROBLEMS (STEP APPROACH)	ASSIGNING VARIABLES AND DEFINING FORMULAS (TRANSLATION FROM ENGLISH INTO MATH)	SOLUTION AND VERIFICATION
Step 1. Carefully read the entire problem. You may have to read it several times. In the left corner of your notebook write what is given. **Step 2.** Ask yourself: "What is the problem looking for?" Assign variables to known and unknown values. If the problem has more than one unknown, represent the second unknown in terms of the same variable. For example, if the problem has two unknowns, *Y* is one unknown. The second	1. The sum of two numbers is 60. The first number is twice as big as the second. Find both numbers. 2. The sum of two numbers is 180. The first number is 5 units greater then $\frac{3}{4}$ of the second. What are the numbers?	1. First number is *A*. Second number is *B*. A is twice *B*: $A = 2B$ $A + B = 60$ Substitute for *A* its equivalent value 2*B*: $2B + B = 60$ $3B = 60$ $B = 20$ $2B = A = 40$ Verify: **LS** = **RS** $40 + 20 = 60$ $60 = 60$

(*continues*)

CHAPTER ORGANIZER AND REFERENCE GUIDE (CONTINUED)

Solving for Unknowns from Word Problems (Step Approach)	Assigning Variables and Defining Formulas (Translation from English into Math)	Solution and Verification
unknown might be 4*Y* (four times the first unknown). You have translated English into math. **Step 3.** Think of a formula, which will help find the solution. Visualize the relationship between unknowns and variables. Set up an equation to solve for unknown(s). **Step 4.** Substitute the known values into the formula and find the unknowns. **Step 5.** Check (verify) the answer to see if it is accurate.		2. The first number is *A*, the second is *B*. $A = \frac{3}{4}B + 5$ $A + B = 180$ Substitute for *A* its equivalent value: $B + \frac{3}{4}B + 5 = 180$ $1\frac{3}{4}B = 175; \quad 1.75B = 175$ $B = 100; A = 80$ Verify: **LS** = **RS** 80 + 100 = 180

Solving for Unknowns from Word Problems | Blueprint Aid | Mathematical Steps to Solve Unknown with Check | Solution and Verification

Situation 1: Number problems, p. 119

U.S. Air reduced its airfare to California by $60. The sale price was $95. What was the original price?

Blueprint Aid

Unknown(s)	Variable(s)	Relationship
Original price	*P*	*P* − $60 = Sale price Sale price = $95

Mathematical steps to solve unknown with check

$P - \$60 = \$95$
$\underline{\quad +60 \qquad +60}$
$P = \$155$

Check
$155 − $60 = $95
$95 = $95

Solution and verification

S = sale price = \$95
D − Amount of discount = \$60
$P = ?\ P - D = S$
$P = S + D;\ P = 60 + 95 = 155$
$P = 155$
Verify:
LS = **RS**
155 − 60 = 95
95 = 95

Situation 2: Finding the whole when part is known, p. 120

K. McCarthy spends $\frac{1}{8}$ of her budget for school. What is the total budget if school costs $5,000?

Blueprint Aid

Unknown(s)	Variable(s)	Relationship
Total budget	*B*	$\frac{1}{8}B$ School = $5,000

Mathematical steps to solve unknown with check

$\frac{1}{8}B = \$5{,}000$
$8\left(\frac{B}{8}\right) = \$5{,}000(8)$
$B = \$40{,}000$

Check
$\frac{1}{8}(\$40{,}000) = \$5{,}000$
$5,000 = $5,000

Solution and verification

B − Total budget
$\frac{1}{8}B$ − Spendings = \$5,000
$\frac{1}{8}B = 5{,}000$
$B = \frac{5{,}000}{\left(\frac{1}{8}\right)} = 5{,}000 \times 8 = 40{,}000$
Verify:
LS = **RS**
$\frac{1}{8} - 40{,}000 = 5{,}000$
5,000 = 5,000

Situation 3: Difference problems, p. 120

Moe sold 8 times as many suitcases as Bill. The difference in their sales is 280 suitcases. How many suitcases did each sell?

