XI. TURBOMACHINERY

This chapter considers the theory and performance characteristics of the mechanical devices associated with the fluid circulation.

General Classification:

Turbomachine - A device which adds or extracts energy from a fluid.

Adds energy:	Pump
Extracts energy:	Turbine

In this context, a pump is a generic classification that includes any device that adds energy to a fluid, e.g. fans, blowers, compressors.

We can classify pumps by operating concept:

1. Positive displacement
2. Dynamic (momentum change)

General Performance Characteristics

<u>Positive Displacement Pumps</u>

1. Delivers pulsating or periodic flow (cavity opens, fluid enters, cavity closes, decreasing volume forces fluid out exit opening).
2. Not sensitive to wide viscosity changes.
3. Delivers a moderate flow rate.
4. Produces a high pressure rise.
5. Small range of flow rate operation (fixed pump speed).

<u>Dynamic Pumps</u>

1. Typically higher flow rates than PDs.
2. Comparatively steady discharge.
3. Moderate to low pressure rise.
4. Large range of flow rate operation.
5. Very sensitive to fluid viscosity.

Typical Performance Curves (at fixed impeller speed)

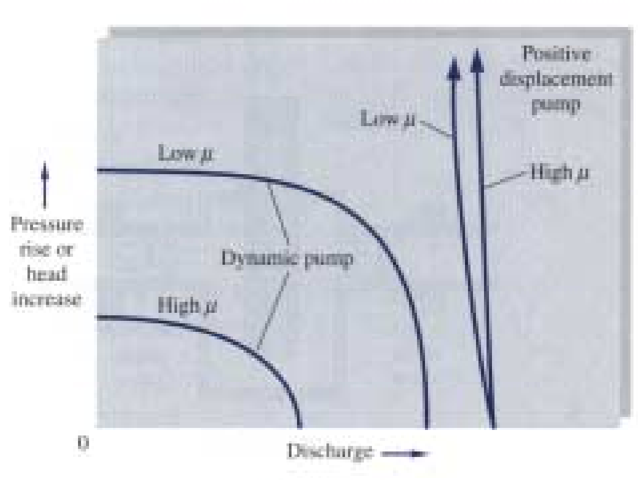


Fig. 11.2 Performance curves for dynamic and positive displacement pumps

Centrifugal Pumps

This is the most common turbomachine used in industry. It includes the general categories of (a) liquid pumps, (b) fans, (c) blowers, etc.

They are momentum change devices and thus fall within the dynamic classification.

Typical schematic shown as

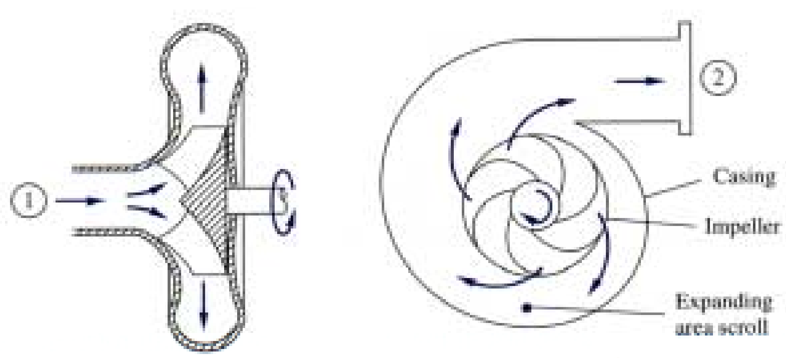


Fig. 11.3 Cutaway schematic of a typical centrifugal pump

Writing the energy equation across the device and solving for $h_p - h_f$,we have

$$H = h_p - h_f = \frac{P_2 - P_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + Z_2 - Z_1$$

where H is the net useful head delivered to the fluid, the head that results in pressure, velocity, and static elevation change.

Since for most pumps (not all), $V_1 = V_2$ and ΔZ is small, we can write

$$H \cong \frac{\Delta P}{\rho g}$$

Since friction losses have already been subtracted, this is the ideal head delivered to the fluid. Note that velocity head has been neglected and can be significant at large flow rates where pressure head is small.

