VI. VISCOUS INTERNAL FLOW

To date, we have considered only problems where the viscous effects were either:

a. known: i.e. - known F_D or h_f

b. negligible: i.e. - inviscid flow

This chapter presents methodologies for predicting viscous effects and viscous flow losses for internal flows in pipes, ducts, and conduits.

Typically, the first step in determining viscous effects is to determine the flow regime at the specified condition.

The two possibilities are:

a. **Laminar flow**

b. **Turbulent flow**

The student should read Section 6.1 in the text, which presents an excellent discussion of the characteristics of laminar and turbulent flow regions.

For steady flow at a known flow rate, these regions exhibit the following:

Laminar flow: A local velocity constant with time, but which varies spatially due to viscous shear and geometry.

Turbulent flow: A local velocity which has a constant mean value but also has a statistically random fluctuating component due to turbulence in the flow.

Typical plots of velocity time histories for laminar flow, turbulent flow, and the region of transition between the two are shown below.

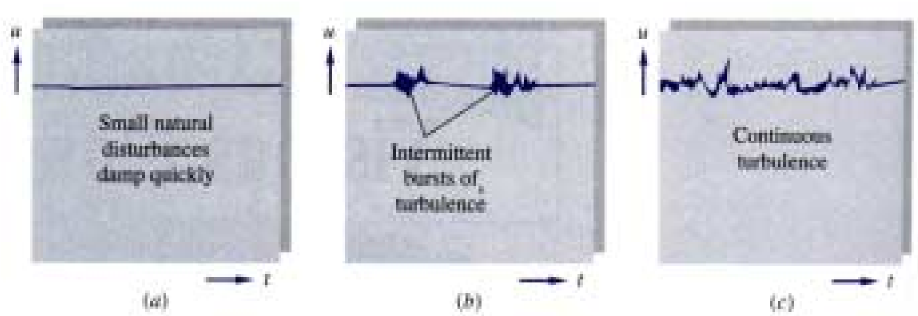


Fig. 6.1 (a) Laminar, (b)transition, and (c) turbulent flow velocity time histories.

Principal parameter used to specify the type of flow regime is the

$$\text{Reynolds number,} \quad \text{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

V - characteristic flow velocity
D - characteristic flow dimension
μ - dynamic viscosity
υ - kinematic viscosity = $\frac{\mu}{\rho}$

We can now define the

Re_{cr} ≡ critical or transition Reynolds number

Re_{cr} ≡ Reynolds number below which the flow is laminar, above which the flow is turbulent

While transition can occur over a range of Re, we will use the following for internal pipe or duct flow:

$$\text{Re}_{cr} \cong 2300 = \left.\frac{\rho V D}{\mu}\right)_{cr} = \left.\frac{VD}{\upsilon}\right)_{cr}$$

Internal Viscous Flow

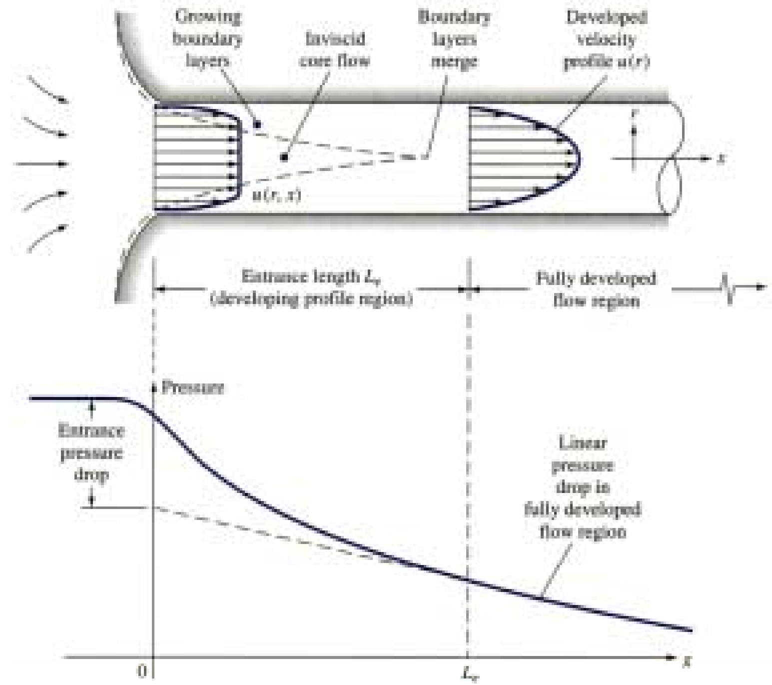
A second classification concerns whether the flow has significant entrance region effects or is fully developed. The following figure indicates the characteristics of the entrance region for internal flows. Note that the slope of the streamwise pressure distribution is greater in the entrance region than in the fully developed region (due to frictional effects plus the acceleration of the core flow as the boundary layer develops).

Typical criteria for the length of the entrance region are given as follows:

Laminar: $\frac{L_e}{D} \cong 0.06\,\text{Re}$

Turbulent $\frac{L_e}{D} \cong 4.4 \text{Re}^{1/6}$

where: L_e = length of the entrance region



Note: **Take care in neglecting entrance region effects**.

In the entrance region, frictional pressure drop/length > the pressure drop/length for the fully developed region. Therefore, if the effects of the entrance region are neglected, the overall predicted pressure drop will be low. This can be significant in a system with short tube lengths, e.g. some heat exchangers.

Fully Developed Pipe Flow

The analysis for **steady, incompressible, fully developed, laminar flow** in a circular horizontal pipe yields the following equations:

$$U(r) = -\frac{R^2}{4\mu}\frac{dP}{dx}\left\{1 - \frac{r^2}{R^2}\right\}$$

$$\frac{U}{U_{\max}} = \left\{1 - \frac{r^2}{R^2}\right\} \quad , \quad U_{\max} = 2V_{avg}$$

and

$$\mathbf{Q = A\ V_{avg} = \pi\ R^2\ V_{avg}}$$

Key Points: Thus for laminar, fully developed pipe flow (**not turbulent**):

a. The velocity profile is parabolic.
b. The maximum local velocity is at the centerline (r = 0).
c. The average velocity is one-half the centerline velocity.
d. The local velocity at any radius varies only with radius, not on the streamwise (x) location (due to the flow being fully developed).

Note: All subsequent equations will use the symbol V (no subscript) to represent the average flow velocity in the flow cross section.

Darcy Friction Factor:

We can now define the Darcy friction factor f as:

$$f \equiv \frac{\left(\frac{D}{L}\right)\Delta P_f}{\rho \frac{V^2}{2}}$$

where ΔP_f = the pressure drop due to friction only.

The general energy equation must still be used to determine total pressure drop.

