## LEVEL ONE

## THIS IS HOW IT ALL STARTS

| LEVEL ONE | LEVEL TWO | LEVEL THREE | LEVEL FOUR |
| :--- | :--- | :--- | :--- |
| This Is How It | Tools for <br> Evaluating <br> All Starts | Making Deci- <br> sions on Real- <br> Alternatives | Rorld Projects <br> Rounding Out |
| the Study |  |  |  |

The foundations of engineering economy are introduced in these four chapters. When you have completed level one, you will be able to understand and work problems that account for the time value of money, cash flows occurring at different times with different amounts, and equivalence at different interest rates. The techniques you master here form the basis of how an engineer in any discipline can take economic value into account in virtually any project environment.

The eight factors commonly used in all engineering economy computations are introduced and applied in this level. Combinations of these factors assist in moving monetary values forward and backward through time and at different interest rates. Also, after these four chapters, you should be comfortable with using many of the Excel spreadsheet functions to solve problems.

## Foundations of Engineering Economy

The need for engineering economy is primarily motivated by the work that engineers do in performing analysis, synthesizing, and coming to a conclusion as they work on projects of all sizes. In other words, engineering economy is at the heart of making decisions. These decisions involve the fundamental elements of cash flows of money, time, and interest rates. This chapter introduces the basic concepts and terminology necessary for an engineer to combine these three essential elements in organized, mathematically correct ways to solve problems that will lead to better decisions. Many of the terms common to economic decision making are introduced here and used in later chapters of the text. Icons in the margins serve as back and forward cross-references to more fundamental and additional material throughout the book.

The case study included after the end-of-chapter problems focuses on the development of engineering economy alternatives.

## LEARNING OBJECTIVES

Purpose: Understand the fundamental concepts of engineering economy.


### 1.1 WHY ENGINEERING ECONOMY IS IMPORTANT TO ENGINEERS (and other professionals)

Decisions made by engineers, managers, corporation presidents, and individuals are commonly the result of choosing one alternative over another. Decisions often reflect a person's educated choice of how to best invest funds, also called capital. The amount of capital is usually restricted, just as the cash available to an individual is usually limited. The decision of how to invest capital will invariably change the future, hopefully for the better; that is, it will be value adding. Engineers play a major role in capital investment decisions based on their analysis, synthesis, and design efforts. The factors considered in making the decision are a combination of economic and noneconomic factors. Additional factors may be intangible, such as convenience, goodwill, friendship, and others.

> Fundamentally, engineering economy involves formulating, estimating, and evaluating the economic outcomes when alternatives to accomplish a defined purpose are available. Another way to define engineering economy is as a collection of mathematical techniques that simplify economic comparison.

Knowing how to correctly apply these techniques is especially important to engineers, since virtually any project will affect costs and/or revenues.

Some of the typical questions that can be addressed using the material in this book are posed below.

## For Engineering Activities

- Should a new bonding technique be incorporated into the manufacture of automobile brake pads?
- If a computer-vision system replaces the human inspector in performing quality tests on an automobile welding line, will operating costs decrease over a time horizon of 5 years?
- Is it an economically wise decision to upgrade the composite material production center of an airplane factory in order to reduce costs by $20 \%$ ?
- Should a highway bypass be constructed around a city of 25,000 people, or should the current roadway through the city be expanded?
- Will we make the required rate of return if we install the newly offered technology onto our medical laser manufacturing line?


## For Public Sector Projects and Government Agencies

- How much new tax revenue does the city need to generate to pay for an upgrade to the electric distribution system?
- Do the benefits outweigh the costs of a bridge over the intracoastal waterway at this point?
- Is it cost-effective for the state to cost-share with a contractor to construct a new toll road?
- Should the state university contract with a local community college to teach foundation-level undergraduate courses or have university faculty teach them?


## For Individuals

- Should I pay off my credit card balance with borrowed money?
- What are graduate studies worth financially over my professional career?
- Are federal income tax deductions for my home mortgage a good deal, or should I accelerate my mortgage payments?
- Exactly what rate of return did we make on our stock investments?
- Should I buy or lease my next car, or keep the one I have now and pay off the loan?


## EXAMPLE 1.1

Two lead engineers with a mechanical design company and a structural analysis firm work together often. They have decided that, due to their joint and frequent commercial airline travel around the region, they should evaluate the purchase of a plane co-owned by the two companies. What are some of the economics-based questions the engineers should answer as they evaluate the alternatives to (1) co-own a plane or (2) continue to fly commercially?

## Solution

Some questions (and what is needed to respond) for each alternative are as follows:

- How much will it cost each year? (Cost estimates are needed.)
- How do we pay for it? (A financing plan is needed.)
- Are there tax advantages? (Tax law and tax rates are needed.)
- What is the basis for selecting an alternative? (A selection criterion is needed.)
- What is the expected rate of return? (Equations are needed.)
- What happens if we fly more or less than we estimate now? (Sensitivity analysis is needed.)


### 1.2 ROLE OF ENGINEERING ECONOMY IN DECISION MAKING

People make decisions; computers, mathematics, and other tools do not. The techniques and models of engineering economy assist people in making decisions. Since decisions affect what will be done, the time frame of engineering economy is primarily the future. Therefore, numbers used in an engineering economic analysis are best estimates of what is expected to occur. These estimates often involve the three essential elements mentioned earlier: cash flows, time of occurrence, and interest rates. These estimates are about the future, and will be somewhat different than what actually occurs, primarily because of changing circumstances and unplanned-for events. In other words, the stochastic nature of estimates will likely make the observed value in the future differ from the estimate made now.

