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APPENDIX A Compound Sum of $1

Period	Percent 1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%
1	1.010	1.020	1.030	1.040	1.050	1.060	1.070	1.080	1.090	1.100	1.110
2	1.020	1.040	1.061	1.082	1.103	1.124	1.145	1.166	1.188	1.210	1.232
3	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	1.331	1.368
4	1.041	1.082	1.126	1.170	1.216	1.262	1.311	1.360	1.412	1.464	1.518
5	1.051	1.104	1.159	1.217	1.276	1.338	1.403	1.469	1.539	1.611	1.685
6	1.062	1.126	1.194	1.265	1.340	1.419	1.501	1.587	1.677	1.772	1.870
7	1.072	1.149	1.230	1.316	1.407	1.504	1.606	1.714	1.828	1.949	2.076
8	1.083	1.172	1.267	1.369	1.477	1.594	1.718	1.851	1.993	2.144	2.305
9	1.094	1.195	1.305	1.423	1.551	1.689	1.838	1.999	2.172	2.358	2.558
10	1.105	1.219	1.344	1.480	1.629	1.791	1.967	2.159	2.367	2.594	2.839
11	1.116	1.243	1.384	1.539	1.710	1.898	2.105	2.332	2.580	2.853	3.152
12	1.127	1.268	1.426	1.601	1.796	2.012	2.252	2.518	2.813	3.138	3.498
13	1.138	1.294	1.469	1.665	1.886	2.133	2.410	2.720	3.066	3.452	3.883
14	1.149	1.319	1.513	1.732	1.980	2.261	2.579	2.937	3.342	3.797	4.310
15	1.161	1.346	1.558	1.801	2.079	2.397	2.759	3.172	3.642	4.177	4.785
16	1.173	1.373	1.605	1.873	2.183	2.540	2.952	3.426	3.970	4.595	5.311
17	1.184	1.400	1.653	1.948	2.292	2.693	3.159	3.700	4.328	5.054	5.895
18	1.196	1.428	1.702	2.206	2.407	2.854	3.380	3.996	4.717	5.560	6.544
19	1.208	1.457	1.754	2.107	2.527	3.026	3.617	4.316	5.142	6.116	7.263
20	1.220	1.486	1.806	2.191	2.653	3.207	3.870	4.661	5.604	6.727	8.062
25	1.282	1.641	2.094	2.666	3.386	4.292	5.427	6.848	8.623	10.835	13.585
30	1.348	1.811	2.427	3.243	4.322	5.743	7.612	10.063	13.268	17.449	22.892
40	1.489	2.208	3.262	4.801	7.040	10.286	14.974	21.725	31.409	42.259	65.001
50	1.645	2.692	4.384	7.107	11.467	18.420	29.457	46.902	74.358	117.39	184.57

APPENDIX A Compound Sum of $1 *(concluded)*

	Percent										
Period	12%	13%	14%	15%	16%	17%	18%	19%	20%	25%	30%
1 . . .	1.120	1.130	1.140	1.150	1.160	1.170	1.180	1.190	1.200	1.250	1.300
2 . . .	1.254	1.277	1.300	1.323	1.346	1.369	1.392	1.416	1.440	1.563	1.690
3 . . .	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685	1.728	1.953	2.197
4 . . .	1.574	1.630	1.689	1.749	1.811	1.874	1.939	2.005	2.074	2.441	2.856
5 . . .	1.762	1.842	1.925	2.011	2.100	2.192	2.288	2.386	2.488	3.052	3.713
6 . . .	1.974	2.082	2.195	2.313	2.436	2.565	2.700	2.840	2.986	3.815	4.827
7 . . .	2.211	2.353	2.502	2.660	2.826	3.001	3.185	3.379	3.583	4.768	6.276
8 . . .	2.476	2.658	2.853	3.059	3.278	3.511	3.759	4.021	4.300	5.960	8.157
9 . . .	2.773	3.004	3.252	3.518	3.803	4.108	4.435	4.785	5.160	7.451	10.604
10 . . .	3.106	3.395	3.707	4.046	4.411	4.807	5.234	5.696	6.192	9.313	13.786
11 . . .	3.479	3.836	4.226	4.652	5.117	5.624	6.176	6.777	7.430	11.642	17.922
12 . . .	3.896	4.335	4.818	5.350	5.936	6.580	7.288	8.064	8.916	14.552	23.298
13 . . .	4.363	4.898	5.492	6.153	6.886	7.699	8.599	9.596	10.699	18.190	30.288
14 . . .	4.887	5.535	6.261	7.076	7.988	9.007	10.147	11.420	12.839	22.737	39.374
15 . . .	5.474	6.254	7.138	8.137	9.266	10.539	11.974	13.590	15.407	28.422	51.186
16 . . .	6.130	7.067	8.137	9.358	10.748	12.330	14.129	16.172	18.488	35.527	66.542
17 . . .	6.866	7.986	9.276	10.761	12.468	14.426	16.672	19.244	22.186	44.409	86.504
18 . . .	7.690	9.024	10.575	12.375	14.463	16.879	19.673	22.091	26.623	55.511	112.46
19 . . .	8.613	10.197	12.056	14.232	16.777	19.748	23.214	27.252	31.948	69.389	146.19
20 . . .	9.646	11.523	13.743	16.367	19.461	23.106	27.393	32.429	38.338	86.736	190.05
25 . . .	17.000	21.231	26.462	32.919	40.874	50.658	62.699	77.388	95.396	264.70	705.64
30 . . .	29.960	39.116	50.950	66.212	85.850	111.07	143.37	184.68	237.38	807.79	2,620.0
40 . . .	93.051	132.78	188.88	267.86	378.72	533.87	750.38	1,051.7	1,469.8	7,523.2	36,119.
50 . . .	289.00	450.74	700.23	1,083.7	1,670.7	2,566.2	3,927.4	5,988.9	9,100.4	70,065.	497,929.

APPENDIX B Compound Sum of an Annuity of $1

	Percent										
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%
1 . . .	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2 . . .	2.010	2.020	2.030	2.040	2.050	2.060	2.070	2.080	2.090	2.100	2.110
3 . . .	3.030	3.060	3.091	3.122	3.153	3.184	3.215	3.246	3.278	3.310	3.342
4 . . .	4.060	4.122	4.184	4.246	4.310	4.375	4.440	4.506	4.573	4.641	4.710
5 . . .	5.101	5.204	5.309	5.416	5.526	5.637	5.751	5.867	5.985	6.105	6.228
6 . . .	6.152	6.308	6.468	6.633	6.802	6.975	7.153	7.336	7.523	7.716	7.913
7 . . .	7.214	7.434	7.662	7.898	8.142	8.394	8.654	8.923	9.200	9.487	9.783
8 . . .	8.286	8.583	8.892	9.214	9.549	9.897	10.260	10.637	11.028	11.436	11.859
9 . . .	9.369	9.755	10.159	10.583	11.027	11.491	11.978	12.488	13.021	13.579	14.164
10 . . .	10.462	10.950	11.464	12.006	12.578	13.181	13.816	14.487	15.193	15.937	16.722
11 . . .	11.567	12.169	12.808	13.486	14.207	14.972	15.784	16.645	17.560	18.531	19.561
12 . . .	12.683	13.412	14.192	15.026	15.917	16.870	17.888	18.977	20.141	21.384	22.713
13 . . .	13.809	14.680	15.618	16.627	17.713	18.882	20.141	21.495	22.953	24.523	26.212
14 . . .	14.947	15.974	17.086	18.292	19.599	21.015	22.550	24.215	26.019	27.975	30.095
15 . . .	16.097	17.293	18.599	20.024	21.579	23.276	25.129	27.152	29.361	31.772	34.405
16 . . .	17.258	18.639	20.157	21.825	23.657	25.673	27.888	30.324	33.003	35.950	39.190
17 . . .	18.430	20.012	21.762	23.698	25.840	20.213	30.840	33.750	36.974	40.545	44.501
18 . . .	19.615	21.412	23.414	25.645	28.132	30.906	33.999	37.450	41.301	45.599	50.396
19 . . .	20.811	22.841	25.117	27.671	30.539	33.760	37.379	41.446	46.018	51.159	56.939
20 . . .	22.019	24.297	26.870	29.778	33.066	36.786	40.995	45.762	51.160	57.275	64.203
25 . . .	28.243	32.030	36.459	41.646	47.727	54.865	63.249	73.106	84.701	98.347	114.41
30 . . .	34.785	40.588	47.575	56.085	66.439	79.058	94.461	113.28	136.31	164.49	199.02
40 . . .	48.886	60.402	75.401	95.026	120.80	154.76	199.64	259.06	337.89	442.59	581.83
50 . . .	64.463	84.579	112.80	152.67	209.35	290.34	406.53	573.77	815.08	1,163.9	1,668.8

