Simulations and Probability

1. Anya invited four friends for an evening of playing board games. What is the probability that at least two of the party of five friends have a birthday in the same month? Simulate this problem using dodecahedral (12-sided) dice. Use at least 200 throws. Tabulate the occurrence of no match, two matches, three matches, four matches, and five matches. Determine the experimental probability of a match. Repeat the activity for a party of four and a party of three.

Challenge: If you couldn't find any 12-sided dice for this question, would it work just as well using five pairs of six-sided dice? Explain.

2. You can find the value of π by throwing sticks onto a striped fabric, such as a bedsheet. You need to find sticks, such as toothpicks, and fabric that has parallel lines that are farther apart than the length of a stick. Measure the length of a stick, and call it *l*. Measure the distance between the parallel lines, and call it *d*. Toss the sticks so that they fall on the bedsheet. Record the number of sticks thrown, and call this *n*. Record the number that fell across one of the lines, and call this *c*. You can calculate π from the formula $\pi = \frac{2ln}{dc}$.

Try this using 100 throws. How close is your result to the value of π? Increase the number of throws until you find a value of π correct to two decimal places.

You can find out more about this process by researching *Buffon's Needle*.

3. An office building has 11 floors and 3 elevators. Each elevator can be programmed to return to its home floor after it has picked up and discharged passengers. It takes each elevator 10 s to move up or down one floor.

The owners of the building want to minimize response time (i.e., how long a passenger must wait, on average, for an elevator to come to him/her from its home floor). Is it better to station all three elevators on the middle floor, or to have one at the top, one in the middle, and one at the bottom?

Design a simulation to test these two scenarios. Devise a random number generator to simulate a call from a floor from 1 to 11, and keep track of response times. Assume that the closest elevator will always respond, and that the elevator has time to return "home" before the next call. Perform a sufficient number of trials to obtain reliable data.

Write a note to the building owners advising them how they should station the elevators.

4. You are a contestant on a game show. There are three doors. Behind one of them is a valuable prize. The other two are empty. You select one of the doors. The host now opens a door that he knows is empty. You are given the option of switching your choice to the remaining door. Should you switch?

Design a simulation to determine the probability of winning if you switch and if you do not switch. This is called the *Monty Hall* problem, named after a popular television game show host.

5. Rhonda claims that selecting the same number in a lottery every time leads to a higher probability of winning than selecting a different number at random each time. Design a simulation to test this assertion. You can use playing cards to simulate the lottery.

Perform a large number of trials. Summarize your findings using a method of your choice. Write a note to Rhonda telling her whether you think she is right or wrong. Include evidence.