


# VISUAL GUIDE: EXPANDING UPON THE TEXT

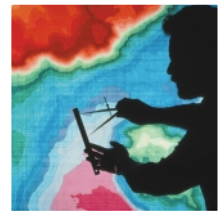
## TOOLS FOR LEARNING

### Real-World Emphasis

Real-world examples are an important aid to the understanding of calculus. We introduce each chapter with a brief application related to the mathematical concepts being developed in order to put each chapter into a larger problem-solving context. Subsequently, both examples within the text and exercises are used to further demonstrate the importance of calculus within the world.



## CHAPTER 3




### APPLICATIONS OF DIFFERENTIATION

One relatively new test available to physicians for diagnosing injuries and disease is the MRI. Magnetic resonance imaging (MRI) is used to visualize internal structures, such as torn cartilage in a knee. The ability to *see* the physical status of a knee or an internal organ without surgery is an invaluable aid to physicians and their patients. However, it still takes an experienced physician to distinguish the important features of an MRI from insignificant ones. If you have ever looked at an MRI or even a conventional x-ray, you have probably been amazed at the details that your physician could quickly identify. In the MRI below, can you identify any damage to the knee? Of course, it always helps to know what you are looking for.

The ability to accurately read graphs is one of the primary goals of this chapter. By the end of section 3.6, you should have a good idea of what the significant features of a graph are. Although we will be looking only at two-dimensional graphs of functions, the language and skills that you acquire here will transfer to plots of seismic readings, sonar mappings of the ocean floor and other graphical displays of information that you may encounter.

Most people do not recognize the vast amount of mathematical computation required to produce a viewable image from an MRI. In an MRI, magnetic fields and pulses of radio waves are used to determine the distribution of hydrogen atoms in the body (see *Visualization* by R. Friedhoff and W. Benzion for more details). The presence of hydrogen atoms, in turn, is deduced from the release of energy during the magnetization process. (This is a long way from a standard x-ray image!) By solving countless equations and performing lengthy calculations, a computer transforms the energy data into an accurate image of the interior of a human body.

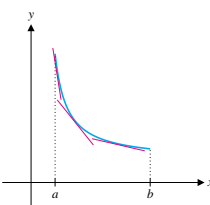
Likewise, it may surprise you how many calculations we must perform to draw an accurate graph of a function. At each stage of the graphing process, we must solve equations to identify significant features of the graph. Because of the



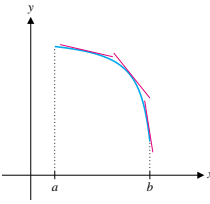
MRI of a knee.

## Definitions, Theorems and Proofs

All formal definitions and theorems are clearly boxed within the text for easy visual reference. Selected proofs are provided for reference. Proofs of some results are found in Appendix A.



**Figure 3.48a**  
Concave up.



**Figure 3.48b**  
Concave down.

increasing) and the one shown in Figure 3.48b is concave down (slopes of tangent lines decreasing). We have the following definition.

**Definition 5.1**  
For a function  $f$  that is differentiable on an interval  $I$ , the graph of  $f$  is

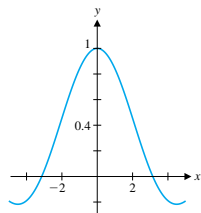
- (i) **concave up** on  $I$ , if  $f'$  is increasing on  $I$  or
- (ii) **concave down** on  $I$ , if  $f'$  is decreasing on  $I$ .

How can you tell when  $f'$  is increasing or decreasing? The derivative of  $f'$  (i.e.,  $f''$ ) yields that information. The following theorem connects this definition with what we already know about increasing and decreasing functions. The proof of the theorem is a straightforward application of Theorem 4.1 to Definition 5.1.

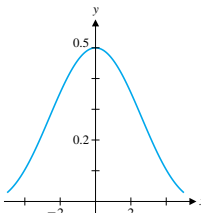
**Theorem 5.1**  
Suppose that  $f''$  exists on an interval  $I$ .

- (i) If  $f''(x) > 0$  on  $I$ , then the graph of  $f$  is concave up on  $I$ .
- (ii) If  $f''(x) < 0$  on  $I$ , then the graph of  $f$  is concave down on  $I$ .

**Example 5.1** Determining Concavity  
Determine where the graph of  $f(x) = 2x^3 + 9x^2 - 24x - 10$  is concave up and concave down and draw a graph showing all significant behavior of the function.



