SUPPLEMENT TO

CHAPTER 4

Reliability

LEARNING OBJECTIVES

After completing this supplement, you should be able to:

1 Define reliability.
2 Perform simple reliability computations.
3 Explain the purpose of redundancy in a system.

SUPPLEMENT OUTLINE

INTRODUCTION

Reliability is a measure of the ability of a product, part, or system to perform its intended function under a prescribed set of conditions. In effect, reliability is a *probability*.

reliability The ability of a product, part, or system to perform its intended function under a prescribed set of conditions.

Suppose that an item has a reliability of .90. This means that it has a 90-percent probability of functioning as intended. The probability it will fail, i.e., its failure rate, is $1 - .90 = .10$, or 10 percent. Hence, it is expected that, on the average, 1 out of every 10 such items will fail or, equivalently, that the item will fail, on average, once in every 10 trials. Similarly, a reliability of .985 implies 15 failures per 1,000 parts or trials.

QUANTIFYING RELIABILITY

Reliability is used in two ways.

1. The probability that the product or system will function when activated (instantaneous reliability).
2. The probability that the product or system will function for a given length of time (continuous reliability).

The first of these focuses on *one point in time* and is often used when a product, part, or system must operate for one time or a relatively few number of times. The second of these focuses on the *length of service.* The distinction will become more apparent as each of these approaches is described in more detail.

The probability that a system, part, or product will operate as planned is an important concept in system and product design. Determining that probability when the product or system consists of a number of *independent* components requires the use of the rules of probability for independent events. Independent events have no relation to the occurrence or nonoccurrence of each other. What follows are three examples illustrating the use of probability rules to determine whether a given product or system will operate successfully.

Rule 1. If two or more events are independent and "success" is defined as the probability that all of the events occur, then the probability of success is equal to the product of the probabilities of the events.

Example. Suppose a room has two lamps, but to have adequate light both lamps must work (success) when turned on. One lamp has a probability of working of .90, and the other has a probability of working of .80. The probability that both will work is $.90 \times .80 = .72$. Note that the order of multiplication is unimportant: $.80 \times .90 = .72$ also. Also note that if the room had three lamps, three probabilities would have been multiplied.

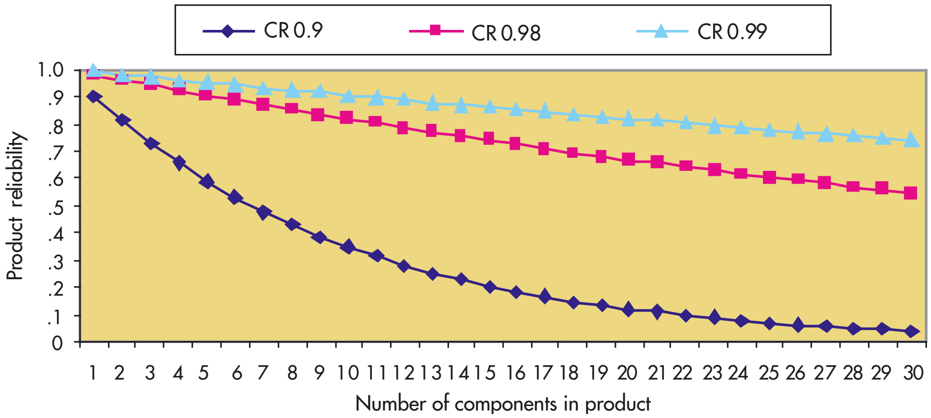
This system can be represented by the following diagram:

Even though the individual components of a system might have high reliability, the system as a whole can have considerably less reliability because all components that are in series (as are the ones in the preceding example) must function. As the number of components in a series increases, the system reliability decreases. For example, a system that has eight components in series, each with a reliability of .99, has a reliability of only $.99^8 = .923$. See Figure 4S–1 for plots of product reliability as a function of number of its components for selected component reliability CR.

Obviously, many products and systems have a large number of component parts that must all operate, and some way to increase overall reliability is needed. One approach

FIGURE 4S-1

Relating product and component reliabilities



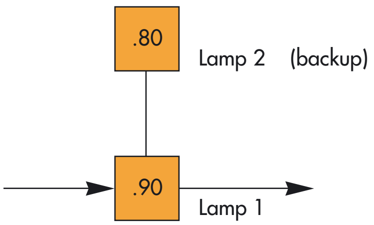
is to overdesign, i.e., enhancing the design to avoid a particular type of failure. For example, using a more durable and higher quality but more expensive material. Another is design simplification, i.e., reducing the number of parts in a product. The third approach is to use **redundancy** in the design. This involves providing backup components for some items.

redundancy The use of backup components to increase reliability.

Rule 2. If two events are independent and "success" is defined as the probability that *at least one* of the events will occur, the probability of success is equal to the probability of either one plus (1.00 − that probability) multiplied by the other probability.

