Appendix 8.A

DERIVING THE CCA TAX SHIELD

Earlier, in Section 8.6, we saw that under Canada's CCA system, the tax shield from investing in an asset can continue in perpetuity. Equation 1, page 255, provided a formula to compute the present value of this perpetual tax shield. In the following discussion, we will explain how Equation 1 was arrived at. We will continue with the notation used earlier; that is,

C = capital cost of an asset acquired at the beginning of Year 1
d = CCA rate for the asset class to which the asset belongs
UCC_t = undepreciated capital cost in Year t after deducting CCA for the year
T_c = the firm's tax rate
r = discount rate
S = salvage amount from the sale of the asset at the end of Year t

We begin by making two simplifying assumptions, which will be relaxed later in the discussion. The assumptions are to ignore the half-year rule and the rules governing the disposal of assets. Suppose you start a new asset class by buying an asset. Our assumptions will enable you to get your first CCA tax shield from the asset in Year 1. The UCC of the class starts out at the beginning of Year 1 and is denoted $UCC_0 = C$.

Table 8.A1 provides details regarding yearly beginning and ending UCC balances and the CCA, which is obtained as the product of the beginning UCC and the CCA rate, d. So, for instance, the CCA in Year 1 (CCA^1) is $d \times UCC_0 = Cd$.

After deducting CCA^1, the UCC at the end of Year 1 becomes

$$\begin{aligned} UCC_1 &= UCC_0 - CCA_1 \\ &= C - Cd \\ &= C(1 - d) \end{aligned}$$

Suppose C is \$200,000 and d is 10 percent, then CCA_1 is \$200,000 × .1 = \$20,000 and UCC_1 is \$200,000 × (1 – .1) = \$180,000. Similarly, the CCA for Year 2 is

$$CCA_2 = d \times UCC_1 = Cd(1 - d),$$

which is

$$\$20{,}000 \times (1 - .1) = \$200{,}000 \times .1 \times (1 - .1) = \$18{,}000$$

for our example. In Year t, you will have taken $t - 1$ previous CCA tax shields, so the UCC at the start of the year is $UCC_{t-1} = C(1 - d)^{t-1}$. The CCA for Year t will be

$$d \times UCC_{t-1} = Cd(1 - d)^{t-1}$$

For instance, if you take t to be 15 years, then, not surprisingly, the UCC balance in Year 15 will be quite small at

$$\$200{,}000 \times .1 \times (1 - .1)^{14} = \$4{,}575.36$$

Until now, given the simplifying assumptions, we show that in a declining balance system, each asset will generate a stream of CCA amounts that will continue in perpetuity. Our algebra-

TABLE 8.A1
UCC and CCA

	Beginning of Year		End of Year
Year	UCC	CCA	UCC
1	UCC_0	Cd	$UCC_1 = C(1 - d)$
2	UCC_1	Cd(1 – d)	$UCC_2 = C(1 - d)^2$
3	UCC_2	$Cd(1 - d)^2$	$UCC_3 = C(1 - d)^3$
–			
t	UCC_t	$Cd(1 - d)^{t-1}$	$UCC_t = C(1 - d)^t$

ic expression also enables us to calculate the CCA and UCC amounts for any given year. Earlier we established that the CCA adds value to a capital investment project by reducing the firm's tax bill. A CCA stream in perpetuity will generate an infinite stream of tax savings. We now need to calculate the present value of this infinite stream of tax savings—that is, the PV of the CCA tax shield. For a given year, the CCA tax shield is simply the product of the year's CCA and the firm's tax rate, or CCA × T_c. A project will therefore generate the following CCA tax shields for years 1 through t:

Year:	**1**	**2**	**3**	**—**	***t***	**—**
CCA tax shield = CCA × T_c	CdT_c	$CdT_c(1-d)$	$CdT_c(1-d)^2$	—	$CdT_c(1-d)^{t-1}$	—

The CCA tax shield forms a perpetuity that is growing at the rate $-d$, which means, in effect, it is declining at this rate. Conceptually, this is like the constant growth dividend we looked at in Section 6.4. Remember, when the dividend grows at a constant rate g, we arrive at the stock price as the present value of expected future dividends as follows:

$$\text{Stock price} = \frac{\text{DIV}_1}{(r-g)}$$

Similar to the stock value, the CCA tax shield has the following present value:

$$\text{PV of CCA tax shield} = \frac{CdT_c}{(r+d)}$$

To determine the value added to an investment project by the tax savings generated by CCA, we can add the present value of the perpetual tax shield to the present value of the "after-tax" revenues and expenses to get the overall present value of the project.

