Two-Dimensional Geometry

1. Old Macdonald had a farm. He used 56 m of fencing to enclose a parallelogram-shaped paddock with area 180 m^2 for his horse. What were the dimensions of the paddock?

Use grid paper to model the parallelogram. Adjust side lengths until you have a parallelogram that matches the measurements given.

Is there more than one solution? Explain.

2. The Greek mathematician Archimedes was trying to find the relationship between the diameter and the circumference of a circle. He approached the problem by drawing a large circle in the sand behind his house and drawing regular polygons inside and outside the circle. He knew that the circumference of the circle must lie between the perimeters of the inner and the outer polygons. You can model Archimedes' method using grid paper.

Draw a circle with diameter 20 cm. Draw a square inside the circle, such that the corners of the square intersect the circumference of the circle. Now, draw another square, this time outside the circle, such that the sides of the square intersect the circumference of the circle. Measure the perimeters of the inner and outer squares.
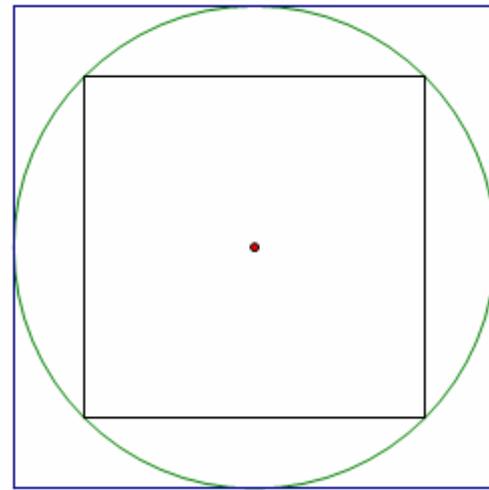
Repeat the above process using pentagons, and then hexagons. You can use a pair of compasses to help you find the side length for each of these that will work.

Set the compasses to a length that you think will work. Pick a point on the circle to start, and work around the circle. If you

arrive back at the starting point after five lengths, you have the right length to draw a pentagon inside the circle. If you are short, use a longer length, and try again. If you are long, use a shorter length and try again. When you have the right length, mark the five points on the circle, and draw the pentagon.

Record your results in a table. Add a column to the table for the mean of the inner and outer polygon perimeters. This will give you an estimate for the circumference of the circle. Finally, add a column to the table for the estimated circumference of the circle divided by the diameter.

Draw two more diagrams, one for a 10-sided polygon, and one for a 12-sided polygon. Hint: You can extend your diagrams for the pentagon and hexagon to draw these additional polygons. Add the appropriate measurements to your table.

How close are your ratios (estimated circumference of the circles divided by the diameter) to π?

3. Ahmed has a supply of patio blocks that are 50 cm long and 25 cm wide. He wants to lay them to make a path in his garden that is 50 cm wide. How many different ways can he make a path that is 25 cm long? 50 cm long? 75 cm long? Use sketches and a table to organize your answers. Continue the table until you can determine a pattern. Use your pattern to determine the number of ways that he can make a path 10 m long.

4. A lattice point is any point in a coordinate system that has integers as coordinates. Pick's theorem says that you can calculate the area of any polygon that has lattice points as vertices by counting up the number of lattice points in the interior, i, and the number of lattice points on the boundary, b. The area, A, is given by the formula $A = i + 0.5b - 1$.

Use grid paper to test Pick's theorem. Draw a triangle, a square, and a parallelogram with areas you can easily calculate from the area formulas you have already learned. Compare the area calculated using Pick's theorem with the area calculated from the formulas you already know.

Try a number of other polygons with different sizes and shapes. You can estimate the area of a polygon by counting the squares on the grid paper.