Blueprint Aid

Unknown(s)	Variable(s)	Relationship
Suitcases sold:		
Moe	8*S*	8*S*
Bill	*S*	− *S*
		280 suitcases

Mathematical steps to solve unknown with check

$8S - S = 280$ (Bill)
$\frac{7S}{7} = \frac{280}{7}$
$S = 40$ (Bill)
$8(40) = 320$ (Moe)

Check
320 − 40 = 280
280 = 280

Solution and verification

Bill's sales − S;
Moe's sales = $8S$
$8S - S$ − difference = 280
$S = ?\ 8S = ?$
$8S - S = 280;\ 7S = 280;$
$S = 40$
$8S = 320$
Verification:
LS = **RS**
8 × 40 − 40 = 280
280 = 280

(*continues*)

CHAPTER ORGANIZER AND REFERENCE GUIDE (CONCLUDED)

<table>
<tr><th>Solving for unknowns from word problems</th><th>Blueprint aid</th><th>Mathematical steps to solve unknown with check</th><th>Solution and verification</th></tr>
<tr>
<td>Situation 4: Calculating unit sales, p. 121

Moe sold 8 times as many suitcases as Bill. Together they sold a total of 360. How many did each sell?</td>
<td><table><tr><th>Unknown(s)</th><th>Variable(s)</th><th>Relationship</th></tr><tr><td>Suitcases sold:
Moe
Bill</td><td>
8S
S</td><td>
8S
+ S
360 suitcases</td></tr></table></td>
<td>$8S + S = 360$
$\frac{9S}{9} = \frac{360}{9}$
$S = 40$ (Bill)
$8S = 320$ (Moe)
Check
$320 + 40 = 360$
$360 = 360$</td>
<td>Bill's sales − S;
Moe's sales = 8S
S + 8S − 360
− Total sales in units
S = ? 8S = ?
S + 8S = 360; 9S = 360; S = 40
8S = 320
Verification:
LS = RS
40 + 320 = 360
360 = 360</td>
</tr>
<tr>
<td>Situation 5: Calculating unit and dollar sales (cost per unit) when total units not given, p. 121

Blue Furniture Company ordered sleepers ($300) and nonsleepers ($200) that cost $8,000. Blue expects sleepers to outsell nonsleepers 2 to 1. How many units of each were ordered? What were dollar costs of each?</td>
<td><table><tr><th>Unknown(s)</th><th>Variable(s)</th><th>Price</th><th>Relationship</th></tr><tr><td>Sleepers
Nonsleepers</td><td>2N
N</td><td>$300
200</td><td>600N
+200N
$8,000 total cost</td></tr></table></td>
<td>$600N + 200N = 8{,}000$
$\frac{800N}{800} = \frac{8{,}000}{800}$
$N = 10$ (nonsleepers)
$2N = 20$ (sleepers)
Check
10 × $200 = $2,000
20 × $300 = 6,000
= $8,000</td>
<td>N − Sales of nonsleepers,
2N − sales of sleepers
200N + 300 × 2 × N = 8,000
− Total cost; N = ? 2N = ?
600N + 200N = 8,000
800N = 8,000; N = 10; 2N = 20
Verification:
LS = RS
200 − 10 + 300 × 20 = 8,000
2,000 + 6,000 = 8,000
8,000 = 8,000</td>
</tr>
<tr>
<td>Situation 6: Calculating unit and dollar sales (cost per unit) when total units given, p. 121

