Chapter 2

Slope of the secant line: $m_{sec} = \frac{f(x_2)-f(x_1)}{x_2-x_1}$

Slope of the tangent line at x=a: $m_{tan} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Distance = rate x time so average velocity: $v_{avg} = \frac{distance}{time}$

Instantaneous velocity at t=a: $v(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

If f(x) is differentiable at x=a then f(x) is continuous at x=a.

$\frac{d}{dx}c = 1$ where c is a constant

$\frac{d}{dx}X=1$

The General Power Rule: For any real number r

$\frac{d}{dx}X^r=rx^{r-1}$

If f(x) and g(x) are differentiable at x and c is any constant then

$\frac{d}{dx}[f(x)+g(x)] = f'(x) + g'(x)$

$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g'(x)$

$\frac{d}{dx}[cf(x)] = c\, f'(x)$

Acceleration is the derivative of velocity.

The Product Rule: Suppose that f and g are differentiable at x. Then

$\frac{d}{dx}[f(x)\, g(x)] = f'(x)\, g(x) + f(x)\, g'(x)$

The Quotient Rule: Suppose that f and g are differentiaable at x. Then

$\frac{d}{dx}[f(x)/g(x)] = \frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$

$\lim_{x \to 0} \sin x = 0$ $\lim_{x \to 0} \cos x = 1$ $\lim_{x \to 0} \frac{\sin x}{x} = 1$ $\lim_{x \to 0} \frac{1-\cos x}{x} = 0$

Derivatives of trigonometric functions:

$\frac{d}{dx} \sin x = \cos x$ $\frac{d}{dx} \cos x = -\sin x$ $\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \sec x = (\sec x)(\tan x)$ $\frac{d}{dx} \csc x = -(\csc x)(\cot x)$ $\frac{d}{dx} \cot x = -(\csc^2 x)$

For any constant a > 0 $\frac{d}{dx}a^x = a^x \ln a$

$\frac{d}{dx}e^x = e^x$ $\frac{d}{dx} e^{-x} = -e^{-x}$ For x > $\frac{d}{dx}\ln x = \frac{1}{x}$

The Chain Rule: If g is differentiable at x and f is differentiable at g(x)

$\frac{d}{dx}[f(g(x))] = f'(g(x))\, g'(x)$ or $\frac{d}{dx}\, g(y) = g'(y)\, y'(x)$

Rolle's Theorem: Suppose that f(x) is continuous on the interval [a,b], differentiable on the interval (a,b) and f(a) = f(b). Then there is a number c in (a,b) such that f'(c) = 0.

For any integer $n > 0$, if $f(x)$ is continuous on the interval $[a,b]$ and differentiable on the interval (a,b) and $f(x) = 0$ has n solutions in $[a,b]$, then $f'(x) = 0$ has at least $(n-1)$ solutions in (a,b).

The Mean Value Theorem: Suppose that f is continuous on the interval $[a,b]$ and differentiaable on the interval (a,b). Then there exists a number c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Suppose that $f'(x) = 0$ for all x in some open interval I. Then $f(x)$ is constant on I.