**Figure 3.7**  
 $y = \frac{\sin x}{x}$ .



**Figure 3.8**  
 $y = \frac{1 - \cos x}{x^2}$ .

**Example 1.5** Revisiting an Old Limit  
Evaluate  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ .

**Solution** Again, this limit has the indeterminate form  $\frac{0}{0}$  and  $f(x) = \sin x$  and  $g(x) = x$  are both continuous and differentiable everywhere. Finally,  $g'(x) = \frac{d}{dx}(x) = 1 \neq 0$ , so that all of the hypotheses of l'Hôpital's Rule are satisfied. From the graph in Figure 3.7, it appears that the limit is approximately 1. We can confirm this suspicion with l'Hôpital's Rule. We have

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{1}{1} = 1,$$

as we proved using a complicated geometric argument in section 2.5.

For some limits, you must apply l'Hôpital's Rule more than once.

**Example 1.6** A Limit Requiring Two Applications of l'Hôpital's Rule  
Evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ .

**Solution** Again, this has the indeterminate form  $\frac{0}{0}$  and it is a simple matter to verify that the hypotheses of l'Hôpital's Rule are satisfied. In this case, the graph in Figure 3.8 indicates the limit to be approximately 0.5. From l'Hôpital's Rule, we have

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{\sin x}{2x},$$

which again has the indeterminate form  $\frac{0}{0}$ . In this case, we can verify that the hypotheses of l'Hôpital's Rule are satisfied for this new limit problem. Applying this again, it then follows that

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{\sin x}{2x} = \lim_{x \rightarrow 0} \frac{\cos x}{2} = \frac{1}{2}.$$

## Examples

Each chapter contains a large number of worked examples, ranging from the simple and concrete to more complex and abstract. A thorough understanding of the initial problem presented and the step-by-step solution will greatly enhance your problem-solving capabilities and your further study of the subject.

## Use of Graphs and Tables

Being able to visualize a problem is an invaluable aid in understanding the concept presented. To this purpose, we have integrated more than 1500 computer-generated graphs throughout the text. You should use them routinely to aid in solving most problems, even if only as a check on the reasonableness of an answer. Each graph and table has been created very carefully to ensure that the ideas presented are clear and accurate. In many places, we have included multiple graphical perspectives, such as with the contour and density plots found in Chapter 12.

# COMMENTARY AND GUIDANCE

In order to help you best interpret the material presented, we have included many side elements and notes for your reference.

### 3.1 LINEAR APPROXIMATIONS AND L'HÔPITAL'S RULE

For what purpose do you use a scientific calculator? If you think about it, you'll discover that there are two distinctly different jobs that calculators do for you. First, they perform arithmetic operations (addition, subtraction, multiplication and division) much faster than any of us could hope to do them. It's not that you don't know how to multiply 1024 by 1673, but rather that it is time-consuming to carry out this (albeit well-understood) calculation with pencil and paper. For such problems, calculators are a tremendous convenience, which none of us would like to live without. Perhaps more significantly, we also use our calculators to compute values of transcendental functions such as sine, cosine, tangent, exponentials and logarithms. In the case of these function evaluations, the calculator is much more than a mere convenience.



If asked to calculate  $\sin(1.2345678)$  without a calculator, you would probably draw a blank. Don't worry, there's nothing wrong with your background. (Also, don't worry that anyone will ever ask you to do this without a calculator.) The problem is that the sine function is not *algebraic*. That is, there is no formula for  $\sin x$  involving only the arithmetic operations. So, how does your calculator "know" that  $\sin(1.2345678) \approx 0.9440056953$ ? In short, it doesn't *know* this at all. Rather, the calculator has a built-in program that generates **approximate** values of the sine and other transcendental functions.

In this section, we take a small step into the (very large) world of approximation by developing a simple approximation method. Although somewhat crude, it points the way

### Technology Guidance

We hope that by using this text you will become proficient at knowing when and how technology is appropriate within calculus. Rather than providing key-punching instructions, we provide advice and guidance on the proper use of technology, empowering you to explore new problems on your own. We also provide comments on possible errors and pitfalls that can be caused due to an over reliance on technology. This guidance is frequently given within an example or explanation of a specific technique. However, an icon is used when this guidance appears within the general discussion.

### Remarks Text

Remarks boxes, found both as marginal and text references, provide a summary or overview of techniques. These boxes also foreshadow future ideas related to the topic being discussed.