Blue Furniture Company ordered 30 sofas (sleepers and nonsleepers) that cost $8,000. The wholesale unit cost was $300 for the sleepers and $200 for the nonsleepers. How many units of each were ordered? What were dollar costs of each?</td>
<td><table><tr><th>Unknown(s)</th><th>Variable(s)</th><th>Price</th><th>Relationship</th></tr><tr><td>Unit cost:
Sleepers
Nonsleepers</td><td>
S
30 − S</td><td>
$300
200</td><td>
300S
+200 (30 − S)
$8,000 total cost</td></tr></table>*When the total units are given, the higher-priced item (sleepers) is assigned to the variable first. This makes the mechanical steps easier to complete.</td>
<td>$300S + 200(30 - S) = 8{,}000$
$300S + 6{,}000 - 200S = 8{,}000$
$100S + 6{,}000 = 8{,}000$
$-6{,}000 \quad -6{,}000$
$\frac{100S}{100} = \frac{2{,}000}{100}$
$S = 20$
Nonsleepers = 30 − 20
= 10
Check
20($300) + 10($200) = $8,000
$6,000 + $2,000 = $8,000
$8,000 = $8,000</td>
<td>S − Sleepers ordered
30 − S − Nonsleepers ordered
300S + 200(30 − S)
= 8,000 Total cost
S = ? 30 − S = ?
300S + 6,000 − 200S = 8,000
100S + 6,000 = 8,000
100S = 2,000; S = 20
30 − S = 30 − 20 = 10
Verification:
LS = RS
300 × 20 + 200 × 10 = 8,000
6,000 + 2,000 = 8,000
8,000 = 8,000</td>
</tr>
<tr>
<td>Key terms</td>
<td>Constants, p. 115
Equation, p. 115
Equivalent equations, p.
Equation in one unknown, p.
Expression, p. 115</td>
<td>Formula, p. 115
Knowns, p. 115
Linear equation, p.
Members of Equation, p.
Root of Equation, p.</td>
<td>Unknown, p. 115
Variables, p. 115
Verification, p.</td>
</tr>
</table>

CRITICAL THINKING DISCUSSION QUESTIONS

1. Explain the difference between a variable and a constant. What would you consider your monthly car payment—a variable or a constant?
2. How does the opposite process rule help solve for the variable in an equation? If a Mercedes costs 3 times as much as a Saab, how could the opposite process rule be used? The selling price of the Mercedes is $60,000.
3. What is the difference between Word Problem Situations 5 and 6 in Learning Unit 5–2? Show why the more expensive item in Word Problem Situation 6 is assigned to the variable first.

END-OF-CHAPTER PROBLEMS

DRILL PROBLEMS (First of Three Sets)

Solve the unknown from the following equations:

5–1. $H + 15 = 70$
5–2. $B + 29 = 75$
5–3. $N + 50 = 290$
5–4. $Q - 60 = 850$
5–5. $5Y = 75$
5–6. $\frac{P}{6} = 92$
5–7. $8Y = 96$
5–8. $\frac{N}{16} = 5$
5–9. $4(P - 9) = 64$
5–10. $3(P - 3) = 27$

DRILL PROBLEMS (Second of Three Sets)

Solve the unknown from the following equations:

5–11. $6B = 420$
5–12. $7(A - 5) = 63$
5–13. $\frac{N}{9} = 7$
5–14. $18(C - 3) = 162$
5–15. $9Y - 10 = 53$
5–16. $7B + 5 = 26$

DRILL PROBLEMS (Third of Three Sets)

Solve the unknown from the following equations:

5–17. $B + 82 - 11 = 190$
5–18. $5Y + 15(Y + 1) = 35$
5–19. $3M + 20 = 2M + 80$
5–20. $20(C - 50) = 19{,}000$

ADDITIONAL DRILL PROBLEMS

5–21. Write equations for the following situations. Use N for the unknown number. Do not solve the equations.

a. Three times a number is 70.
b. A number increased by 13 equals 25.
c. Seven less than a number is 5.
d. Fifty-seven decreased by 3 times a number is 21.
e. Fourteen added to one-third of a number is 18.
f. Twice the sum of a number and 4 is 32.
g. Three-fourths of a number is 9.
h. Two times a number plus 3 times the same number plus 8 is 68.

