Chapter 4

Suppose that F and G are both antiderivatives of f on an interval [a,b]. Then, G(x) = F(x) + c, for some constant c.

The Power Rule: For any rational power, $r \neq -1$, $\int x^r\,dx = \frac{x^{r+1}}{r+1} + c$.

$\int \sin x\,dx = -\cos x + c$

$\int \cos x\,dx = \sin x + c$

$\int \sec^2 x\,dx = \tan x + c$

$\int \csc^2 x\,dx = -\cot x + c$

$\int \sec x \tan dx = \sec x + c$

$\int \csc x \cot x\,dx = -\csc x + c$

$\int e^x\,dx = e^x + c$

$\int e^{-x}\,dx = -e^{-x} + c$

Suppose that f(x) and g(x) have antiderivatives. Then for any constants a and b,
$\int [a\,f(x) + b\,g(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx$.

If $\int f(x)\,dx = F(x) + c$, then for any constant, a, $\int f(ax)\,dx = \frac{1}{a}F(ax) + c$.

For $x \neq 0$ $\frac{d}{dx} \ln |x| + c$

$\int \frac{1}{x}\,dx = \ln |x| + c$

$\int \frac{f'(x)}{f(x)}\,dx = \ln |f(x)| + c$

If n is any positive integer and c is any constant, then
(i) $\sum_{i=1}^{n} c = cn$ (ii) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ (iii) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

For any constants c and, $\sum_{i=1}^{n} (ca_i + db_i) = c\sum_{i=1}^{n} a_i + d\sum_{i=1}^{n} b_i$

For a function f defined on the interval [a,b] and $f(x) \geq 0$ on [a,b] the **area A under the curve** y = f(x) on [a,b] is given by $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$.

Let $\{x_0, x_1,\ldots,x_n\}$ be a regular partition of the interval [a,b], with $x_i - x_{i-1} = \Delta x = \frac{b-a}{n}$, for all i. Pick points $c_1, c_2, \ldots c_n$, where c_i is any point in the subinterval $[x_{i-1}, x_i]$, for i = 1,2,...,n. **The Riemann sum** for this partition and set of evaluation points is $\sum_{i=1}^{n} f(x_i)\,\Delta x$.

The **definite integral** of f from a to b is $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$, for any function f defined on [a,b] for which the limit exists and is the same for any choice of evauation points $c_1, c_2, \ldots c_n$. When the limit exists, we say that f is <u>integrable</u> on [a,b].

Suppose that $f(x) \geq 0$ on the interval [a,b] and A_1 is the area bounded between the curve $y = f(x)$ and the x-axis the $a \leq x \leq b$. Further, suppose that $f(x) \leq 0$ on the interval [b,c] amd A_2 is the area bounded between the curve $y = f(x)$ and the x-axis for $b \leq x \leq c$. The signed area between $y = f(x)$ and the x-axis for $a \leq x \leq c$ is $A_1 - A_2$ and the total area between $y = f(x)$ and the x-axis $a \leq x \leq c$ is $A_1 + A_2$.

If f(x) is continuous on the closed interval [a,b], then f is integrable on [a,b].

I f and g are integrable on [a,b] and c is any constant, then the following are true

(i) $\int_a^b [f(x) + g(x)]\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$

(ii) $\int_a^b [f(x) - g(x)]\, dx = \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$

(iii) $\int_a^b c\, f(x)\, dx = c \int_a^b f(x)\, dx$

(iv) $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$ for any c in [a,b].

Average value of a function: $f_{avg} = \lim_{n\to\infty} [\frac{1}{b-a} \sum_{i=1}^{n} f(x_i)\, \Delta x] = \frac{1}{b-a} \int_a^b f(x)\, dx.$

If f is continuous on [a,b], there is a number c in (a,b) for which $f(c) = \frac{1}{b-a} \int_a^b f(x)\, dx$.

The Fundamental Theorem of Calculus, Part 1: If f is continuous on [a,b] and F(x) is any antiderivative of f, then $\int_a^b f(x)\, dx = F(b) - F(a)$.

The Fundamental Theorem of Calculus, Part 2: If f is continuous on [a,b] and $F(x) = \int_a^b f(t)\, dt$, then $F'(x) = f(x)$, on [a,b].

The Midpoint Rule: $\int_a^b f(x)\, dx \approx \sum_{i=1}^{n} f(c_i)\, \Delta x$ where c_i is the midpoint of the subinterval $[x_{i-1}, x_i]$.

The Trapezoidal Rule: $\int_a^b f(x)dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)] = T_n$

Simpson's Rule: $\int_a^b f(x)\, dx \approx \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)] = S_n(f)$.