Therefore, we obtain

$$\Delta P_f = \rho g h_f = f \frac{L}{D} \rho \frac{V^2}{2}$$

and the friction head loss h_f is given as

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Note: The definitions for f and h_f are **valid for either laminar or turbulent flow**. However, you must evaluate f for the correct flow regime, laminar or turbulent.

Key Point: It is common in industry to define and use a "fanning" friction factor f_f in viscous pipe flow analyses. The fanning friction factor differs from the Darcy friction factor by a factor of 4.

Thus, care should be taken when using unfamiliar equations or data since use of f_f in equations that were developed for the Darcy friction factor will result in significant errors (a factor of 4).

Your employer will not be happy if you order a 10 hp motor for a 2.5 hp application. The equation suitable for use with f_f is

fanning friction factor only:

$$h_f = 4\, f_f \frac{L}{D} \frac{V^2}{2g}$$

Laminar flow:

Application of the results for the laminar flow velocity profile to the definition of the Darcy friction factor yields the following expression:

$$f = \frac{64}{\text{Re}}$$

laminar flow only (**Re < 2300**)

Thus, with the value of the Reynolds number, the friction factor for laminar flow is easily determined.

Turbulent flow:

A similar analysis is not readily available for turbulent flow. However, the Colebrook equation, shown below, provides an excellent representation for the variation of the Darcy friction factor in the turbulent flow regime. Note that the equation depends on both the pipe Reynolds number and the roughness ratio, is transcendental, and cannot be expressed explicitly for f.

$$f = -2\log\left[\frac{2.51}{\text{Re}\, f^{1/2}} + \frac{\varepsilon / D}{3.7}\right]$$

turbulent flow only (**Re > 2300**)

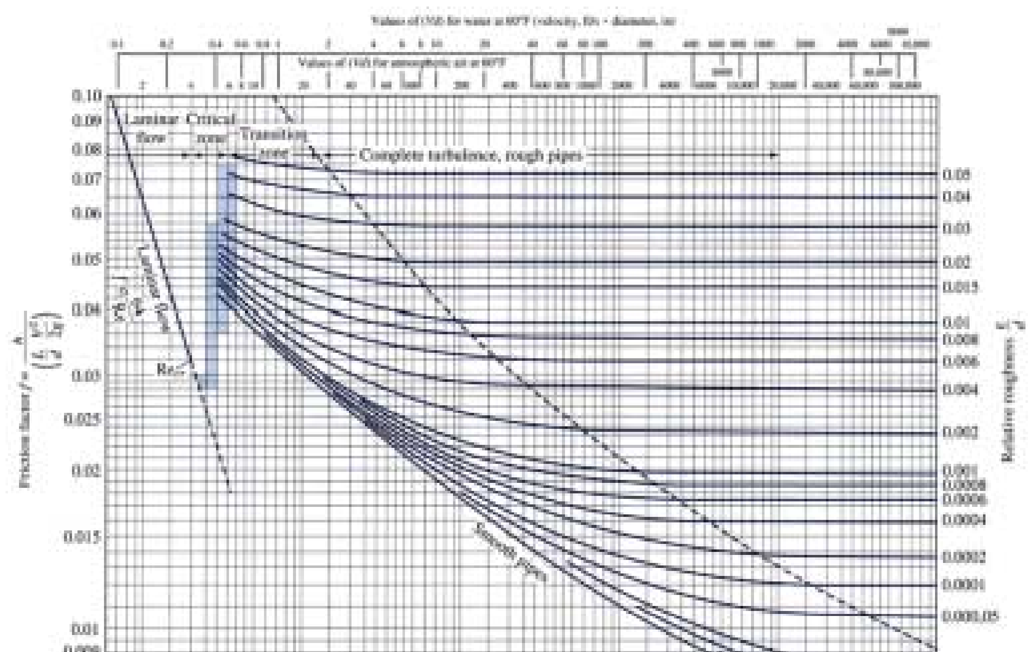
where ε = nominal roughness of pipe or duct being used. (Table 6.1, text)

(**Note**: Take care with units for ε; ε /D must be non-dimensional)

A good approximate equation for the turbulent region of the Moody chart is given by Haaland's equation:

$$f = \left\{ -1.8 \log \left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon / D}{3.7} \right)^{1.11} \right] \right\}^{-2}$$

Note again the roughness ratio ε/D must be non-dimensional in both equations. Graphically, the results for both laminar and turbulent flow pipe friction are represented by the Moody chart as shown below.



Typical roughness values are shown in the following table:

Table 6.1 Average roughness values of commercial pipe

		ϵ		
Material	**Condition**	**ft**	**mm**	**Uncertainty, %**
Steel	Sheet metal, new	0.00016	0.05	± 60
	Stainless, new	0.000007	0.002	± 50
	Commercial, new	0.00015	0.046	± 30
	Riveted	0.01	3.0	± 70
	Rusted	0.007	2.0	± 50
Iron	Cast, new	0.00085	0.26	± 50
	Wrought, new	0.00015	0.046	± 20
	Galvanized, new	0.0005	0.15	± 40
	Asphalted cast	0.0004	0.12	± 50
Brass	Drawn, new	0.000007	0.002	± 50
Plastic	Drawn tubing	0.000005	0.0015	± 60
Glass	—	Smooth	Smooth	
Concrete	Smoothed	0.00013	0.04	± 60
	Rough	0.007	2.0	± 50
Rubber	Smoothed	0.000033	0.01	± 60
Wood	Stave	0.0016	0.5	± 40

Haaland's equation is valid for turbulent flow (Re > 2300) and is easily set up on a computer, spreadsheet, etc.

Key fluid system design considerations for laminar and turbulent flow

a. Most internal flow problems of engineering significance are turbulent, not laminar. Typically, a very low flow rate is required for internal pipe flow to be laminar. If you open your kitchen faucet and the outlet flow stream is larger than a kitchen match, the flow is probably turbulent. Thus, check your work carefully if your analysis indicates laminar flow.

b. The following can be easily shown:

Laminar flow:
$$\Delta P_f \propto \left\{\mu, L, Q, D^{-4}\right\}$$
$$\dot{W}_f \propto \left\{\mu, L, Q^2, D^{-4}\right\}$$

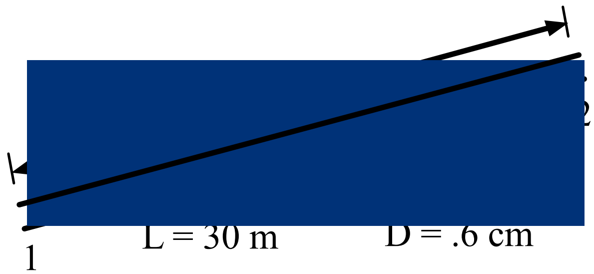
Turbulent flow:
$$\Delta P_f \propto \left\{\rho^{3/4}, \mu^{1/4}, L, Q^{1.75}, D^{-4.75}\right\}$$
$$\dot{W}_f \propto \left\{\rho^{3/4}, \mu^{1/4}, L, Q^{2.75}, D^{-4.75}\right\}$$

The subscript 'f' on each of these terms indicates that these represent the **effects due to friction only** and are not the total pressure drop or power requirements.