#### Remark 3.1

Fermat's Theorem says that local extrema can only occur at critical numbers. This does not say that there is a local extremum at **every** critical number. In fact, this is false, as we illustrate in examples 3.8 and 3.9.

absolute minimum). You should try to use your graphing utility to produce a graph of  $y = f(x)$ . If you get only half of the graph displayed in Figure 3.26, you have discovered one of the dangers in relying too heavily on technology. The algorithms used by most calculators and many computers will return a complex number (or an error) when asked to compute certain fractional powers of negative numbers. This annoying shortcoming presents only occasional difficulties. We mention this here only to make you aware that technology has limitations.

#### A Horizontal Tangent at a Point That Is Not a Local Extremum

#### Example 3.8

Find the critical numbers and local extrema of  $f(x) = x^3$ .

**Solution** It should be clear from Figure 3.27 that  $f$  has no local extrema. However,  $f'(x) = 3x^2 = 0$  for  $x = 0$  (the only critical number of  $f$ ). In this case,  $f$  has a horizontal tangent line at  $x = 0$ , but does not have a local extremum there (or anywhere else).



Although it seemed to be very efficient in the last two examples, Newton's method does not always work. We urge you to make sure that the values coming from the method are getting progressively closer and closer together (zeroing-in, we hope, on the desired solution). Don't stop until you've reached the limits of accuracy of your computing device. Also, be sure to compute the value of the function at the suspected approximate zero. If the function value is not close to zero, do not accept the value as an approximate zero.

As we illustrate in the following example, Newton's method requires a good initial guess in order to find an accurate approximation.

#### Example 2.3 The Effect of a Bad Guess on Newton's Method

Use Newton's method to find an approximate zero of  $f(x) = x^3 - 3x^2 + x - 1$ .

**Solution** From the graph in Figure 3.14, there appears to be a zero on the interval  $(2, 3)$ . If you were to use the (not particularly good) initial guess  $x_0 = 1$ , you would get  $x_1 = 0$ ,  $x_2 = 1$ ,  $x_3 = 0$  and so on. Try this for yourself. Newton's method is sensitive to the initial guess and you just made a bad initial guess. If you had instead started with the improved initial guess  $x_0 = 2$ , Newton's method would have quickly converged to the approximate zero 2.769292354. (Again, try this for yourself.)

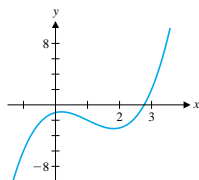


Figure 3.14

$$y = x^3 - 3x^2 + x - 1.$$

### Lightbulb Icon

A lightbulb icon indicates the relaying of an important idea in clear, nontechnical language. This icon is also used to highlight a special problem-solving tip within the text.

**HISTORICAL NOTES**



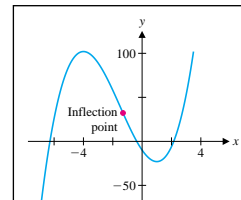
**Sir Isaac Newton (1642–1727)**  
 An English mathematician and scientist known as the co-inventor of calculus. In a 2-year period from 1665 to 1667, Newton made major discoveries in several areas of calculus, as well as optics and the law of gravitation. Newton's mathematical results were not published in a timely fashion. Instead, techniques such as Newton's method were quietly introduced as useful tools in his scientific papers. Newton's *Mathematical Principles of Natural Philosophy* is widely regarded as one of the greatest achievements of the human mind.

**Historical Notes**

These notes provide a historical context for the development of calculus. Biographical information about prominent mathematicians and their contributions to the development of calculus are given to put the subject matter in perspective.

**Notes Text**

Notes boxes, found within the margin, serve to step back from a concept to gain perspective by adding details, making connections to previous material or noting when an example is indicative of a general pattern.



**Figure 3.49**

$y = 2x^3 + 9x^2 - 24x - 10.$

**NOTES**

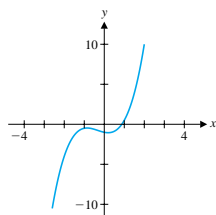
If  $(c, f(c))$  is an inflection point, then either  $f''(c) = 0$  or  $f''(c)$  is undefined. So, finding all points where  $f''(x)$  is zero or is undefined gives you all possible candidates for inflection points. But beware: not all points where  $f''(x)$  is zero or undefined correspond to inflection points.

**CONCEPTUAL UNDERSTANDING THROUGH PRACTICE**

We have written this text with a strong problem-solving emphasis, including the introduction of many topics from graphical, numerical and algebraic points of view. In many instances, the emphasis on graphical and numerical methods for solving problems frees us to consider more realistic and complex problems than are usually presented in calculus. Furthering this emphasis, we have included a variety of exercise types to strengthen your problem-solving skills.