Example. There are two lamps in a room. When turned on, one has a probability of working of .90 and the other has a probability of working of .80. Only a single lamp is needed to light for success. If one fails to light when turned on, the other lamp is turned on. Hence, one of the lamps is a backup in case the other one fails. Either lamp can be treated as the backup; the probability of success will be the same. The probability of success is .90 + (1 − .90) × .80 = .98. If the .80 lamp is first, the calculation would be .80 + (1 − .80) × .90 = .98.

This system can be represented by the following diagram.

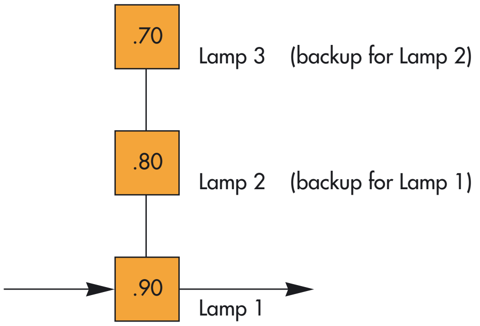


Rule 3. If three events are involved and success is defined as the probability that at least one of them occurs, the probability of success is equal to the probability of the first one (any of the events), plus the product of (1.00 − that probability) and the probability of the second event (any of the remaining events), plus the product of (1.00 − the first probability) and (1 − the second probability) and the probability of the third event. This rule can be expanded to cover more than three events.

Example. Three lamps have probabilities of .90, .80, and .70 of lighting when turned on. Only one lighted lamp is needed for success; hence, two of the lamps are considered to be backups. The probability of success is:

[#1 operates]		[#1 fails	and	#2 operates]		[#1 fails	and	#2 fails	and	#3 operates]	
.90	+	(1 − .90)	×	.80	+	(1.00 − .90)	×	(1.00 − .80)	×	.70	= .994

This system can be represented by the following diagram:

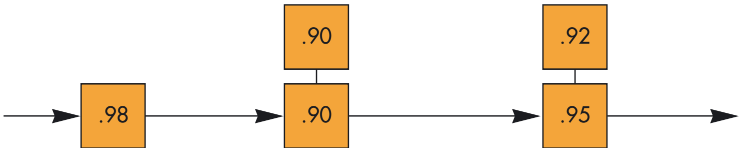


Alternatively, the system reliability = 1 − probability that none of the components work

$$= 1 - (1 - .90)(1 - .80)(1 - .70) = .994$$

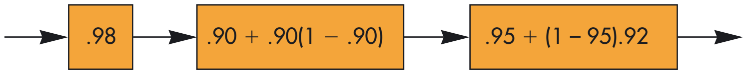
Example S–1

Determine the reliability of the system shown below.



Solution

The system can be reduced to a series of three components:



The system reliability is, then, the product of these:

$.98 \times .99 \times .996 = .966$

The second way of looking at reliability considers the incorporation of a time dimension: probabilities are determined relative to a specified length of time. This approach is commonly used in product warranties, which pertain to a given period of time after purchase of a product.

In this case, failure rate per unit is defined as the number of failures divided by total operating time.

Example S–2

Two hundred units of a particular component were subjected to an accelerated life test equivalent to 2,500 hours of normal use. One failed after 1,000 hours and another after 2,000 hours. All others were working at the conclusion of the test.

The failure rate per unit = 2/[198 (2,500) + 1000 + 2000] = 0.000004 per hour

Note that this formula assumes constant failure rate over time.

A typical profile of product failure rate over time is illustrated in Figure 4S–2. Because of its shape, it is sometimes referred to as a bathtub curve. Usually, a number of products fail shortly after they are put into service, not because they wear out, but because they are defective to begin with. Examples include electronics components such as capacitators. The rate of failures decreases rapidly as the truly defective items are weeded out. During the second phase, there are fewer failures because most of the defective items have been eliminated, and it is too soon to encounter items that fail because they have worn out. In some cases, this phase covers a relatively long period of time. In the third phase, failures occur because the products are worn out, and the failure rate increases.

FIGURE 4S-2

Failure rate is generally a function of time (the bathtub curve)

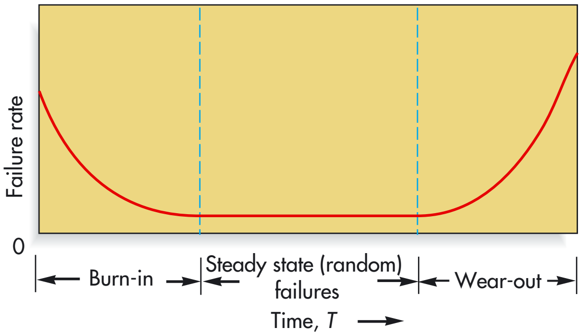


FIGURE 4S-3

Number of light bulbs remaining over time

Source: S. M. Shafer and J. R. Meredith. *Operations Management* (New York: John Wiley & Sons Inc., 1998), p. 781. Reprinted with permission of John Wiley & Sons Inc.