Thus, both pressure drop and pump power are <u>very dependent</u> on flow rate and pipe/conduit diameter. Small changes in diameter and/or flow rate can significantly change circuit pressure drop and power requirements.

Example (Laminar flow):

Water, 20°C flows through a 0.6 cm tube, 30 m long, at a flow rate of 0.34 liters/min. If the pipe discharges to the atmosphere, determine the supply pressure if the tube is inclined 10° above the horizontal in the flow direction.



Water Properties:

$\rho = 998\ kg/m^3 \quad \rho g = 9790\ N/m^3$

$\nu = 1.005\ E\text{-}6\ m^2/s$

Energy Equation (neglecting α)

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + Z_2 - Z_1 + h_f - h_p$$

which for steady-flow in a constant diameter pipe with $P_2 = 0$ gage becomes,

$$\frac{P_1}{\rho g} = Z_2 - Z_1 + h_f = L \sin 10^{\circ} + h_f$$

$$V = \frac{Q}{A} = \frac{0.34E^{-3}\ m^3/\min * 1\min/60s}{\pi (0.3/100)^2\ m^2} = 0.2\ m/s$$

$$\text{Re} = \frac{VD}{\upsilon} = \frac{0.2 * 0.006}{1.005E^{-6}} = 1197 \quad \rightarrow \quad \text{laminar flow}$$

$$\text{f} = \frac{64}{\text{Re}} = \frac{64}{1197} = 0.0535$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.0535 * \frac{30m}{0.006m}\ \frac{0.2^2}{2 * 9.807m/s^2} = 0.545\,m$$

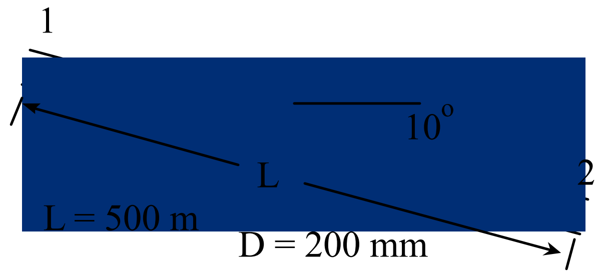
$$\frac{P_1}{\rho g} = 30 * \sin 10\text{Þ} + 0.545 = 5.21 + 0.545 = 5.75m$$

gravity head	**friction head**	**total head loss**

$P_1 = 9790\ N/m^3 * 5.75\ m = 56.34\ kN/m^3$ (kPa) ~ 8.2 psig **ans.**

Example (turbulent flow):

Oil, ρ = 900 kg/m^3, ν = 1 E-5 m^2/s, flows at 0.2 m^3/s through a 500 m length of 200 mm diameter, cast iron pipe. If the pipe slopes downward 10^o in the flow direction, compute hf, total head loss, pressure drop, and power required to overcome these losses.



The energy equation for α = 1 can be written as follows where h_t = total head loss.

$$\frac{P_1 - P_2}{\rho g} = h_t = \frac{V_2^2 - V_1^2}{2g} + Z_2 - Z_1 + h_f - h_p$$

which reduces to

$$h_t = Z_2 - Z_1 + h_f$$

$$V = \frac{Q}{A} = \frac{0.2\,m^3/s}{\pi(.1)^2\,m^2} = 6.4\ m/s$$

Table 6.1, cast iron, ε = 0.26 mm

$$\text{Re} = \frac{V\,D}{\upsilon} = \frac{6.4*.2}{1E^{-5}} = 128{,}000 \rightarrow \text{turbulent flow,} \qquad \frac{\varepsilon}{D} = \frac{0.26}{200} = 0.0013$$

Since flow is turbulent, use Haaland's equation to determine friction factor (check your work using the Moody chart).

$$f = \left\{-1.8\log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]\right\}^{-2}, \quad f = \left\{-1.8\log\left[\frac{6.9}{128{,}000} + \left(\frac{0.0013}{3.7}\right)^{1.11}\right]\right\}^{-2}$$

$$f = 0.02257 \quad h_f = f\,\frac{L}{D}\frac{V^2}{2g} = 0.02257*\frac{500m}{0.2m}\,\frac{6.4^2}{2*9.807m/s^2} = 116.6\,m$$ **ans.**

$h_t = Z_2 - Z_1 + h_f$ = - 500 sin ^{0} + 116.6 = - 86.8 + 116.6 = 29.8 m **ans.**

Note that for this problem, there is a negative gravity head loss (i.e. a head increase) and a positive frictional head loss resulting in the net head loss of 29.8 m.

$$\Delta P = \rho g h_t = 900\, kg/m^3 * 9.807\, m/s^2 * 29.8\, m = 263\, kPa$$ **ans.**

$$\dot{W} = \rho Q g h_t = Q \Delta P = 0.2\, m^3/s \cdot 263\, kN/m^2 = 52.6\, kW$$ **ans.**

Note that this is not necessarily the power required to drive a pump, as the pump efficiency will typically be less than 100%.

These problems are easily set up for solution in a spreadsheet as shown below. Make sure that the calculation for friction factor includes a test for laminar or turbulent flow (i.e. Re > or < 2300) with the analysis then using the correct equation for friction factor, f.

Always verify any computer solution with problems having a known solution.

FRICTIONAL HEAD LOSS CALCULATION

All Data are entered in std. S.I. Units e.g. (m, sec., kg), except ε ε

Ex. 6.7

	Input Data			Calculated Results		
L =	500	(m)		V =	6.37	(m/sec)
D =	0.2	(m)		Re =	127324	
ε =	0.26	(mm)		ε/D =	0.0013	
ρ =	900	(kg/m^3)		f=	0.02258	
ν =	1.00E-05	(m^2/sec)		hf =	116.6	
Q =	0.2	(m^3/sec)		sum Ki =	0.00	
D1 =	0.2	(m)		hm =	0.00	(m)
D2 =	0.2	(m)				
d KE =	0.00	(m)		ht =	29.80	(m)
d Z =	-86.8	(m)				(m)
				P1-P2 =	263.0	(kPa)

Solution Summary:

To solve basic pipe flow frictional head loss problem, use the following procedure:

1. Use known flow rate to determine Reynolds number.
2. Identify whether flow is laminar or turbulent.
3. Use correct expression to determine friction factor (with ε/D if necessary).
4. Use definition of hf to determine friction head loss.
5. Use general energy equation to determine total pressure drop.