**EXERCISES 3.4**

- Suppose that  $f(0) = 2$  and  $f(x)$  is an increasing function. To sketch the graph of  $y = f(x)$ , you could start by plotting the point  $(0, 2)$ . Filling in the graph to the left, would you move your pencil up or down? How does this fit with the definition of increasing?
- Suppose you travel east on an east-west interstate highway. You reach your destination, stay a while and then return home. Explain the First Derivative Test in terms of your velocities (positive and negative) on this trip.
- Suppose that you have a differentiable function  $f(x)$  with two critical numbers. Your computer has shown you a graph that looks like the one below.



Discuss the possibility that this is a representative graph: that is, is it possible that there are any important points not shown in this window?

- Suppose that the function in exercise 3 has three critical numbers. Explain why the graph is not a representative graph. Explain how you would change the graphing window to show the rest of the graph.

In exercises 15–34, find the  $x$ -coordinates of all extrema and sketch a graph.

- |                               |                                    |
|-------------------------------|------------------------------------|
| 15. $y = x^3 + 2x^2 - x - 1$  | 16. $y = x^3 + 4x - 2$             |
| 17. $y = x^4 + 2x^2 - x + 2$  | 18. $y = x^5 + 2x^4 - x^2 + 1$     |
| 19. $y = x\sqrt{x^2 + 1}$     | 20. $y = \frac{x}{\sqrt{x^2 + 1}}$ |
| 21. $y = xe^{-2x}$            | 22. $y = x^2e^{-x}$                |
| 23. $y = \ln x^2$             | 24. $y = e^{-x^2}$                 |
| 25. $y = \frac{x}{x^2 - 1}$   | 26. $y = \frac{x^2}{x^2 - 1}$      |
| 27. $y = \frac{x^3}{x^2 - 1}$ | 28. $y = \frac{x^2}{x^2 + 1}$      |
| 29. $y = \sin x + \cos x$     | 30. $y = \cos x - x$               |
| 31. $y = \sqrt{x^3 + 3x^2}$   | 32. $y = 2x^{1/2} - 4x^{-1/2}$     |
| 33. $y = x^{2/3} - 2x^{-1/3}$ | 34. $y = x^{4/3} + 4x^{1/3}$       |

In exercises 35–42, find the  $x$ -coordinates of all extrema and sketch graphs showing global and local behavior of the function.


- $y = x^3 - 13x^2 - 10x + 1$
- $y = x^3 + 15x^2 - 70x + 2$
- $y = x^4 - 15x^3 - 2x^2 + 40x - 2$
- $y = x^4 - 16x^3 - 0.1x^2 + 0.5x - 1$


**End-of-Section Exercises**

Each exercise set has been carefully constructed to reinforce both the concepts and mechanics of calculus, while encouraging individual exploration. Our goal is to create original and imaginative exercises that provide an appropriate review of the topics covered in each section while reinforcing the basic skills needed to master the concept.

**Writing Exercises**

Each exercise set begins with a variety of writing exercises. These exercises can be used as springboards for discussion and are intended to give you an opportunity to carefully consider important mathematical concepts and ideas and express these in your own words. Learning to verbalize mathematical structures is a key skill in mastering concepts.

60.  In this exercise, we look at the ability of fireflies to synchronize their flashes. (To see a remarkable demonstration of this ability, see David Attenborough's video series *Trials of Life*.) Let the function  $f(t)$  represent an individual firefly's rhythm, so that the firefly flashes whenever  $f(t)$  equals an integer. Let  $e(t)$  represent the rhythm of a neighboring firefly, where again  $e(t) = n$ , for some integer  $n$ , whenever the neighbor flashes. One model of the interaction between fireflies is  $f'(t) = \omega + A \sin[e(t) - f(t)]$  for constants  $\omega$  and  $A$ . If the fireflies are synchronized ( $e(t) = f(t)$ ), then  $f'(t) = \omega$ , so the fireflies flash every  $1/\omega$  time units. Assume that the difference between  $e(t)$  and  $f(t)$  is less than  $\pi$ . Show that if  $f(t) < e(t)$ , then  $f'(t) > \omega$ . Explain why this means that the individual firefly is speeding up its flash to match its neighbor. Similarly, discuss what happens if  $f(t) > e(t)$ .