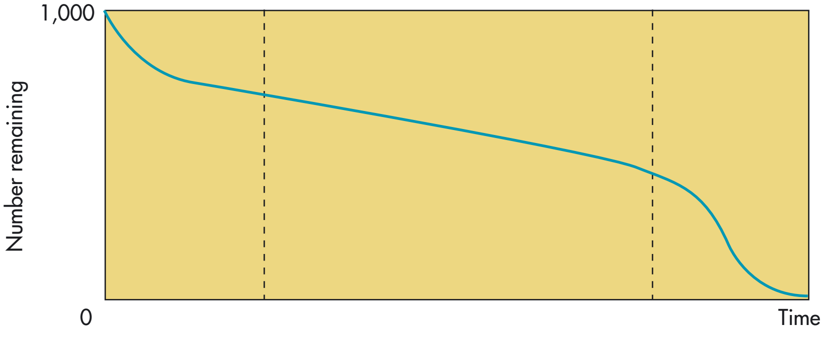
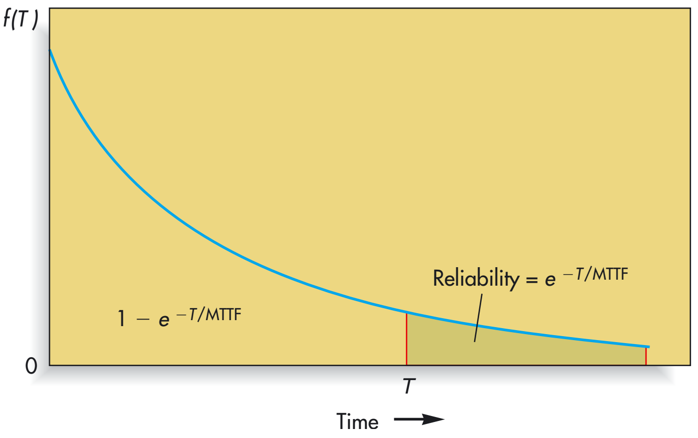


FIGURE 4S-4

An Exponential distribution



If 1,000 light bulbs are tested, the number remaining over time will be as shown in Figure 4S–3, where initially and at the end there are sharp drops in the number of bulbs working.

mean time to failure (MTTF) The average length of time before failure of a product or component.

The inverse of failure rate per unit time is **Mean Time to Failure (MTTF)**. For data in Example S–2,

MTTF = 1/Failure rate per unit time = 1/0.000004 = 250,000 hours.

Note that this formula assumes that failure rate is constant and there is no wear-out period. In many cases, the useful life of the component restricts MTTF.

For repairable items, a similar term, Mean Time between Failure (MTBF), is usually used. MTBF is the average time from a failure to the next failure. It includes the repair time.

After the burn-in period and before the wear-out, it often turns out that the time to failure can be modelled by an Exponential distribution with an average equal to MTTF, such as that depicted in Figure 4S–4. Equipment failures as well as product failures may occur in this pattern. The probability that equipment or a product put into service at time 0 will fail *before* some specified time, T, is equal to the area under the curve between 0 and T. Reliability is specified as the probability that a product will last *at least until* time T; reliability is equal to the area under the curve *beyond* T. (Note that the total area under the curve is 100 percent.) Observe that as the specified length of service increases, the area under the curve to the right of that point (i.e., the reliability) decreases.

Determining values for the area under a curve to the right of a given point, T, becomes a relatively simple matter using a table of Exponential values. An Exponential distribution is completely described using a single parameter, the distribution mean, the mean time to failure. Using the symbol T to represent length of service, the probability that failure will not occur before time T (i.e., the area in the right tail) is easily determined:

$$P(\text{no failure before } T) = e^{-T/\text{MTTF}}$$

where

$e = 2.7183$

T = Length of service before failure

MTTF = Mean time to failure

The probability that failure will occur before time T is 1.00 minus that amount:

$$P(\text{failure before } T) = 1 - e^{-T/\text{MTTF}}$$

Selected values of $e^{-T/\text{MTTF}}$ are listed in Table 4S–1.

TABLE 4S–1

Values of $e^{-T/MTTF}$

T/MTTF	$e^{-T/MTTF}$	T/MTTF	$e^{-T/MTTF}$	T/MTTF	$e^{-T/MTTF}$
.10	.9048	2.60	.0743	5.10	.0061
.20	.8187	2.70	.0672	5.20	.0055
.30	.7408	2.80	.0608	5.30	.0050
.40	.6703	2.90	.0550	5.40	.0045
.50	.6065	3.00	.0498	5.50	.0041
.60	.5488	3.10	.0450	5.60	.0037
.70	.4966	3.20	.0408	5.70	.0033
.80	.4493	3.30	.0369	5.80	.0030
.90	.4066	3.40	.0334	5.90	.0027
1.00	.3679	3.50	.0302	6.00	.0025
1.10	.3329	3.60	.0273	6.10	.0022
1.20	.3012	3.70	.0247	6.20	.0020
1.30	.2725	3.80	.0224	6.30	.0018
1.40	.2466	3.90	.0202	6.40	.0017
1.50	.2231	4.00	.0183	6.50	.0015
1.60	.2019	4.10	.0166	6.60	.0014
1.70	.1827	4.20	.0150	6.70	.0012
1.80	.1653	4.30	.0136	6.80	.0011
1.90	.1496	4.40	.0123	6.90	.0010
2.00	.1353	4.50	.0111	7.00	.0009
2.10	.1255	4.60	.0101		
2.20	.1108	4.70	.0091		
2.30	.1003	4.80	.0082		
2.40	.0907	4.90	.0074		
2.50	.0821	5.00	.0067		

Example S-3

By means of extensive testing, a manufacturer has determined that its Super Sucker Vacuum Cleaner models have an expected life that is Exponential with a mean of four years and insignificant burn-in period. Find the probability that one of these cleaners will have a life that ends:

a. After the initial four years of service.

b. Before four years of service are completed.

c. Not before six years of service.

