Chapter 6

The **natural logarithm** , $\ln x = \int_1^x \frac{1}{t}\, dt$, for $t > 0$

$\frac{d}{dx} \ln x = \frac{1}{x}$, for $x > 0$

$\int \frac{1}{x} dx = \ln |x| + c$

For any real numbers a, b > 0 and any rational number r, (i) $\ln 1 = 0$ (ii) $\ln(ab) = \ln a + \ln b$ (iii) $\ln (a/b) = \ln a - \ln b$ (iv) $\ln(a^r) = r \ln a$.

Assume that f and g have domains A and B, respectively, and that f(g(x)) is defined for all x in B and g(f(x)) is defined for all x in A. If f(g(x)) = x, for all x in B and g(f(x)) = x, for all x in A, we say that g is the **inverse of f**, written $g = f^{-1}$. Equivalently, f is the inverse of g, $f= g^{-1}$.

A function f is callled **one-to-one** when for every y in the range of f, there is exactly one x in the domain of f for which y = f(x).

A function f has an inverse if and only it it is one-to-one.

Suppose that f is a one-to-one continuous function. Then, f^{1} is also continuous.

Suppose that f is a one-to-one and differentiable function. Then, as long as $f'(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$.

For r,s any real numbers and t any rational number, (i) $e^r e^s = e^{r+s}$ (ii) $e^r/e^s = e^{r-s}$ (iii) $(e^r)^t = e^{rt}$.

$\frac{d}{dx}(e^x) = e$ $\quad \int e^x\, dx = e^x + c$

For any base a > 0 ($a \neq 1$) and any x > 0, $\log_a x = \frac{\ln x}{\ln a}$.

Exponential growth: $y(t) = A\, e^{kt}$

Newton's Law of Cooling: $y(t) = A\, e^{kt} + T_a$

Continuously compounding interest: $M = \$P\, e^{rt}$

The Logistic growth equation: $y = \frac{AMe^{kMt}}{1+Ae^{kMt}}$

Euler's Method: $y(x_{i+1}) \approx y_i + hf(x_i,y_i)$, for i = 0,1,2,...

$y = \sin^{-1} x$ if and only if $\sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$ if and only if $\cos y = x$ and $0 \le y \le \pi$
$y = \tan^{-1} x$ if and only if $\tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \sec^{-1} x$ if and only if $\sec y = x$ and $y \in [0,\frac{\pi}{2}) \cup (\frac{\pi}{2},\pi)$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + c$

$\int \frac{1}{\sqrt{1-x^2}} dx = -\cos^{-1}x + c$

$\int \frac{1}{1-x^2} dx = \tan^{-1}x + c$

$\int \frac{1}{|X| \sqrt{x^2-1}} dx = \sec^{-1}x + c$

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\quad$ $\cosh x = \frac{e^x + e^x}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\tanh x = \frac{\sinh x}{\cosh x}$ $\quad$ $\coth x = \frac{\cosh x}{\sinh x}$ $\quad$ $\operatorname{sech} x = \frac{1}{\cosh x}$ $\quad$ $\operatorname{csch} x = \frac{1}{\sinh x}$

$\frac{d}{dx} \sinh x = \cosh x$ $\quad$ $\frac{d}{dx} \cosh x = \sinh x$ $\quad$ $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$

$\frac{d}{dx} \coth x = -\operatorname{csch} x$ $\quad$ $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$ $\quad$ $\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

$y = \sinh^{-1}x$ if and only if $\sinh y = x$

For $x \geq 1$ $y = \cosh^{-1} x$ if and only if $\cosh y = x$ and $y \geq 0$

For x in $(-1,1)$ $y = \tanh^{-1} x$ if and only if $\tanh y = x$

$\frac{d}{dx} \sinh^{-1}x = \frac{1}{\sqrt{1+x^2}}$ $\quad$ $\frac{d}{dx} \cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$ $\quad$ $\frac{d}{dx} \tanh^{-1}x = \frac{1}{1-x^2}$

$\frac{d}{dx} \coth^{-1}x = \frac{1}{1-x^2}$ $\quad$ $\frac{d}{dx} \operatorname{sech}^{-1}x = \frac{-1}{x\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx} \operatorname{csch}^{-1}x = \frac{-1}{|x| \sqrt{1+x^2}}$