61.  The HIV virus attacks specialized T cells that trigger the human immune system response to a foreign substance. If  $T(t)$  is the population of uninfected T cells at time  $t$  (days) and  $V(t)$  is the population of infectious HIV in the bloodstream, a model that has been used to study AIDS is given by the following differential equation that describes the rate at which the population of T cells changes.

$$T'(t) = 10 \left[ 1 + \frac{1}{1 + V(t)} \right] - 0.027T(t) + 0.01 \frac{T(t)V(t)}{100 + V(t)} - 0.0000247(t)V(t).$$

## Exploratory Exercises

Each exercise set concludes with exploratory exercises that are project-like assignments designed for you to obtain a sense of the ongoing, exciting nature of mathematics-related research. These exercises offer excellent opportunities for group work or discussion, depending on the structure of your individual calculus course.

## Chapter Review Exercises

Chapter Review Exercise sets are provided as an overview of the chapter and will test your understanding prior to continuing with the text.

## CHAPTER REVIEW EXERCISES

In exercises 1 and 2, find the linear approximation to  $f(x)$  at  $x_0$ .

1.  $f(x) = e^{3x}$ ,  $x_0 = 0$       2.  $f(x) = \sqrt{x^2 + 3}$ ,  $x_0 = 1$

In exercises 3 and 4, use a linear approximation to estimate the quantity.

3.  $\sqrt[3]{7.96}$       4.  $\sin 3$

In exercises 5 and 6, use Newton's method to find an approximate root.

5.  $x^3 + 5x - 1 = 0$       6.  $x^3 = e^{-x}$

7. Explain why Newton's method fails on  $x^3 - 3x + 2 = 0$  with  $x_0 = 1$ .

8. Show that the approximation  $\frac{1}{(1-x)} \approx 1+x$  is valid for "small"  $x$ .

In exercises 9–18, do the following by hand. (a) Find all critical numbers, (b) identify all intervals of increase and decrease, (c) determine whether each critical number represents a local maximum, local minimum or neither, (d) determine all intervals of concavity and (e) find all inflection points.

9.  $f(x) = x^3 + 3x^2 - 9x$       10.  $f(x) = x^4 - 4x + 1$   
 11.  $f(x) = x^4 - 4x^3 + 2$       12.  $f(x) = x^3 - 3x^2 - 24x$   
 13.  $f(x) = xe^{-4x}$       14.  $f(x) = x^2 \ln x$   
 15.  $f(x) = x\sqrt{x^2 - 4}$       16.  $f(x) = (x^2 - 1)^{2/3}$   
 17.  $f(x) = \frac{x}{x^2 + 4}$       18.  $f(x) = \frac{x}{\sqrt{x^2 + 2}}$

In exercises 19–22, find the absolute extrema of the function on the interval.

19.  $f(x) = x^3 + 3x^2 - 9x$  on  $[0, 4]$   
 20.  $f(x) = x^3 + 3x^2 - 9x$  on  $[-4, 0]$   
 21.  $f(x) = x^{4/5}$  on  $[-2, 3]$   
 22.  $f(x) = x^2e^{-x}$  on  $[-1, 4]$

In exercises 23–26, find the  $x$ -coordinates of all local extrema.

23.  $f(x) = x^3 + 4x^2 + 2x$       24.  $f(x) = x^4 - 3x^2 + 2x$   
 25.  $f(x) = x^5 - 2x^2 + x$       26.  $f(x) = x^5 + 4x^2 - 4x$

27. Sketch a graph of a function with  $f(-1) = 2$ ,  $f(1) = -2$ ,  $f'(x) < 0$  for  $-2 < x < 2$ ,  $f'(x) > 0$  for  $x < -2$  and  $x > 2$ .


28. Sketch a graph of a function with  $f'(x) > 0$  for  $x \neq 0$ ,  $f'(0)$  undefined,  $f''(x) > 0$  for  $x < 0$  and  $f''(x) < 0$  for  $x > 0$ .

In exercises 29–38, sketch a graph of the function and completely discuss the graph.