5–22. Solve for the unknown number:

a. $B + 12 = 38$
b. $29 + M = 44$
c. $D - 77 = 98$
d. $7N = 63$
e. $\frac{X}{12} = 11$
f. $3Q + 4Q + 2Q = 108$
g. $H + 5H + 3 = 57$
h. $2(N - 3) = 62$
i. $\frac{3R}{4} = 27$
j. $E - 32 = 41$
k. $5(2T - 2) = 120$
l. $12W - 5W = 98$
m. $49 - X = 37$
n. $12(V + 2) = 84$
o. $7D + 4 = 5D + 14$
p. $7(T - 2) = 2T - 9$

WORD PROBLEMS (First of Three Sets)

5–23. The *Omaha World-Herald,* on January 23, 2001, ran an article titled "Lending Agency Convicted of Predatory Lending Practices." A loan company took the title to an elderly Bellevue widow's home by paying taxes of about $22,200. The market value of the house is $3\frac{1}{2}$ times the tax. What was the market value? Round to the nearest ten thousands.

5–24. A statistician compared the price of a 1955 Ford Thunderbird to that of a 2002 Ford Thunderbird. The 2002 Thunderbird's price is $34,595. This is 13 times as much as the selling price for the 1955 Thunderbird. What was the selling price of the 1955 Ford Thunderbird? (Round to the nearest hundred.)

5–25. Joe Sullivan and Hugh Kee sell cars for a Ford dealer. Over the past year, they sold 300 cars. Joe sells 5 times as many cars as Hugh. How many cars did each sell?

5–26. Nanda Yueh and Lane Zuriff sell homes for ERA Realty. Over the past 6 months they sold 120 homes. Nanda sold 3 times as many homes as Lane. How many homes did each sell?

5–27. Dots sells T-shirts ($2) and shorts ($4). In April, total sales were $600. People bought 4 times as many T-shirts as shorts. How many T-shirts and shorts did Dots sell? Check your answer.

5–28. Dots sells 250 T-shirts ($2) and shorts ($4). In April, total sales were $600. How many T-shirts and shorts did Dots sell? Check your answer. *Hint:* Let S = Shorts.

WORD PROBLEMS (SECOND OF THREE SETS)

5–29. On a flight from Vancouver to Toronto, Charter Airlines reduced its Internet price $130. The sale price was $299.50. What was the original price?

5–30. Fay, an employee at the Gap, budgets $\frac{1}{5}$ of her yearly salary for clothing. Fay's total clothing bill for the year is $8,000. What is her yearly salary?

5–31. Bill's Roast Beef sells 5 times as many sandwiches as Pete's Deli. The difference between their sales is 360 sandwiches. How many sandwiches did each sell?

5–32. A researcher compared the number of calories for the food sold by several fast-food chains. McDonald's "Big N' Tasty with Cheese" has almost 600 calories. This is 4 times as many as are in the "McSalad Shaker Chef Salad." How many calories are in the salad?

5–33. Computer City sells batteries ($3) and small boxes of pens ($5). In August, total sales were $960. Customers bought 5 times as many batteries as boxes of pens. How many of each did Computer City sell? Check your answer.

5–34. Staples sells cartons of pens ($10) and rubber bands ($4). Leona ordered a total of 24 cartons for $210. How many cartons of each did Leona order? Check your answer. *Hint:* Let P = Pens.

WORD PROBLEMS (THIRD OF THREE SETS)

5–35. On December 7, 2000, the *Chicago Sun-Times* ran an article comparing major media ad outlays in the millions of dollars (TV, print, radio, outdoor). In the year 2003, the expected outlay in North America will be $168.6. This is $1\frac{1}{4}$ times more than was spent in 1999. What was the total dollar outlay (in millions) during 1999? Round to nearest tenth.

5–36. At General Electric, shift $1\frac{1}{4}$ produced 4 times as much as shift 2. General Electric's total production for July was 5,500 jet engines. What was the output for each shift?