Unknown Flow Rate and Diameter Problems

Problems involving unknown flow rate and diameter in general require iterative/ trial & error solutions due to the complex dependence of Re, friction factor, and head loss on velocity and pipe size.

Unknown Flow Rate:

For the special case of known friction loss, hf , a closed form solution can be obtained for the problem of unknown Q.

The solution proceeds as follows:

Given: Known values for D, L, h_f, ρ, and μ , calculate V or Q.

(**Note**: It is the friction head loss, hf, not the total head loss, ht, used in the solution.

Define solution parameter:

$$\varsigma = \frac{1}{2} f \, \mathrm{Re}_D^2 = \frac{g D^3 h_f}{L \upsilon^2}$$

Note that this solution does not contain velocity and the parameter ζ can be calculated from known values for D, L, h_f, ρ, and μ. The Reynolds number and subsequently the velocity can be determined from ζ and the following equations:

Turbulent: $$\text{Re}_D = -(8\varsigma)^{1/2} \log\left\{ \frac{\varepsilon / D}{3.7} + \frac{1.775}{\sqrt{\zeta}} \right\}$$

Laminar: $$\text{Re}_D = \frac{\varsigma}{32}$$

and laminar to turbulent transition can be assumed to occur approximately at $\zeta = 73{,}600$ (check Re at end of calculation to confirm).

Note that this procedure is not valid (except perhaps for initial estimates) for problems involving significant minor losses where the head loss due only to pipe friction is not known.

For these problems a trial and error solution using a computer is best. For example, using the previous spreadsheet, assume values for Q until the correct ht is obtained.

Example 6.9

Oil, with ρ = 950 kg/m^3 and ν = 2 E-5 m^2/s, flows through 100 m of a 30 cm diameter pipe. The pipe is known to have a head loss of 8 m and a roughness ratio ε/D = 0.0002. Determine the flow rate and oil velocity possible for these conditions.

Without any information to the contrary, we will neglect minor losses and KE head changes. With these assumptions, we can write:

$$\varsigma = \frac{g D^3 h_f}{L \upsilon^2} = \frac{9.807\, m/s^2 * 0.30^3\, m^3 * 8.0\, m}{100\, m * (2\, E{-}5\, m^2/s)^2} = 5.3\, E7 > \text{ 73,600; turbulent}$$

$$\text{Re}_D = -(8 * 5.3\, E7)^{1/2} \log\left\{ \frac{0.0002}{3.7} + \frac{1.775}{\sqrt{5.3\, E7}} \right\} = 72{,}600 \quad \text{checks, turbulent}$$

$$\text{Re}_D = \frac{V D}{\upsilon}, \quad V = \frac{72{,}600 * 2\, E{-}5\, m^2/s}{0.3\, m} = 4.84 \frac{m}{s}$$ **ans.**

$$Q = A\,V = \pi\,.15^2\ m^2\, 4.84\ m/s = 0.342\, m^3/s$$ **ans.**

This is the maximum flow rate and oil velocity that could be obtained through the given pipe and given conditions (hf = 8 m).

Again, this problem could have also been solved using a computer based trial and error procedure in which a value is assumed for the fluid flow rate until a flow rate is found which results in the specified head loss.

Note also that with a more general computer based procedure, the problem being solved can include the effects of minor losses, KE, and PE changes with no additional difficulty.

Unknown Pipe Diameter:

A similar difficulty arises for problems involving unknown pipe difficulty, except a closed form, analytical solution is not generally available. Again, a trial and error solution is appropriate for use to obtain the solution and the problem can again include losses due to KE, PE, and piping components with no additional difficulty. This procedure is shown in the following example using the previous spreadsheet based solution.

Ex. 6.11 A fluid with the indicated properties is known to flow through a 100 m long pipe with ε = .06 mm and a flow rate of 0.342 m^3/s and a frictional head loss, hf = 8 m. What diameter pipe is required to provide these conditions?

Again, assume values of D until the value of hf = 8 m is obtained in the solution.

FRICTIONAL HEAD LOSS CALCULATION

All Data are entered in std. S.I. Units e.g. (m, sec., kg), except ε a

Ex. 6.11

	Input Data			Calculated Results		
L =	100	(m)		V =	4.87	(m/sec)
D =	0.30	(m)		Re =	72817	
ε =	0.06	(mm)		ε/D =	0.000201	
ρ =	950	(kg/m^3)		f=	0.0198	
ν =	2.00E-05	(m^2/sec)		hf =	8.0	
Q =	0.342	(m^3/sec)		sum Ki =	0.00	
D1 =	0.30	(m)		hm =	0.00	(m)
D2 =	0.30	(m)				
d KE =	0.00	(m)		ht =	8.0	(m)
d Z =	0.0	(m)				(m)
				P1-P2 =	74.7	(kPa)

Non-Circular Ducts:

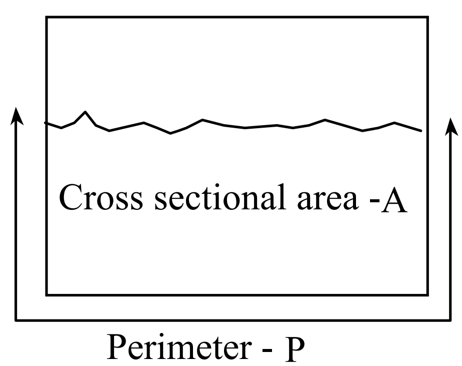
For flow in non-circular ducts or ducts for which the flow does not fill the entire cross-section, we can define the hydraulic diameter D_h as

$$D_h = \frac{4A}{P}$$

where

A = cross-sectional area of actual flow,

P = wetted perimeter, i.e. the perimeter on which viscous shear acts



With this definition, **<u>all previous equations</u>** for the Reynolds number, Re, friction factor, f, and head loss, h_f, are valid as previously defined and can be used on both circular and non-circular flow cross-sections.

Minor Losses

In addition to frictional losses for a length L of pipe, we must also consider losses due to various fittings (valves, unions, elbows, tees, etc.). These losses are typically expressed as

$$h_m = K_i \frac{V^2}{2g}$$

where

h_m = the equivalent head loss across the fitting or flow component

V = average flow velocity for the pipe size of the fitting

K_i = the minor loss coefficient for given flow component; valve, union, etc. See Sec. 6.7, Table 6.5, 6.6, Fig. 6.19, 6.20, 6.21, 6.22, etc.

i.e. Loss Coefficient $K = \frac{h_m}{V^2/(2g)} = \frac{\Delta P}{1/2 \rho V^2}$

Table 6.5 shows minor loss K values for several common types of valves, fully open, and for elbows and tees.

