CHAPTER 9: Optimizing Measurements
9.4 Maximize the Volume of a Square-Based Prism
Maximizing the Volume of a Square-Based Prism

The maximum volume for a given surface area of a square-based prism always occurs when the prism is a cube.

The surface are of a cube is given by the formula $SA = 6s^2$, where s is the side length of the cube. When you are given the surface area, solve for s to find the dimensions of the square-based prism with maximum volume.

Example:

a) Edwin is planning to sell his home-made raisins at the local fair. To keep costs down, he will make his own boxes, budgeting 240 cm^2 of cardboard for each box. Show that he can maximize the volume of the box by using a cube with a side length of 6.3 cm.

b) Raneeta is planning to hold a "guess the number of gumballs" contest in her candy store. She will make a square-based Lucite box to hold the gumballs for display in the window. The box will have a bottom and sides, but no top. If she has 420 cm^2 of Lucite, find the dimensions of the square-based box that maximum the volume.

Solution:

a) The volume will be a maximum when a cube is used.

$$SA = 6s^2$$
$$240 = 6s^2$$
$$\frac{240}{6} = \frac{6s^2}{6}$$
$$40 = s^2$$
$$6.3 = s$$

The side length of the required cube is 6.3 cm.

b) Use pencil and paper, a graphing calculator, a spreadsheet, or other aid to make a table of possible dimensions. The volume is a maximum with a base length of 11.8 cm and a height of 5.9 cm.

Base (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
11.5	6.3	827.28	420
11.6	6.2	827.78	420
11.7	6.0	828.10	420
11.8	5.9	828.24	420
11.9	5.8	828.21	420
12.0	5.8	828.00	420

Practice:

1. a) A package of gift wrap covers an area of 3 m^2. Renata wants to wrap a present in the shape of a square-based prism. What is the maximum volume of the present that she can wrap?

b) Nigel is weaving the basket for a hot air balloon in the shape of a square-based prism. He estimates that he has enough willow branches to weave 20 m^2. The basket will have a bottom and four sides, but no top. Find the dimensions that maximum the volume of the basket.

Answers:

1. a) 0.35 m^3 b) b = 2.6 m, h = 1.3 m