APPENDIX B Compound Sum of an Annuity of $1 *(concluded)*

Period	Percent 12%	13%	14%	15%	16%	17%	18%	19%	20%	25%	30%
1 . . .	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2 . . .	2.120	2.130	2.140	2.150	2.160	2.170	2.180	2.190	2.200	2.250	2.300
3 . . .	3.374	3.407	3.440	3.473	3.506	3.539	3.572	3.606	3.640	3.813	3.990
4 . . .	4.779	4.850	4.921	4.993	5.066	5.141	5.215	5.291	5.368	5.766	6.187
5 . . .	6.353	6.480	6.610	6.742	6.877	7.014	7.154	7.297	7.442	8.207	9.043
6 . . .	8.115	8.323	8.536	9.754	8.977	9.207	9.442	0.683	9.930	11.259	12.756
7 . . .	10.089	10.405	10.730	11.067	11.414	11.772	12.142	12.523	12.916	15.073	17.583
8 . . .	12.300	12.757	13.233	13.727	14.240	14.773	15.327	15.902	16.499	19.842	23.858
9 . . .	14.776	15.416	16.085	16.786	17.519	18.285	19.086	19.923	20.799	25.802	32.015
10 . . .	17.549	18.420	19.337	20.304	21.321	22.393	23.521	24.701	25.959	33.253	42.619
11 . . .	20.655	21.814	23.045	24.349	25.733	27.200	28.755	30.404	32.150	42.566	56.405
12 . . .	24.133	25.650	27.271	29.002	30.850	32.824	34.931	37.180	39.581	54.208	74.327
13 . . .	28.029	29.985	32.089	34.352	36.786	39.404	42.219	45.244	48.497	68.760	97.625
14 . . .	32.393	34.883	37.581	40.505	43.672	47.103	50.818	54.841	59.196	86.949	127.91
15 . . .	37.280	40.417	43.842	47.580	51.660	56.110	60.965	66.261	72.035	109.69	167.29
16 . . .	42.753	46.672	50.980	55.717	60.925	66.649	72.939	79.850	87.442	138.11	218.47
17 . . .	48.884	53.739	59.118	65.075	71.673	78.979	87.068	96.022	105.93	173.64	285.01
18 . . .	55.750	61.725	68.394	75.836	84.141	93.406	103.74	115.27	128.12	218.05	371.52
19 . . .	63.440	70.749	78.969	88.212	98.603	110.29	123.41	138.17	154.74	273.56	483.97
20 . . .	72.052	80.947	91.025	102.44	115.38	130.03	146.63	165.42	186.69	342.95	630.17
25 . . .	133.33	155.62	181.87	212.79	249.21	292.11	342.60	402.04	471.98	1,054.8	2,348.80
30 . . .	241.33	293.20	356.79	434.75	530.31	647.44	790.95	966.7	1,181.9	3,227.2	8,730.0
40 . . .	767.09	1,013.7	1,342.0	1,779.1	2,360.8	3,134.5	4,163.21	5,529.8	7,343.9	30,089.	120,393.
50 . . .	2,400.0	3,459.5	4,994.5	7,217.7	10,436.	15,090.	21,813.	31,515.	45,497.	280,256.	1,659,731.

APPENDIX C Present Value of $1

	Percent											
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	0.901	0.893
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826	0.812	0.797
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751	0.731	0.712
4	0.961	0.924	0.885	0.855	0.823	0.792	0.763	0.735	0.708	0.683	0.659	0.636
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621	0.593	0.567
6	0.942	0.888	0.837	0.790	0.746	0.705	0.666	0.630	0.596	0.564	0.535	0.507
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513	0.482	0.452
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467	0.434	0.404
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.500	0.460	0.424	0.391	0.361
10	0.905	0.820	0.744	0.676	0.614	0.558	0.508	0.463	0.422	0.386	0.352	0.322
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350	0.317	0.287
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319	0.286	0.257
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290	0.258	0.229
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263	0.232	0.205
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239	0.209	0.183
16	0.853	0.728	0.623	0.534	0.458	0.394	0.339	0.292	0.252	0.218	0.188	0.163
17	0.844	0.714	0.605	0.513	0.436	0.371	0.317	0.270	0.231	0.198	0.170	0.146
18	0.836	0.700	0.587	0.494	0.416	0.350	0.296	0.250	0.212	0.180	0.153	0.130
19	0.828	0.686	0.570	0.475	0.396	0.331	0.277	0.232	0.194	0.164	0.138	0.116
20	0.820	0.673	0.554	0.456	0.377	0.312	0.258	0.215	0.178	0.149	0.124	0.104
25	0.780	0.610	0.478	0.375	0.295	0.233	0.184	0.146	0.116	0.092	0.074	0.059
30	0.742	0.552	0.412	0.308	0.231	0.174	0.131	0.099	0.075	0.057	0.044	0.033
40	0.672	0.453	0.307	0.208	0.142	0.097	0.067	0.046	0.032	0.022	0.015	0.011
50	0.608	0.372	0.228	0.141	0.087	0.054	0.034	0.021	0.013	0.009	0.005	0.003