Solution

MTTF = 4 years

a. $T = 4$ years:

$$T/\text{MTTF} = \frac{4\text{ years}}{4\text{ years}} = 1.0$$

From Table 4S–1, $e^{-1.0} = .3679$.

b. The probability of failure before $T = 4$ years is $1 - e^{-1}$, or $1 - .3679 = .6321$.

c. $T = 6$ years:

$$T/\text{MTTF} = \frac{6\text{ years}}{4\text{ years}} = 1.50$$

From Table 4S–1, $e^{-1.5} = .2231$.

Mechanical items such as ball bearings, valves, and springs tend to have little burn-in and steady-state periods, and start to wear out right away.

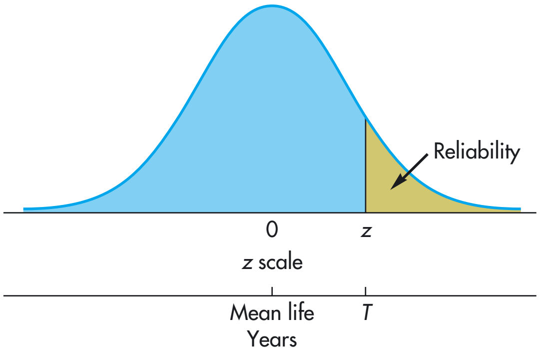
Product failure due to wear-out can sometimes be modelled by a Normal distribution. Obtaining probabilities involves the use of a table (refer to Appendix B, Table B). The table provides areas under a Normal curve from the left end of the curve to a specified point z, where z is a *standardized* value computed using the formula:

$$z = \frac{T - \text{Mean wear-out time}}{\text{Standard deviation of wear-out time}}$$

Thus, to work with the Normal distribution, it is necessary to know the mean of the distribution and its standard deviation. A Normal distribution is illustrated in Figure 4S–5. Appendix B, Table B contains Normal probabilities (i.e., the area that lies to the left of z). To obtain a probability that service life will not exceed some value T, compute z and refer to the table. To find the reliability for time T, subtract this probability from 100 percent. To obtain the value of T that will provide a given probability, locate the nearest probability under the curve *to the left* in Table B. Then use the corresponding z in the preceding formula and solve for T.

FIGURE 4S-5

A Normal curve



Example S–4

The mean life of a certain ball bearing can be modelled using a Normal distribution with a mean of six years and a standard deviation of one year. Determine each of the following:

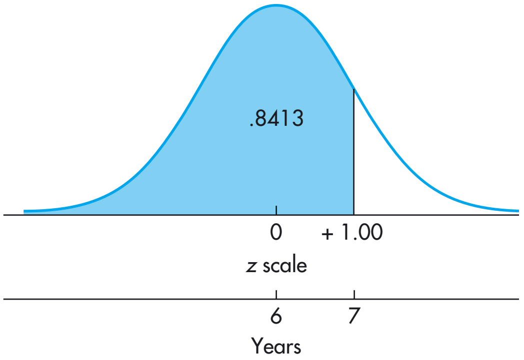
a. The probability that a ball bearing will fail *before* seven years of service.
b. The probability that a ball bearing will fail *after* seven years of service (i.e., find its reliability).
c. The service life that will provide a failure probability of 10 percent.

Solution

Wear-out life mean = 6 years
Wear-out life standard deviation = 1 year
Wear-out life is Normally distributed

a. Compute z and use it to obtain the probability directly from Appendix B, Table B (see diagram).

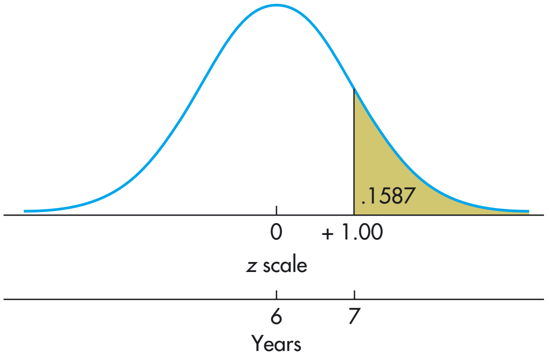
$$z = \frac{7-6}{1} = +1.00$$



Thus, $P(T < 7) = .8413$.