Let us now relax our two simplifying assumptions. Until now, we had ignored the half-year rule of Canadian tax law requiring that, in the year of purchase of a depreciable asset, only half of the asset's value is added to the asset class balance. The remaining half is added in the following year. When we consider the half-year rule, the present value of the CCA tax shield has to be computed for the two halves separately and then added up. To see how this is done, let us find the CCA stream on the two halves of the asset value, as follows:

Year:	**1**	**2**	**3**	**...**	***t***	**...**
CCA on first half of asset value	$Cd/2$	$Cd(1-d)/2$	$Cd(1-d)^2/2$	...	$Cd(1-d)^t/2$	...
CCA on second half of asset value	0	$Cd/2$	$Cd(1-\text{d})/2$	...	$Cd(1-\text{d})^{t-1}/2$	...

If we multiply by T_c then the PV of perpetual CCA tax shields for the first half becomes

$$\frac{1}{2}\left[\frac{CdT_c}{1+r} + \frac{CdT_c(1-d)}{(1+r)^2} + \frac{CdT_c(1-d)^2}{(1+r)^3} + \cdots t\,\frac{CdT_c(1-d)^{t-1}}{(1+r)^t}\right] = \frac{1}{2}\left[\frac{CdT_c}{r+d}\right]$$

Notice that in the second half below, the same CCA stream (and CCA tax shields) are deferred one year.

$$\frac{1}{2}\left[\frac{CdT_c}{(1+r)^2} + \frac{CdT_c(1-d)}{(1+r)^3} + \frac{CdT_c(1-d)^2}{(1+r)^4} + \cdots t\,\frac{CdT_c(1-d)^{t-2}}{(1+r)^t}\right]$$

To find the present value of the second half, we have to discount the CCA tax shields back to time 0:

$$\frac{1}{2}\left[\frac{CdT_c}{r+d}\right]\left[\frac{1}{1+r}\right]$$

When we add up the two present values we get the total present value of the CCA tax shield under the half-year rule:

$$\frac{1}{2}\left[\frac{CdT_c}{r+d}\right] + \frac{1}{2}\left[\frac{CdT_c}{r+d}\right]\left[\frac{1}{1+r}\right]$$

Using a bit of algebra, this formula can be simplified to provide Equation 1, which we discussed earlier:

$$= \frac{1}{2}\left[\frac{CdT_c}{r+d}\right]\left[1 + \frac{1}{1+r}\right]$$

$$= \frac{1}{2}\left[\frac{CdT_c}{r+d}\right]\left[\frac{1+r+1}{1+r}\right]$$

$$= \frac{CdT_c}{r+d}\left[\frac{1+.5r}{1+r}\right]$$

This general formula enables us to compute the present value of the tax shield on CCA under the half-year rule. Relaxing our second assumption, when a residual or salvage value arising from the sale of an asset is introduced into the discussion, we get the present value of lost CCA tax shields due to salvage value, as discussed in Section 8.6 and shown as Equation 2, page 255.

When we combine equations 1 and 2, we get a general formula for the present value of the CCA tax shield, which we presented earlier as Equation 3, page 255:

Present value of CCA tax shield =

$$\left[\frac{CdT_c}{r+d}\right]\left[\frac{1+.5r}{1+r}\right] - \left[\frac{SdT_c}{(d+r)}\right]\left[\frac{1}{(1+r)^t}\right]$$

Solutions to Excel Spreadsheet Model Questions

1.