APPENDIX C Present Value of $1 *(concluded)*

Period	13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	35%	40%	50%
						Percent							
1	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	0.800	0.769	0.741	0.714	0.667
2	0.783	0.769	0.756	0.743	0.731	0.718	0.706	0.694	0.640	0.592	0.549	0.510	0.444
3	0.693	0.675	0.658	0.641	0.624	0.609	0.593	0.579	0.512	0.455	0.406	0.364	0.296
4	0.613	0.592	0.572	0.552	0.534	0.515	0.499	0.482	0.410	0.350	0.301	0.260	0.198
5	0.543	0.519	0.497	0.476	0.456	0.437	0.419	0.402	0.320	0.269	0.223	0.186	0.132
6	0.480	0.456	0.432	0.410	0.390	0.370	0.352	0.335	0.262	0.207	0.165	0.133	0.088
7	0.425	0.400	0.376	0.354	0.333	0.314	0.296	0.279	0.210	0.159	0.122	0.095	0.059
8	0.376	0.351	0.327	0.305	0.285	0.266	0.249	0.233	0.168	0.123	0.091	0.068	0.039
9	0.333	0.300	0.284	0.263	0.243	0.225	0.209	0.194	0.134	0.094	0.067	0.048	0.026
10	0.295	0.270	0.247	0.227	0.208	0.191	0.176	0.162	0.107	0.073	0.050	0.035	0.017
11	0.261	0.237	0.215	0.195	0.178	0.162	0.148	0.135	0.086	0.056	0.037	0.025	0.012
12	0.231	0.208	0.187	0.168	0.152	0.137	0.124	0.112	0.069	0.043	0.027	0.018	0.008
13	0.204	0.182	0.163	0.145	0.130	0.116	0.104	0.093	0.055	0.033	0.020	0.013	0.005
14	0.181	0.160	0.141	0.125	0.111	0.099	0.088	0.078	0.044	0.025	0.015	0.009	0.003
15	0.160	0.140	0.123	0.108	0.095	0.084	0.074	0.065	0.035	0.020	0.011	0.006	0.002
16	0.141	0.123	0.107	0.093	0.081	0.071	0.062	0.054	0.028	0.015	0.008	0.005	0.002
17	0.125	0.108	0.093	0.080	0.069	0.060	0.052	0.045	0.023	0.012	0.006	0.003	0.001
18	0.111	0.095	0.081	0.069	0.059	0.051	0.044	0.038	0.018	0.009	0.005	0.002	0.001
19	0.098	0.083	0.070	0.060	0.051	0.043	0.037	0.031	0.014	0.007	0.003	0.002	0
20	0.087	0.073	0.061	0.051	0.043	0.037	0.031	0.026	0.012	0.005	0.002	0.001	0
25	0.047	0.038	0.030	0.024	0.020	0.016	0.013	0.010	0.004	0.001	0.001	0	0
30	0.026	0.020	0.015	0.012	0.009	0.007	0.005	0.004	0.001	0	0	0	0
40	0.008	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0	0	0	0	0
50	0.002	0.001	0.001	0.001	0	0	0	0	0	0	0	0	0

APPENDIX D Present Value of an Annuity of $1

	Percent											
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	0.901	0.893
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736	1.713	1.690
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487	2.444	2.402
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170	3.102	3.037
5	4.853	4.715	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791	3.696	3.605
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355	4.231	4.111
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868	4.712	4.564
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335	5.146	4.968
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759	5.537	5.328
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145	5.889	5.650
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495	6.207	5.938
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814	6.492	6.194
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103	6.750	6.424
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367	6.982	6.628
15	13.865	12.849	11.939	11.118	10.380	9.712	9.108	8.559	8.061	7.606	7.191	6.811
16	14.718	13.578	12.561	11.652	10.838	10.106	9.447	8.851	8.313	7.824	7.379	6.974
17	15.562	14.292	13.166	12.166	11.274	10.477	9.763	9.122	8.544	8.022	7.549	7.102
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.372	8.756	8.201	7.702	7.250
19	17.226	15.678	14.324	13.134	12.085	11.158	10.336	9.604	8.950	8.365	7.839	7.366
20	18.046	16.351	14.877	13.590	12.462	11.470	10.594	9.818	9.129	8.514	7.963	7.469
25	22.023	19.523	17.413	15.622	14.094	12.783	11.654	10.675	9.823	9.077	8.422	7.843
30	25.808	22.396	19.600	17.292	15.372	13.765	12.409	11.258	10.274	9.427	8.694	8.055
40	32.835	27.355	23.115	19.793	17.160	15.046	13.332	11.925	10.757	9.779	8.951	8.244
50	39.196	31.424	25.730	21.482	18.256	15.762	13.801	12.233	10.962	9.915	9.042	8.304

APPENDIX D Present Value of an Annuity of $1 *(concluded)*

	Percent												
Period	13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	35%	40%	50%
1	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	0.800	0.769	0.741	0.714	0.667
2	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528	1.440	1.361	1.289	1.224	1.111
3	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106	1.952	1.816	1.696	1.589	1.407
4	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589	2.362	2.166	1.997	1.849	1.605
5	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991	2.689	2.436	2.220	2.035	1.737
6	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326	2.951	2.643	2.385	2.168	1.824
7	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605	3.161	2.802	2.508	2.263	1.883
8	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837	3.329	2.925	2.598	2.331	1.922
9	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031	3.463	3.019	2.665	2.379	1.948
10	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192	3.571	3.092	2.715	2.414	1.965
11	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327	3.656	3.147	2.752	2.438	1.977
12	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439	3.725	3.190	2.779	2.456	1.985
13	6.122	5.842	5.583	5.342	5.118	4.910	4.715	4.533	3.780	3.223	2.799	2.469	1.990
14	6.302	6.002	5.724	5.468	5.229	5.008	4.802	4.611	3.824	3.249	2.814	2.478	1.993
15	6.462	6.142	5.847	5.575	5.324	5.092	4.876	4.675	3.859	3.268	2.825	2.484	1.995
16	6.604	6.265	5.954	5.668	5.405	5.162	4.938	4.730	3.887	3.283	2.834	2.489	1.997
17	6.729	6.373	6.047	5.749	5.475	5.222	4.988	4.775	3.910	3.295	2.840	2.492	1.998
18	6.840	6.467	6.128	5.818	5.534	5.273	5.003	4.812	3.928	3.304	2.844	2.494	1.999
19	6.938	6.550	6.198	5.877	5.584	5.316	5.070	4.843	3.942	3.311	2.848	2.496	1.999
20	7.025	6.623	6.259	5.929	5.628	5.353	5.101	4.870	3.954	3.316	2.850	2.497	1.999
25	7.330	6.873	6.464	6.097	5.766	5.467	5.195	4.948	3.985	3.329	2.856	2.499	2.000
30	7.496	7.003	6.566	6.177	5.829	5.517	5.235	4.979	3.995	3.332	2.857	2.500	2.000
40	7.634	7.105	6.642	6.233	5.871	5.548	5.258	4.997	3.999	3.333	2.857	2.500	2.000
50	7.675	7.133	6.661	6.246	5.880	5.554	5.262	4.999	4.000	3.333	2.857	2.500	2.000

appendix E

Time Value of Money and Investment Applications

OVERVIEW

Many applications for the time value of money exist. Applications use either the compound sum (sometimes referred to as *future value*) or the present value. Additionally some cash flows are annuities. An **annuity** represents cash flows that are equally spaced in time and are constant dollar amounts. Car payments, mortgage payments, and bond interest payments are examples of annuities. Annuities can either be present value annuities or compound sum annuities. In the next section, we present the concept of compound sum and develop common applications related to investments.

COMPOUND SUM

Compound Sum: Single Amount

In determining the **compound sum,** we measure the future value of an amount that is allowed to grow at a given rate over a period of time. Assume an investor buys an asset worth \$1,000. This asset (gold, diamonds, art, real estate, etc.) is expected to increase in value by 10 percent per year, and the investor wants to know what it will be worth after the fourth year. At the end of the first year, the investor will have \$1,000 × (1 + 0.10), or \$1,100. By the end of year two, the \$1,100 will have grown by another 10 percent to \$1,210 (\$1,100 × 1.10). The four-year pattern is indicated below:

1st year: \$1,000 × 1.10 = \$1,100
2nd year: \$1,100 × 1.10 = \$1,210
3rd year: \$1,210 × 1.10 = \$1,331
4th year: \$1,331 × 1.10 = \$1,464

After the fourth year, the investor has accumulated \$1,464. Because compounding problems often cover a long time, a generalized formula is necessary to describe the compounding process. We shall let:

S = Compound sum
P = Principal or present value
i = Interest rate, growth rate, or rate of return
n = Number of periods compounded

The simple formula is:

$$S = P(1 + i)^n \quad \text{(E–1)}$$

In the preceding example, the beginning amount, P, was equal to \$1,000; the growth rate, i, equaled 10 percent; and the number of periods, n, equaled 4, so we get:

$$S = \$1,000\ (1.10)^4, \text{ or } \$1,000 \times 1.464 = \$1,464$$

The term $(1.10)^4$ is found to equal 1.464 by multiplying 1.10 four times itself. This mathematical calculation is called an exponential, where you take (1.10) to the fourth power. On your calculator, you would have an exponential key y^x where y represents (1.10) and x represents 4. For students with calculators, we have prepared Appendix F for both Hewlett-Packard and Texas Instruments calculators.

