Surface Area and Volume of Prisms and Pyramids

The surface area is the sum of the areas of the faces.
The volume of a prism equals the area of the base times the height.
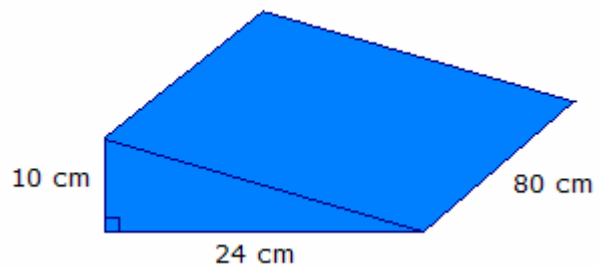
The volume of a pyramid equals $\dfrac{1}{3}$ the area of the base times the height.

Example:

a) A DVD player came packaged in a box with a base measuring 30 cm by 40 cm, and a height of 20 cm. Find the volume and the surface area of the box.
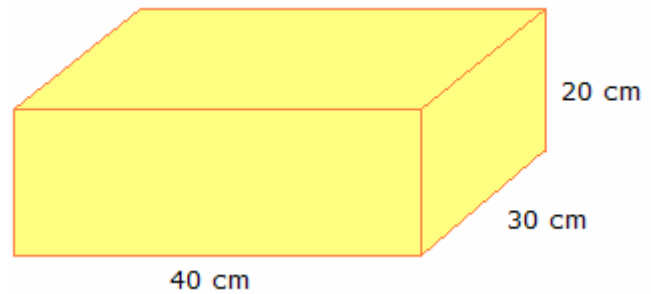
b) Sam made a small ramp to allow him to roll a dolly over a doorstep more easily, as shown. Find the volume and the surface area of the ramp.

c) Renata bought a "change tent" in the shape of a pyramid to use at track meets, with dimensions as shown. Find the volume of the tent, and the surface area.

Solution:

a)

$$V = (\text{area of base}) \times (\text{height})$$
$$= lwh$$
$$= 40 \times 30 \times 20$$
$$= 24\,000 \text{ cm}^3$$

$$SA = \text{sum of the areas of the faces}$$
$$= 2(lw + wh + lh)$$
$$= 2(40 \times 30 + 30 \times 20 + 40 \times 20)$$
$$= 5200 \text{ cm}^2$$

The volume is 24 000 cm³, and the surface area is 5200 cm².

b)

$$V = (\text{area of side}) \times (\text{length})$$
$$= \frac{1}{2}bhl$$
$$= \frac{1}{2} \times 24 \times 10 \times 80$$
$$= 9600 \text{ cm}^3$$

Let the length of the hypotenuse be c.

$$c^2 = b^2 + h^2$$
$$= 24^2 + 10^2$$
$$= 676$$
$$c = 26 \text{ cm}$$

$$SA = \text{area of sides} + \text{area of top} + \text{area of bottom} + \text{area of back}$$
$$= 2\left(\frac{1}{2}bh\right) + cl + hl + bl$$
$$= 24 \times 10 + 26 \times 80 + 10 \times 80 + 24 \times 80$$
$$= 5040 \text{ cm}^2$$

The volume is 9600 cm³, and the surface area is 5040 cm².

c) Let the height of the pyramid be *h*. The height is related to half the base and the slant height as shown.
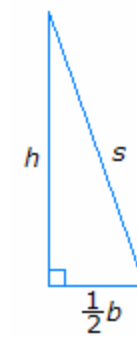
$$s^2 = \left(\frac{1}{2}b\right)^2 + h^2$$

$$2.5 = 0.7^2 + h^2$$

$$6.25 = 0.49 + h^2$$

$$6.25 - 0.49 = 0.49 + h^2 - 0.49$$

$$5.76 = h^2$$

$$2.4 = h$$

$$V = \frac{1}{3}(\text{area of base}) \times (\text{height})$$

$$= \frac{1}{3}b^2 h$$

$$= \frac{1}{3} \times 1.4^2 \times 2.4$$

$$= 1.568 \text{ m}^3$$

$$SA = \text{sum of the areas of the faces}$$
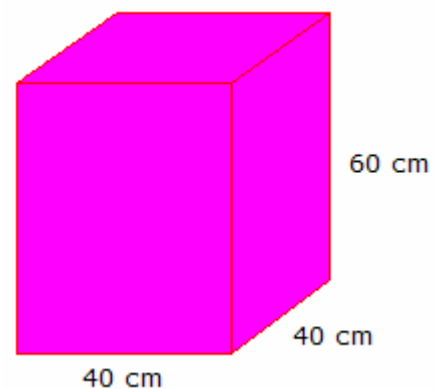
$$= b^2 + 4\left(\frac{1}{2}bs\right)$$

$$= 1.4^2 + 2 \times 1.4 \times 2.5$$

$$= 8.96 \text{ m}^2$$

The volume is 1.568 m^3, and the surface area is 8.96 m^2.

Practice:

1. a) The stand for a porcelain vase was in the shape of a wooden box, with a base measuring 40 cm by 40 cm, and a height of 60 cm. Find the volume and the surface area of the box.

60 cm

40 cm

40 cm

b) A glass prism used to break light into a spectrum had dimensions as shown. Find the volume and the surface area of the prism.

4 cm

4 cm

8 cm

4 cm
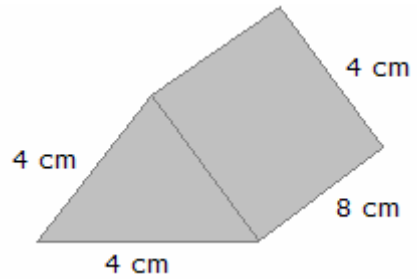
c) Anwar opened a new middle-eastern restaurant in town. He built a decorative pyramid beside the front door, with a square base measuring 2.4 m on a side, and a height of 1.6 m. Find the volume and the surface area of the pyramid.

1.6 m

2.4 m

2.4 m
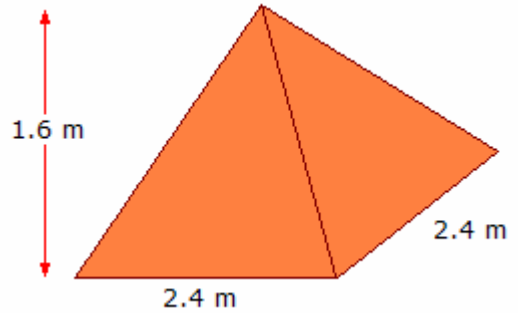
Answers:

1. a) $V = 96\,000$ cm^3 b) $V = 55.4$ cm^3 c) $V = 3.1$ m^3

   $SA = 12\,800$ cm^2 $SA = 109.9$ cm^2 $SA = 15.4$ m^2