The ideal power to the fluid is given by $P_w = \rho\ Q\ g\ H$

The pump efficiency is given by

$$\eta = \frac{P_w}{BHP} = \frac{\rho Q g H}{BHP} = \frac{\rho Q g H}{\omega T}$$

where BHP = shaft power necessary to drive the pump

ω = angular speed of shaft

T = torque delivered to pump shaft

Note that from the efficiency equation, pump efficiency is zero at zero flow rate Q and at zero pump head, H.

Basic Pump Theory

Development of basic pump theory begins with application of the integral conservation equation for moment-of-momentum previously presented in Ch. III.

Applying this equation to a centrifugal pump with one inlet, one exit, and uniform properties at each inlet and exit, we obtain

$$\bar{T} = \dot{m}_e \, \bar{r} \, x \, \bar{V}_e - \dot{m}_i \, \bar{r} \, x \, \bar{V}_i$$

where $\bar{T}$ is the shaft torque needed to drive the pump

$\bar{V}_i$, $\bar{V}_e$ are the absolute velocities at the inlet and exit of the pump

Thus, the applied torque is equal to the change of angular momentum across the device.

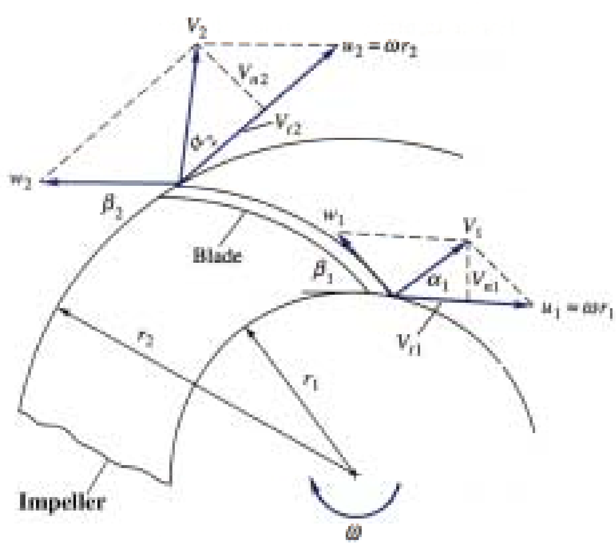
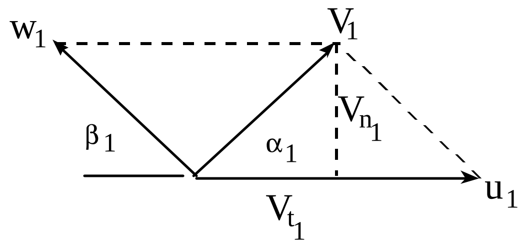


Fig. 11.4 Inlet and exit velocity diagrams for an idealized pump impeller

Since the velocity diagram is key to the analysis of the device, we will discuss the elements of the diagram in detail.

1. At the inner radius r_1 we have two velocity components:



a. the circumferential velocity due to the impeller rotation

$u_1 = r_1 \omega$ blade tip speed at inner radius

b. relative flow velocity tangent to the blade

w_1 tangent to the blade angle β_1

These combine to yield the absolute inlet velocity V_1 at angle α_1

The absolute velocity can be resolved into two absolute velocity components:

1. Normal (radial) component:

$$V_{n_1} = V_1 \sin\alpha_1 = w_1 \sin\beta_1$$

Note that for ideal pump design,

$$V_{n_1} = V_1 \quad and \quad \alpha_1 = 90^o$$

2. Absolute tangential velocity:

$$V_{t_1} = V_1 \cos\alpha_1 = u_1 - w_1 \cos\beta_1$$

again, ideally $V_{t_1} = 0$

It is also important to note that V_{n_1} is used to determine the inlet flow rate, i.e.,

$$Q = A_1 V_{n_1} = 2\pi r_1 b_1 V_{n_1}$$

where b_1 is the inlet blade width.