For those not proficient with calculators or who have calculators without financial functions, Table E–1 is a shortened version of the compound sum table found in Appendix A. The table tells us the amount \$1 would grow to if it were invested for any number of periods at a given rate of return. Using this table for our previous example, we find an interest factor for the compound sum in the row where $n = 4$ and the column where $i = 10$ percent. The factor is 1.464, the same as previously calculated. We multiply this factor times any beginning amount to determine the compound sum.

When using compound sum tables to calculate the compound sum, we shorten our formula from $S = P(1 + i)^n$ to:

$$S = P \times S_{IF} \quad \text{(E–2)}$$

TABLE E–1 Compound Sum of \$1 (S_{IF})

Periods	1%	2%	3%	4%	6%	8%	10%
1	1.010	1.020	1.030	1.040	1.060	1.080	1.100
2	1.020	1.040	1.061	1.082	1.124	1.166	1.210
3	1.030	1.061	1.093	1.125	1.191	1.260	1.331
4	1.041	1.082	1.126	1.170	1.262	1.360	1.464
5	1.051	1.104	1.159	1.217	1.338	1.469	1.611
10	1.105	1.219	1.344	1.480	1.791	2.159	2.594
20	1.220	1.486	1.806	2.191	3.207	4.661	6.727
30	1.348	1.811	2.427	3.243	5.743	10.063	13.268

where S_{IF} equals the interest factor for the compound sum found in Table E–1 or Appendix A. Using a new example, assume $5,000 is invested for 20 years at 6 percent. Using Table E–1, the interest factor for the compound sum would be 3.207, and the total value would be:

$$S = P \times S_{IF}\,(n = 20, i = 6\%)$$
$$= \$5{,}000 \times 3.207$$
$$= \$16{,}035$$

Example—Compound Sum, Single Amount

Problem: Mike Donegan receives a bonus from his employer of $3,200. He will invest the money at a 12 percent rate of return for the next eight years. How much will he have after eight years?

Solution: Compound sum, single amount:

$$S = P \times S_{IF} \quad (n = 8, i = 12\%) \quad \text{Appendix A}$$
$$= \$3{,}200 \times 2.476 = \$7{,}923.20$$

Compound Sum: Annuity

Our previous example was a one-time single investment. Let us examine a **compound sum of an annuity** where constant payments are made at equally spaced periods and grow to a future value. The normal assumption for a compound sum of an annuity is that the payments are made at the end of each period, so the last payment does not compound or earn a rate of return.

Figure E–1 demonstrates the timing and compounding process when $1,000 per year is contributed to a fund for four consecutive years. The $1,000 for each period is multiplied by the compound sum factors for the appropriate periods of compounding. The first $1,000 comes in at the end of the first period and has three periods to compound; the second $1,000 at the end of the second period, with two periods to compound; the third payment has one period to compound; and the last payment is multiplied by a factor of 1.00 showing no compounding at all.

Because compounding the individual values is tedious, compound sum of annuity tables can be used. These tables simply add up the interest factors from the compound sum tables for a single amount. Table E–2 is a shortened version of Appendix B, the compound sum of an annuity table showing the compound sum factors for a specified period and rate of return. Notice that all the way across the table, the factor in period one is 1.00. This reflects the fact that the last payment does not compound.

One example of the compound sum of an annuity applies to the individual retirement account (IRA) and Keogh retirement plans. The IRA allows workers to invest $2,000 per year in a tax-free account and the Keogh allows a maximum of $30,000 per year to be invested in a retirement account for self-employed individuals.[1] Assume Dr. Piotrowski shelters $30,000 per year from age 35 to 65. If she makes 30 payments of $30,000 and earns a rate of return of 8 percent, her Keogh account at retirement would be more than $3 million.

[1]The annual allowable deductibles are scheduled to increase between 2001 and 2011.

FIGURE E–1 Compounding Process for Annuity

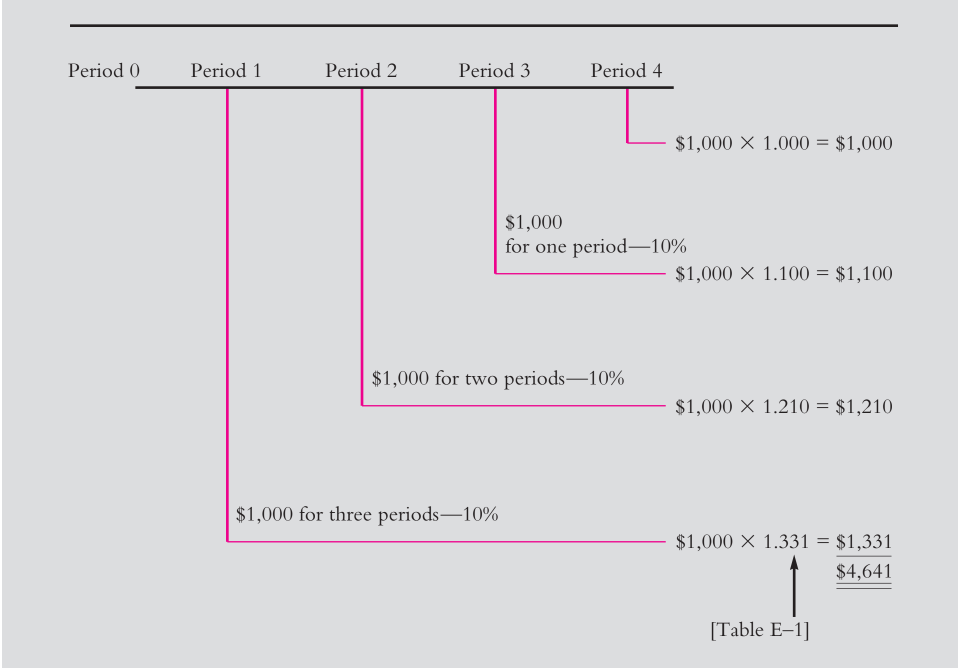

TABLE E–2 Compound Sum of an Annuity of $1 ($SA_{IF}$)

Periods	1%	2%	3%	4%	6%	8%	10%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.010	2.020	2.030	2.040	2.060	2.080	2.100
3	3.030	3.060	3.091	3.122	3.184	3.246	3.310
4	4.060	4.122	4.184	4.246	4.375	4.506	4.641
5	5.101	5.204	5.309	5.416	5.637	5.867	6.105
10	10.462	10.950	11.464	12.006	13.181	14.487	15.937
20	22.019	24.297	26.870	29.778	36.786	45.762	57.275
30	34.785	40.588	47.575	56.085	79.058	113.280	164.490

$$S = R \times SA_{IF}\,(n = 30, i = 8\%\text{ return}) \qquad \text{(E–3)}$$

$$= \$30{,}000 \times 113.280$$
$$= \$3{,}398{,}400$$

While this seems like a lot of money in today's world, we need to measure what it will buy 30 years from now after inflation is considered. One way to examine this is to calculate what the $30,000 payments would have to be if they only kept up with inflation. Let's assume inflation of 3 percent over the next 30 years and recalculate the sum of the annuity:

$$S = R \times SA_{IF}\,(n = 30, i = 3\%\text{ inflation})$$
$$= \$30{,}000 \times 47.575$$
$$= \$1{,}427{,}250$$

To maintain the purchasing power of each $30,000 contribution, Dr. Piotrowski needs to accumulate $1,427,250 at the estimated 3 percent rate of inflation. Since her rate of return of 8 percent is 5 percentage points higher than the inflation rate, she is adding additional purchasing power to her portfolio.

