

5 Linear Systems

Many amusement parks offer two kinds of daily admission. You can buy an all-inclusive pass for unlimited use of the rides and attractions, or you can pay a lower price for entry and then buy tickets for the rides and attractions. How can you decide which is the better value? In this chapter you will use systems of linear equations to solve problems like this one.

In this chapter, you will

- determine graphically the point of intersection of two linear relations
- solve systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination
- solve problems that arise from realistic situations described in words or represented by given linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method



Key Terms

elimination method
linear system

point of intersection
substitution method

Literacy Link

Use a Venn diagram to help you choose which method to use to solve a linear system. How would you draw the Venn diagram?

Gordana owns her own aesthetics business. She has to set the prices for the services she provides so that her business will be profitable. How might Gordana use linear systems to help set prices?



Get Ready!

Algebraic Expressions

1. Simplify by collecting like terms. The first part has been done for you.

a) $2d + 5 - 4d - 9$
 $= 2d - 4d + 5 - 9$
 $= -2d - 4$

b) $3x + 4 + 2x - 1$

c) $11y - 5 + 2y + 8$

d) $7m - 3m + 5 + 11$

e) $3c + 5 - 2c - 10$

f) $5v + 3 - 4v + 7$

Manipulate and Solve Equations

2. Rearrange each equation to isolate y . The first part has been done for you.

a) $-3x + 8y = 11$
 $-3x + 8y + 3x = 3x + 11$
 $8y = 3x + 11$
 $\frac{8y}{8} = \frac{3x}{8} + \frac{11}{8}$
 $y = \frac{3x}{8} + \frac{11}{8}$

b) $3x - y = 4$

c) $4x + y = 9$

d) $4x - 2y = 14$

e) $5x - 2y = 6$

f) $2x + 3y - 1 = 0$

g) $4x + 6y - 9 = 0$

3. For each equation, find the value of y when $x = 3$. The first part has been done for you.

a) $2x + y = 8$
 $2(3) + y = 8$
 $6 + y = 8$
 $6 + y - 6 = 8 - 6$
 $y = 2$

b) $y = 3x + 1$

c) $y = 4x - 3$

d) $y = 8x + 1$

e) $x - y = 9$

f) $3x - 2y = 8$

g) $12x + 5y = 17$

4. For each equation, find the value of x when $y = 4$.

a) $3y + 3x = 1$

b) $4y = 2x + 7$

c) $x - 2y = 9$

d) $y = 6x + 1$

e) $3x - 2y = 16$

f) $y = 3x + 1$

g) $2x - 2y = 9$

h) $y = 13x - 19$

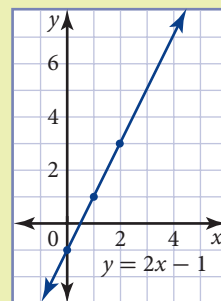
Graph Linear Relations

5. Graph each linear relation. The first part has been done for you.

a) $y = 2x - 1$

The y-intercept is -1 and the slope is 2 .

Plot the point $(0, -1)$, then move up 2 units and right 1 unit to find two other points on the line.



b) $y = 4x + 6$

c) $y = -x - 3$

d) $y = -3x + 2$

e) $y = -5x + 1$

f) $y = 4x + 7$

g) $y = -2x + 6$

Chapter Problem

Logan volunteers for a dog rescue organization. He is planning a series of events to raise money for the organization. In this chapter, you will learn how to calculate the break-even point, and the amount Logan should charge for each event to make sure that he covers his costs.



6. Graph each linear relation. The first part has been done for you.

a) $2x + y = 4$

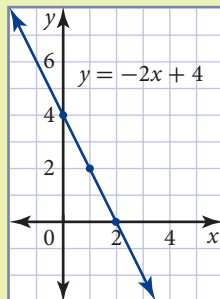
Rewrite the equation in slope y-intercept form.

$$2x + y - 2x = -2x + 4$$

$$y = -2x + 4$$

The y-intercept is 4 and the slope is -2 .

Plot the point $(0, 4)$, then move down 2 units and right 1 unit to find two other points on the line.



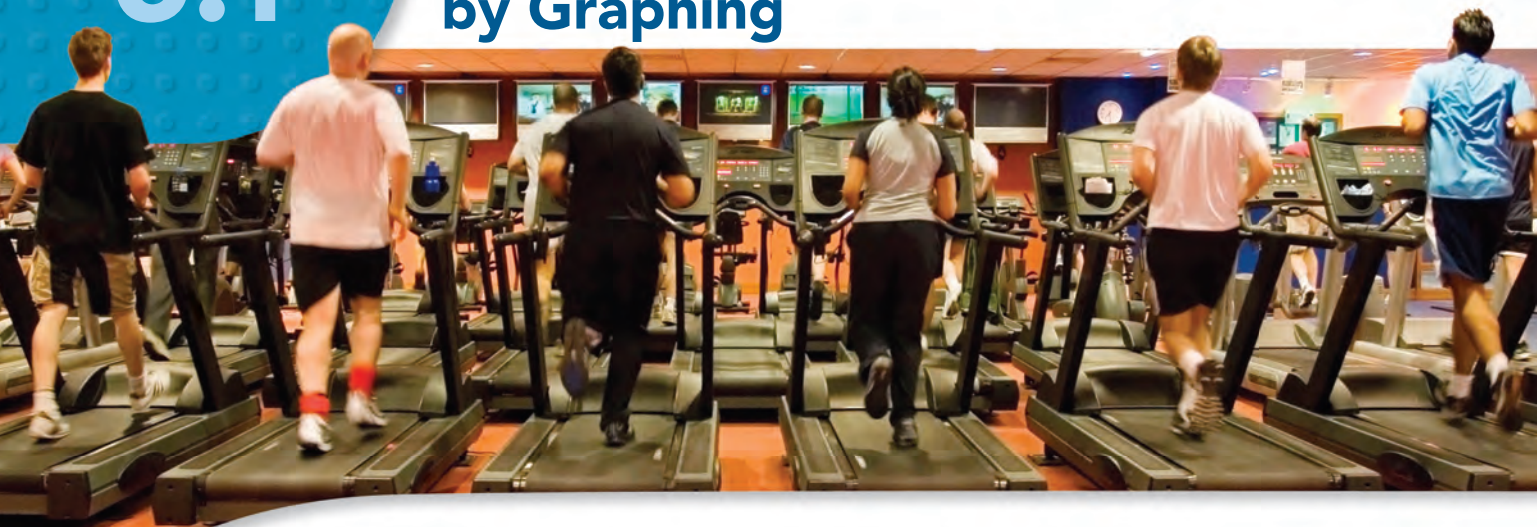
- b) $4x - y = 8$
c) $-x + y = -3$
d) $-3x - y = 3$
e) $-5x + y - 10 = 0$
f) $-3x - y = 9$
g) $y = 4x + 6$

Translate Words to Algebra

7. Write an equation to represent each situation. The first part has been done for you.
- a) The cost to have flyers delivered is \$50 plus \$0.10 per flyer. Callum spends a total of \$400 to have flyers delivered. *Let n represent the number of flyers Callum has delivered.*
 $400 = 0.1n + 50$
- b) Ron earns \$8.50/h. Last week, he earned \$52.00.
- c) The total mass of a metal storage drum is 90 kg. The mass of an empty drum is 18 kg and the mass of the liquid stored in the drum is 2.3 kg/L.
- d) Bohdan pays \$300 to rent a car. He pays a flat fee of \$125 plus \$0.35/km.
- e) Linnea earns \$270 for the first 30 h she works in a week. After that, she earns \$12/h. Last week, Linnea earned \$450.

5.1

Solve Linear Systems by Graphing



Eirlys is planning to join a gym. She is trying to decide between two different payment plans. How can Eirlys use linear equations to help her decide?

Investigate

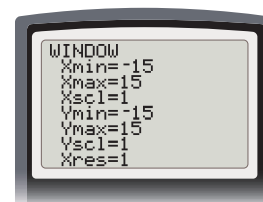
Tools

- graphing calculator

Compare Taxi Fares

Trip Taxi charges \$2.50 plus \$0.20/km. Comfort Taxi charges \$0.50/km.

1. Let x represent the distance travelled in kilometres and y represent the total cost of a trip. Write an equation to represent the total cost for each taxi company.
2. Suppose you plan to travel 12 km. Which taxi company should you choose? Why?
3. Suppose you plan to travel 35 km. Which taxi company should you choose? Why?
4. Use a graphing calculator.
 - Press **WINDOW**. Enter the window settings shown.
 - Press **Y=**. Enter the equation for Trip Taxi as Y1 and the equation for Comfort Taxi as Y2.
 - Press **2nd** **[CALC]** **5** **ENTER** **ENTER** **ENTER**. Record the coordinates of the point of intersection. What do these coordinates mean in terms of this situation?



linear system

- a set of two or more linear equations that are considered at the same time

point of intersection

- the point at which two lines cross
- the coordinates of the point of intersection satisfies both equations

5. Check that the coordinates from question 4 lie on the line representing the total cost of a trip with Trip Taxi and on the line representing the total cost of a trip with Comfort Taxi. How did you check?
6. Are there any other points that lie on both lines? How do you know?
7. **Reflect** What is the significance of the point of intersection of two linear equations?