Table 6.5 Minor loss coefficient for common valves and piping components

	Nominal diameter, in								
	Screwed				Flanged				
	½	1	2	4	1	2	4	8	20
Valves (fully open):									
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8	5.5
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07	0.03
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0	2.0
Elbows:									
45° regular	0.39	0.32	0.30	0.29					
45° long radius					0.21	0.20	0.19	0.16	0.14
90° regular	2.0	1.5	0.95	0.64	0.50	0.39	0.30	0.26	0.21
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15	0.10
180° regular	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25	0.20
180° long radius					0.40	0.30	0.21	0.15	0.10
Tees:									
Line flow	0.90	0.90	0.90	0.90	0.24	0.19	0.14	0.10	0.07
Branch flow	2.4	1.8	1.4	1.1	1.0	0.80	0.64	0.58	0.41

Figure 6.18 shows minor loss K values for several types of common valves.

Note that the K valves shown here are for the indicated fractional opening. Also, the fully open values may not be consistent with values indicated in Table 6.5 for fully open valves or for the valve of a particular manufacturer. In general, use specific manufacturer's data when available.

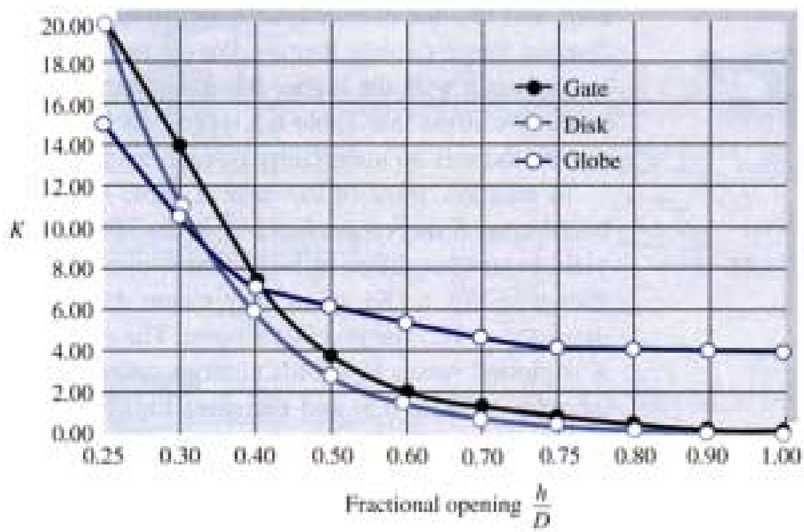


Fig. 6.18 Average loss coefficients for partially open valves

Note that exit losses are K ≅ 1 for all <u>submerged</u> exits, e.g. fluid discharged into a tank at a level below the fluid surface.

Also, for an open pipe discharge to the atmosphere, there is no loss coefficient when the energy equation is written only to the end of the pipe.

In general, do not take point 1 for an analysis to be in the plane of an inlet having an inlet loss. You do not know what fraction

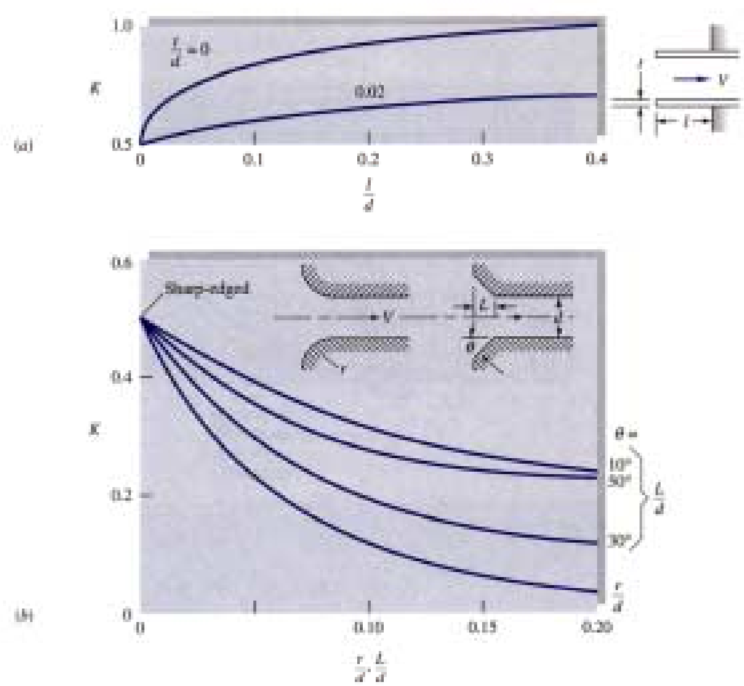


Fig. 6.21 Entrance and exit loss coefficients

of the inlet loss to consider. (a) reentrant inlets; (b) rounded and beveled inlets

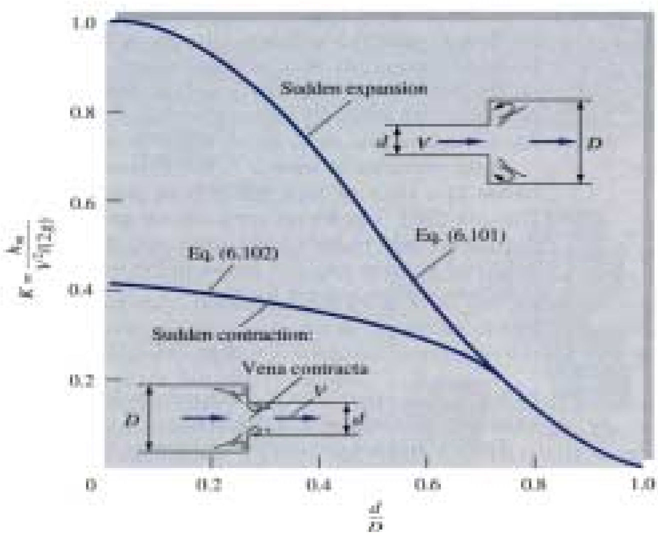


Fig. 6.22 Sudden contraction and expansion losses.

Note that the losses shown in Fig. 6.22 do not represent losses associated with pipe unions or reducers. These must be found in other sources in the literature.

Also note that the loss coefficient is always based on the velocity in the smaller diameter (d) of the pipe, irrespective of the direction of flow.

Assume that this is also true for reducers and similar area change fittings.

These and other sources of data now provide the ability to determine frictional losses for both the pipe and other piping/duct flow components.