Example—Compound Sum, Annuity

Problem: Sonny Outlook invests $2,000 in an IRA at the end of each year for the next 40 years. With an anticipated rate of return of 11 percent, how much will the funds grow to after 40 years?

Solution: Compound sum, annuity:

$$S = R \times SA_{IF} (n = 40, i = 11\%) \qquad \text{Appendix B}$$
$$= \$2{,}000 \times 581.83 = \$1{,}163.660$$

PRESENT VALUE CONCEPT

Present Value: Single Amount

The **present value** is the exact opposite of the compound sum. A future value is discounted to the present. For example, earlier we determined the compound sum of $1,000 for four periods at 10 percent was $1,464. We could reverse the process to state that $1,464 received four years from today is worth only $1,000 today if one can earn a 10 percent return on money during the four years. This $1,000 value is called its present value. The relationship is depicted in Figure E–2.

The formula for present value is derived from the original formula for the compound sum. As the following two formulas demonstrate, the present value is simply the inverse of the compound sum.

FIGURE E–2 Relationship of Present Value and Compound Sum

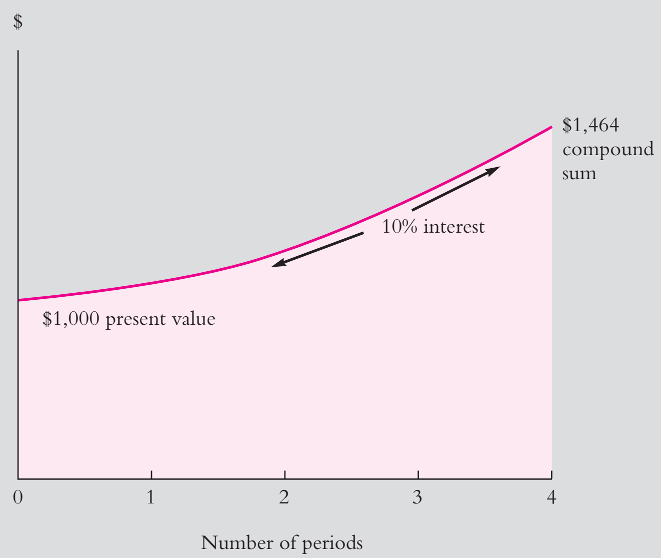

TABLE E–3 Present Value of \$1 (PV_{IF})

Periods	1%	2%	3%	4%	6%	8%	10%
1	0.990	0.980	0.971	0.962	0.943	0.926	0.909
2	0.980	0.961	0.943	0.925	0.890	0.857	0.826
3	0.971	0.942	0.915	0.889	0.840	0.794	0.751
4	0.961	0.924	0.888	0.855	0.792	0.735	0.683
5	0.951	0.906	0.863	0.822	0.747	0.681	0.621
10	0.905	0.820	0.744	0.676	0.558	0.463	0.386
20	0.820	0.673	0.554	0.456	0.312	0.215	0.149
30	0.742	0.552	0.412	0.308	0.174	0.099	0.057

$$S = P(1 + i)^n \text{ Compound sum}$$
$$P = S \times 1/(1 + i)^n \text{ Present value} \quad \textbf{(E–4)}$$

The present value can be determined by solving for a mathematical solution to the above formula, or by using Table E–3, the Present Value of \$1. When we use Table E–3, the present value interest factor $1/(1 + i)^n$ is found in the table and represented by PV_{IF}. We substitute it into the formula above:

$$P = S \times PV_{IF} \quad \textbf{(E–5)}$$

Let's demonstrate that the present value of \$1,464, based on our assumptions, is worth \$1,000 today:

$$P = S \times PV_{IF}\,(n = 4, i = 10\%) \text{ Table E–3 or Appendix C}$$
$$= \$1{,}464 \times 0.683$$
$$= \$1{,}000$$

Present value becomes very important in determining the value of investments. Assume you think a certain piece of land will be worth \$500,000 10 years from now. If you can earn a 10 percent rate of return on investments of similar risk, what would you be willing to pay for this land?

$$P = S \times PV_{IF}\,(n = 10, i = 10\%)$$
$$= \$500{,}000 \times 0.386$$
$$= \$193{,}000$$

This land's present value to you today would be \$193,000. What would you have 10 years from today if you invested \$193,000 at a 10 percent return? For this answer, we go to the compound sum factor from Table E–1:

$$S = P \times S_{IF}\,(n = 10, i = 10\%)$$
$$= \$193{,}000 \times 2.594$$
$$= \$500{,}642$$

The compound sum would be \$500,642. The two answers do not equal \$500,000 because of the mathematical rounding used to construct tables with three decimal points. If we carry out the interest factors to four places, 0.386 becomes 0.3855 and 2.594 becomes 2.5937 and the two answers will be quite similar.

Near the end of the compound sum of an annuity section, we showed that Dr. Piotrowski could accumulate \$3,398,400 by the time she retired in 30 years.

What would be the present value of this future sum if we brought it back to the present at the rate of inflation of 3 percent?

$$P = S \times PV_{IF}\,(n = 30, i = 3\%)$$
$$= \$3{,}398{,}400 \times 0.412$$
$$= \$1{,}400{,}141$$

The amount she will have accumulated will be worth $1,400,141 in today's dollars. If the rate of inflation averaged 6 percent over this time, the amount would fall to $591,322 ($3,398,400 × 0.174). Notice how sensitive the present value is to a 3 percentage point change in the inflation rate. Another concern is being able to forecast inflation correctly. These examples are simply meant to heighten your awareness that money has a time value and financial decisions require this to be considered.

Example—Present Value, Single Amount

Problem: Barbara Samuels received a trust fund at birth that will be paid out to her at age 18. If the fund will accumulate to $400,000 by then and the discount rate is 9 percent, what is the present value of her future accumulation?

Solution: Present value, single amount:

$$P = S \times PV_{IF}\,(n = 18, i = 9\%) \qquad \text{Appendix C}$$
$$= \$400{,}000 \times 0.212 = \$84{,}800$$

Present Value: Annuity

To find the **present value of an annuity,** we are simply finding the present value of an equal cash flow for several periods instead of one single cash payment. The analysis is the same as taking the present value of several cash flows and adding them. Since we are dealing with an annuity (equal dollar amounts), we can save time by creating tables that add up the interest factors for the present value of single amounts and make present value annuity factors. We do this in Table E–4, a shortened version of Appendix D. Before using Table E–4, let's compute the present value of $1,000 to be received each year for five years at 6 percent. We could use the present value of five single amounts and Table E–3.

Period	Receipt	*IF* @ 6%	
1	$1,000 ×	0.943 =	$ 943
2	$1,000 ×	0.890 =	$ 890
3	$1,000 ×	0.840 =	$ 840
4	$1,000 ×	0.792 =	$ 792
5	$1,000 ×	0.747 =	$ 747
		4.212	$4,212 Present value

Another way to get the same value is to use Table E–4. The present value annuity factor under 6 percent and 5 periods is equal to 4.212, or the same value we got from adding the individual present value factors for a single amount. We can simply calculate the answer as follows:

where:

A = the present value of an annuity

R = the annuity amount

PVA_{IF} = the interest factor from Table E–4

TABLE E–4 Present Value of an Annuity of \$1 (PVA_{IF})

Periods	1%	2%	3%	4%	6%	8%	10%
1	0.990	0.980	0.971	0.962	0.943	0.926	0.909
2	1.970	1.942	1.913	1.886	1.833	1.783	1.736
3	2.941	2.884	2.829	2.775	2.673	2.577	2.487
4	3.902	3.808	3.717	3.630	3.465	3.312	3.170
5	4.853	4.713	4.580	4.452	4.212	3.993	3.791
8	7.652	7.325	7.020	6.773	6.210	5.747	5.335
10	9.471	8.983	8.530	8.111	7.360	6.710	6.145
20	18.046	16.351	14.877	13.590	11.470	9.818	8.514
30	25.808	22.396	19.600	17.292	13.765	11.258	9.427

$$A = R \times PVA_{IF}\,(n = 5, i = 6\%) \qquad \textbf{(E–6)}$$

$$= \$1{,}000 \times 4.212$$
$$= \$4{,}212$$

Present value of annuities applies to many financial products such as mortgages, car payments, and retirement benefits. Some financial products such as bonds are a combination of an annuity and a single payment. Interest payments from bonds are annuities, and the principal repayment at maturity is a single payment. Both cash flows determine the present value of a bond.