A **linear system** is a set of two or more linear relations considered at the same time. Graphically, the solution to a linear system is the **point of intersection** of the relations.

Example**1****Solve a Linear System by Graphing**

Solve this linear system, then check.

$$y + 2x = -5 \quad \textcircled{1} \qquad y = \frac{2}{3}x + 3 \quad \textcircled{2}$$

Solution

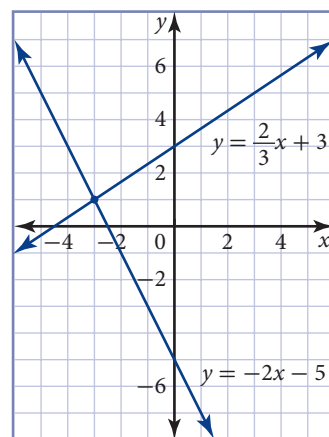
Rearrange equation $\textcircled{1}$ into slope y -intercept form.

$$\begin{aligned} y + 2x &= -5 \\ y + 2x - 2x &= -5 - 2x \\ y &= -5 - 2x \\ y &= -2x - 5 \end{aligned}$$

The slope is -2 and the y -intercept is -5 .

Equation $\textcircled{2}$ is in slope y -intercept form. The slope is $\frac{2}{3}$ and the y -intercept is 3 .

Use the slope and y -intercept to graph each line on the same coordinate grid. Label each line with its equation.



From the graph, the point of intersection is $(-3, 1)$. Check these coordinates in each of the original equations.

Equation ①

$$\begin{aligned} \text{LS} &= y + 2x & \text{RS} &= -5 \\ &= 1 + 2(-3) \\ &= 1 - 6 \\ &= -5 \\ &= \text{RS} \end{aligned}$$

Equation ②

$$\begin{aligned} \text{LS} &= y & \text{RS} &= \frac{2}{3}x + 3 \\ &= 1 & &= \frac{2}{3}(-3) + 3 \\ & & &= -2 + 3 \\ & & &= 1 \\ & & &= \text{LS} \end{aligned}$$

The solution to this linear system is $(-3, 1)$.

Example

2

Choose a Fitness Club

KC Fitness Club charges a flat fee of \$25 per month plus \$5 per visit. Workout Zone charges a flat fee of \$35 per month plus \$3 per visit. For how many visits per month is the total cost the same for both fitness clubs?

Solution

Let x represent the number of visits per month.

Let y represent the total cost for the month in dollars.

KC Fitness Club

$$y = 5x + 25$$

The total cost for the month is \$25 plus \$5 for each visit.

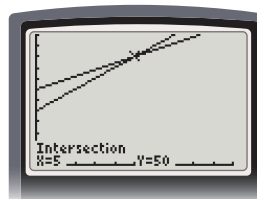
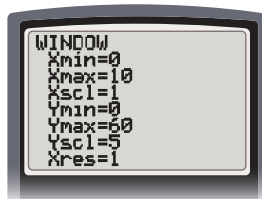
Workout Zone

$$y = 3x + 35$$

The total cost for the month is \$35 plus \$3 for each visit.

Use a graphing calculator. Graph both equations in the same window.

- Press $\boxed{2\text{nd}} \boxed{[\text{CALC}]} \boxed{5} \boxed{\text{ENTER}} \boxed{\text{ENTER}} \boxed{\text{ENTER}}$.



The point of intersection is $(5, 50)$. This means that the total cost for both fitness clubs is \$50 when 5 visits are made in one month.

Key Concepts

- The solution to a linear system is the point of intersection of the lines.
- A linear system can be solved by graphing the lines, then reading the point of intersection from the graph.
- To check the solution to a linear system, substitute the coordinates of the point of intersection into the original equations.

Discuss the Concepts

- D1. a)** Explain, in your own words, what it means to solve a system of equations.
- b)** Compare your answer from part a) with that of one of your classmates.
- D2.** Is it possible for a linear system to have no solution? Explain your reasoning.
- D3.** Describe in words how you would solve the linear system $y = 3x + 1$ and $y = -2x + 3$.

Practise the Concepts

A

For help with questions 1 to 4, refer to Example 1 or Example 2.

- 1.** Find the point of intersection of each linear system.

Check your answers.

a) $y = 2x + 3$

$y = 4x - 1$

c) $y = -x + 5$

$y = x + 1$

b) $y = -x - 7$

$y = 3x + 5$

d) $y = x + 4$

$y = -x - 2$

- 2.** Solve each linear system by graphing.

a) $y = 2x$

$y = 4x + 1$

c) $y = 2x - 3$

$y = \frac{5}{2}x + 1$

b) $y = \frac{1}{2}x - 2$

$y = \frac{3}{4}x + 4$

d) $y = 4x - 5$

$y = \frac{9}{5}x - 10$



3. Solve each linear system by graphing. Check your answers.

a) $3x + y = 7$
 $-x + 2y = 7$

b) $y = 7x + 3$
 $x - y = 3$

c) $2x + 3y = 8$
 $x - 2y = -3$

d) $5x - y = -4$
 $2x - 3y = 1$

4. Find the point of intersection by graphing. Check your answers.

a) $-2x - y = -8$
 $x + y = 9$

b) $x + 2y = -5$
 $3x - y = -1$

c) $y = 2x + 1$
 $2x + y = 1$

d) $x + y = 7$
 $x - y = -1$

Apply the Concepts **B**

For help with questions 5 to 7, refer to Example 2.

5. Sherwood Tennis Club charges a \$150 initial fee to join the club, and then a \$20 monthly fee. Coronation Tennis Club charges an initial fee of \$100 and \$30 per month.



a) Write an equation to represent the cost to be a member of the Sherwood Tennis Club.

b) Write an equation to represent the cost to be a member of Coronation Tennis Club.

c) Graph the equations from parts a) and b).

d) Find the point of intersection of the lines. What does this point represent?

e) If you were planning to join for a year, which club should you join? Why?

6. FunNGames Video rents game machines for \$10 and video games for \$3 each. Big Vid rents game machines for \$7 and video games for \$4 each. Let y be the total rental cost and x the number of games rented.

a) Write an equation to represent the total cost for a rental from FunNGames Video.

b) Write an equation to represent the total cost for a rental from Big Vid.

c) Find the point of intersection.

d) What does this point of intersection represent?

7. Katrin is looking at banquet halls for her parents' anniversary party. Moonlight Hall charges a fixed cost of \$1000 plus \$75 per guest. Riverside Hall charges \$1500 plus \$50 per guest. Let C represent the total cost, and n represent the number of guests.
- Write an equation to represent the total cost for Moonlight Hall.
 - Write an equation to represent the total cost for Riverside Hall.
 - Find the number of guests for which the total cost is the same at both halls.
8. Solve this linear system by graphing.
- $$4x - y - 7 = 0 \quad \textcircled{1} \qquad -2x - y - 1 = 0 \quad \textcircled{2}$$
9. During the winter months, Don uses his pickup truck to clear snow from driveways. Don charges \$15 per driveway. Morgan's Snow Removal charges \$150 for the season.

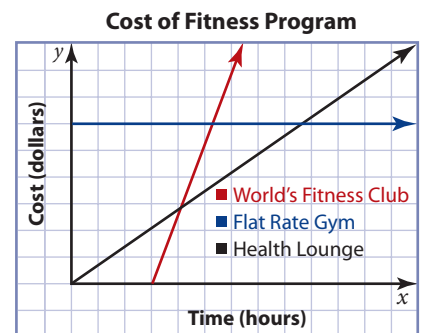


- Write an equation to represent the total cost for Don to clear your driveway for the season.
- Write an equation to represent the total cost for Morgan's Snow Removal to clear your driveway for the season.
- Explain how you would decide who to hire to clear your driveway this winter.

Literacy Connect

Achievement Check

10. This graph shows the costs of fitness programs at three different facilities. Describe the meaning of the intersections of the lines. What advice would you give to someone based on the graph?



- 11.** Gabriella works in a leather clothing factory. She is paid \$80 per day plus \$10 for each jacket she makes. Chan also works in the factory. He makes a flat fee of \$110 per day.
- Write an equation to represent the total amount Gabriella earns in one day.
 - Write an equation to represent the total amount Chan earns in one day.
 - How many jackets must Gabriella make in order to make as much in a day as Chan? Show your work.
- 12.** Find the point of intersection of $3x - 2y = 14$ and $4x + y = 15$ by graphing.

Chapter Problem

Math Connect

The break-even point is the point at which the cost is exactly equal to the revenue.

