Assignment 3: Solving Equations (0.4) **Name**______________________

Please provide a handwritten response.

1a. One way to solve algebraic equations in *Mathematica* is to use the **`Solve`** command. For example, we can find the zeros of $f(x) = x^2 - 3x + 2$ as in Example 4.1 by executing the command **`f[x_] = x^2 – 3x + 2`** followed by

`Solve[f[x] == 0, x]`

Record the result below. (The double equal sign **`==`** indicates an equation in *Mathematica*. You can write "replacement rules" such as `{{x->1},{x->2}}` in *Mathematica*'s output as simply "$x = 1$ or $x = 2$".)

1b. The **`Solve`** command can be used on more complicated equations, such as the one studied in Example 4.3; execute the commands **`Clear[f]`** and

`f[x_] = x^3 – x^2 – 2x + 2`

followed by **`Solve[f[x] == 0, x]`** to find the zeros of $f(x) = x^3 - x^2 - 2x + 2$, and record the result below.

1c. Once again, *Mathematica* did not give a completely decimal answer. We can achieve a decimal answer by giving a name, say **`solns`** , to the solutions *Mathematica* finds, and then applying the **`N`** command to those solutions. Execute the command

`solns = Solve[f[x] == 0, x]`

followed by **`N[solns]`** , and record the result below.

2a. Sometimes the **`Solve`** command is unable to solve an equation algebraically; in this case we can try to solve it numerically, as mentioned in Example 4.5, using the **`FindRoot`** command. **`FindRoot`** requires, however, that an approximate value of the solution be known in advance, and this can usually be found by graphing. As an example, execute the command

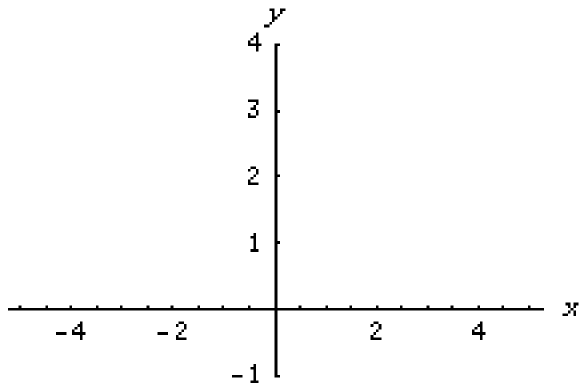
`Solve[Cos[x] == x^2 – 1, x]`

to try to solve the equation $\cos x = x^2 - 1$ in Exercise 39 of the text. (We will learn how to use *Mathematica* with trigonometric functions in general later.) Record the output below; did we get our answer?

2b. To use **`FindRoot`** instead, we will begin with a graph to show approximately where the solution(s), if any, might be found. Execute the command

```
Plot[{Cos[x], x^2 - 1}, {x, -5, 5}]
```

to plot each side of our equation as a function of *x* over the domain $-5 \le x \le 5$, and sketch the result on the axes at right. It seems from this graph that there are solutions at roughly $x = \pm 1$, and we can now use this information in the **`FindRoot`** command.



2c. Because **`FindRoot`** strictly speaking only finds zeros of functions, we will consider ourselves to be finding zeros of the function $\cos x - \left(x^2 - 1\right)$ rather than solving the equation $\cos x = x^2 - 1$, although these of course amount to the same thing. Execute the command

```
FindRoot[Cos[x] – (x^2 – 1), {x, 1}]
```

to find an accurate value of the solution of the equation near $x = 1$, and likewise execute

```
FindRoot[Cos[x] – (x^2 – 1), {x, -1}]
```

to do the same near $x = -1$; record the results below.

2d. Now change parts **b** and **c** so as to solve the equation $\cos x = x^2 - 5$ instead; remember to replace the **`1`** in **`{x, 1}`** to an appropriate starting value suggested by your graph, and similarly for **`{x, -1}`**. Record the solutions below.

3a. *Mathematica* can perform many other algebraic operations. For example, the **`Expand`** command expands algebraic expressions; execute the command **`Expand [(x + y)^7]`** to expand the binomial expression $(x + y)^7$, and record the result below.

3b. Likewise the **`Factor`** command factors expressions; execute the command

```
Factor[x^4 – 3x^2 + 2]
```

to find the factors of $x^4 - 3x^2 + 2$, and record the result below. (See Exercise 18.)