Example—Present Value, Annuity

Problem: Ross "The Hoss" Sullivan has just renewed his contract with the Chicago Bears for an annual payment of \$3 million per year for the next eight years. The newspapers report the deal is worth \$24 million. If the discount rate is 14 percent, what is the true present value of the contract?

Solution: Present value, annuity:

$$A = R \times PVA_{IF}\,(n = 8, i = 14\%) \qquad \text{Appendix D}$$
$$= \$3{,}000{,}000 \times 4.639 = \$13{,}917{,}000$$

Present Value: Uneven Cash Flow

Many investments are a series of uneven cash flows. For example, buying common stock generally implies an uneven cash flow from future dividends and the sale price. We hope to buy common stock in companies that are growing and have increasing dividends. Assume you want to purchase Caravan Motors common stock on January 1, 2001. You expect to hold the stock for five years and then sell it at \$60 in December 2005. You also expect to receive dividends of \$1.60, \$2.00, \$2.00, \$2.50, and \$3.00 during those five years.

What would you be willing to pay for the common stock if your required return on a stock of this risk is 14 percent. Let's set up a present value analysis for an uneven cash flow using Appendix C, the present value of a single amount. Since this is not an annuity, each cash flow must be evaluated separately. For simplicity, we assume all cash flows come at the end of the year. Also, the cash flow in year 2005 combines the \$3.00 dividend and expected \$60 sale price.

Year	Cash Flow	PV_{IF} 14%	Present Value
2001	$ 1.60	0.877	$ 1.40
2002	2.00	0.769	1.54
2003	2.00	0.675	1.35
2004	2.50	0.592	1.48
2005	63.00	0.519	32.70
Present value of Caravan Motors under these assumptions:			$38.47

If you were satisfied that your assumptions were reasonably accurate, you would be willing to buy Caravan at any price equal to or less than $38.47. This price will provide you with a 14 percent return if all your forecasts come true.

Example—Present Value, Uneven Cash Flow

Problem: Joann Zinke buys stock in Collins Publishing Company. She will receive dividends of $2.00, $2.40, $2.88, and $3.12 for the next four years. She assumes she can sell the stock for $50 after the last dividend payment (at the end of four years). If the discount rate is 12 percent, what is the present value of the future cash flows? (Round all values to two places to the right of the decimal point.) The present value of future cash flows is assumed to equal the value of the stock.

Solution: Present value, uneven cash flow:

Year	Cash Flow	PV_{IF} 12%	Present Value
1	$ 2.00	0.893	$ 1.79
2	2.40	0.797	1.91
3	2.88	0.712	2.05
4	53.12	0.636	33.78
			$39.53

The present value of the cash flows is $39.53.

Example—Present Value, Uneven Cash Flow

Problem: Sherman Lollar wins a malpractice suit against his accounting professor, and the judgment provides him with $3,000 a year for the next 40 years, plus a single lump-sum payment of $10,000 after 50 years. With a discount rate of 10 percent, what is the present value of his future benefits?

Solution: Present value, annuity plus a single amount:

Annuity

$$A = R \times PVA_{IF}(n = 40, i = 10\%) \qquad \text{Appendix D}$$
$$= \$3{,}000 \times 9.779 = \$29{,}337$$

Single amount

$$P = S \times PV_{IF}(n = 50, i = 10\%) \qquad \text{Appendix C}$$
$$= \$10{,}000 \times 0.009 = \$90$$

Total present value = $29,337 + $90 = $29,427

appendix F

Using Calculators for Financial Analysis

This appendix is designed to help you use either an algebraic calculator (Texas Instruments BA-35 Student Business Analyst) or the Hewlett-Packard 12C Financial Calculator. We realize that most calculators come with comprehensive instructions, and this appendix is only meant to provide basic instructions for commonly used financial calculations.

There are always two things to do before starting your calculations as indicated in the first table: clear the calculator, and set the decimal point. If you do not want to lose data stored in memory, do not perform steps 2 and 3 in the first box below.

Each step is listed vertically as a number followed by a decimal point. After each step you will find either a number or a calculator function denoted by a box [ ]. Entering the number on your calculator is one step and entering the function is another. Notice that the HP 12C is color coded. When two boxes are found one after another, you may have an [f] or a [g] in the first box. An [f] is orange coded and refers to the orange functions above the keys. After typing the [f] function, you will automatically look for an orange-coded key to punch. For example, after [f] in the first Hewlett-Packard box (right-hand panel), you will punch in the orange-color-coded [REG]. If the [f] function is not followed by another box, you merely type in [f] and the value indicated.

	Texas Instruments BA-35	Hewlett-Packard 12C
First clear the calculator.	1. [ON/C] [ON/C]	1. [CLX] Clears Screen
	2. 0	2. [f]
	3. [STO] Clears memory	3. [REG] Clears Memory
Set the decimal point. The TI BA-35 has two choices: 2 decimal points or variable decimal points. The screen will indicate Dec 2 or the decimal will be variable. The HP 12C allows you to choose the number of decimal points. If you are uncertain, just provide the indicated input exactly as shown on the right.	1. [2nd]	1. [f]
	2. [STO]	2. 4 (# of decimals)

The [g] is coded blue and refers to the functions on the bottom of the function keys. After the [g] function key, you will automatically look for blue-coded keys. This first occurs on page 668 of this appendix.

Familiarize yourself with the keyboard before you start. In the more complicated calculations, keystrokes will be combined into one step.

In the first four calculations on this page and on page 665 we simply instruct you on how to get the interest factors for Appendices A, B, C, and D. We have chosen to use examples as our method of instruction.

	Texas Instruments BA-35	Hewlett-Packard 12C
A. Appendix A Compound Sum of $1	To Find Interest Factor	To Find Interest Factor
	1. 1	1. 1
i = 9% or 0.09; n = 5 years	2. +	2. [enter]
$S_{IF} = (1 + i)^n$	3. 0.09 (interest rate)	3. 0.09 (interest rate)
Sum = Present Value × S_{IF}	4. [=]	4. [+]
$S = P \times S_{IF}$	5. [y^x]	5. 5 (# of periods)
Check the answer against the number in Appendix A. Numbers in the appendix are rounded. Try different rates and years.	6. 5 (# of periods)	6. [y^x] answer 1.5386
	7. = answer 1.538624	
B. Appendix B Compound Sum of an Annuity of $1	To Find Interest Factor	To Find Interest Factor
	Repeat steps 1 through 7 in part A of this section. Continue with step 8.	Repeat steps 1 through 6 in part A of this section. Continue with step 7.
i = 9% or 0.09; n = 5 years		
$SA_{IF} = \frac{(1 + i)^n - 1}{1}$	8. [−]	7. 1
Sum = Receipt × SA_{IF}	9. 1	8. [−]
$S = R \times SA_{IF}$	10. [−]	9. 0.09
Check your answer with Appendix B. Repeat example using different numbers and check your results with the number in Appendix B. Numbers in appendix are rounded.	11. [÷]	10. [÷] answer 5.9847
	12. 0.09	
	13. [=] answer 5.9847106	