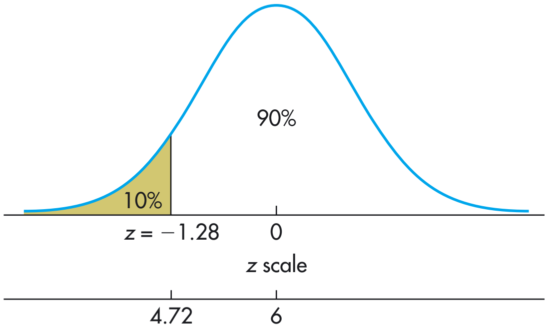
b. Subtract the probability determined in part *a* from 100 percent (see diagram).

$$1.00 - .8413 = .1587$$



c. Use the Normal table and find the value of z that corresponds to an area under the curve (starting from the left side) of 10% (see diagram).

$$z = -1.28 = \frac{T-6}{1}$$



Solving for T, we find $T = 4.72$ years.

AVAILABILITY

availability The fraction of time a piece of equipment is expected to be available for operation.

A related measure of importance to customers, and hence to designers, is **availability**. It measures the fraction of time an equipment or a repairable product is expected to be operational (as opposed to being down for repairs). Availability can range from zero (never available) to 1.00 (always available). Companies that can offer equipment with high availability have a competitive advantage over companies that offer equipment with lower availability. Availability is a function of both the mean time to failure and the mean time to repair. We assume that there is little delay before a failed item begins to be repaired. The availability factor can be computed using the following formula:

$$Availability = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

mean time to repair The average length of time to repair a failed item.

where

MTTF = Mean time to failure

MTTR = **Mean time to repair**

Example S-5

A copier is expected to be able to operate for 200 hours after repair, and the mean repair time is expected to be two hours. Determine the availability of the copier.

Solution

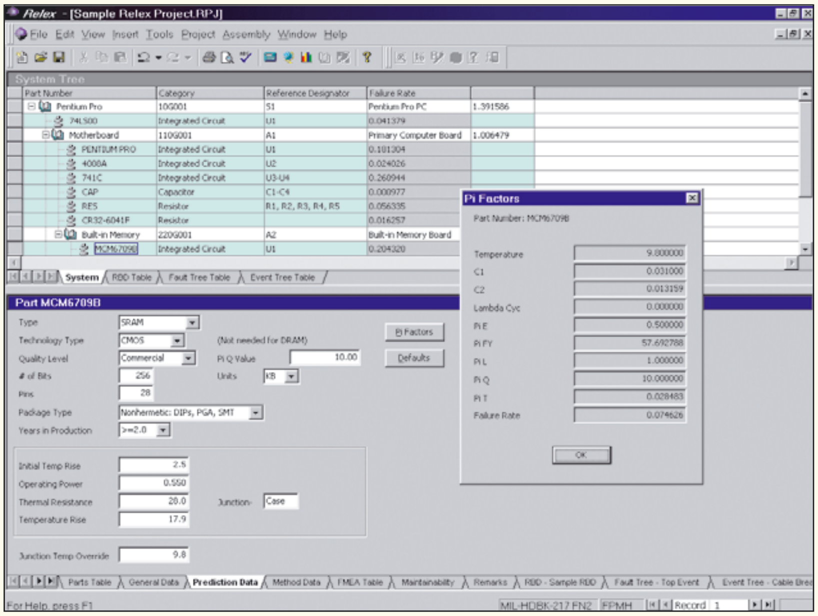
MTTF = 200 hours, and MTTR = 2 hours

$$Availability = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} = \frac{200}{200 + 2} = .99$$

Relex Software

Relex is a reliability software company based in Greensburg, Pennsylvania. Its software assists in designing products and parts in order to reduce chance of failure, reliability data collection, estimating failure rate and mean time between failures, finding the cause of failure, and many other reliability activities. Below is a screenshot of the bill of materials of the processing unit of a computer, with component failure rate (per million hour) provided from the library of the software, based on quality and expected operating conditions such as temperature.

Source: www.relexsoftware.com/customers/cs/images/sciatl_pred_lg.gif. Image courtesy Relex Software Corporation.



Two implications for design are revealed by the availability formula. One is that availability increases as the mean time to failure increases. The other is that availability also increases as the mean time to repair decreases. It would seem obvious that designers would want to design products that have a long time to failure. In addition, some design options enhance repairability. Laser printers, for example, are designed with print cartridges that can be easily replaced.

KEY TERMS

Solved Problems

Problem 1

A product design engineer must decide if a redundant component is cost-justified in a certain system. The system in question has a critical component with a probability of .98 of operating. System failure would involve a cost of \$20,000. For a cost of \$100, a switch and backup component could be added that would automatically transfer the control to the backup component in the event of a failure. Should the backup be added if the backup operating probability is also .98?

Solution

Because no probability is given for the switch, we will assume that its probability of operating when needed is 100 percent. The expected cost of failure (i.e., without the backup) is \$20,000 (1 − .98) = \$400.

With the backup, the probability of *not* failing would be:

$$.98 + .02(.98) = .9996$$

Hence, the probability of failure would be 1 − .9996 = .0004. The expected cost of failure with the backup would be the added cost of the backup plus the failure cost:

$$\$100 + \$20{,}000(.0004) = \$108$$

Because this is less than the cost without the backup, adding the backup is definitely cost justified.