<u>The total frictional loss now becomes</u>

$$h_f = f\frac{L}{D}\frac{V^2}{2g} + \sum K_i \frac{V^2}{2g}$$

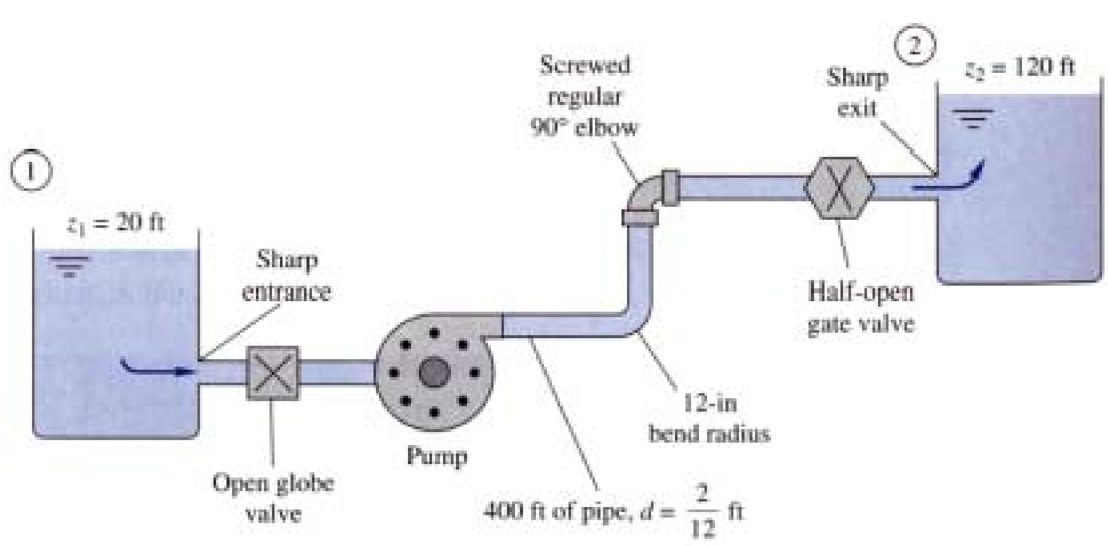
or

$$h_f = \left\{ f\frac{L}{D} + \sum K_i \right\} \frac{V^2}{2g}$$

These equations would be appropriate for a single pipe size (with average velocity V). For multiple pipe/duct sizes, this term must be repeated for each pipe size.

Key Point: The energy equation must still be used to determine <u>the total head loss and pressure drop</u> from all possible contributions.

Example 6.16



Water, ρ = 1.94 slugs/ft^3 and ν = 1.1 E-5 ft^2/s, is pumped between two reservoirs at 0.2 ft^3/s through 400 ft of 2–in diameter pipe with ε/D = 0.001 having the indicated minor losses. Compute the pump horsepower (motor size) required.

Writing the energy equation between points 1 and 2 (the free surfaces of the two reservoirs), we obtain

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + Z_2 - Z_1 + h_f - h_p$$

For this problem, the pressure ($P_1 = P_2$) and velocity ($V_1 = V_2 = 0$) head terms are zero and the equation reduces to

$$h_p = Z_2 - Z_1 + h_f = Z_2 - Z_1 + \left\{ f\frac{L}{D} + \sum K_i \right\} \frac{V^2}{2g}$$

For a flow rate Q = 0.2 ft^3/s we obtain

$$V = \frac{Q}{A} = \frac{0.2\ ft^3/s}{\pi(1/12)^2\ ft^2} = 9.17\ ft/s$$

With ε/D = 0.001 and $\mathrm{Re} = \dfrac{VD}{\nu} = \dfrac{9.17\ ft/s(2/12)\ ft}{1.1E{-}5\ ft^2/s} = 139{,}000$

The flow is turbulent and Haaland's equation can be used to determine the friction factor:

$$f = \left\{-1.8\log\left[\frac{6.9}{139{,}000} + \left(\frac{.001}{3.7}\right)^{1.11}\right]\right\}^{-2} = 0.0214$$

The minor losses for the problem are summarized in the following table:

Note: The loss for a pipe bend is not the same as for an elbow fitting.

If there were no tank at the pipe discharge and point 2 were at the pipe exit, there would be no exit loss coefficient. However, there would be an exit K.E. term.

Loss element	Ki
Sharp entrance (Fig. 6.21)	0.5
Open globe valve (Table 6.5)	6.9
12 " bend, R/D = 12/6 = 2 (Fig. 6.19)	0.15
Threaded, 90Þ, reg. elbow, (Table 6.5)	0.95
Gate valve, 1/2 closed (Fig. 6.18)	2.7
Submerged exit (Fig. 6.20)	1
Σ Ki =	12.2

Substituting in the energy equation we obtain

$$h_p = (120-20) + \left\{0.0214\frac{400}{2/12}\frac{9.17^2}{64.4}\right\} + \left\{12.2\frac{9.17^2}{64.4}\right\}$$

$$h_p = 100 + 67.1 + 15.9 = 183\,\mathrm{ft}$$

Note the distribution of the total loss between static, pipe friction, and minor losses.

The power required to be delivered to the fluid is given by

$$P_f = \rho Q g h_p = 1.94\frac{slug}{ft^3}\,32.2\frac{ft}{s^2}\,0.2\frac{ft^3}{s}\,183\ ft = 2286\ ft\ lbf$$

$$P_f = \frac{2286 ft lbf}{550 ft lbf/s/hp} = 4.2 hp$$

If the pump has an efficiency of 70%, the power requirements would be specified by

$$\dot{W}_p = \frac{4.2\, hp}{0.70} = 6\ hp$$

Solution Summary:

To solve a basic pipe flow pressure drop problem, use the following procedure:

1. Use known flow rate to determine Reynolds number.
2. Identify whether flow is laminar or turbulent.
3. Use appropriate expression to find friction factor (with ε/D if necessary).
4. Use definition of h_f to determine friction head loss.
5. Tabulate and sum minor loss coefficients for piping components.

6a. Use general energy equation to determine total pressure drop, or

6b. Determine pump head requirements as appropriate.

7. Determine pump power and motor size if required.

Multiple-Pipe Systems

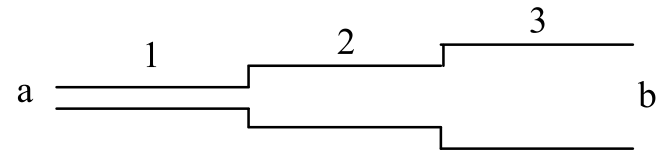
Basic concepts of pipe system analysis apply also to multiple pipe systems. However, the solution procedure is more involved and can be iterative.

Consider the following:

a. Multiple pipes in series
b. Multiple pipes in parallel

<u>Series Pipe System</u>:

The indicated pipe system has a steady flow rate Q through three pipes with diameters D_1, D_2, & D_3.



Two important rules apply to this problem.