5–37. Ivy Corporation gave 84 people a bonus. If Ivy had given 2 more people bonuses, Ivy would have rewarded $\frac{2}{3}$ of the workforce. How large is Ivy's workforce?

5–38. Jim Murray and Phyllis Lowe received a total of $50,000 from a deceased relative's estate. They decided to put $10,000 in a trust for their nephew and divide the remainder. Phyllis received $\frac{3}{4}$ of the remainder; Jim received $\frac{1}{4}$. How much did Jim and Phyllis receive?

5–39. The first shift of GME Corporation produced $1\frac{1}{2}$ times as many lanterns as the second shift. GME produced 5,600 lanterns in November. How many lanterns did GME produce on each shift?

5–40. Wal-Mart sells thermometers ($2) and hot-water bottles ($6). In December, Wal-Mart's total sales were $1,200. Customers bought 7 times as many thermometers as hot-water bottles. How many of each did Wal-Mart sell? Check your answer.

5–41. Ace Hardware sells cartons of wrenches ($100) and hammers ($300). Howard ordered 40 cartons of wrenches and hammers for $8,400. How many cartons of each are in the order? Check your answer.

ADDITIONAL WORD PROBLEMS

5–42. A blue denim shirt at the Gap was marked down $15. The sale price was $30. What was the original price?

Unknown(s)	Variable(s)	Relationship

5–43. Goodwin's Corporation found that $\frac{2}{3}$ of its employees were vested in their retirement plan. If 124 employees are vested, what is the total number of employees at Goodwin's? Use the step approach. Verify your answer.

5–44. Eileen Haskin's utility and telephone bills for the month totalled $180. The utility bill was 3 times as much as the telephone bill. How much was each bill?

Unknown(s)	Variable(s)	Relationship

5–45. Ryan and his friends went to the golf course to hunt for golf balls. Ryan found 15 more than $\frac{1}{3}$ of the total number of golf balls that were found. How many golf balls were found if Ryan found 75 golf balls? Use the step approach. Verify your answer.

5–46. Linda Mills and Sherry Somers sold 459 tickets for the Advertising Club's raffle. If Linda sold 8 times as many tickets as Sherry, how many tickets did each one sell?

Unknown(s)	Variable(s)	Relationship

5–47. Jason Mazzola wanted to buy a suit at Giblee's. Jason did not have enough money with him, so Mr. Giblee told him he would hold the suit if Jason gave him a deposit of $\frac{1}{5}$ of the cost of the suit. Jason agreed and gave Mr. Giblee $79. What was the price of the suit? Use the step approach. Verify your answer.

5–48. Peter sold watches ($7) and necklaces ($4) at a flea market. Total sales were $300. People bought 3 times as many watches as necklaces. How many of each did Peter sell? What were the total dollar sales of each?

Unknown(s)	Variable(s)	Price	Relationship

5–49. Peter sold watches ($7) and necklaces ($4) at a flea market. Total sales for 48 watches and necklaces were $300. How many of each did Peter sell? What were the total dollar sales of each?

Unknown(s)	Variable(s)	Price	Relationship

5–50. A 3,000 piece of direct mailing cost $1,435. Printing cost is $550, about $3\frac{1}{2}$ times the cost of typesetting. How much did the typesetting cost? Round to the nearest cent.

Unknown(s)	Variable(s)	Price	Relationship

5–51. In 2003, Tony Rigato, owner of MRM, saw an increase in sales to $13.5 million. Rigato states that since 2000, sales have more than tripled. What were his sales in 2000? Use the step approach. Verify your answer.

CHALLENGE PROBLEMS

5–52. Jack Barney and Michelle Denny sold a total of 450 home entertainment centres at Circuit City. Jack sold 2 times as many units as Michelle. Michelle had $97,500 in total sales, which was $1\frac{1}{2}$ times as much as Jack's sales. **(a)** How many units did Michelle sell? **(b)** How many units did Jack sell? **(c)** What was the total dollar amount Jack sold? **(d)** What was the average amount of sales for Jack? **(e)** What was the average amount of sales for Michelle? Round both averages to the nearest dollar.