- 13.** Logan is renting a theatre for a fundraising screening of *Best in Show*. The theatre rental is \$675 plus \$2 per person for operating the snack bar. Let C represent the cost for the event if it is held at this theatre. Let n represent the number of tickets sold by the students.
- Write an equation to show the cost for the theatre.
 - Write an equation to show the revenue (the total amount from ticket sales) if Logan charges \$8.50 per ticket.
 - How many tickets must Logan sell in order to break even?

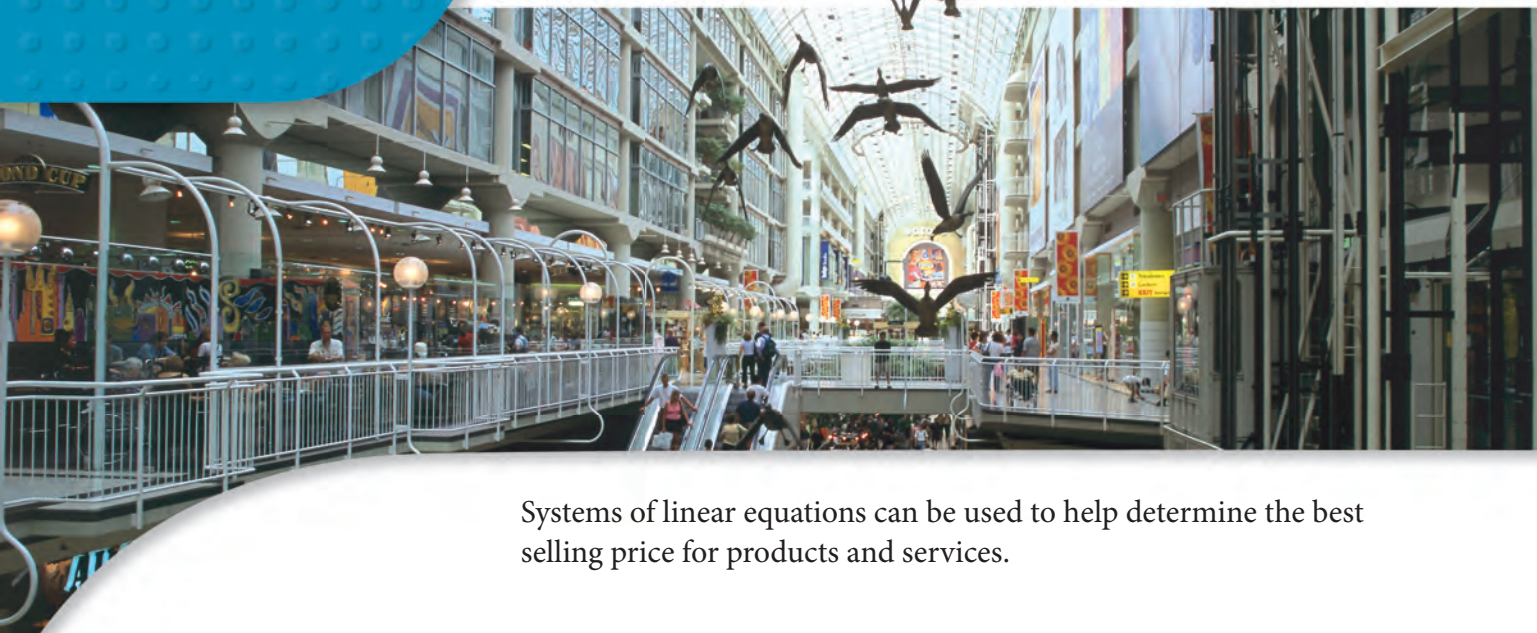
Extend the Concepts



- 14.** Use a graphing calculator. Graph $y = 2x + 4$ and $y = 2x - 5$ in the same window.
- From the graph, does it appear that these lines will intersect?
 - What is the result when you use the Intersect feature to find the point of intersection? Explain.
- 15. a)** Find the slope and y -intercept for each equation.
- $$y = 3x - 4 \quad \textcircled{1}$$
- $$6x - 2y = 8 \quad \textcircled{2}$$
- What do you notice?
- On grid paper, graph the linear system from part a). What do you notice about the lines?
 - How many solutions does this linear system have? Explain.
- 16.** When two equations represent the same line, the lines are coincidental. How many points of intersection does a system of coincidental lines have? Explain.
- 17.** Is it possible for a linear system to have exactly two solutions? Explain.

5.2

Solve Linear Systems by Substitution



Systems of linear equations can be used to help determine the best selling price for products and services.

Investigate

Tools

- graphing calculator or grid paper

A Class Problem

Each morning at the beginning of class, Ms. Edwards gives her math students a puzzle to solve. Here is today's puzzle:

The sum of Jane's age and her mother's age is 60. Jane's mother is 3 times as old as Jane. How old is Jane? How old is her mother?

Work with a partner.

1. Let x represent Jane's age in years and y represent her mother's age in years.
 - a) Write an equation to represent the sum of their ages.
 - b) Write an equation to represent the statement "Jane's mother is three times as old as Jane."
2. Use your equations from question 1 to write a single equation in terms of the variable x .
 - a) Substitute the expression for y from the second equation into the first equation. This is called the **substitution method**.
 - b) Solve the equation that results. Interpret the meaning of the solution in terms of this situation.
3. Use your result from question 2. Find the value of y . What does this value represent?

substitution method

- an algebraic method of solving a system of linear equations
- one equation is solved for one variable, then that value is substituted into the other equation

4. Graph your equations from question 1 on the same grid or in the same window. Find the point of intersection of the lines. How do the coordinates of the point of intersection compare to your results for questions 2 and 3? Explain.

Example

1

Solve by Substitution

Use the substitution method to solve this linear system.

$$4x - 7y = 20 \quad \textcircled{1}$$

$$x - 3y = 10 \quad \textcircled{2}$$

Solution

The solution to a linear system is the point of intersection of the lines. At the point of intersection, the value of x and the value of y are the same for both equations.

$$4x - 7y = 20 \quad \textcircled{1}$$

$$x - 3y = 10 \quad \textcircled{2}$$

Solve equation $\textcircled{2}$ for x .

$$4x - 7y = 20 \quad \textcircled{1}$$

$$x = 3y + 10 \quad \textcircled{2}$$

$$4(3y + 10) - 7y = 20$$

Substitute $x = 3y + 10$ from equation $\textcircled{2}$ into equation $\textcircled{1}$.

$$12y + 40 - 7y = 20$$

$$5y + 40 = 20$$

$$5y + 40 - 40 = 20 - 40$$

$$5y = -20$$

$$\frac{5y}{5} = \frac{-20}{5}$$

$$y = -4$$

$$x - 3y = 10$$

$$x - 3(-4) = 10$$

$$x + 12 = 10$$

$$x + 12 - 12 = 10 - 12$$

$$x = -2$$

Substitute the value $y = -4$ into equation $\textcircled{2}$ to find the value of x .

Check the solution $(-2, -4)$ in equation $\textcircled{1}$.

$$\text{LS} = 4x - 7y \qquad \text{RS} = 20$$

$$= 4(-2) - 7(-4)$$

$$= -8 + 28$$

$$= 20$$

$$= \text{RS}$$

The solution to the linear system is $(-2, -4)$.

Example**2****Supply and Demand**

Rachelle is an economist. She evaluates the effect of changing the price on the supply and the demand for a product. The selling price in dollars, y , of a product is related to the number of units sold, x , according to these equations:

Demand: $y + 0.4x = 10$

Supply: $y = 0.6x + 2$

Solve this system algebraically. What does the solution represent?

Solution

The solution to a linear system is the point of intersection of the lines.

Demand: $y + 0.4x = 10$ ①

Supply: $y = 0.6x + 2$ ②

Substitute $y = 0.6x + 2$ from equation ② into equation ①.

$$(0.6x + 2) + 0.4x = 10$$

$$x + 2 = 10$$

$$x + 2 - 2 = 10 - 2$$

$$x = 8$$

$$y = 0.6x + 2$$

$$y = 0.6(8) + 2$$

$$y = 4.8 + 2$$

$$y = 6.8$$

Substitute the value $x = 8$ into equation ② to find the value of y .

Check the solution $(8, 6.8)$ in equation ①.

LS = $y + 0.4x$ RS = 10

$$= 6.8 + 0.4(8)$$

$$= 6.8 + 3.2$$

$$= 10$$

$$= \text{RS}$$

The solution to the linear system is $(8, 6.8)$. When the price of the product is \$6.80, 8 units are sold.

Example**3****Find the Break-Even Point**

Katherine is selling T-shirts to raise money for diabetes research. The supplier charges a \$210 design fee plus \$3 per T-shirt. Katherine plans to sell the T-shirts for \$10 each. How many T-shirts does Katherine need to sell in order to break even?

Solution

Let C represent the total cost of the T-shirts, in dollars, and n represent the number of T-shirts. Use the information to write a system of equations.

$$C = 3n + 210 \quad \textcircled{1}$$

$$C = 10n \quad \textcircled{2}$$

The total cost of the T-shirts is \$210 plus \$3 per T-shirt.