1. The flow rate is the same through each pipe section. For incompressible flow, this is expressed as

$$Q_1 = Q_2 = Q_3 = Q \quad \text{or} \quad D_1^2 V_1 = D_2^2 V_2 = D_3^2 V_3$$

2. The total frictional head loss is the sum of the head losses through the various sections.

$$h_{f,a-b} = h_{f,1} + h_{f,2} + h_{f,3}$$

$$h_{f,a-b} = \left(f\frac{L}{D} + \sum K_i \right)_{D_1} \frac{V_1^2}{2g} + \left(f\frac{L}{D} + \sum K_i \right)_{D_2} \frac{V_2^2}{2g} + \left(f\frac{L}{D} + \sum K_i \right)_{D_3} \frac{V_3^2}{2g}$$

Note: Be careful how you evaluate the transitions from one section to the next. In general, loss coefficients for transition sections are based on the velocity of the smaller section.

Example: Given a pipe system as shown in the previous figure. The total pressure drop is $P_a - P_b = 150$ kPa and the elevation change is $Z_b - Z_a = -5$ m. Given the following data, determine the flow rate of water through the section.

Pipe	L (m)	D (cm)	e (mm)	e/D
1	100	8	0.24	0.003
2	150	6	0.12	0.02
3	80	4	0.2	0.005

The energy equation is written as

$$\frac{P_a - P_b}{\rho g} = \frac{V_b^2 - V_a^2}{2g} + Z_b - Z_a + h_f - h_p$$

where hf is given by the sum of the total frictional losses for the three pipe sections.

With no pump; hp is 0, Z_b - Z_a =

- 5 m, ht = 15.3 m for ΔP = 150 kPa , and hf(net) = 20.08 m (including KE effects).

$$h_t = \frac{P_a - P_b}{\rho g} = \frac{150{,}000\, N/m^2}{9790\, N/m^2} = 15.3\, m$$

Since the flow rate Q and velocity are the only remaining variables, the solution is easily obtained from a spreadsheet by assuming Q until ΔP = 150 kPa.

Fluid	
ρ(kg/m^3)=	1000
ν(m^2/s)=	1.02E-06
inlet & exit	
dZ (m)=	-5
Da(m)=	0.08
Db(m)=	0.04
Assume	
Q (m^3/s)=	0.00283
Va(m/s)=	0.56
Vb(m/s)=	2.25
dKE(m)=	0.24
hf(net)=	20.08
	Actual
Pa-Pb (kPa)	150

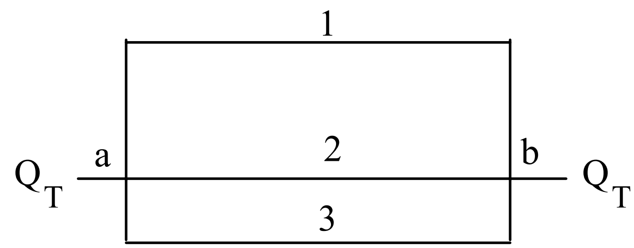
	1	2	3
L(m)	100	150	80
D(m)=	0.08	0.06	0.04
ε(mm)=	0.24	0.12	0.20
ε/D=	0.003	0.002	0.005
V(m/s)=	0.56	1.00	2.25
Re=	44082.8	58777.1	88165.6
f=	0.02872	0.02591	0.03139
hf=	0.58	3.30	16.18
? Ki	0	0	0
hm=	0	0	0
hf(calc)=	0.58	3.30	16.18
Calculated			
150.00	Q (m^3/hr)=	10.17	

Thus it is seen that a flow rate of 10.17 m^3/hr produces the indicated head loss through each section and a net total ΔP = 150 kPa.

A solution can also be obtained by writing all terms explicitly in terms of a single velocity, however, the algebra is quite complex (unless the flow is laminar), and an iterative solution is still required. All equations used to obtain the solution are the same as those presented in previous sections.

<u>Parallel Pipe Systems</u>

A flow rate Q_T enters the indicated parallel pipe system. The total flow splits and flows through 3 parallel pipe sections, each with different diameters and lengths.



Two basic rules apply to parallel pipe systems:

1. The total flow entering the parallel section is equal to the sum of the flow rates through the individual sections
2. The total pressure drop across the parallel section is equal to the pressure drop across each individual parallel segment.

Note that if a common junction is used for the start and end of the parallel section, the velocity and elevation change is also the same for each section. Thus, the flow rate through each section must be such that the frictional loss is the same for each and the sum of the flow rates equals the total flow.

For the special case of no kinetic or potential energy change across the sections, we obtain:

$$h_t = (h_f + h_m)_1 = (h_f + h_m)_2 = (h_f + h_m)_3$$

and

$$Q_T = Q_1 + Q_2 + Q_3$$

Again, the equation used for both the pipe friction and minor losses is the same as previously presented. The flow and pipe dimensions used for the previous example are now applied to the parallel circuit shown above.
Example: A parallel pipe section consists of three parallel pipe segments with the lengths and diameters shown below. The total pressure drop is 150 kPa and the parallel section has an elevation drop of 5 m. Neglecting minor losses and kinetic energy changes, determine the flow rate of water through each pipe section.

The solution is iterative and is again presented in a spreadsheet. The net friction head loss of 20.3 m now occurs across each of the three parallel sections.

Fluid	
ρ(kg/m^3) =	1000
ν(m^2/s) =	1.02E-06
inlet & exit	
dZ (m) =	-5
Da(m) =	0.08
Db(m) =	0.04
Q (m^3/hr)=	99.91
Assume	
Q1 (m^3/s)=	62.54
Q2(m^3/s)=	25.95
Pa - Pb (kPa)	150.13
ht(m) =	15.31

	1	2	3
L(m)	100	150	80
D(m) =	0.08	0.06	0.04
ε(mm) =	0.24	0.12	0.20
ε/D=	0.003	0.002	0.005
Q(m^3/hr) =	62.54	25.95	11.41
V(m/s)=	3.46	2.55	2.52
Re =	271083.5	149977.8	98919.7
f=	0.02666	0.02450	0.03129
hf =	20.30	20.30	20.30
ΣKi =	0	0	0
hm=	0.00	0.00	0.00
hf,net(m) =	20.30	20.30	20.30
hf +ΔZ=	15.30	15.30	15.30
Q(m^3/hr) =	62.54	25.95	11.41