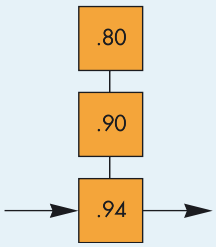
Problem 2

Due to the extreme cost of interrupting production, a firm has two standby machines available in case a particular machine breaks down. The machine in use has a reliability of .94, and the backups have reliabilities of .90 and .80. In the event of a failure, either backup can be brought into service. If one fails, the other backup can be used. Compute the system reliability.

Solution

$$R1 = .94, \quad R2 = .90, \quad \text{and} \quad R3 = .80$$

The system can be depicted in this way:



$$R_{\text{system}} = R_1 + R_2(1 - R_1) + R_3(1 - R_2)(1 - R_1)$$
$$= .94 + .90(1 - .94) + .80(1 - .90)(1 - .94) = .9988$$

Problem 3

A hospital has three *independent* fire alarm systems, with reliabilities of .95, .97, and .99. In the event of a fire, what is the probability that a warning would be given?

Solution

A warning would *not* be given if all three alarms failed. The probability that at least one alarm would operate is 1 − P(none operate):

$$P(\text{none operate}) = (1 - .95)(1 - .97)(1 - .99) = .000015$$
$$P(\text{warning}) = 1 - .000015 = .999985$$

Alternatively, $P(\text{warning}) = .95 + .97(1 - .95) + .99(1 - .95)(1 - .97) = .999985$

Problem 4

A weather satellite has an expected life of 10 years from the time it is placed into earth's orbit. Determine its probability of failure after each of the following lengths of service. (Assume Exponential distribution is appropriate.)

a. 5 years.
b. 12 years.
c. 20 years.
d. 30 years.

Solution

MTTF = 10 years

Compute the ratio T/MTTF for $T = 5$, 12, 20, and 30, and obtain the values of $e^{-T/\text{MTTF}}$ from Table 4S–1. The solutions are in the right column of the following table.

	T	MTTF	T/MTTF	$e^{-T/\text{MTTF}}$
a.	5	10	.50	.6065
b.	12	10	1.20	.3012
c.	20	10	2.00	.1353
d.	30	10	3.00	.0498

Problem 5

What is the probability that the satellite described in Solved Problem 4 will fail between 5 and 12 years after being placed into earth's orbit?

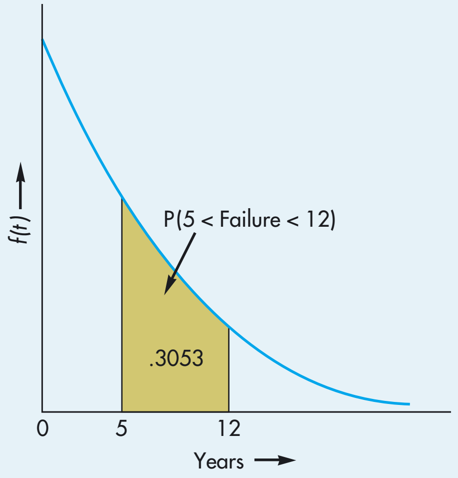
Solution

$$P(5 \text{ years} < \text{failure} < 12 \text{ years}) = P(\text{failure after 5 years}) - P(\text{failure after 12 years})$$

Using the probabilities shown in the previous solution, you obtain:

P(failure after 5 years) = .6065
$-P$(failure after 12 years) = .3012
.3053

The corresponding area under the curve is illustrated as follows:



Problem 6

One line of radial tires produced by a large company has a wear-out life that can be modelled using a Normal distribution with a mean of 25,000 km and a standard deviation of 2,000 km. Determine each of the following:

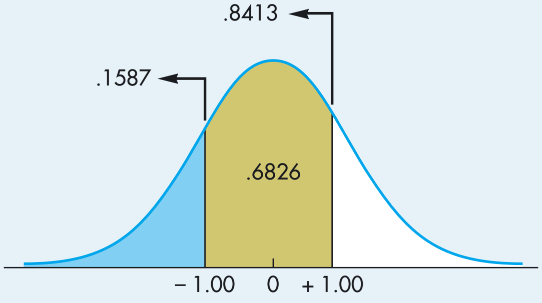
a. The percentage of tires that can be expected to wear out within ±2,000 km of the average (i.e., between 23,000 km and 27,000 km).
b. The percentage of tires that can be expected to fail between 26,000 km and 29,000 km.

