

Chapter 1

$\lim_{x \rightarrow a^+} f(x) = L$ if as x gets closer and closer to a from the right $f(x)$ gets closer and closer to L .

$\lim_{x \rightarrow a^-} f(x) = L$ if as x gets closer and closer to a from the left $f(x)$ gets closer and closer to L .

$\lim_{x \rightarrow a} f(x) = L$ if and only if both one sided limits equal L .

Computing limits: (i) $\lim_{x \rightarrow a} c = c$ (ii) $\lim_{x \rightarrow a} x = a$ (iii) $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$

(iv) $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$ (v) $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$

(vi) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ $\lim_{x \rightarrow a} g(x) \neq 0$ (vii) $\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$ (viii) $\lim_{x \rightarrow a} x^n = a^n$

More limits: (i) $\lim_{x \rightarrow a} \sin x = \sin a$ (ii) $\lim_{x \rightarrow a} \cos x = \cos a$ (iii) $\lim_{x \rightarrow a} e^x = e^a$ (iv) $\lim_{x \rightarrow a} \ln x = \ln a$ $a > 0$

Squeeze Theorem: If $f(x) < g(x) < h(x)$ for all x in some interval (c,d) except possibly at the point a and $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$ then $\lim_{x \rightarrow a} g(x) = L$

A function is **continuous** on an interval if its graph can be drawn without lifting your pencil.

A function is **continuous** at $x = a$ if (i) $f(a)$ is defined (ii) $\lim_{x \rightarrow a} f(x)$ exists and (iii) $\lim_{x \rightarrow a} f(x) = f(a)$
Otherwise f is discontinuous at $x = a$.

A discontinuity is **removable** if $\lim_{x \rightarrow a} f(x)$ exists.

If f and g are continuous at $x = a$ then (i) $(f + g)$ is continuous at $x = a$ (ii) (fg) is continuous at $x = a$ (iii) (f/g) is continuous at $x = a$. If $g(a) \neq 0$

If $\lim_{x \rightarrow a} g(x) = L$ and f is continuous at L , then $\lim_{x \rightarrow a} f(g(x)) = f(\lim_{x \rightarrow a} g(x)) = f(L)$

If f is continuous at every point on an open interval (a,b) we say that f is **continuous on** (a,b)
 f is **continuous on the closed interval** $[a,b]$ if $\lim_{x \rightarrow a^+} f(x) = f(a)$ and $\lim_{x \rightarrow b^-} f(x) = f(b)$

The Intermediate Value Theorem : Suppose that f is continuous on the closed interval $[a,b]$ and W is any number between $f(a)$ and $f(b)$. Then, there is a number c in $[a,b]$ for which $f(c) = W$.

A **vertical asymptote** is a vertical line where a function approaches

A **horizontal asymptote** is a horizontal line that $f(x)$ approaches as x goes to

For any rational number $t > 0$ $\lim_{x \rightarrow \pm\infty} \frac{1}{x^t} = 0$

Precise Definition of a Limit: For a function f defined in some open interval containing a (but not necessarily at a itself), we say $\lim_{x \rightarrow a} f(x) = L$ if given any number $\varepsilon > 0$, there is another number $\delta > 0$, such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.