Katherine will break even when the total sales of T-shirts is equal to the total cost to produce the T-shirts.

$$C = 3n + 210 \quad \textcircled{1}$$

$$C = 10n \quad \textcircled{2}$$

$$10n = 3n + 210$$

$$10n - 3n = 3n + 210 - 3n$$

$$7n = 210$$

$$\frac{7n}{7} = \frac{210}{7}$$

$$n = 30$$

Substitute $C = 10n$ from equation $\textcircled{2}$ into equation $\textcircled{1}$.

$$C = 10n$$

$$C = 10(30)$$

$$C = 300$$

Substitute the value $n = 30$ into equation $\textcircled{2}$ to find the value of C .

Check the solution (30, 300) in equation $\textcircled{1}$.

$$\text{LS} = C$$

$$= 300$$

$$\text{RS} = 3n + 210$$

$$= 3(30) + 210$$

$$= 90 + 210$$

$$= 300$$

$$= \text{LS}$$

Katherine needs to sell 30 T-shirts to break even.

Key Concepts

- A system of linear equations can be solved algebraically using the substitution method.
- To solve a linear system by substitution, one equation is solved for one variable, then that value is substituted into the other equation.
- The break-even point is the point at which the cost to produce an item is equal to its selling price.

Discuss the Concepts

- D1.** Explain why you can solve the system of equations $y = 3x + 1$ and $x + y = 3$ by substituting $3x + 1$ for y in the second equation.
- D2.** Describe how to use the substitution method to solve the system $2x + y = 8$ and $4x + 3y = 12$.
- D3.** How is solving a linear system by substitution the same as solving by graphing? How is it different?

Practise the Concepts **A**

For help with questions 1 to 3, refer to Example 1.

1. Use the substitution method to solve each linear system.

a) $3x + 2y - 1 = 0$

$y = -x + 3$

c) $x + 4y = 5$

$x + 2y = 7$

e) $2x + 3y = -1$

$x + y = 1$

g) $6x + 5y = 7$

$x - y = 3$

b) $3x - y = 4$

$x + y = 8$

d) $2x + y = 3$

$4x - 3y = 1$

f) $x - 3y = -2$

$2x + 5y = 7$

h) $x - 3y = 5$

$7x + 2y = 12$

2. Solve each linear system.

a) $2x - y = 5$

$3x + y = -9$

c) $6x + 3y = 5$

$x - 2y = 0$

b) $4x + 2y = 7$

$-x - y = 6$

d) $8x - y = 10$

$3x - y = 9$

3. Solve each linear system using the substitution method.

a) $x + y = -2$

$x - y = 6$

c) $2x + y = 2$

$3x + 2y = 5$

b) $x - y = 9$

$x + y = 3$

d) $2x - 3y = 6$

$2x - y = 7$

4. Malcolm is twice as old as Sundeeep. The sum of their ages is 39.
- Write an equation to represent the information in the first sentence.
 - Write an equation to represent the information in the second sentence.
 - Use the method of substitution to find the ages of the boys.

For help with question 5, refer to Example 2.

5. For Nina's retirement party, her family decides to rent a hall for a dinner. Regal Hall costs \$500 for the hall rental and \$15 per guest, and Party Place charges \$410 for the hall and \$18 per guest.
- Write a system of linear equations to represent the situation.
 - How many guests must attend for the charges to be the same? Solve by substitution.

For help with question 6, refer to Example 3.

6. Carly rents a theatre for a spring concert. The theatre charges \$825 plus \$2 per person. Carly plans to charge \$7 per person. Let C represent the cost for the event. Let n represent the number of people attending.
- Write a system of linear equations to represent the situation.
 - How many tickets must Carly sell in order to break even?
 - What is the price per ticket?

Literacy Connect



7. When solving a linear system by substitution, how do you decide which variable to solve for first?
8. Dmitri plays hockey. He earns 1 point for every goal he scores and 1 point for every assist. This season he earned 63 points. He scored 17 fewer goals than assists.
- Write a system of linear equations to represent this information.
 - How many goals did Dmitri score this season? How many assists did he have?
9. Sam makes two types of quilts. The first type costs \$25 for fabric and \$40 per hour for hand quilting. The second type costs \$50 for fabric and \$22 per hour for machine quilting. For what number of hours are the costs the same?
10. Solve the linear system $3x - y = 19$ and $4x + 3y = 12$ by substitution.

- 11.** A band held a concert in its hometown. A total of 15 000 people attended. The tickets cost \$8.50 per student and \$12.50 per adult. The concert took in a total of \$162 500. How many adults came to the concert?
- 12.** Vito wants to hire a truck to do some moving. Athena's Garage charges \$80 for the day plus \$0.22/km. City Truck Rental charges \$100 per day and \$0.12/km.
- Write a linear system to represent this problem.
 - Solve the system of equations. Interpret the solution in terms of the problem.
 - Which company should Vito hire? Explain.
- 13.** Minden Karate Club has a competition for the students. If you win a grappling match you are awarded 5 points. If you tie, you are awarded 2 points. Rebecca grappled 15 times with 3 losses and her score was 42 points. How many grapples did Rebecca win?



Chapter Problem

- 14.** Logan is selling dog tags to raise money for the dog rescue organization. The company that makes the tags charges a flat fee of \$348 plus \$2 per tag. Logan plans to sell the tags for \$5 each.
- Write an equation to show the total cost for the dog tags.
 - Write an equation to show the revenue.
 - How many dog tags must Logan sell in order to break even?

Extend the Concepts



- 15.** Is $(3, -5)$ a solution to the linear system $2x + 5y = 19$ and $6y - 8x = -54$? Explain.

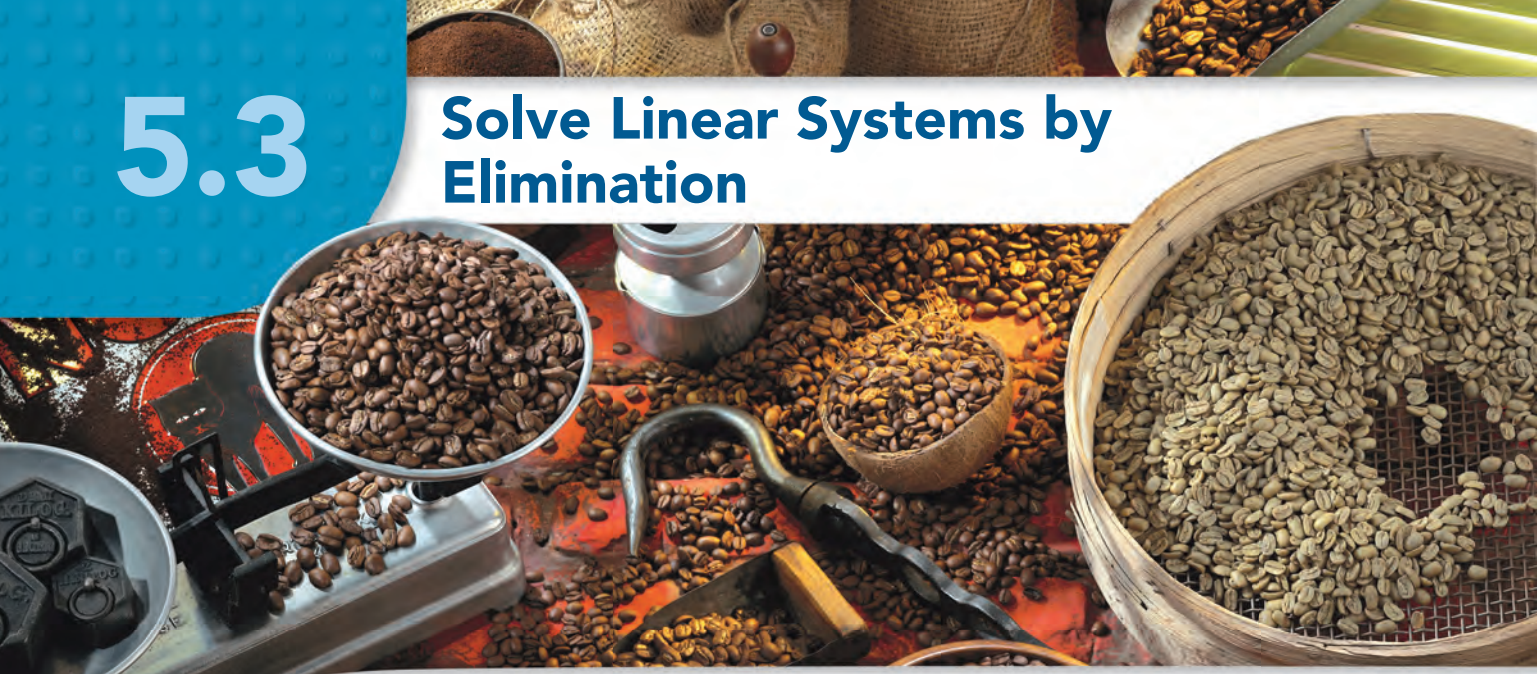
Math Connect

Remember, parallel lines have the same slope.