	Texas Instruments BA-35	Hewlett-Packard 12C
C. Appendix C Present Value of $1	To Find Interest Factor	To Find Interest Factor
$i = 9\%$ or 0.09; $n = 5$ years	Repeat steps 1 through 7 in part A of this section. Continue with step 8.	Repeat steps 1 through 6 in part A of this section. Continue with step 7.
$PV_{IF} = (1 + i)^n$		
	8. [1/x] answer 0.6499314	7. [1/x] answer 0.6499
Present Value $=$ Sum $\times PV_{IF}$		
$P = S \times PV_{IF}$		
Check the answer against the number in Appendix C. Numbers in the appendix are rounded.		
D. Appendix D Present Value of an Annuity of $1	To Find Interest Factor	To Find Interest Factor
$i = 9\%$ or 0.09; $n = 5$ years	Repeat steps 1 through 8 in parts A and C. Continue with step 9.	Repeat steps 1 through 7 in parts A and C. Continue with step 8.
$PV_{IF} = \dfrac{1 - [(1/(1 + i)^n]}{i}$	9. [−]	8. 1
Present Value $=$ Annuity $\times PVA_{IF}$	10. 1	9. [−]
	11. [ ]	10. [CH5]
$A = R \times PVA_{IF}$		
	12. [+/−]	11. 0.09
Check your answer with Appendix D. Repeat example using different numbers and check your results with the number in Appendix D. Numbers in appendix are rounded.	13. [÷]	12. [÷] answer 3.8897
	14. 0.09	
	15. [=] answer 3.8896513	

On the following pages, you can determine bond valuation, yield to maturity, net present value of an annuity, net present value of an uneven cash flow, internal rate of return for an annuity, and internal rate of return for an uneven cash flow.

BOND VALUATION USING BOTH THE TI BA-35 AND THE HP 12C

Solve for V = Price of the bond, given:

C_t = \$80 annual coupon payments or 8 percent coupon (\$40 semiannually)
P_n = \$1,000 principal (par value)
n = 10 years to maturity (20 periods semiannually)
i = 9.0 percent rate in the market (4.5 percent semiannually)

You may choose to refer to Chapter 12 for a complete discussion of bond valuation.

	Texas Instruments BA-35	Hewlett-Packard 12C
Bond Valuation	Set Finance Mode [2nd] [FIN]	Clear Memory [f] [REG]
All steps begin with number 1. Numbers following each step are keystrokes followed by a box [ ]. Each box represents a keystroke and indicates which calculator function is performed.	Set decimal to 2 places	Set decimal to 3 places
	Decimal	[f] 3
	[2nd] [STO]	1. 9.0 (yield to maturity)
	1. 40 (semiannual coupon)	2. [i]
The Texas Instruments calculator requires that data be adjusted for semiannual compounding; otherwise it assumes annual compounding.	2. [PMT]	3. 8.0 (coupon in percent)
	3. 4.5 (yield to maturity) semiannual basis	4. [PMT]
The Hewlett-Packard 12C internally assumes that semiannual compounding is used and requires annual data to be entered. The HP 12C is more detailed in that it requires the actual day, month, and year. If you want an answer for a problem that requires a given number of years (e.g., 10 years), simply start on a date of your choice and end on the same date 10 years later, as in the example.	4. [%i]	5. 1.092002 (today's date month-day-year)*
	5. 1000 principal	6. [enter]
	6. [FV]	7. 1.092012 (maturity date month-day-year)*
	7. 20 (semiannual periods to maturity)	8. [f]
	8. [N]	9. [Price] Answer 93.496
	9. [CPT]	
	10. [PV] answer 934.96	Answer is given as % of par value and equals \$934.96.
	Answer is given in dollars, rather than % of par value.	If Error message occurs, clear memory and start over.
		*See instructions in the third paragraph of the first column.

YIELD TO MATURITY ON BOTH THE TI BA-35 AND HP 12C

Solve for Y = Yield to maturity, given:

V = \$895.50 price of bond
C_t = \$80 annual coupon payments or 8 percent coupon (\$40 semiannually)
P_n = \$1,000 principal (par value)
n = 10 years to maturity (20 periods semiannually)

You may choose to refer to Chapter 12 for a complete discussion of yield to maturity.

Yield to Maturity

All steps are numbered. All numbers following each step are keystrokes followed by a box [ ]. Each box represents a keystroke and indicates which calculator function is performed.

The Texas Instruments BA-35 does not internally compute to a semiannual rate, so that data must be adjusted to reflect semiannual payments and periods. The answer received in step 10 is a semiannual rate, which must be multiplied by 2 to reflect an annual yield.

The Hewlett-Packard 12C internally assumes that semiannual payments are made and, therefore, the answer in step 9 is the annual yield to maturity based on semiannual coupons. If you want an answer on the HP for a given number of years (e.g., 10 years), simply start on a date of your choice and end on the same date 10 years later, as in the example.

Texas Instruments BA-35

Set Finance Mode [2nd] [FIN]

Set decimal to 2 places

Decimal

[2nd] [STO]

1. 20 (semiannual periods)
2. [N]
3. 1000 (par value)
4. [FV]
5. 40 (semiannual coupon)
6. [PMT]
7. 895.50 (bond price)
8. [PV]
9. [CPT]
10. [%i] answer 4.83%
11. [×]
12. 2
13. [=] answer 9.65% (annual rate)

Hewlett-Packard 12C

Clear Memory [f] [REG]

Set decimal [f] 2

1. 89.55 (bond price as a percent of par)
2. [PV]
3. 8.0 (coupon in %)
4. [PMT]
5. 1.092002 (today's date)*
6. [enter]
7. 1.092012 (maturity date)*
8. [f]
9. [YTM] answer 9.65%

In case you receive an Error message, you have probably made a keystroke error. Clear the memory

[f] [REG]

and start over.

*See instructions in the third paragraph of the first column.

NET PRESENT VALUE OF AN ANNUITY ON BOTH THE TI BA-35 AND THE HP 12C

Solve for A = Present value of annuity, given:

n = 10 years (number of years cash flow will continue)
PMT = \$5,000 per year (amount of the annuity)
i = 12 percent (cost of capital K_a)
Cost = \$20,000

	Texas Instruments BA-35	Hewlett-Packard 12C
Net Present Value of an Annuity	Set Finance Mode [2nd] [FIN]	Set decimal to 2 places
All steps are numbered and some steps include keystrokes. All numbers following each step are keystrokes followed by a box [ ]. Each box represents a keystroke and indicates which calculator function is performed on that number.	Set decimal to 2 places	[f] 2
	Decimal	[f] [REG] clears memory
	[2nd] [STO]	1. 20000 (cash outflow)
	1. 10 (years of cash flow)	2. [CHS] changes sign
	2. [N]	3. [g]
The calculation for the present value of an annuity on the TI BA-35 requires that the project cost be subtracted from the present value of the cash inflows.	3. 5000 (annual payments)	4. [CFo]
	4. [PMT]	5. 5000 (annual payments)
	5. 12 (cost of capital)	6. [g] [Cfj]
The HP 12C could solve the problem exactly with the same keystrokes as the TI. However, since the HP uses a similar method to solve uneven cash flows, we elected to use the method that requires more keystrokes but which includes a negative cash outflow for the cost of the capital budgeting project.	6. [%i]	7. 10 [g] [N_j] (years)
	7. [CPT]	8. 12 [i] (cost of capital)
	8. [PV]	9. [f] [NPV]
	9. [−]	answer \$8,251.12
	10. 20,000	If an Error message appears, start over by clearing the memory with
To conserve space, several keystrokes have been put into one step.	11. [=] answer \$8,251.12	[f] [REG]

NET PRESENT VALUE OF AN UNEVEN CASH FLOW ON BOTH THE TI BA-35 AND HP 12C

Solve for NPV = Net present value, given:

n = 5 years (number of years cash flow will continue)
PMT = \$5,000 (yr. 1); 6,000 (yr. 2); 7,000 (yr. 3); 8,000 (yr. 4); 9,000 (yr. 5)
i = 12 percent (cost of capital K_a)
Cost = \$25,000

Net Present Value of an Uneven Cash Flow

All steps are numbered and some steps include several keystrokes. All numbers following each step are keystrokes followed by a box [ ]. Each box represents a keystroke and indicates which calculator function is performed on that number.