Likewise for the outer radius r_2 we have the following:

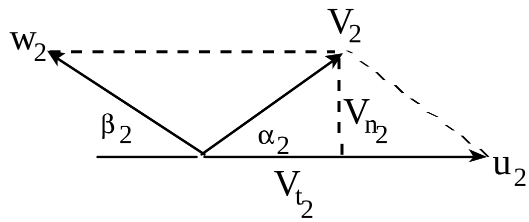
a. the circumferential velocity due to the impeller rotation

$u_2 = r_2\,\omega$ blade tip speed at outer radius

b. relative flow velocity tangent to the blade

w_2 tangent to the blade angle β_2

These again combine to yield the absolute outlet velocity V_2 at angle α_2



The exit absolute velocity can also be resolved into two absolute velocity components:

1. Normal (radial) component:

$$V_{n2} = V_2 \sin\alpha_2 = w_2 \sin\beta_2 = \frac{Q}{2\pi\, r_2\, b_2}$$

Note that Q is the same as for the inlet flow rate

2. Absolute tangential velocity:

$$V_{t2} = V_2 \cos\alpha_2 = u_2 - w_2 \cos\beta_2$$

$$V_{t2} = u_2 - \frac{V_{n2}}{\tan\beta_2} = u_2 - \frac{Q}{2\pi\, r_2\, b_2 \tan\beta_2}$$

where $Q = A_1 V_{n1} = 2\pi\, r_1\, b_1\, V_{n1} = A_2 V_{n2} = 2\pi\, r_2\, b_2\, V_{n2}$

Again, each of the above expressions follows easily from the velocity diagram, and the student should draw and use the diagram with each pump theory problem.

We can now apply the moment - of – momentum equation.

$$\overline{T} = \rho Q \{ r_2 * V_{t_2} - r_1 * V_{t_1} \}$$ (again V_{t1} is zero for the ideal design)

For a sign convention, we have assumed that V_{t1} and V_{t2} are positive in the direction of impeller rotation.

The " ideal" power supplied to the fluid is given by

$$P_w = \omega \overline{T} = \rho Q \{ \omega r_2 \; V_{t_2} - \omega r_1 \; V_{t_1} \}$$

or

$$P_w = \omega \overline{T} = \rho Q \{ u_2 \; V_{t_2} - u_1 \; V_{t_1} \} = \rho Q g H$$

Since these are ideal values, the shaft power required to drive a non-ideal pump is given by

$$BHP = \frac{P_w}{\eta_p}$$

The head delivered to the fluid is

$$H = \frac{\rho Q \{ u_2 \; V_{t_2} - u_1 \; V_{t_1} \}}{\rho Q g} = \frac{\{ u_2 \; V_{t_2} - u_1 \; V_{t_1} \}}{g}$$

For the special case of purely radial inlet flow

$$H^* = \frac{u_2 \; V_{t_2}}{g}$$

From the exit velocity diagram, substituting for V_{t2} we can show that

$$H = \frac{u_2^2}{g} - \frac{\omega Q}{2\pi b_2 g \tan\beta_2} \qquad \text{has the form} \qquad C_1 - C_2 Q$$

where: $C_1 = \frac{u_2^2}{g}$ C_1=shutoff head, the head produced at zero flow, Q = 0

Example 11.1:

A centrifugal water pump operates at the following conditions:

speed = 1440 rpm, r_1 = 4 in, r_2 = 7 in, $\beta_1 = 30^o$, $\beta_2 = 20^o$, $b_1 = b_2$ = 1.75 in

Assuming the inlet flow enters normal to the impeller (zero absolute tangential velocity):

find: (a) Q, (b) T, (c) W_p, (d) h_p, (e) ΔP

$$\omega = 1440 \frac{\text{rev}}{\text{min}} \frac{2\pi}{60} = 150.8 \frac{\text{rad}}{\text{s}}$$

Calculate blade tip velocities:

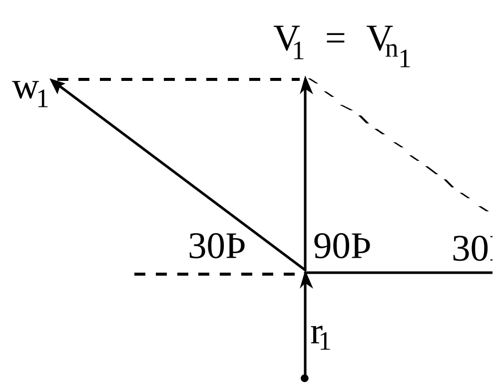
$$u_1 = r_1 \omega = \frac{4}{12} \text{ft} \, 150.8 \frac{\text{rad}}{\text{s}} = 50.3 \frac{\text{ft}}{\text{s}} \qquad u_2 = r_2 \omega = \frac{7}{12} \text{ft} \, 150.8 \frac{\text{rad}}{\text{s}} = 88 \frac{\text{ft}}{\text{s}}$$