Year:	0	1	2	3	4	5	6	…∞
Capital investment	10,000							
Working capital	1,000	3,550	3,728	3,914	4,110	3,039	0	
Change in working capital	1,000	2,550	178	186	196	–1,071	–3,039	
Revenues		15,000	15,750	16,538	17,364	18,233		
Expenses		10,000	10,500	11,025	11,576	12,155		
Profit before tax (excluding CCA tax shield)		5,000	5,250	5,513	5,788	6,078		
Tax (35%)		1,750	1,838	1,929	2,026	2,127		
Operating cash flows (excluding CCA tax shield)		3,250	3,413	3,583	3,762	3,950		
Salvage value							0	
Total cash flow (excluding CCA tax shield)	–11,000	700	3,235	3,397	3,567	5,021	3,039	
PV of cash flow (excluding CCA tax shield)	–11,000	625	2,579	2,418	2,267	2,849	1,540	
Present value (excluding CCA tax shield) (A)	1,277							
CCA[1]		1,500	2,550	1,785	1,250	875	612	→ 0
CCA tax shield[1]		525	893	625	437	306	214	→ 0
PV of CCA tax shield[1] (B)	2,366							
Net present value (A) + (B)	3,643							

Notes: [1] The CCA and the CCA tax shield will continue even after Year 6. The PV of the CCA tax shield has been calculated assuming that the CCA and the CCA tax shield will continue in perpetuity.

2.

Year:	0	1	2	3	4	5	6	…∞
Capital investment	10,000							
Working capital	1,500	4,000	4,000	4,000	4,000	2,500	0	
Change in working capital	1,500	2,500	0	0	0	–1,500	–2,500	
Revenues		15,000	15,000	15,000	15,000	15,000		
Expenses		10,000	10,000	10,000	10,000	10,000		
Profit before tax (excluding CCA tax shield)		5,000	5,000	5,000	5,000	5,000		
Tax (35%)		1,750	1,750	1,750	1,750	1,750		
Operating cash flows (excluding CCA tax shield)		3,250	3,250	3,250	3,250	3,250		
Salvage value							0	
Total cash flow (excluding CCA tax shield)	–11,500	750	3,250	3,250	3,250	4,750	2,500	
PV of cash flow (excluding CCA tax shield)	–11,500	701	2,839	2,653	2,479	3,387	1,666	
Present value (excluding CCA tax shield) (A)	2,225							
CCA[1]		1,500	2,550	1,785	1,250	875	612	→ 0
CCA tax shield[1]		525	893	625	437	306	214	→ 0
PV of CCA tax shield[1] (B)	2,745							
Net present value (A) + (B)	4,970							

Notes: [1] The CCA and the CCA tax shield will continue even after Year 6. The PV of the CCA tax shield has been calculated assuming that the CCA and the CCA tax shield will continue in perpetuity.

Although the real discount rate is barely affected by the change in inflation, the real value of CCA and the present value of the CCA tax shield increase, which increases project NPV.

3.

Year:	0	1	2	3	4	5	6
Capital investment	10,000						
Working capital	1,500	4,075	4,279	4,493	4,717	3,039	0
Change in working capital	1,500	2,575	204	214	225	–1,679	–3,039
Revenues		15,000	15,750	16,538	17,364	18,233	
Expenses		10,000	10,500	11,015	11,576	12,155	
Profit before tax (excluding depreciation tax shield)		5,000	5,250	5,513	5,788	6,078	
Tax (35%)		1,750	1,838	1,930	2,026	2,127	
Operating cash flows (excluding depreciation tax shield)		3,250	3,412	3,583	3,762	3,950	
Salvage value							0
Total cash flow (excluding depreciation tax shield)	–11,500	675	3,209	3,369	3,538	5,629	3,039
PV of cash flow (excluding depreciation tax shield)	–11,500	603	2,558	2,398	2,248	3,194	1,540
Present value (excluding depreciation tax shield) (A)	1,041						
Depreciation		2,000	2,000	2,000	2,000	2,000	
Depreciation tax shield	0	700	700	700	700	700	0
PV of depreciation tax shield	0	625	558	498	445	396	0
Total present value of depreciation tax shield (B)	2,523						
Net present value (A) + (B)	3,564						

It is worthwhile noting that often large corporations keep two sets of books, one for shareholders and one for Canada Revenue Agency (CRA). It is common to use straight-line depreciation on the shareholder books and the CCA system on the tax books. Only the CCA recorded in the tax books is relevant in capital budgeting.