Because we are dealing with uneven cash flows, each number must be entered. The TI BA-35 requires that you make use of the memory. In step 2, you enter the future cash inflow in year 1, and in step 3, you determine its present value, which is stored in memory. After the first 1-year calculation, following year present values are calculated in the same way and added to the stored value using the [SUM] key. Finally, the recall key [RCL] is used to recall the present value of the total cash inflows.

The HP 12C requires each cash flow to be entered in order. The [CFo] key represents the cash flow in time period 0. The [CFj] key automatically counts the year of the cash flow in the order entered and so no years need to be entered. Finally, the cost of capital of 12% is entered and the [f] key and [NPV] key are used to complete the problem.

Texas Instruments BA-35

Clear memory [ON/C] 0 [STO]

Set decimal to 2 places

Decimal

[2nd] [STO]

Set finance mode

[2nd] [FIN]

1. 12 [%i]
2. 5000 [FV]
3. 1 [N] [CPT] [PV] [SUM]
4. 6000 [FV]
5. 2 [N] [CPT] [PV] [SUM]
6. 7000 [FV]
7. 3 [N] [CPT] [PV] [SUM]
8. 8000 [FV]
9. 4 [N] [CPT] [PV] [SUM]
10. 9000 [FV]
11. 5 [N] [CPT] [PV] [SUM]
12. [RCL] (answer 24420.90
13. [−]
14. 25000 (cash outflow)
15. [=] answer −\$579.10

Negative Net Present Value

Hewlett-Packard 12C

Set decimal to 2 places

[f] 2

[f] [REG] clears memory

1. 25000 (cash outflow)
2. [CHS] changes sign
3. [g] [CFo]
4. 5000 [g] [CFj]
5. 6000 [g] [CFj]
6. 7000 [g] [CFj]
7. 8000 [g] [CFj]
8. 9000 [g] [CFj]
9. 12 [i]
10. [f] [NPV]

answer −\$579.10
Negative Net Present Value

If you receive an Error message, you have probably made a keystroke error. Clear memory with [f] [REG] and start over with step 1.

INTERNAL RATE OF RETURN FOR AN ANNUITY ON BOTH THE TI BA-35 AND HP 12C

Solve for IRR = Internal rate of return, given:

$$n = 10 \text{ years (number of years cash flow will continue)}$$
$$PMT = \$10{,}000 \text{ per year (amount of the annuity)}$$
$$\text{Cost} = \$50{,}000 \text{ (this is the present value of the annuity)}$$

	Texas Instruments BA-35	Hewlett-Packard 12C
Internal Rate of Return of an Annuity	Clear memory [ON/C] 0 [STO]	Set decimal to 2 places
	Set Finance Mode [2nd] [FIN]	[f] 2
All steps are numbered and some steps include several keystrokes. All numbers following each step are keystrokes followed by a box [ ]. Each box represents a keystroke and indicates which calculator function is performed on that number.	Decimal	[f] [REG] clears memory
	[2nd] [STO]	1. 5000 (cash outflow)
	1. 10 (years of cash flow)	2. [CHS] changes sign
	2. [N]	3. [g]
The calculation for the internal rate of return on an annuity using the TI BA-35 requires relatively few keystrokes.	3. 10000 (annual payments)	4. [CFo]
	4. [PMT]	5. 10000 (annual payments)
	5. 50000 (present value)	6. [g] [Cfj]
The HP 12C requires more keystrokes than the TI BA-35, because it needs to use the function keys [f] and [g] to enter data into the internal programs. The HP method requires that the cash outflow be expressed as a negative, while the TI BA-35 uses a positive number for the cash outflow.	6. [PV]	7. 10 [g] [Nj] (years)
	7. [CPT]	8. [f] [IRR]
	8. [%i]	answer is 15.10%
	answer is 15.10%	If an Error message appears, start over by clearing the memory with
To conserve space, several keystrokes have been put into one step.	At an internal rate of return of 15.10%, the present value of the $50,000 outflow is equal to the present value of $10,000 cash inflows over the next 10 years.	[f] [REG]

INTERNAL RATE OF RETURN WITH AN UNEVEN CASH FLOW ON BOTH THE TI BA-35 AND HP 12C

Solve for *IRR* = Internal rate of return (return which causes present value of outflows to equal present value of the inflows), given:

n = 5 years (number of years cash flow will continue)
PMT = $5,000 (yr. 1); 6,000 (yr. 2); 7,000 (yr. 3); 8,000 (yr. 4); 9,000 (yr. 5)
Cost = $25,000

	Texas Instruments BA-35	Hewlett-Packard 12C
Internal Rate of an Uneven Cash Flow All steps are numbered and some steps include several keystrokes. All numbers following each step are keystrokes followed by a box [ ]. Each box represents a keystroke and indicates which calculator function is performed on that number. Because we are dealing with uneven cash flows, the mathematics of solving this problem with the TI BA-35 is not possible. A more advanced algebraic calculator would be required. However, for the student willing to use trial and error, the student can use the NPV method and try different discount rates until the NPV equals zero. Check Chapter 12 on methods for approximating the IRR. This will provide a start. The HP 12C requires each cash flow to be entered in order. The [CFo] key represents the cash flow in time period 0. The [CFj] key automatically counts the year of the cash flow in the order entered and so no years need to be entered. To find the internal rate of return, use the [f] [IRR] keys and complete the problem.	Clear memory [ON/C] 0 [STO] Set decimal to 2 places Decimal [2nd] [STO] Set finance mode [2nd] [FIN] 1. 12 [%i] (your IRR est.) 2. 5000 [FV] 3. 1 [N] [CPT] [PV] [STO] 4. 6000 [FV] 5. 2 [N] [CPT] [PV] [SUM] 6. 7000 [FV] 7. 3 [N] [CPT] [PV] [SUM] 8. 8000 [FV] 9. 4 [N] [CPT] [PV] [SUM] 10. 9000 [FV] 11. 5 [N] [CPT] [PV] [SUM] 12. [RCL] (answer 24,420.90) 13. [−] 14. 25000 (cash outflow) 15. [=] answer −$579.10 Negative NPV Start over with a lower discount rate (try 11.15). Answer is 24999.75. With a cash outflow of $25,000, the IRR would be 11.15%	Set decimal to 2 places [f] 2 [f] [REG] clears memory 1. 25000 (cash outflow) 2. [CHS] changes sign 3. [g] [CFo] 4. 5000 [g] [CFj] 5. 6000 [g] [CFj] 6. 7000 [g] [CFj] 7. 8000 [g] [CFj] 8. 9000 [g] [CFj] 9. [f] [IRR] answer 11.15% If you receive an Error message, you have probably made a keystroke error. Clear memory with [f] [REG] and start over with step 1.