Since the design is ideal, at the inlet

$\alpha_1 = 90^o$, $V_{t1} = 0$

$V_{n1} = U_1 \tan 30^0 = 50.3 \tan 30^o = 29.04$ ft/s

$$Q = 2\pi r_1 b_1 V_{n1}$$



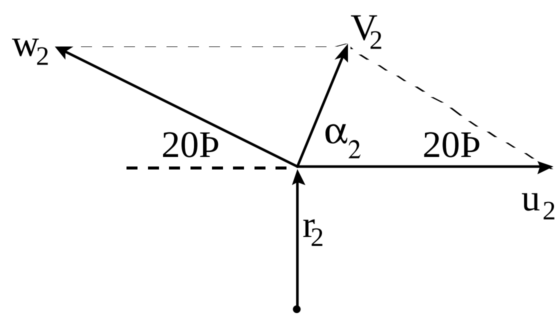
$$Q = 2\pi \frac{4}{12}\,\text{ft}\,1.75\,\text{ft}\,29.04\frac{\text{ft}}{\text{s}} = 8.87\frac{\text{ft}^3}{\text{s}}$$

$$Q = 8.87\frac{\text{ft}^3}{\text{s}}\,60\frac{s}{\text{min}}\,7.48\frac{\text{gal}}{\text{ft}^3} = 3981\frac{\text{gal}}{\text{min}}$$

This is the flow rate for ideal design or $Vt_1 = 0$ and $a_1 = 90^\circ$.

Repeat for the outlet:

$$V_{n2} = \frac{Q}{2\pi\, r_2\, b_2} = \frac{8.87\frac{\text{ft}^3}{\text{s}}}{2\pi\frac{7}{12}\,\text{ft}\,\frac{1.75}{12}\,\text{ft}}$$



$$V_{n2} = 16.6\frac{\text{ft}}{\text{s}}$$

$$w_2 = \frac{V_{n_2}}{\sin 20^o} = \frac{16.6\,\text{ft/s}}{\sin 20^o} = 48.54\frac{\text{ft}}{\text{s}}$$

$$V_{t_2} = u_2 - w_2\cos\beta_2 = 88 \quad - 48.54\cos 20^o = 42.4\frac{\text{ft}}{\text{s}}$$

We are now able to determine the pump performance parameters. Since for the centrifugal pump, the moment arm r_1 at the inlet is zero, the momentum equation becomes

$$T = \rho Q\left\{r_2 * V_{t_2}\right\} = 1.938\frac{\text{slug}}{\text{ft}^3}\,8.87\frac{\text{ft}^3}{\text{s}}\,\frac{7}{12}\,\text{ft}\,42.4\frac{\text{ft}}{\text{s}} = 425.1\,\text{ft}-\text{lbf}$$

This is the ideal torque delivered to the fluid.

Ideal power delivered to the fluid:

$$P = \omega T = 150.8\frac{\text{rad}}{\text{s}}\,425.1\,\text{ft}-\text{lbf} = 64{,}103\frac{\text{ft}-\text{lbf}}{\text{s}} = 116.5\,\text{hp}$$

Note that for a real (non-ideal) pump the input power (motor size) required would be greater proportional to the efficiency of the pump.

Head produced by the pump (ideal):

$$H = \frac{P}{\rho g Q} = \frac{64{,}103\,\text{ft} - \text{lbf/s}}{62.4\,\frac{\text{lbf}}{\text{ft}^3}\,8.87\,\frac{\text{ft}^3}{\text{s}}} = 115.9\,\text{ft}$$

Pressure increase produced by the pump:

$$\Delta P = \rho g H = 62.4\,\frac{\text{ft}^3}{\text{s}}\,115.9\,\text{ft} = 7226\,\text{psf} = 50.2\,\text{psi}$$