29.  $f(x) = x^4 + 4x^3$       30.  $f(x) = x^4 + 4x^2$   
 31.  $f(x) = x^4 + 4x$       32.  $f(x) = x^4 - 4x^2$   
 33.  $f(x) = \frac{x}{x^2 + 1}$       34.  $f(x) = \frac{x}{x^2 - 1}$   
 35.  $f(x) = \frac{x^2}{x^2 + 1}$       36.  $f(x) = \frac{x^2}{x^2 - 1}$   
 37.  $f(x) = \frac{x^3}{x^2 - 1}$       38.  $f(x) = \frac{4}{x^2 - 1}$

39. Find the point on the graph of  $y = \dots$
40. Show that the line through the two perpendicular to the tangent line to  $y = \dots$
41. A city is building a highway from 4 miles east and 6 miles south of point A is swamp land, where the way is \$6 million per mile. On dry land is \$1 million per mile. Find the point on the boundary to which the highway should be built to minimize the total cost.
42. Repeat exercise 41 with a cost of \$16 million per mile on swamp land. Explain why the optimal point in this exercise is west of the optimal point found in exercise 41.
43. A soda can in the shape of a cylinder is to hold 16 ounces. Find the dimensions of the can that minimizes the surface area of the can.
44. Suppose that  $C(x) = 0.02x^2 + 4x + 1200$  is the cost of manufacturing  $x$  items. Show that  $C'(x) > 0$  and explain in business terms why this has to be true. Show that  $C''(x) > 0$  and explain why this indicates that the manufacturing process is not very efficient.
45. The charge in an electrical circuit at time  $t$  is given by  $Q(t) = e^{-3t} \sin 2t$  coulombs. Find the current.
46. If the concentration  $x(t)$  of a chemical in a reaction changes according to the equation  $x'(t) = 0.3x(t)[4 - x(t)]$ , find the

where  $a$  and  $b$  are positive constants. If  $n(0) = a/b$ , what is  $n'(0)$ ? Based on this calculation, would  $n(t)$  increase, decrease or neither? If  $n(0) > a/b$ , is  $n'(0)$  positive or negative? Based on this calculation, would  $n(t)$  increase, decrease or neither? If  $n(0) < a/b$ , is  $n'(0)$  positive or negative? Based on this calculation, would  $n(t)$  increase, decrease or neither? Putting this information together, conjecture the limit of  $n(t)$  as  $t \rightarrow \infty$ . Repeat this analysis under the assumption that  $a < 0$ . [Hint: Because of its definition,  $n(t)$  is positive, so ignore any negative values of  $n(t)$ .]

52.  One way of numerically approximating a derivative is by computing the slope of a secant line. For example,  $f'(a) \approx \frac{f(b) - f(a)}{b - a}$ , if  $b$  is close enough to  $a$ . In this exercise, we will develop an analogous approximation to the second derivative. Graphically, we can think of the secant line as an approximation of the tangent line. Similarly, we can match the second derivative behavior (concavity) with a parabola. Instead of finding the secant line through two points on the curve, we find the parabola through three points on the curve. The second derivative of this approximating parabola will serve as an approximation of the second derivative of the curve. The first step