- 16. a)** Try to use the substitution method to solve the system $y = 3x$ and $y = 3x - 7$. What do you notice?
- b)** Graph the system. What do you notice?
- c)** Explain why there is no solution for this linear system.
- 17. a)** Try to use the substitution method to solve the system $3y - 6x = 15$ and $y = 2x + 5$. What do you notice?
- b)** Graph the system. What do you notice?
- c)** Explain why there is more than one solution for this linear system.

5.3

Solve Linear Systems by Elimination



Jemma owns a coffee shop. She mixes different kinds of coffee beans to make different blends of coffee.

Investigate A

Add or Subtract Linear Equations

Tools

- grid paper

Work in a group of four.

1. a) Each person chooses a different linear system.

System A

$$x - y = 1 \quad \textcircled{1}$$

$$2x + 5y = 16 \quad \textcircled{2}$$

System C

$$x - y = 2 \quad \textcircled{1}$$

$$2x + 3y = 14 \quad \textcircled{2}$$

System B

$$x - 3y = 6 \quad \textcircled{1}$$

$$3x + 4y = -21 \quad \textcircled{2}$$

System D

$$x + y = 5 \quad \textcircled{1}$$

$$2x - 5y = 17 \quad \textcircled{2}$$

- b) On grid paper, graph the linear system. Label each equation. What is the point of intersection?
2. a) Add the equations in your linear system. Label the resulting equation $\textcircled{3}$.
- b) Graph equation $\textcircled{3}$ on the same set of axes as you used in question 1. What do you notice?
3. a) Subtract one equation from the other. Label the resulting equation $\textcircled{4}$.
- b) Graph equation $\textcircled{4}$ on the same set of axes as you used in questions 1 and 2. What do you notice?

4. Compare your results with students in other groups who graphed the same linear system.
5. Compare your results with other members of your group.
6. **Reflect** Make a statement about the equation that results from adding or subtracting the equations in a linear system.

Investigate

B

Multiply a Linear Equation by a Constant

Tools

- grid paper

Work in a group of four.

1. **a)** Each person chooses a different linear equation.

Equation A

$$y = 4x + 3$$

Equation C

$$y = -x - 2$$

Equation B

$$2x + y = 5$$

Equation D

$$3x - y = 1$$

- b)** Use grid paper. Graph the line and label it with its equation.
2. **a)** Choose a number between -5 and 5 . Multiply each term in your equation by the number you chose.
 - b)** Graph the resulting equation on the same set of axes. What do you notice?
 - c)** Choose a different number. Multiply each term in your original equation by the number and graph the result on the same set of axes.
4. Compare your results with students in other groups who graphed the same line.
5. Compare your results with other members of your group.
6. **Reflect** Make a statement about the equation that results from multiplying each term in an equation by a constant.

elimination method

- an algebraic method of solving a system of linear equations
- the equations are added or subtracted to eliminate one variable

When two linear equations are added or subtracted, the resulting equation passes through the same point of intersection. When each term in a linear equation is multiplied by a constant, the resulting equation gives the same line. In this section, you will apply these principles to use another algebraic method to solve linear systems, the **elimination method**.

Example**1****Solve by Elimination**

Use the elimination method to solve the linear system
 $3x + y = 19$ and $4x - y = 2$.

Solution

Write the equations so like terms appear in a column.

$$3x + y = 19 \quad \textcircled{1}$$

$$+ 4x - y = 2 \quad \textcircled{2}$$

$$\hline 7x = 21 \quad \textcircled{3}$$

$$\frac{7x}{7} = \frac{21}{7}$$

$$x = 3$$

Add the equations to eliminate y .

Solve equation $\textcircled{3}$ for x .

$$3x + y = 19$$

$$3(\mathbf{3}) + y = 19$$

$$9 + y = 19$$

$$9 + y - 9 = 19 - 9$$

$$y = 10$$

Substitute the value $x = 3$ into equation $\textcircled{1}$ to find the value of y .

Check the solution (3, 10) in equation $\textcircled{2}$.

$$\text{LS} = 4x - y \qquad \qquad \qquad \text{RS} = 2$$

$$= 4(\mathbf{3}) - (\mathbf{10})$$

$$= 12 - 10$$

$$= 2$$

$$= \text{RS}$$

The solution is (3, 10).

Example**2****Solve a Linear System**

Solve the linear system $4x - 2y = 6$ and $x + y = 6$.

Solution

$$4x - 2y = 6 \quad \textcircled{1}$$

$$x + y = 6 \quad \textcircled{2}$$

$$4x - 2y = 6 \quad \textcircled{1}$$

$$+ 2x + 2y = 12 \quad \textcircled{3}$$

$$\hline 6x = 18$$

$$\frac{6x}{6} = \frac{18}{6}$$

$$x = 3$$

Multiply each term in equation $\textcircled{2}$ by 2.

Label the new equation $\textcircled{3}$.

Add $\textcircled{1} + \textcircled{3}$.

$$x + y = 6$$

$$3 + y = 6$$

$$3 + y - 3 = 6 - 3$$

$$y = 3$$

Check the solution (3, 3) in equation ①.

$$\begin{aligned} \text{LS} &= 4x - 2y & \text{RS} &= 6 \\ &= 4(3) - 2(3) \\ &= 12 - 6 \\ &= 6 \\ &= \text{RS} \end{aligned}$$

The solution is (3, 3).

Substitute the value $x = 3$ into equation ② to find the value of y .

Example

3

Make a Coffee Blend

Jemma is making 120 kg of a new blend of coffee that will sell for \$15/kg. The blend is made from two kinds of coffee: one that sells for \$18/kg, and another that sells for \$10/kg. How much of each type of coffee should Jemma use to make the new blend?

Solution

Let x represent the mass, in kilograms, of the coffee that sells for \$18/kg.

Let y represent the mass, in kilograms, of the coffee that sells for \$10/kg.

$$x + y = 120 \quad \text{①}$$

$$18x + 10y = 1800 \quad \text{②}$$

$$10x + 10y = 1200 \quad \text{③}$$

$$\begin{array}{r} 8x \qquad \qquad = 600 \end{array}$$

$$\frac{8x}{8} = \frac{600}{8}$$

$$x = 75$$

$$x + y = 120$$

$$75 + y = 120$$

$$75 + y - 75 = 120 - 75$$

$$y = 45$$

Jemma will make 120 kg of the blend.

The blend will sell for \$15/kg, so the cost for 120 kg is \$1800.

Multiply each term in equation ① by 10. Label the new equation ③.

Subtract ② – ③.

Substitute the value $x = 75$ into equation ① to find the value of y .

$$x + y = 6$$

$$3 + y = 6$$

$$3 + y - 3 = 6 - 3$$

$$y = 3$$

Check the solution (3, 3) in equation ①.

$$\begin{aligned} \text{LS} &= 4x - 2y & \text{RS} &= 6 \\ &= 4(3) - 2(3) \\ &= 12 - 6 \\ &= 6 \\ &= \text{RS} \end{aligned}$$

The solution is (3, 3).

Substitute the value $x = 3$ into equation ② to find the value of y .

Example

3

Make a Coffee Blend

Jemma is making 120 kg of a new blend of coffee that will sell for \$15/kg. The blend is made from two kinds of coffee: one that sells for \$18/kg, and another that sells for \$10/kg. How much of each type of coffee should Jemma use to make the new blend?

Solution

Let x represent the mass, in kilograms, of the coffee that sells for \$18/kg.

Let y represent the mass, in kilograms, of the coffee that sells for \$10/kg.

$$x + y = 120 \quad \text{①}$$

$$18x + 10y = 1800 \quad \text{②}$$

$$10x + 10y = 1200 \quad \text{③}$$

$$\begin{array}{r} 8x \qquad \qquad = 600 \end{array}$$

$$\frac{8x}{8} = \frac{600}{8}$$

$$x = 75$$

$$x + y = 120$$

$$75 + y = 120$$

$$75 + y - 75 = 120 - 75$$

$$y = 45$$

Jemma will make 120 kg of the blend.

The blend will sell for \$15/kg, so the cost for 120 kg is \$1800.

Multiply each term in equation ① by 10. Label the new equation ③.

Subtract ② – ③.

Substitute the value $x = 75$ into equation ① to find the value of y .

Check the solution (75, 45) in equation ②.

$$\begin{aligned} \text{LS} &= 18x - 10y & \text{RS} &= 1800 \\ &= 18(75) - 10(45) \\ &= 1350 - 450 \\ &= 1800 \\ &= \text{RS} \end{aligned}$$

Jemma should use 75 kg of the coffee that sells for \$18/kg and 45 kg of the coffee (that sells for \$10/kg) to make the blend.

Key Concepts

- A system of linear equations can be solved algebraically using the elimination method.
- To solve a linear system by elimination, the equations are added or subtracted to eliminate one variable.
- When each term in an equation is multiplied by a constant, the resulting equation produces the same line when graphed.