Solution

Notes: (1) Kilometres are analogous to time and are handled in exactly the same way; (2) *percentage* is probability times 100.

a. The phrase "within ±2,000 km of the average" translates to within one standard deviation of the mean since the standard deviation equals 2,000 km. Therefore the range of z is $z = -1.00$ to $z = +1.00$, and the area under the curve between those points is found as the difference between $P(z < +1.00)$ and $P(z < -1.00)$, using values obtained from Appendix B, Table B.

$$P(z < +1.00) = .8413$$
$$-P(z < -1.00) = .1587$$
$$P(-1.00 < z < +1.00) = .6826$$



b. Wear-out mean = 25,000 km

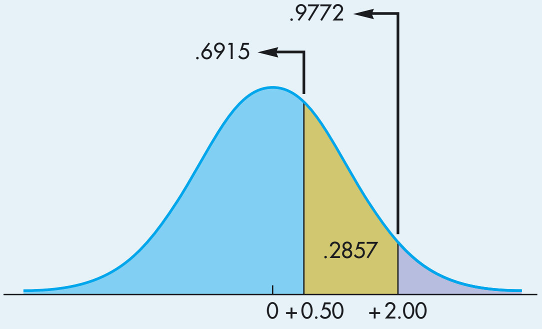
Wear-out standard deviation = 2,000 km

$$P(26{,}000 < \text{Wear-out} < 29{,}000) = P(z < z_{29{,}000}) - P(z < z_{26{,}000})$$

$$z_{29{,}000} = \frac{29{,}000 - 25{,}000}{2{,}000} = +2.00 \rightarrow .9772$$

$$z_{26{,}000} = \frac{26{,}000 - 25{,}000}{2{,}000} = +.50 \rightarrow .6915$$

The difference is .9772 − .6915 = .2857, which means 28.57 percent of tires will wear out between 26,000 km and 29,000 km.



DISCUSSION AND REVIEW QUESTIONS

1. Define the term *reliability*.
2. Explain why a product or system might have an overall reliability that is low even though its components have fairly high reliabilities.
3. What is redundancy and how can it improve product reliability?

INTERNET EXERCISES

1. Visit either www.smrp.org/jobs.asp or www.sre.org/current/current.htm, pick a job announcement, and briefly summarize the duties involved.
2. Visit www.relex.com, choose a case (under "our clients"), and summarize the benefits of Relex software to the company. If there are new concepts or terms in the reading, use the Glossary under "Resources" on the Web page.

PROBLEMS

1. Consider the following system:

Determine the probability that the system will operate under each of these conditions:
a. The system as shown.
b. Each system component has a backup with a reliability of .90 and a switch that is 100 percent reliable.
c. Backups with .90 reliability and a switch that is 99 percent reliable.

2. A product is composed of four parts. In order for the product to function properly in a given situation, each of the parts must function. Two of the parts each have a .96 probability of functioning, and the other two each have a probability of .99. What is the overall probability that the product will function properly?
3. A system consists of three identical components. In order for the system to perform as intended, all of the components must perform. Each has the same probability of performance. If the system is to have a .92 probability of performing, what is the probability of performance needed by each of the individual components?
4. A product engineer has developed the following equation for the cost of a system component: $C = (10P)^2$, where C is the cost in dollars and P is the probability that the component will operate as expected. The system is composed of two identical components, both of which must operate for the system to operate. The engineer can spend $173 for the two components. To the nearest two decimal places, what is the largest component reliability that can be achieved?
5. The guidance system of a ship is controlled by a computer that has three major modules. In order for the computer to function properly, all three modules must function. Two of the modules have reliabilities of .97, and the other has a reliability of .99.
   a. What is the reliability of the computer?
   b. A backup computer identical to the one being used can be installed to improve overall reliability. Assuming that the new computer can automatically function if the main one fails, determine the resulting reliability.
   c. If the backup computer must be activated by a switch in the event that the first computer fails, and the switch has a reliability of .98, what is the overall reliability of the system? (*Both* the switch and the backup computer must function in order for the backup to take over.)
6. One of the industrial robots designed by a leading producer of servomechanisms has four major components. Components' reliabilities are .98, .95, .94, and .90. All of the components must function in order for the robot to operate effectively.
   a. Calculate the reliability of the robot.
   b. Designers want to improve the reliability by adding a backup component. Due to space limitations, only one backup can be added. The backup for any component will have the same reliability as the unit for which it is the backup. Which component should get the backup in order to achieve the highest reliability of robot?
   c. If one backup with a reliability of .92 can be added to any one of the main components, which component should get it to obtain the highest overall reliability?
7. A production line has three machines A, B, and C, with reliabilities of .99, .96, and .93, respectively. The machines are arranged so that if one breaks down, the others must shut down. Engineers are weighing two alternative designs for increasing the line's reliability. Plan 1 involves adding an identical backup line, and plan 2 involves providing a backup for each machine. In either case, three additional machines (A, B, and C) would be used with reliabilities equal to the original three.
   a. Which plan will provide the higher reliability?
   b. Explain why the two reliabilities are not the same.
   c. What other factors might enter into the decision of which plan to adopt?
8. Refer to the previous problem.
   a. Assume that the single switch used in plan 1 is 98 percent reliable, while reliabilities of the machines remain the same. Recalculate the reliability of plan 1. Compare the reliability of this plan with the reliability of the plan 1 calculated in solving the original problem. How much did reliability of plan 1 decrease as a result of a 98 percent reliable switch?

b. Assume that the three switches used in plan 2 are all 98 percent reliable, while reliabilities of the machines remain the same. Recalculate the reliability of plan 2. Compare the reliability of this plan with the reliability of the plan 2 calculated in solving the original problem. How much did reliability of plan 2 decrease?