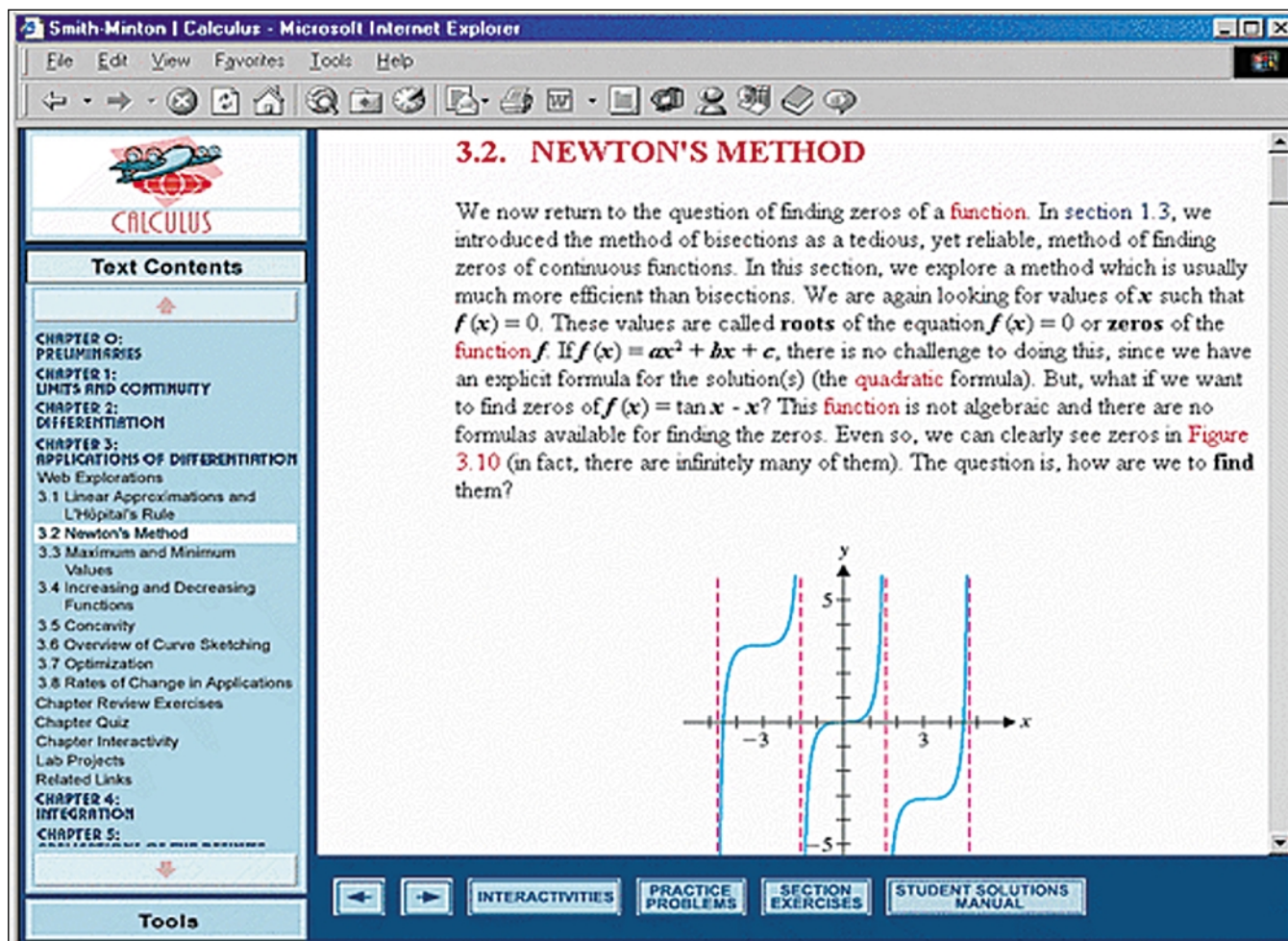
# BEYOND THE TEXT

*Access to Calculus: An Interactive Text is included with each copy of this book. Based on the text itself, this program takes key examples and figures from the text and puts them into an interactive format for further practice. Examples and figures that are expanded on are indicated by an icon  within the text. We guarantee that this program will become one of the best study partners that you can find.*

*Within Calculus: An Interactive Text you will find:*

## Online Text

This study partner provides your entire *Calculus* text online for easy reference from any computer. In addition to the textual elements themselves, all figures, examples, theorems, and definitions have been compiled into individual libraries for quick access. A complete online glossary of terms has also been provided. These items can be accessed individually or through links within the text, where appropriate.



Smith-Minton | Calculus - Microsoft Internet Explorer

File Edit View Favorites Tools Help

**3.2. NEWTON'S METHOD**

We now return to the question of finding zeros of a **function**. In section 1.3, we introduced the method of bisections as a tedious, yet reliable, method of finding zeros of continuous functions. In this section, we explore a method which is usually much more efficient than bisections. We are again looking for values of  $x$  such that  $f(x) = 0$ . These values are called **roots** of the equation  $f(x) = 0$  or **zeros** of the **function**  $f$ . If  $f(x) = ax^2 + bx + c$ , there is no challenge to doing this, since we have an explicit formula for the solution(s) (the **quadratic** formula). But, what if we want to find zeros of  $f(x) = \tan x - x$ ? This **function** is not algebraic and there are no formulas available for finding the zeros. Even so, we can clearly see zeros in **Figure 3.10** (in fact, there are infinitely many of them). The question is, how are we to **find** them?