Discuss the Concepts

- D1.** Consider the linear system $x + y = 5$ and $x - y = 7$.
- To eliminate the x terms, would you add or subtract the two equations? Explain.
 - To eliminate the y terms, would you add or subtract the two equations? Explain.
 - Will you end up with the same point of intersection if you add or subtract the two equations? Explain.
- D2.** What is the main difference between the method of substitution and the method of elimination?

Practise the Concepts **A**

For help with question 1, refer to Example 1.

- 1.** Use the elimination method to solve each linear system.

a) $x + y = 2$
 $3x - y = 2$

b) $x - y = -1$
 $3x + y = -7$

c) $2x + y = 8$
 $4x - y = 4$

d) $2x - y = -6$
 $4x + y = -6$

e) $2x + y = -5$
 $-2x + y = -1$

f) $4x - y = -1$
 $-4x - 3y = -19$

For help with question 2, refer to Example 2.

2. Solve each linear system by elimination. Check your answer.

a) $2x + y = 7$
 $x - y = -1$

b) $3x + 2y = -1$
 $-3x + 4y = 7$

c) $x - y = 3$
 $2x + y = 3$

d) $3x + 2y = 5$
 $x - 2y = -1$

e) $2x + 5y = 3$
 $2x - y = -3$

f) $2x - y = 3$
 $4x - y = -1$

3. Solve each linear system.

a) $x + 2y = 2$
 $3x + 5y = 4$

b) $3x + 5y = 12$
 $2x - y = -5$

c) $2x - 3y = -12$
 $6x + 5y = -8$

d) $4x - 7y = 19$
 $3x - 2y = 11$

4. Solve each linear system.

a) $4x + 3y = 4$
 $8x - y = 1$

b) $5x - 3y = 2$
 $10x + 3y = 5$

c) $5x + 2y = 48$
 $x + y = 15$

d) $2x + 3y = 8$
 $x - 2y = -3$

Apply the Concepts **B**

Math Connect

Spice mixes are blends of spices or herbs. Many are pre-mixed and packaged for sale. For example, lemon pepper, chili powder, curry powder, Chinese five-spice powder, and garlic salt.

For help with question 5, refer to Example 3.

5. Abby mixes cinnamon and nutmeg to make 25 g of a spice mix. Cinnamon costs 9¢/g and nutmeg costs 12.5¢/g. The spice mix costs 9.7¢/g. How much of each spice does Abby need to use?
6. Tickets for a play cost \$5 for adults and \$3 for children. A total of 800 tickets are sold and total sales are \$3600.
- a) Write a system of linear equations to represent the situation.
b) How many adult tickets are sold?
7. When flying into the wind, an airplane travels at an average speed of 540 km/h. When flying with the wind, the airplane travels at an average speed of 680 km/h. Let s represent the speed of the airplane with no wind and w represent the wind speed.
- a) Write a system of linear equations to represent the situation.
b) Describe how you would calculate the wind speed.

Literacy Connect

8. Eleni rents a car on two separate occasions. The first time, she pays \$180 for 3 days and 150 km. The next time, she pays \$180 for 2 days and 400 km.
- What is the average cost per day?
 - What is the average cost per kilometre?
9. Logan's next fundraising event is dog grooming. A local dog groomer will charge a flat fee of \$120 plus \$8 per dog. Logan plans to charge customers \$16 per dog.

Chapter Problem



- Write a linear system to represent this situation.
- What is the minimum number of customers Logan needs to make money from this event?

Extend the Concepts **C**

10. An inlet pipe on a storage tank will fill the tank in 9 h. An outlet pipe will empty the same tank in 12 h. Suppose both pipes are open. How long will it take to fill the tank?
11. Sarah can paint a fence in 5 h. Wesley can paint the fence in 6 h. They decide to work together. How long will it take Sarah and Wesley to paint the fence?



12. Songi's car's radiator has a capacity of 25 L. It is currently full of water mixed with antifreeze. One-fifth of the volume of the solution is antifreeze. The weather is getting colder and Songi wants to change the mixture so that the solution will be three-fifths antifreeze. How much of the solution in her radiator does Songi need to drain and replace with antifreeze?

5.4

Solve Problems Involving Linear Systems



The Snowbirds Demonstration Team is a group of pilots and technicians from the Canadian Forces. They fly more than 50 different formations and manoeuvres at air shows all over North America. Linear equations can be used to model the flight paths of the Snowbirds for some of their manoeuvres.

Investigate

Tools

- grid paper or graphing calculator

Choose a Method to Solve a Linear System

1. Use a graph to solve the linear system $y = 3x + 1$ and $y = 4x - 3$.
2. Use a graph to solve the linear system $x + y = 101$ and $300x - y = 200$.
3. Compare your results from questions 1 and 2.
 - a) Why was it difficult to solve the linear system in question 2 by graphing?
 - b) Which method might have been easier to use? Why?
4.
 - a) Use the substitution method to solve the linear system from question 1.
 - b) Use the elimination method to solve the linear system from question 1.
 - c) Compare your answers from questions 1, 4a), and 4b). What do you notice?
 - d) Which method did you find easiest to use in solving the linear system $y = 3x + 1$ and $y = 4x - 3$? Why?

5. **a)** Use the substitution method to solve the linear system from question 2.
- b)** Use the elimination method to solve the linear system from question 2.
- c)** Compare your answers from questions 2, 5a), and 5b). What do you notice?
- d)** Which method did you find easiest to use in solving the linear system $x + y = 101$ and $300x - y = 200$? Why?
6. **Reflect** When might you choose to solve a system of equations each way?
- a)** by graphing
- b)** by substitution
- c)** by elimination

Example

1

Find the Number of Cars and Trucks

Neil's little brother has a total of 8 cars and trucks to play with. For his birthday, he wants to double the number of cars he has. If he does, he will have a total of 11 cars and trucks. How many cars does Neil's brother have now? How many trucks?



Solution

Let x represent the number of cars Neil's brother has now and y the number of trucks he has now.

$$2x + y = 11 \quad \textcircled{1}$$

$$x + y = 8 \quad \textcircled{2}$$

The values of x and y that solve this system must be whole numbers. It is not possible to have part of a car or truck.

Method 1: Solve by Substitution

Rearrange one equation to isolate y .

$$2x + y = 11 \quad \textcircled{1}$$

$$y = 8 - x \quad \textcircled{2}$$

$$2x + (8 - x) = 11$$

$$2x + 8 - x = 11$$

$$x + 8 = 11$$

$$x + 8 - 8 = 11 - 8$$

$$x = 3$$

Substitute $8 - x$ for y into equation $\textcircled{1}$.

$$y = 8 - x$$

$$y = 8 - 3$$

$$y = 5$$

Substitute $x = 3$ into equation $\textcircled{2}$ and solve for y .

Check the solution $(3, 5)$ in equation $\textcircled{1}$.

$$\text{LS} = 2x + y$$

$$= 2(3) + 5$$

$$= 6 + 5$$

$$= 11$$

$$= \text{RS}$$

$$\text{RS} = 11$$

The solution is $(3, 5)$. Neil's brother has 3 cars and 5 trucks now.

Method 2: Solve by Graphing Using Technology

Rearrange each equation to isolate y .

$$2x + y = 11$$

$$2x + y - 2x = 11 - 2x$$

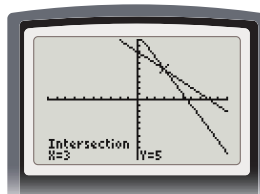
$$y = 11 - 2x$$

$$x + y = 8$$

$$x + y - x = 8 - x$$

$$y = 8 - x$$

- Use a graphing calculator. Press $\boxed{Y=}$. Enter the equations $Y1 = 11 - 2x$ and $Y2 = 8 - x$.
- Use standard window settings. Press $\boxed{\text{GRAPH}}$.



- Press $\boxed{2nd}$ $\boxed{[CALC]}$ $\boxed{5}$ \boxed{ENTER} \boxed{ENTER} \boxed{ENTER} .

The solution is $(3, 5)$. Neil's brother has 3 cars and 5 trucks now.

In Example 1, the problem can be solved by graphing, or algebraically using the substitution method or the elimination method. Choose the method you find easiest to work with.

Example

2

Investing

Mari invests \$3000 in two funds. The education savings plan pays interest at a rate of 7% per year and the guaranteed investment certificate (GIC) pays interest at 5% per year. At the end of the year, she has earned \$190 in interest. How much did Mari invest at each rate?

Solution

Let e represent the amount invested in the education savings plan.
Let g represent the amount invested in the GIC.

$$e + g = 3000 \quad \textcircled{1}$$

Mari invests a total of \$3000.

$$0.07e + 0.05g = 190 \quad \textcircled{2}$$

Express each percent as a decimal.