5–53. Bessy has 6 times as much money as Bob, but when each earns $6, Bessy will have 3 times as much money as Bob. How much does each have before and after earning the $6?

SUMMARY PRACTICE TEST

1. Railway Company reduced its round-trip first-class ticket price from Toronto to Montreal by $95. The sale price was $205.99. What was the original price? (p. 120)
2. Al Ring is an employee at Amazon.com. He budgets $\frac{1}{6}$ of his salary for clothing. If Al's total clothing for the year is $9,000, what is his yearly salary? (p. 120)
3. A local Best Buy sells 6 times as many DVDs as Future Shop. The difference between their sales is 150 DVDs. How many DVDs did each sell? (p. 120)
4. Working at Sharper Image, Joy Allen and Flo Ring sold a total of 900 scooters. Joy sold 4 times as many scooters as Flo. How many did each sell? (p. 121)
5. Kitchen Etc. sells sets of pots ($19) and dishes ($13) at a local charity. On the Canada Day weekend, Kitchen Etc.'s total sales were $1,780. People bought 4 times as many pots as dishes. How many of each did Kitchen Etc. sell? Check your answer. (p. 121)
6. Dominos sold a total of 1,300 small pizzas ($7) and hamburgers ($9) during the Super Bowl. How many of each did Dominos sell if total sales were $11,000? Check your answer. (p. 121)

SOLUTIONS TO PRACTICE QUIZZES

LU 5–1

1. a. $\frac{1}{2}Q - 9 = 14$ **b.** $8(Q + 31) = 50$ **c.** $10 - 2Q = 2$

d. $8Q - 2 = 21$ **e.** $4Q + 2 = 15$ **f.** $2Q - 8 = 4$

2. a.
$$\begin{aligned} B + 24 &= 60 \\ -24 &\quad -24 \\ B &= \mathbf{36} \end{aligned}$$

b.
$$\frac{\cancel{4}D}{\cancel{4}} = \frac{240}{4}$$
$$D = \mathbf{60}$$

c.
$$\frac{\cancel{12}B}{\cancel{12}} = \frac{144}{12}$$
$$B = \mathbf{12}$$

d.
$$\cancel{6}\left(\frac{B}{\cancel{6}}\right) = 50(6)$$
$$B = \mathbf{300}$$

e.
$$\begin{aligned} \frac{B}{4} + 4 &= 16 \\ -4 &\quad -4 \\ \frac{B}{4} &= 12 \\ \cancel{4}\left(\frac{B}{\cancel{4}}\right) &= 12(4) \\ B &= \mathbf{48} \end{aligned}$$

f.
$$\begin{aligned} 3(B - 8) &= 18 \\ 3B - 24 &= 18 \\ +24 &\quad +24 \\ \frac{\cancel{3}B}{\cancel{3}} &= \frac{42}{3} \\ B &= \mathbf{14} \end{aligned}$$

LU 5–2

1.

Unknown(s)	Variable(s)	Relationship
Original price	P*	P − $30 = Sale price Sale price = $90

*P = Original price.

MATHEMATICAL STEPS

$$\begin{aligned} P - \$30 &= \$90 \\ +30 &\quad +30 \\ P &= \mathbf{\$120} \end{aligned}$$

STEPS 1. & 2. *P:* Original price
D: Amount of reduction/discount = $30
S − Sale price = $90

STEP 3. $P - D = S$

STEP 4. $P - 30 = 90$
$P = 90 + 30$
$P = 120$

STEP 5. Verify the answer. The left side must equal the right side.
Left Side = Right Side
120 − 30 = 90
90 = 90

THE ANSWER IS CORRECT

2.

Unknown(s)	Variable(s)	Relationship
Yearly salary	S*	$\frac{1}{8}S$ Entertainment = $6,500

*S = Salary.