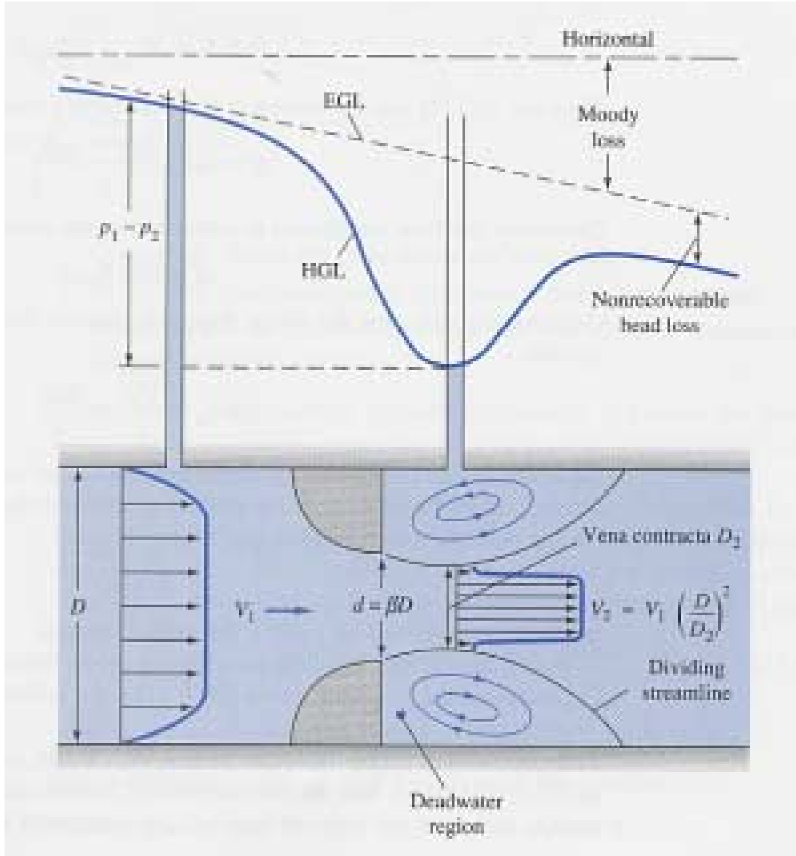
Total Flow, Qt(m^3/hr) = 99.91

The strong effect of diameter can be seen with the smallest diameter having the lowest flow rate, even though it also has the shortest length of pipe.

Again the total flow is distributed between the three parallel sections such that the head loss across each section is the same, in this case 20.3 m.

Bernoulli Obstruction Theory

An obstruction meter is a device that is used to measure flow rate using the pressure drop caused by an obstruction in the flow.



Given a basic obstruction meter as shown in the accompanying figure, write Bernoulli's equation between points 1 & 2.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 =$$

$$\frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

Assume:

1. Steady - flow
2. incompressible flow
3. no friction losses
4. uniform velocity profiles at 1 & 2
5. $Z_1 = Z_2$

For these conditions, Bernoulli's Equation and conservation of mass become:

$$\frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2} = \frac{V_2^2}{2}\left\{1 - \frac{V_1^2}{V_2^2}\right\} \qquad \frac{V_1}{V_2} = \frac{A_2}{A_1}$$

Substituting and solving for V_2, we obtain:

$$V_2 = \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho_f}}$$

The volumetric flow rate is now given by

$$Q = A_2 V_2 = \frac{A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho_f}}$$

The above equation predicts the flow for ideal (frictionless) flow conditions.

Define: C_d = discharge or flow coefficient

$$C_d = \frac{actual\ flow\ rate}{ideal\ flow\ rate}$$

Can now write the equation for the actual flow rate as:

$$Q_a = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho_f}}$$

Define: E = Velocity of approach factor

$$E = \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} = \frac{1}{\sqrt{1-\beta^4}} \quad \text{where} \quad \beta = \frac{D_2}{D_1}$$

β = ratio of the throat-minimum flow diameter to the main flow diameter

Also: define Y = expansion factor = factor which accounts for compressibility effects of the flow (usually small). Y = 1 no compressibility effects, Y < 1, some compressibility effects.

Therefore, the final equation now becomes:

$$Q_a = Y C_d \, E \, A_2 \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho_f}}$$

or:

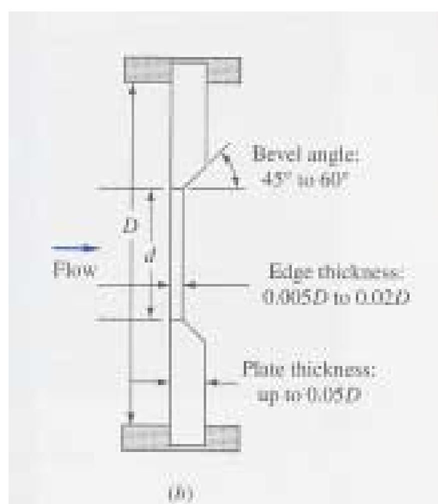
$$\dot{m} = \rho \, Q_a = Y \, C_d \, E \, A_2 \sqrt{2 \, \rho_f \left(P_1 - P_2\right)}$$

For gases, the Expansion Factor, Y, is given by the expression:

$$Y = 1 - \left(0.41 + 0.35\,\beta^4\right)\frac{P_1 - P_2}{k\,P_1}$$

Orifice Flow Rate Analysis

We will now outline the procedure required to use a given thin plate orifice to determine the flow rate based on orifice geometry and measured pressure drop.



Data: D, β, d, P_{1g}, ΔP, T_1, P_{bar}

1. Calculate orifice constants: γ, A_t, E

γ (fluid), $A_t = \pi d^2/4$, $E = \frac{1}{\sqrt{1-\beta^4}}$

2. If fluid is a gas, calculate the Expansion factor, Y. Otherwise, set Y = 1.

$$X = \frac{\Delta P}{P_1} \qquad Y = 1 - \left(.41 + .35\beta^4\right)\frac{X}{\gamma}$$

3. Calculate the Discharge Coefficient

$$C_d = f(\beta) + 91.71\beta^{2.5}\,\mathrm{Re}_D^{-.75} + \frac{0.09\beta^4}{1-\beta^4}F_1 - 0.0337\beta^3 F_2$$

where

$$f(\beta) = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8$$

The factors F_1 and F_2 vary with pressure tap position.

corner taps:	$F_1 = 0$	$F_2 = 0$		
D & 1/2 D taps:	$F_1 = 0.4333$	$F_2 = 0.47$		
Flange taps:	$F_2 = \frac{1}{D(in)}$	D > 2.3 in	$F_1 = \frac{1}{D(in)}$
		$2.0 \le D \le 2.3$ in	$F_1 = 0.4333$

4. Calculate Mass Flow Rate

$$\dot{m} = \rho Q_a = Y C_d E A_2 \sqrt{2\rho_f (P_1 - P_2)}$$

Where for ideal gases: $\rho = P_1/RT_1$

5. Calculate the actual volumetric flow rate

$$Q_a = \frac{\dot{m}}{\rho}$$