Chapter 6 SUMMARY

- The second law of thermodynamics leads to the definition of a new property called *entropy*, which is a quantitative measure of microscopic disorder for a system.

- The definition of entropy is based on the *Clausius inequality*, given by

$$\oint \frac{\delta Q}{T} \leq 0 \qquad \text{(kJ/K)}$$

  where the equality holds for internally or totally reversible processes and the inequality for irreversible processes.

- Any quantity whose cyclic integral is zero is a property, and entropy is defined as

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \qquad \text{(kJ/K)}$$

- For the special case of an internally reversible, isothermal process, it gives

$$\Delta S = \frac{Q}{T_o} \qquad \text{(kJ/K)}$$

- The inequality part of the Clausius inequality combined with the definition of entropy yields an inequality known as the *increase of entropy principle*, expressed as

$$S_{gen} \geq 0 \qquad \text{(kJ/K)}$$

  where S_{gen} is the *entropy generated* during the process.

- Entropy change is caused by heat transfer, mass flow, and irreversibilities. Heat transfer to a system increases the entropy, and heat transfer from a system decreases it. The effect of irreversibilities is always to increase the entropy.

- Entropy is a property, and it can be expressed in terms of more familiar properties through the *T ds* relations, expressed as

$$Tds = du + Pdv$$

and

$$Tds = dh - vdP$$

- These two relations have many uses in thermodynamics and serve as the starting point in developing entropy-change relations for processes. The successful use of *T ds* relations depends on the availability of property relations. Such relations do not exist for a general pure substance but are available for incompressible substances (solids, liquids) and ideal gases.

- The *entropy-change* and *isentropic relations* for a process can be summarized as follows:

1. *Pure substances:*

Any process: $\Delta s = s_2 - s_1$ (kJ/kg·K)

Isentropic process: $s_2 = s_1$

2. Incompressible substances

Any process: $s_2 - s_1 = C_{av} \ln \frac{T_2}{T_1}$ (kJ/kg·K)

Isentropic process: $T_2 = T_1$

3. *Ideal gases:*

a. Constant specific heats (approximate treatment):

Any process:

$$s_2 - s_1 = C_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{(kJ/kg·K)}$$

and

$$s_2 - s_1 = C_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{(kJ/kg·K)}$$

Or, on a unit-mole basis,

$$\bar{s}_2 - \bar{s}_1 = \overline{C}_{v,av} \ln\frac{T_2}{T_1} + R_u \ln\frac{v_2}{v_1} \qquad \text{(kJ/kmol·K)}$$

and

$$\bar{s}_2 - \bar{s}_1 = \overline{C}_{p,av} \ln\frac{T_2}{T_1} - R_u \ln\frac{P_2}{P_1} \qquad \text{(kJ/kmol·K)}$$

Isentropic process:

$$\left(\frac{T_2}{T_1}\right)_{s=const.} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=const.} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1}\right)_{s=const.} = \left(\frac{v_1}{v_2}\right)^{k}$$

b. Variable specific heats (exact treatment):

Any process:

$$s_2 - s_1 = s_2^o - s_1^o - R\ln\frac{P_2}{P_1} \qquad \text{(kJ/kg·K)}$$

or

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^o - \bar{s}_1^o - R_u \ln\frac{P_2}{P_1} \qquad \text{(kJ/kmol·K)}$$

Isentropic process:

$$s_2^o = s_1^o + R\ln\frac{P_2}{P_1} \qquad \text{(kJ/kg·K)}$$

$$\left(\frac{P_2}{P_1}\right)_{s = const.} = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_{s = const.} = \frac{v_{r2}}{v_{r1}}$$

where P_r is the *relative pressure* and v_r is the *relative specific volume.* The function s^o depends on temperature only.

- The *steady-flow work* for a reversible process can be expressed in terms of the fluid properties as

$$w_{rev} = -\int_1^2 v\,dP - \Delta ke - \Delta pe \qquad \text{(kJ/kg)}$$

- For incompressible substances *(v* = constant) *steady-flow work* for a reversible process simplifies to

$$w_{rev} = -v\,(P_2 - P_1) - \Delta ke - \Delta pe \qquad \text{(kJ/kg)}$$

- The work done during a steady-flow process is proportional to the specific volume. Therefore, *v* should be kept as small as possible during a compression process to minimize the work input and as large as possible during an expansion process to maximize the work output.

- The reversible work inputs to a compressor compressing an ideal gas from T_1, P_1, to P_2 in an isentropic (Pv^k = constant), polytropic (Pv^n = constant), or isothermal (Pv = constant) manner, are determined by integration for each case with the following results:

  Isentropic:

$$w_{comp,\,in} = \frac{kR(T_2 - T_1)}{k-1} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

Polytropic:

$$w_{comp,in} = \frac{nR(T_2 - T_1)}{n-1} = \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Isothermal:

$$w_{comp,in} = RT \ln \frac{P_2}{P_1} \qquad \text{(kJ/kg)}$$

- The work input to a compressor can be reduced by using multistage compression with intercooling. For maximum savings from the work input, the pressure ratio across each stage of the compressor must be the same.

- Most steady-flow devices operate under adiabatic conditions, and the ideal process for these devices is the isentropic process.

- The parameter that describes how efficiently a device approximates a corresponding isentropic device is called *isentropic* or *adiabatic efficiency*. It is expressed for turbines, compressors, and nozzles as follows:

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic Ke at nozzle exit}} = \frac{\vec{V}_{2a}^2}{\vec{V}_{2s}^2} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

In the relations above, h_{2a} and h_{2s} are the enthalpy values at the exit state for actual and isentropic processes, respectively.

- The entropy balance for any system undergoing any process can be expressed in the general form as

$$\underbrace{S_{in} - S_{out}}_{\substack{\text{Net entropy transfer} \\ \text{by heat and mass}}} + \underbrace{S_{gen}}_{\substack{\text{Entropy} \\ \text{generation}}} = \underbrace{\Delta S_{system}}_{\substack{\text{Change in} \\ \text{entropy}}} \qquad \text{(kJ/K)}$$

- The entropy balance for any system undergoing any process can be expressed in the general *rate form,* as

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\substack{\text{Rate of net entropy transfer} \\ \text{by heat and mass}}} + \underbrace{\dot{S}_{gen}}_{\substack{\text{Rate of entropy} \\ \text{generation}}} = \underbrace{\Delta \dot{S}_{system}}_{\substack{\text{Rate of change} \\ \text{of entropy}}} \qquad \text{(kW/K)}$$

- For a general *steady-flow process* the entropy balance simplifies to

$$\dot{S}_{gen} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$