**Text Contents**

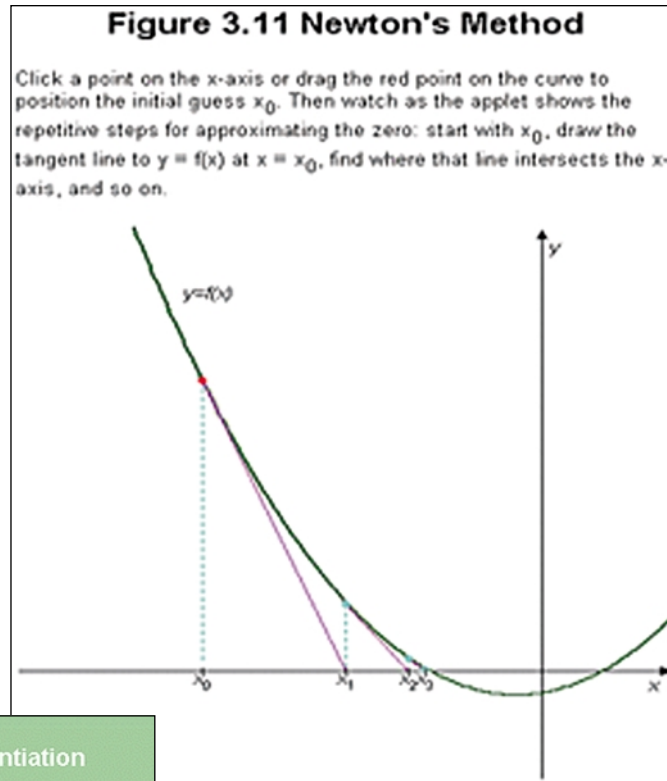
- CHAPTER 0: PRELIMINARIES
- CHAPTER 1: LIMITS AND CONTINUITY
- CHAPTER 2: DIFFERENTIATION
- CHAPTER 3: APPLICATIONS OF DIFFERENTIATION
  - Web Explorations
  - 3.1 Linear Approximations and L'Hôpital's Rule
  - 3.2 Newton's Method**
  - 3.3 Maximum and Minimum Values
  - 3.4 Increasing and Decreasing Functions
  - 3.5 Concavity
  - 3.6 Overview of Curve Sketching
  - 3.7 Optimization
  - 3.8 Rates of Change in Applications
  - Chapter Review Exercises
  - Chapter Quiz
  - Chapter Interactivity
  - Lab Projects
  - Related Links
- CHAPTER 4: INTEGRATION
- CHAPTER 5: VECTOR CALCULUS

**Tools**

← → INTERACTIVITIES PRACTICE PROBLEMS SECTION EXERCISES STUDENT SOLUTIONS MANUAL

## Interactivities

Key figures within each section have been turned into interactive Java applets for your further exploration. By interacting with the figure, you will be able to clearly see the usefulness and limitations of the concept being presented. Additionally, a Chapter Interactivity has been provided to help conceptualize the material developed throughout the chapter. These Chapter Interactivities often use real-world visualization to enhance the meaning behind the concepts being presented. There are over 200 Java applets provided within this program for your use.



**Calculus**  
**Chapter 3 Applications of Differentiation**

**Section 3.2 Newton's Method**

In the following equation, use Newton's method with the given  $x_0$  to compute  $x_2$ , rounding to the nearest thousandth.

*Problem 5*

$$x^3 + 2x^4 + 1 = 0, x_0 = 1$$

Your Answer:

GIVE ME  
A HINT

SHOW ME  
THE SOLUTION

PRINT THIS  
PROBLEM

VIEW NEXT  
PROBLEM

## Practice Problems

Several key examples per section have been used to create algorithmically-generated practice problems, developed to further your problem-solving skills and to clarify your understanding of concepts. If you are having difficulties understanding an example these practice problems will walk you through the problem, providing hints and if necessary, will provide you with the fully worked solution and textual reference to help explain the concept. You can then request a similar problem to try again until you feel comfortable with your understanding. A Chapter Quiz consisting of a random sampling of problems from each section can be used to test your understanding prior to a quiz or test. Rather than walking you through the solution of the problems, this quiz will record your answers and provide detailed feedback at the end of the quiz. There are over 400 practice problems available for your use.

## Additional Features

### Student Solutions Manual

Fully worked solutions for select odd-numbered exercises within the text can be accessed directly through the End-of-Section and End-of-Chapter exercises within the online text. A print version of this manual is available for purchase for those who choose that option.

### Lab Projects

A technology-based lab project is provided for your further exploration. These real-world projects will enable you to practice your problem-solving skills by applying technology as appropriate within a controlled environment.

### Web Explorations and Web Links

The World Wide Web has become an invaluable research tool, expanding real-world emphasis within calculus much further than could ever be done within a text. We have provided fun exploratory exercises accessing interesting real-world data as well as monthly updated chapter-specific links for further research of topics covered within the chapter.

### Tools

A fully functional online graphing calculator is provided for quick access while working with the program. In addition, access to NetTutor™—your online tutorial service—and the text-specific Online Learning Center is provided.