Appendix 5.A

DURATION: MEASURING THE LIFE OF A BOND

Bond prices rise when interest rates fall, and prices go down when interest rates rise. This gives rise to interest rate risk. But as Figure 5.8 illustrated, not all bonds are equally affected by changes in interest rates. We found that the 30-year bond's price was more sensitive to a change in the interest rate than the otherwise identical 3-year bond's price. We concluded that longer-term bonds have more interest rate risk than shorter-term bonds.

Up until now, we have considered the number of years to maturity as the measure of the length of a bond's life. This measure works well if the bond is a zero-coupon bond. A five-year, zero-coupon bond makes one payment after five years. However, if the bond pays coupons, it makes payments each year for five years. To say it has a five-year life is misleading. The average time to the bond's cash flow is less than five years. Would you say that a 5-year bond with a 10 percent coupon rate has an equal life to an otherwise identical 5-year bond with only a 5 percent coupon rate? With the 10 percent coupon, you receive greater payments during years 1 to 4 than with the 5 percent coupon. The higher the coupon, the sooner you get payment and the shorter the effective life of the bond.

duration A measurement of the life of a bond based on the average time to the bond's payments.

We need a new measure of the bond's life that recognizes that some of payments are made sooner than others. Bond analysts often use the concept of **duration** to describe the life of a bond.[1]

Duration is a weighted average of the time to each payment. The weight for the time of each payment equals the present value of the payment as a fraction of the bond's total value. If P_0 is the current value of an n-year bond, C_1, C_2, …,C_n are the cash flows for years 1 to n, the bond's duration is

$$\textbf{Duration} = 1 \times \frac{PV(C_1)}{P_0} + 2 \times \frac{PV(C_2)}{P_0} + 3 \times \frac{PV(C_3)}{P_0} + \ldots + n \times \frac{PV(C_n)}{P_0} \qquad (5.A1)$$

Table 5.A1 shows the duration calculation for a 5 percent, 5-year bond paying interest annually and a current price of \$957.88. The bond's yield to maturity is 6 percent. Column 3 shows the present value of each cash flow and column 4 shows its fraction of the bond's value. For example, the present value of the \$50 payment due in one year is \$47.17 (\$50/1.06). This payment represents 4.9 percent of the bond's present value, \$41.17/\$957.88. In other words, at the end of one year, 4.9 percent of the bond's current value will be received. The present value of the fifth-year payment is \$784.62, almost 82 percent of the bond's value.

To calculate a bond's duration, multiply the number of years to each payment by its proportion of the bond's value and add up the terms. The duration of the 5-year, 5 percent coupon bond is about 4.5 years. By contrast, the duration of a 5-year, 10 percent coupon bond with a current price of \$1,165.8 and a 6 percent yield to maturity is only 4.2 years. As expected, the higher the coupon rate, the shorter the effective maturity of the bond.

TABLE 5.A1
Calculation of duration for a 5 percent, 5-year bond paying interest annually. Bond current price is \$957.88 and its yield is 6 percent.

Year	Cash Flow	Present Value of Cash Flow	Present Value of Cash Flow as Proportion of Total Value	Proportion of Total Value × Time
1	50	50/(1.06) = 47.17	47.17/957.88 = 0.049	1 × .049 = 0.049
2	50	$50/(1.06)^2$ = 44.50	45.50/957.88 = 0.046	2 × .046 = 0.093
3	50	$50/(1.06)^3$ = 41.98	41.98/957.88 = 0.044	3 × .044 = 0.131
4	50	$50/(1.06)^4$ = 39.60	39.60/957.88 = 0.041	4 × .041 = 0.165
5	1,050	$1{,}050/(1.06)^5$ = 784.62	784.62/957.88 = 0.819	5 × .819 = 4.096
	Total =	957.88	1.00	4.535

[1] This measure is also known as Macaulay duration after its inventor. A good book on the topic of duration and bond yields is *Financial Markets Rates & Flows*, 6th ed., by James Van Horne (Prentice-Hall, 2001).

Checkpoint 5.A1

Following Table 5.A1, verify that the duration of a 5-year, 10 percent coupon bond with a current price of $1,165.8 and a 6 percent yield to maturity is 4.2 years.

Duration provides a useful way to compare the interest rate risk of bonds. The greater a bond's duration, the greater the bond's interest rate risk. Consider again the 5 percent and 10 percent coupon bonds. In Table 5.A2, the prices of each bond are shown as interest rates change from the current 6 percent. For a 50 basis point increase in the discount rate to 6.5 percent, the 5 percent coupon bond's price decreases 2.11 percent but the 10 percent coupon bond's price decreases only 2.01 percent. Likewise, a 50 basis point decrease in the discount rate results in a bigger percentage price increase for the 5 percent coupon bond. The results show that the longer a bond's duration, the more interest rate risk it bears.

TABLE 5.A2
Comparing the interest rate risk of bonds with different coupons but identical maturity. The 5% coupon bond's duration is greater than the 10% coupon bond. The 5% bond's interest rate sensitivity is also greater than the 10% bond's.

	5-year, 5% coupon rate bond (Duration = 4.5 years)		5-year, 10% coupon rate (Duration = 4.2 years)	
Discount Rate	**Bond Price**	**Percentage Change from Current Price**	**Bond Price**	**Percentage Change from Current Price**
5.5%	978.65	+2.17%	1,192.16	+2.03%
6% (current rate)	957.88	0	1,168.49	0
6.5%	937.66	–2.11%	1,145.45	–2.01%
Total percentage change		4.28%		4.04%

Duration is an important concept used in the management of interest rate risk. Suppose your firm has promised to make pension payments to retired employees. The discounted present value of these pension payments is $4 million. To meet the pension obligations, the company sets aside $4 million now and invests it in government bonds. However, as interest rates change, the present value of the pension liability and also of the bond investment change. To ensure that the value of the bonds in the fund is always sufficient to meet the pension liability, the bonds are selected to create a bond portfolio with the same duration as the pension obligation. This strategy of matching the duration of assets to liabilities is sometimes called portfolio immunization.

Questions and Problems

5.A1 Calculating Duration. The following bond prices were recorded on June 1, 2005. Assume each bond pays interest annually on June 1.

Bond	Price
Canada 7s of 2010	116.58
Canada 6.5s of 2011	111.56

a. What is the yield to maturity on each bond?

b. What is the duration of each bond?

c. Which bond has the greater interest rate risk?

5.A2 Calculating Duration. A star hockey player has negotiated a contract that pays him $5 million in the first year, $7 million in the second year, and $8 million in the third year. If his personal borrowing rate is 8 percent, calculate the duration of the contract.

Solutions to Check Point 5.A1

Year	Cash flow	Present Value of Cash Flow	Present Value of Cash Flow as Proportion of Total Value	Proportion of Total Value × Time
1	100	$100/(1.06) = 94.34$	94.34/ 1,168.49 = 0.081	1 × .081 = 0.081
2	100	$100/(1.06)^2 = 89.00$	89.00/ 1,168.49 = 0.076	2 × .076 = 0.152
3	100	$100/(1.06)^3 = 83.96$	83.96/ 1,168.49 = 0.072	3 × .072 = 0.216
4	100	$100/(1.06)^4 = 79.21$	79.21/ 1,168.49 = 0.068	4 × .068 = 0.271
5	1,100	$1{,}100/(1.06)^5 = 821.98$	821.98/ 1,168.49 = 0.703	5 × .703 = 3.517
	Total =	1168.49	1.000	4.237