9. A Web server has five major components which must all function in order for it to operate as intended. Assuming that each component of the system has the same reliability, what is the reliability each one must have in order for the overall system to have a reliability of .98?
10. Repeat Problem 9 under the condition that one of the components will have a backup with a reliability equal to that of any one of the other components.
11. Hoping to increase the chances of reaching a performance goal, the director of a research project has assigned three separate research teams the same task. The director estimates that the team probabilities are .9, .8, and .7 for successfully completing the task in the allotted time. Assuming that the teams work independently, what is the probability that the task will not be completed in time?
12. An electronic chess game has a useful life that is Exponential with a mean of 30 months. Determine each of the following:
    a. The probability that any given unit will operate for at least (1) 39 months, (2) 48 months, (3) 60 months.
    b. The probability that any given unit will fail sooner than (1) 33 months, (2) 15 months, (3) 6 months.
    c. The length of service time after which the percentage of failed units will approximately equal (1) 50 percent, (2) 85 percent, (3) 95 percent, (4) 99 percent.
13. A manufacturer of programmable calculators is attempting to determine a reasonable free-service period for a model it will introduce shortly. The manager of product testing has indicated that the calculators have an expected life of 30 months. Assume product life can be described by an Exponential distribution.
    a. If service contracts are offered for the expected life of the calculator, what percentage of those sold would be expected to fail during the service period?
    b. What service period would result in a failure chance of approximately 10 percent?
14. Lucky Lumen lightbulbs have a life that is Exponentially distributed with a mean of 5,000 hours. Determine the probability that one of these lightbulbs will last:
    a. At least 6,000 hours.
    b. No longer than 1,000 hours.
    c. Between 1,000 hours and 6,000 hours.
15. Planetary Communications, Inc., intends to launch a satellite that will enhance reception of television programs in Alaska. According to its designers, the satellite will have an expected life of six years. Assume that Exponential distribution applies. Determine the probability that it will function for each of the following time periods:
    a. More than 9 years.
    b. Less than 12 years.
    c. More than 9 years but less than 12 years.
    d. At least 21 years.
16. An office manager has received a report from a consultant that includes a section on equipment replacement. The report indicates that scanners have a service life that is Normally distributed with a mean of 41 months and a standard deviation of 4 months. On the basis of this information, determine the percentage of scanners that can be expected to fail in the following time periods:
    a. Before 38 months of service.
    b. Between 40 and 45 months of service.
    c. Within 2 months of the mean life.
17. A major television manufacturer has determined that its 19-inch colour TV picture tubes have a service life that can be modelled by a Normal distribution with a mean of six years and a standard deviation of a half year.
    a. What probability can you assign to service lives of (1) at least five years? (2) at least six years? (3) at most seven and a half years?
    b. If the manufacturer offers service contracts of four years on these picture tubes, what percentage can be expected to fail during the service period?

18. Refer to Problem 17. What service period would correspond to percentage failure of:
    a. 2 percent?
    b. 5 percent?
19. Determine the availability for each of these cases:
    a. MTTF = 40 days, average repair time = 3 days.
    b. MTTF = 300 hours, average repair time = 6 hours.
20. A machine can operate for an average of 50 days before it needs to be overhauled, a process which takes two days. Calculate the availability of this machine.
21. A manager must decide between two machines. Machine A has a projected average operating time of 142 hours and a projected average repair time of seven hours. Projected times for machine B are an average operating time of 65 hours and a repair time of 2 hours. What are the projected availabilities of each machine?
22. A designer estimates that she can (*a*) increase the average time to failure of a part by 5 percent at a cost of $450, or (*b*) reduce the average repair time by 10 percent at a cost of $200. Which option would be more cost-effective? Currently, the average time to failure is 100 hours and the average repair time is four hours.
23. Auto Batteries battery life is Normally distributed with a mean of 4.7 years and a standard deviation of .3 year. The batteries are warranteed to operate for a minimum of four years. If a battery fails within the warranty period, it will be replaced with a new battery at no charge.
    a. What percentage of batteries would you expect to fail before the warranty period expires?
    b. A competitor is offering a warranty of 54 months on its premium battery. The manager of Auto Batteries is toying with the idea of using the same battery with a different exterior, labelling it as a premium battery, and offering a 54-month warranty on it. How much more would the company have to charge on its "premium" battery to offset the additional cost of replacing batteries?
24. In practice, for a series system, the failure rate of a system is estimated by adding the failure rate of its components. For a system made of n identical components in series, each having a probability of failure = P_f, probability of system failure is approximately $n(P_f)$ provided that P_f is sufficiently small. Choose a value of $n > 1$ and $P_f < .05$ and show the above result.
25. The MTTF of the central processing unit (CPU) of a single board computer is 150,000 hours (Source: www.relex.com/customers/cs/versatel.asp). You can assume Exponential distribution for operating time of this component until failure. What is the probability that this component will operate without failure for
    a. 2.5 years
    b. 5 years
    c. 10 years
    (*Hint:* You may use a calculator, instead of Table 4S–1, to obtain more accurate probabilities.)