Chapter 16 SUMMARY

- In this chapter the thermodynamic aspects of high-speed fluid flow are examined. For high-speed flows, it is convenient to combine the enthalpy and the kinetic energy of the fluid into a single term called *stagnation* (or *total) enthalpy* h_0, defined as

$$h_0 = h + \frac{\vec{V}^2}{2} \qquad \textbf{(kJ/kg)}$$

  The properties of a fluid at the stagnation state are called *stagnation properties* and are indicated by the subscript zero.

- The *stagnation temperature* of an ideal gas with constant specific heats is

$$T_0 = T + \frac{\vec{V}^2}{2C_p}$$

  which represents the temperature an ideal gas will attain when it is brought to rest adiabatically.

- The (isentropic) stagnation properties of an ideal gas are related to the static properties of the fluid by

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)}$$

- When stagnation enthalpies are used, the conservation of energy equation for a single-stream, steady-flow device can be expressed as

$$q_{\text{in}} + w_{\text{in}} + (h_{01} + gz_1) = q_{\text{out}} + w_{\text{out}} + (h_{02} + gz_2)$$

  where h_{01} and h_{02} are the stagnation enthalpies at states 1 and 2, respectively.

- The velocity at which an infinitesimally small pressure wave travels through a medium is the *velocity of sound* (or the *sonic velocity).* It is expressed as

$$C^2 = \left(\frac{\partial P}{\partial \rho} \right)_s$$

- For an ideal gas the velocity of sound becomes

$$C = \sqrt{KRT}$$

- The *Mach number* is the ratio of the actual velocity of the fluid to the velocity of sound at the same state:

$$M = \frac{\vec{V}}{C}$$

- The flow is called *sonic* when $M = 1$, *subsonic* when $M < 1$, *supersonic* when $M > 1$, *hypersonic* when $M >> 1$, and *transsonic* when $M \cong 1$.

- The nozzles whose flow area decreases in the flow direction are called *converging* nozzles. Nozzles whose flow area first decreases and then increases are called *converging-diverging nozzles.* The location of the smallest flow area of a nozzle is called the *throat.*

- The highest velocity to which a fluid can be accelerated in a convergent nozzle is the sonic velocity. Accelerating a fluid to supersonic velocities is only possible in converging-diverging nozzles. In all supersonic converging-diverging nozzles, the flow velocity at the throat is the velocity of sound.

- The ratios of the stagnation to static properties for ideal gases with constant specific heats can be expressed in terms of the Mach number as

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2} \right) M^2$$

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2} \right) M^2 \right]^{k/(k-1)}$$

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2} \right) M^2 \right]^{1/(k-1)}$$

- When M = 1, the resulting static-to-stagnation property ratios for the temperature, pressure, and density are called *critical ratios* and are denoted by the superscript asterisk:

$$\frac{T^*}{T_0} = \left(\frac{2}{k+1} \right)$$

$$\frac{P_0}{P} = \left(\frac{2}{k+1} \right)^{k/(k-1)}$$

$$\frac{\rho_0}{\rho} = \left(\frac{2}{k+1} \right)^{1/(k-1)}$$

- The pressure outside the exit plane of a nozzle is called the *back pressure*. For all back pressures lower than P^*, the pressure at the exit plane of the converging nozzle is equal to P^*, the Mach number at the exit plane is unity, and the mass flow rate is the maximum (or choked) flow rate.

- Under steady-flow conditions, the mass flow rate through the nozzle is constant and can be expressed as

$$\dot{m} = \frac{AMP_0 \sqrt{k/(RT_0)}}{\left[1 + (k-1)M^2/2\right]^{(k+1)/[2(k-1)]}}$$

- The variation of flow area A through the nozzle relative to the throat area A^* for the same mass flow rate and stagnation properties of a particular ideal gas is

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}M^2\right)\right]^{(k+1)/[2(k-1)]}$$

- The parameter M* is defined as the ratio of the local velocity to the velocity of sound at the throat *(M* = 1):

$$M^* = \frac{\vec{V}}{C^*}$$

It can also be expressed as

$$M^* = M\sqrt{\frac{k+1}{2+(k-1)M^2}}$$

- In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging-diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a *normal shock,* which causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The properties of an ideal gas with constant specific heats before (subscript *x*) and after (subscript *y)* a shock are related by

$$T_{0x} = T_{0y}$$

$$\frac{T_y}{T_x} = \frac{1+M_x^2(k-1)/2}{1+M_y^2(k-1)/2}$$

$$\frac{P_y}{P_x} = \frac{M_x\sqrt{1+M_x^2(k-1)/2}}{M_y\sqrt{1+M_y^2(k-1)/2}}$$

$$M_y^2 = \frac{M_x^2+2/(k-1)}{2M_x^2k/(k-1)-1}$$

- The entropy change across the shock is obtained by applying the entropy-change equation for an ideal gas across the shock:

$$s_y - s_x = C_p \ln\frac{T_y}{T_x} - R\ln\frac{P_y}{P_x}$$

- The deviation of actual nozzles from isentropic ones is expressed in terms of the *nozzle efficiency* η_N, *nozzle velocity coefficient* C_V, and the *coefficient of discharge* C_D**,** which are defined as

$$\eta_{\rm N} = \frac{\vec{V}_2^2/2}{\vec{V}_{2s}^2/2} = \frac{\text{Actual kinetic energy at nozzle exit}}{\text{Kinetic energy at nozzle exit for isentropic flow}} = \frac{h_{01}-h_2}{h_{01}-h_{2s}}$$

$$\text{from the same inlet state to the same exit pressure}$$

$$C_{\rm V} = \frac{\vec{V}_2}{\vec{V}_{2s}} = \frac{\text{Actual velocity at nozzle exit}}{\text{Velocity at nozzle exit for isentropic flow}} = \sqrt{\eta_{\rm N}}$$

$$\text{from the same inlet state to the same exit pressure}$$

$$C_{\rm D} = \frac{\dot{m}}{\dot{m}_s} = \frac{\text{Actual mass flow rate through nozzle}}{\text{Mass flow rate through nozzle for isentropic flow}}$$

$$\text{from the same inlet state to the same exit pressure}$$

where h_{01} is the stagnation enthalpy of the fluid at the nozzle inlet, h_2 is the actual enthalpy at the nozzle exit, and h_{2s} is the exit enthalpy under isentropic conditions for the same exit pressure.

- The performance of a diffuser is expressed in terms of the *diffuser efficiency* $\eta_{\rm D}$ the *pressure recovery factor* $F_{\rm P}$, and the *pressure rise coefficient* $C_{\rm PR}$. They are defined as

$$\eta_{\rm D} = \frac{\Delta h_s}{\vec{V}_1^2/2} == \frac{h_{02s}-h_1}{h_{01}-h_1}$$

$$F_{\rm P} = \frac{\text{Actual stagnation pressure at diffuser exit}}{\text{Isentropic stagnation pressure}} = \frac{P_{02}}{P_{01}}$$

$$C_{\rm PR} = \frac{\text{Actual pressure rise}}{\text{Isentropic pressure rise}} = \frac{P_2-P_1}{P_{01}-P_1}$$

- Steam often deviates considerably from ideal-gas behavior, and no simple property relations are available for it. Thus it is often necessary to use steam tables instead of ideal-gas relations. The critical-pressure ratio of steam is often taken to be 0.546, which corresponds to a specific heat ratio of k = 1.3 for superheated steam.

- At high velocities, steam does not start condensing when it encounters the saturation line, and it exists as a *supersaturated substance*. Supersaturation states are nonequilibrium (or metastable) states, and care should be exercised in dealing with them.