This system of equations would be difficult to solve by graphing. Solve by elimination.

$$0.07e - 0.05g = 190 \quad \textcircled{2}$$

$$0.05e + 0.05g = 150 \quad \textcircled{3}$$

Multiply each term in equation ① by 0.05 to get equation ③, then subtract ② - ③.

$$\underline{0.02e} \qquad = 40$$

$$\frac{0.02e}{0.02} = \frac{40}{0.02}$$

$$e = 2000$$

$$e + g = 3000$$

$$2000 + g = 3000$$

Substitute $e = 2000$ into equation ① and solve for g .

$$2000 + g - 2000 = 3000 - 2000$$

$$g = 1000$$

Check the solution (2000, 1000) in equation ②.

$$\text{LS} = 0.07e + 0.05g \qquad \text{RS} = 190$$

$$= 0.07(2000) + 0.05(1000)$$

$$= 140 + 50$$

$$= 190$$

$$= \text{RS}$$

Mari invested \$2000 in the education savings plan at 7% and \$1000 in the GIC at 5%.

Key Concepts

- A linear system can be solved by graphing or algebraically.
- Different methods of solving a linear system should give the same solution.

Discuss the Concepts

- D1.** Describe a situation in which you would solve a linear system by graphing. Give an example.
- D2.** Give an example of a linear system you would solve using the substitution method. Explain why you would use substitution to solve the system.
- D3.** Give an example of a linear system you would solve using the elimination method. Explain why you would use elimination to solve the system.

Practise the Concepts **A**



For help with question 1, refer to Example 1.

- 1.** Julia's annual salary in dollars, S , can be represented by the equation $S = 30\,500 + 500n$ where n is the number of years she has worked for the company. Aysha works for another company. Her annual salary can be represented by the equation $S = 26\,000 + 1000n$ where n represents the number of years Aysha has been working there. After how many years will the two women have the same salary? What will that salary be?

For help with questions 2 and 3, refer to Example 2.

- 2.** Silvio invests \$8000 for his children's education. He invests part of the money in a high-risk bond that pays 5% interest per year, and the rest in a lower-risk bond that pays 3.25% per year. After one year, he has a total of \$312.50 in interest. How much did Silvio invest at each rate?
- 3.** Gareth plans to go to college in a year and needs to save for tuition. He invests his summer earnings of \$3050, part at 8% interest per year, and part at 7.5% per year. After one year, Gareth has earned a total of \$234 in interest. How much did he invest at each rate?



4. To join Jungle Gymnastics Club, Sonja will pay a monthly fee of \$25 and an initiation fee of \$200. If she chooses to join Brant Gymnastics Club, she will pay an initial fee of only \$100 but \$35 per month.
- After how many months are the costs the same?
 - If Sonja plans to do gymnastics for 6 months, which club should she join? Why?
 - If Sonja plans to do gymnastics for more than a year, which club should she join? Why?
5. Solve the linear system $y = 2x - 1$ and $3x - y = 5$. Which method did you use? Why?
6. For a basketball tournament, Marcus orders T-shirts for all the participants. The medium-sized shirts cost \$4 per shirt, and the large-sized shirts cost \$5 per shirt. Marcus orders a total of 70 shirts. He spends \$320. How many of the shirts are medium-sized?
7. Students hold a car wash to raise money for a school trip to the west coast. They charge \$7 per car and \$10 per van. If they washed a total of 52 cars and vans and earned \$457, how many cars did they wash? How many vans did they wash?



Achievement Check

8. Meredith drove 255 km from Kingston to Toronto in $2\frac{3}{4}$ h. She drove part of the way at 100 km/h and the rest of the way at 60 km/h.
- Suppose the equation for time is $\frac{x}{100} + \frac{y}{60} = 2.75$. What do the variables represent?
 - Write an equation for the total distance.
 - Solve the linear system to find the distance Meredith drove at each speed.

Literacy Connect

9. Which method would you use to solve the system $3x - y = 8$ and $4x - y = -15$? Why?
10. For a contest, students have to solve this problem:
The length of a rectangle is 6 cm more than its width. The perimeter of the rectangle is 84 cm. What are the dimensions of the rectangle?
- a) One student says the dimensions of the rectangle are 39 cm by 45 cm. Is the student correct? How do you know?
- b) What is the answer to the contest problem?

Extend the Concepts

11. The ordered pairs $(1, 3)$ and $(-2, -9)$ are both solutions to the linear system $y = 4x - 1$ and $8x - 2y = 2$. Explain how this is possible.
12. Students are planning a ski trip. They have a choice between two packages. The first package costs \$630 per student. It includes 2 meals a day and accommodation for 9 days. The second package costs \$720 per student. It includes 3 meals a day and accommodation for 9 days.

Math Connect

The 48 000 m² Kempinski Hotel Mall in Dubai, opened in 2006. The hotel is attached to the world's third-largest indoor ski resort. It offers a unique chance to ski in the middle of the desert.



- a) What is the cost per meal?
- b) What is the cost per day for accommodation?
13. Harvinder drives 400 km in 5.5 h. For the first part of his trip, his average speed is 80 km/h. For the second part of his trip, his average speed is 60 km/h.
- a) Let x represent the distance Harvinder travels at 80 km/h. Let y represent the distance he drove at 60 km/h. Write a system of equations to represent this situation.
- b) How far does Harvinder drive at each speed?

5

Chapter 5 Review

Review of Key Terms

1. Explain the meaning of each term in your own words.
 - a) linear system
 - b) point of intersection
 - c) substitution method
 - d) elimination method

5.1 Solve Linear Systems by Graphing, pages 198 to 204

2. Solve each linear system by graphing.

a) $y = 2x + 3$
 $y = x + 4$

b) $2x - y = -13$
 $x - y = -10$

c) $x + y = 7$
 $3x - y = 5$

d) $2x - y = 2$
 $4x + y = 10$

3. Use a graphing calculator to solve each linear system. Round answers to two decimal places where necessary.

a) $y = -3x + 4$
 $y = 4x + 13$

b) $y = 5x - 6$
 $y = \frac{2}{5}x - \frac{3}{5}$

c) $y = -4x + \frac{2}{3}$
 $y = 2x + \frac{8}{3}$

d) $y = 6x - 13$
 $y = -\frac{3}{4}x - \frac{5}{2}$

5.2 Solve Linear Systems by Substitution, pages 205 to 211

4. Solve each linear system by substitution.

a) $y = 7x + 1$
 $2x + y = 10$

b) $2x + y = 4$
 $4x - y = -1$

c) $x - y = 7$
 $3x + y = 5$

d) $x + y = 5$
 $x - y = -1$

5. On his farm, Maddock plants a total of 20 ha. He plants both corn and canola. If he plants three times as much corn as canola, how many hectares of each type of plant does he plant?



6. A teacher plans to buy books for her class. She has 28 students and wants to buy a book for each one of them. The books cost \$5 each for softcovers and \$8 each for hardcovers. The teacher has \$173 to spend. How many of each type of book can she buy?

5.3 Solve Linear Systems by Elimination, pages 212 to 218

7. Solve each linear system by elimination.

a) $x + 2y = 3$
 $x - 4y = 0$

b) $3x + 2y = 18$
 $x - 3y = -5$

c) $2x - 3y = -19$
 $4x + 6y = 28$

d) $3x + y = 4$
 $6x - y = -1$

8. Isabella rode her motorcycle at constant speed. It took her 2 hours to travel 216 km with the wind behind her. The return trip took her 3 hours riding into the wind.



a) Let s represent the speed of the motorcycle and w represent the speed of the wind. Write a linear system to represent this situation.

b) Find the speed of the motorcycle and the speed of the wind.

9. The Athletic Council wants to buy a total of 45 volleyballs and basketballs. The council has \$435 to spend. Each volleyball costs \$8 and each basketball costs \$11. How many of each type of ball can be purchased?

5.4 Solve Problems Involving Linear Systems, pages 219 to 225

10. Solve each linear system. Which method did you use each time? Why?

a) $x + 3y = 7$
 $2x + 4y = 11$

b) $2x - y = 27$
 $x + y = 12$

c) $y = -x + 8$
 $y = 6x + 1$

d) $y = 2x - 8$
 $x - y = 4$

11. Anna has a total of \$6000 to invest. She puts part of it in an investment paying 8% per year, and the rest in an investment paying 6% per year. At the end of one year, Anna earned \$440 in interest. How much did she invest at each rate?

12. Doug is buying a new cell phone. He has narrowed his choices down to two plans. The first plan costs \$40 per month with unlimited calling. The second plan costs \$10 per month plus \$0.10 per minute. Which plan should Doug choose? Explain your choice.

13. Carlie has a jar of coins. She tells her sister that the jar has 45 quarters and dimes altogether and the value of the coins is \$6.30. Find the number of each type of coin in the jar.