MATHEMATICAL STEPS

$$\frac{1}{8}S = \$6{,}500$$
$$8\left(\frac{S}{8}\right) = \$6{,}500(8)$$
$$S = \mathbf{\$52{,}000}$$

STEPS 1. & 2. *S:* Yearly salary = ?
$\frac{1}{8}S$: Entertainment budget
$\frac{1}{8}S = \$6{,}500$

STEP 3. $\frac{1}{8}S = 6{,}500$

STEP 4. $S = 6{,}500 \div \left(\frac{1}{8}\right)$
$S = 6{,}500 \times 8 = 52{,}000$

STEP 5. Verify the answer. The left side must equal the right side.
Left Side = Right Side
$\frac{1}{8} \times 52{,}000 = 6{,}500$
6,500 = 6,500

THE ANSWER IS CORRECT

3.

Unknown(s)	Variable(s)	Relationship
Micro	$5M$*	$5M$
Morse	M	$-\ M$
		20 computers

*M = Computers.

MATHEMATICAL STEPS

$$5M - M = 20$$

$$\frac{4M}{4} = \frac{20}{4}$$

$$M = 5 \text{ (Morse)}$$

$$5M = 25 \text{ (Micro)}$$

STEPS 1. & 2. *M:* Morse's sales
$5M$: Micro sales
$5M - M$: Difference in sales = 20
M = ?
$5M$ = ?

STEP 3. $5M - M = 20$

STEP 4. $4M = 20$
$M = 5$
$5M = 25$

STEP 5. Verify the answer. The left side must equal the right side.
Left Side = Right Side
25 − 5 = 20
20 = 20

THE ANSWER IS CORRECT

4.

Unknown(s)	Variable(s)	Relationship
Stoves sold:		
Susie	$5C$*	$5C$
Cara	C	$+\ C$
		180 stoves

*C = Stoves.

MATHEMATICAL STEPS

$$5C + C = 180$$

$$\frac{6C}{6} = \frac{180}{6}$$

$$C = 30 \text{ (Cara)}$$

$$5C = 150 \text{ (Susie)}$$

STEPS 1. & 2. *C:* Cara's sales
$5C$: Susie's sales
$C + 5C$: Total sales in units = 180

STEP 3. $C + 5C = 180$

STEP 4. $6C = 180$
$C = 30$
$5C = 150$

STEP 5. Verify the answer. The left side must equal the right side.
Left Side = Right Side
30 + 150 = 180
180 = 180

THE ANSWER IS CORRECT

5.

Unknown(s)	Variable(s)	Price	Relationship
Meatball	M	$6	$6M$
Cheese	$2M$	5	$+\ 10M$
			$1,600 total sales

MATHEMATICAL STEPS

$$6M + 10M = 1{,}600$$

$$\frac{16M}{16} = \frac{1{,}600}{16}$$

$$M = 100 \text{ (meatball)}$$

$$2M = 200 \text{ (cheese)}$$

CHECK

(100 × $6) + (200 × $5) = $1,600
$600 + $1,000 = $1,600
$1,600 = $1,600

STEPS 1. & 2. *M:* Sales of meatball pizzas
$2M$: Sales of cheese pizzas
$6M + 5 \times 2M$: Total sales in $ = $1,600
M = ?
$2M$ = ?

STEP 3. $6M + 10M = 1{,}600$

STEP 4. $16M = 1{,}600$
$M = 100$
$2M = 200$

STEP 5. Verify the answer. The left side must equal the right side.
Left Side = Right Side
6 × 100 + 5 × 200 = 1,600
1,600 = 1,600

THE ANSWER IS CORRECT

6.

Unknown(s)	Variable(s)	Price	Relationship
Unit sales:			
Meatball	M*	\$6	$6M$
Cheese	$300 - M$	5	$+ 5(300 - M)$
			\$1,600 total sales

*We assign the variable to the most expensive to make the mechanical steps easier to complete.