- Solve by graphing.
 - $y = 2x + 3$
 $y = x + 4$
 - $y = 3x - 4$
 $y = \frac{1}{2}x + 1$
 - $y = -5x + 2$
 $y = x + 8$
- Solve by substitution.
 - $2x - y = 3$
 $x - y = 4$
 - $x = 4y - 3$
 $2x + y = 6$
 - $x - 2y = 7$
 $2x = 3y + 13$
- Solve by elimination.
 - $2x - y = -13$
 $x - y = -10$
 - $2x - y = -2$
 $x + 2y = 9$
 - $4x - 3y = 11$
 $2x + 3y = -1$
- Solve the linear system. Which method did you use? Why?
 $4x + 5y = 3$
 $2x - 3y = 7$
- Explain how you would solve this linear system.
 $y = 7x + 1$
 $2x + y = 10$
 - What is the solution?
- Nathan and Vivek have part-time jobs at the same company. Nathan is paid \$10 per shift and \$4 for each item he makes in his shift. Vivek is paid \$40 per shift and \$1 per item he makes.
 - Write a system of linear equations to represent this situation.
 - How many items must each person make in one shift to earn the same amount of money?
 - How much will they each earn for that shift?
- Ramona is planning to have graduation shirts made. Company A charges a \$40 set-up fee plus \$5 per shirt for printing. Company B charges a \$100 set-up fee plus \$2 per shirt for printing.
 - Write a system of linear equations to represent this situation.
 - Find the number of shirts for which the cost is the same for both companies.
 - Under what circumstances should Ramona choose company A? Company B? Explain.

Chapter Problem Wrap-Up

To wrap-up his fundraising campaign, Logan organized a dog show. The cost to rent the park was \$1300 plus \$2.50 per person for the snack bar fee. Logan charged \$5 for each student ticket and \$8 for each adult ticket. His total ticket sales were \$3585 and the total attendance was 525 people.

- a) How many students attended the dog show?
- b) How many adults attended the dog show?
- c) How much money did Logan raise with this event for the dog rescue organization?



8. Snowbound Adventures charges a \$5 flat fee plus \$1/h to rent snowboarding equipment. Shred-Zone charges a \$7 flat fee plus \$0.50/h to rent snowboarding equipment.
 - a) Write a linear equation to represent the total cost for each company.
 - b) What is the point of intersection of the linear system? What do the coordinates of the point of intersection mean in terms of this situation?
 - c) Describe a situation in which it was cheaper to rent equipment from Snowbound Adventures.
9. The school sold 108 tickets for the spring concert. Student tickets cost \$2 each and adult tickets cost \$5 each.
 - a) The concert proceeds were \$351. How many students attended?
 - b) If each of the 108 concert attendees paid an average of \$3 for refreshments, find the total revenue from the concert.
10. Barbara and her daughter Linda are choosing a new cell phone plan. CellularPlus charges \$10 per month and \$0.35 per minute for any minutes used during the month. CheapCell charges \$20 per month and \$0.15 per minute for every minute used in the month. For what number of minutes per month will the plans cost the same amount?

Task

Charity Fundraising

The Student Council at your school plans to raise money for a local charity. The fundraising committee has two options and needs to determine which option will raise the greater amount of money.



Option A

60/40 Raffle

Raffle tickets are sold for \$4 each. The person whose ticket is drawn wins 40% of the total amount collected from ticket sales. The remaining 60% goes to the charity.

Option B

Battle of the Bands

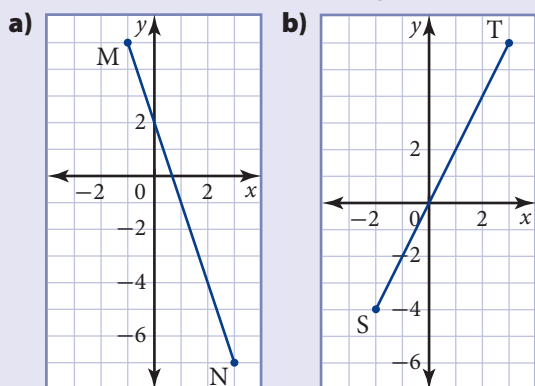
The committee rents the local community theatre to hold a battle of the bands contest. The theatre always charges the same fixed rental fee. Two other schools ran similar fundraisers earlier in the year. One school sold 100 tickets and raised \$50 for charity. The second school sold 300 tickets and raised \$1050 for charity. Each of the schools charged the same price for tickets.

1. For Option A, let x represent the number of raffle tickets sold and y represent the amount that goes to the charity. Write an algebraic equation to model the amount raised for charity by Option A.
2. **a)** For Option B, let p represent the price of one ticket and r represent the fixed rental fee at the community theatre. Use the information about the fundraisers held by the other two schools to determine the fixed rental fee and the ticket price.
b) Use your answer from part a). Let x represent the number of tickets sold for the battle of the bands and y represent the amount raised for charity. Write an algebraic equation to model the amount raised for charity by Option B.
3. **a)** Write and then solve a system of linear equations to determine the number of tickets that must be sold for Options A and B to raise the same amount of money.
b) Under what conditions would you recommend using Option A? Option B?
4. **a)** Suppose 200 tickets are sold. Which option would raise more money? How much more?
b) Without calculating the difference, will the difference between the amount raised using each option increase or decrease if only 100 tickets are sold? Explain your reasoning.



Chapter 3: Linear Relations

1. Find the slope of each line segment.



2. A taxi driver charges a \$2 flat fee plus \$0.35/km. The cost can be modelled by the equation $C = 0.35d + 2$, where C is the cost in dollars and d is the distance in kilometres.

- Make a table of values showing the cost for trips from 0 to 10 km.
 - Graph the costs from part a).
 - Find the slope of the line. What does the slope represent?
3. a) Graph the relation $y = -3x + 1$ using a graphing calculator with the standard window settings. Sketch the graph in your notebook.
- b) Calculate the rate of change by looking in the table of values.
- c) Determine the value of y when $x = 0$.

4. Write an equation for each line given the slope and y -intercept.

- slope: 2, y -intercept: 4
- slope: $\frac{3}{2}$, y -intercept: $-\frac{1}{4}$
- slope: -3 , y -intercept: 0
- slope: 0, y -intercept: $\frac{1}{2}$

5. a) Graph the relation $y = -3x + 1$ using a graphing calculator with the standard window settings. Sketch the graph in your notebook.
- b) Calculate the rate of change by using the table of values.
- c) Determine the value of y when $x = 0$.
6. Determine the equation of each line.
- slope of -4 , through $(-1, -6)$
 - through $(3, 4)$ and $(1, -6)$
 - a vertical line through $(1, -6)$

Chapter 4: Linear Equations

7. Solve each linear equation.

- $3x - 8 = 7$
- $\frac{x}{3} + 2 = 6$
- $6 - 4x = 2x + 12$
- $2(x + 1) = 3x + 6$
- $\frac{2(t + 3)}{4} = t - 2$
- $\frac{x + 2}{3} = \frac{2x + 1}{4}$

- 8.** Ji Hwan joined the local wellness centre. He paid an initial fee of \$100 and will pay \$5 per visit. The relation can be modelled using the equation $C = 100 + 5x$, where C is the total cost in dollars and x is the number of visits.
- What will be the total cost if Ji Hwan uses the centre 26 times?
 - How many times can he use the centre for \$400?
- 9.** Rearrange each formula to solve for the indicated variable.
- $P = 2l + 2w$, for l
 - $A = \pi r^2$, for r
 - $S = 2\pi rh$, for h
- 10.** Use the simple interest formula $I = Prt$ to find the principal amount that was invested to earn \$600 in interest after 6 years at 2%.
- 11.** Rearrange each equation into slope y -intercept form, then state the slope and y -intercept.
- $2x + y - 6 = 0$
 - $3x - y + 4 = 0$
 - $4x - 3y - 6 = 0$
- 12.** The line $2x + By - 8 = 0$ passes through the point (1, 3). Find the value of B .
- 13.** Graph to solve each system of equations.
- $y = 2x - 2$
 $y = 3x - 3$
 - $2x + y = -4$
 $-x - 2y = 5$
- 14.** Solve each system of linear equations by substitution. Check your solution.
- $y = 3x + 2$
 $x + 2y = 11$
 - $3x + y = -9$
 $x - 2y = 4$
- 15.** Solve each system of linear equations by elimination. Check your solution.
- $x + 3y = 2$
 $3x + 2y = -1$
 - $2x - y = -3$
 $6x + 4y = 12$
- 16.** Angela is seeking a venue for her school's athletic council banquet. Primo Banquet Hall charges \$2000 plus \$50 per person. The Lookout Banquet Hall charges \$1500 plus \$75 per person.
- Write an equation to represent the total cost for each banquet hall.
 - Find the point of intersection for the linear system.
 - What does this point represent?
- 17.** Joan needs to rent a theatre for a concert. The theatre charges \$700 plus \$3 per person. Joan plans to sell tickets for \$10 per person. Let C represent the cost of the event and n represent the number of people attending the concert.
- Write a linear system to represent the situation.
 - How many tickets must be sold to break even?
 - What will be the total cost?

Chapter 5: Linear Systems