MATHEMATICAL STEPS

$$6M + 5(300 - M) = 1{,}600$$
$$6M + 1{,}500 - 5M = 1{,}600$$
$$M + 1{,}500 = 1{,}600$$
$$-1{,}500 \quad -1{,}500$$
$$M = 100$$

Meatball = 100

Cheese = 300 − 100 = 200

CHECK

100(\$6) + 200(\$5) = \$600 + \$1,000
= \$1,600

STEPS 1. & 2. M: Sales of meatball pizzas
$300 - M$: Sales of cheese pizzas
$6M + 5(300 - M)$: Total sales in \$ = \$1,600
$M = ?$
$300 - M = ?$

STEP 3. $6M + (300 - M) = 1{,}600$

STEP 4. $6M + 1{,}500 - 5M = 1{,}600$
$M + 1{,}500 = 1{,}600$
$M = 100$
$2M = 200$

STEP 5. Verify the answer. The left side must equal the right side.

Left Side	=	Right Side
$6 \times 100 + 5 \times (300 - 100)$	=	1,600
1,600	=	1,600

THE ANSWER IS CORRECT

Business Math Scrapbook

WITH INTERNET APPLICATION

Putting Your Skills to Work

Eager in the East

Top five advertisers in Vietnam, January through June.

COMPANY	PRODUCT	AD SPENDING (millions)
Unilever	Soap and shampoo	$10.70
Coca-Cola	Soft drinks	1.80
Procter & Gamble	Soap and shampoo	1.40
PepsiCo	Soft drinks	0.96
LG Group	Televisions	0.95

Source: ACNielsen estimates

PROJECT A

If Coca-Cola increases its advertising spending in Vietnam by 3 times for the next 6 months, what will it spend for the year?

Go to the Coca-Cola Web site and try to determine what Coca-Cola is spending on advertising today.

Goodyear to Cut Total of 650 Jobs At U.K. Facility

By Patricia Davis

Staff Reporter of The Wall Street Journal

Goodyear Tire & Rubber Co. said it will discontinue commercial truck-tire and mold production at the Dunlop Tyres U.K. plant in Birmingham, England, resulting in the loss of 650 jobs there.

The Akron, Ohio, company will integrate truck-tire and mold production within its recently completed joint venture with **Sumitomo Rubber Industries** Ltd. in Europe, which makes the Dunlop brand. Truck-tire production will be moved to joint-venture plants in the U.K., Germany and France. Mold production will shift to Goodyear's facility in Luxembourg.

The company said the action will result in substantial cost savings and synergies for the Goodyear and Dunlop joint venture.

PROJECT B

If the total number of employees at Goodyear's Birmingham plant were 11,581 after the layoff, what was the total employment before the layoff?

Go to the Goodyear Web site to see how their business is doing today.

Juice Processors Make a Bet on India's Market

By Rasul Bailay

Staff Reporter of The Wall Street Journal

NEW DELHI—Breakfast in India rarely includes orange juice. Most people prefer their wake-up tea with milk and believe that citrus and milk mixed in the same meal are bad for the stomach and sour a person's mood. And later in the day, most Indians would rather stop at an open-air stall than buy a bottle or carton from a supermarket.

Despite these cultural hurdles, juice processors are betting they have identified a potentially profitable niche and their products, appearing in stores nationwide, are aimed at India's growing middle class.

"The market is in its infancy," says Abhay Manglik, country manager at Tropicana Beverages Co., the local unit of Bradenton, Fla.-based Tropicana Products Inc. "But the potentials are huge. We expect the Indian market to grow at least five times by the year 2002."

PROJECT C

If the total market for orange juice in India is $35 million, what should the Indian market grow to in the year 2002?

Go to the Web and look up New Delhi. Try to find other cultural differences that may affect business corporations.

Internet Projects: See text website (www.mcgrawhill.ca/college/slater) and the Internet Resource Guide on the Student CD-ROM.