Commonly, sensitivity analysis is performed during the engineering economic study to determine how the decision might change based on varying estimates,

especially those that may vary widely. For example, an engineer who expects initial software development costs to vary as much as $\pm 20 \%$ from an estimated $\$ 250,000$ should perform the economic analysis for first-cost estimates of $\$ 200,000, \$ 250,000$, and $\$ 300,000$. Other uncertain estimates about the project can be "tweaked" using sensitivity analysis. (Sensitivity analysis is quite easy to perform using electronic spreadsheets. Tabular and graphical displays make analysis possible by simply changing the estimated values. The power of spreadsheets is used to advantage throughout this text and on the supporting website.)

Engineering economy can be used equally well to analyze outcomes of the past. Observed data are evaluated to determine if the outcomes have met or not met a specified criterion, such as a rate of return requirement. For example, suppose that 5 years ago, a United States-based engineering design company initiated a detailed-design service in Asia for automobile chassis. Now, the company president wants to know if the actual return on the investment has exceeded $15 \%$ per year.

There is an important procedure used to address the development and selection of alternatives. Commonly referred to as the problem-solving approach or the decision-making process, the steps in the approach follow.

1. Understand the problem and define the objective.
2. Collect relevant information.
3. Define the feasible alternative solutions and make realistic estimates.
4. Identify the criteria for decision making using one or more attributes.
5. Evaluate each alternative, using sensitivity analysis to enhance the evaluation.
6. Select the best alternative.
7. Implement the solution and monitor the results.

Engineering economy has a major role in all steps and is primary to steps 2 through 6. Steps 2 and 3 establish the alternatives and make the estimates for each one. Step 4 requires the analyst to identify attributes for alternative selection. This sets the stage for the technique to apply. Step 5 utilizes engineering economy models to complete the evaluation and perform any sensitivity analysis upon which a decision is based (step 6).

## EXAMPLE 1.2

Reconsider the questions presented for the engineers in the previous example about co-owning an airplane. State some ways in which engineering economy contributes to decision making between the two alternatives.

## Solution

Assume that the objective is the same for each engineer-available, reliable transportation that minimizes total cost. Use the steps above.

Steps 2 and 3: The framework for an engineering economy study assists in identifying what should be estimated or collected. For alternative 1 (buy the plane), estimate the purchase cost, financing method and interest rate, annual operating costs, possible
increase in annual sales revenue, and income tax deductions. For alternative 2 (fly commercial) estimate commercial transportation costs, number of trips, annual sales revenue, and other relevant data.

Step 4: The selection criterion is a numerically valued attribute called a measure of worth. Some measures of worth are

| Present worth (PW) | Future worth (FW) | Payback period |
| :--- | :--- | :--- |
| Annual worth (AW) | Rate of return (ROR) | Economic value added |
| Benefit/cost ratio (B/C) | Capitalized cost (CC) |  |

When determining a measure of worth, the fact that money today is worth a different amount in the future is considered; that is, the time value of money is accounted for.

There are many noneconomic attributes-social, environmental, legal, political, personal, to name a few. This multiple-attribute environment may result in less reliance placed on the economic results in step 6 . But this is exactly why the decision maker must have adequate information for all factors-economic and noneconomic-to make an informed selection. In our case, the economic analysis may favor the co-owned plane (alternative 1), but because of noneconomic factors, one or both engineers may select alternative 2 .

Steps 5 and 6: The actual computations, sensitivity analysis, and alternative selection are accomplished here.

The concept of the time value of money was mentioned above. It is often said that money makes money. The statement is indeed true, for if we elect to invest money today, we inherently expect to have more money in the future. If a person or company borrows money today, by tomorrow more than the original loan principal will be owed. This fact is also explained by the time value of money.

## The change in the amount of money over a given time period is called the time value of money; it is the most important concept in engineering economy.

### 1.3 PERFORMING AN ENGINEERING ECONOMY STUDY

Consider the terms engineering economy, engineering economic analysis, economic decision making, capital allocation study, economic analysis, and similar terms to be synonymous throughout this book. There is a general approach, called the Engineering Economy Study Approach, that provides an overview of the engineering economic study. It is outlined in Figure 1-1 for two alternatives. The decision-making process steps are keyed to the blocks in Figure 1-1.

Alternative Description The result of decision-making process step 1 is a basic understanding of what the problem requires for solution. There may initially be many alternatives, but only a few will be feasible and actually evaluated. If


Figure 1-1
Engineering economy study approach.

Step 1

Step 2

Step 3

Step 4
Step 5


| 0 |  | 0 |
| :--- | :---: | :--- |
| 0 |  | 0 |
| 0 | Implement | 0 |
| 0 | alternative 1 | 0 |
| 0 |  | 0 |
| 0 |  | 0 |

alternatives $A, B$, and $C$ have been identified for analysis, when method $D$, though not recognized as an alternative, is the most attractive, the wrong decision is certain to be made.

Alternatives are stand-alone options that involve a word description and best estimates of parameters, such as first cost (including purchase price, development, installation), useful life, estimated annual incomes and expenses, salvage value (resale or trade-in value), an interest rate (rate of return), and possibly inflation and income tax effects. Estimates of annual expenses are usually lumped together and called annual operating costs (AOCs) or maintenance and operation ( $\mathrm{M} \& \mathrm{O}$ ) costs.

Cash Flows The estimated inflows (revenues) and outflows (costs) of money are called cash flows. These estimates are made for each alternative (step 3). Without cash flow estimates over a stated time period, no engineering economy study can be conducted. Expected variation in cash flows indicates a real need for sensitivity analysis in step 5 .

Analysis Using Engineering Economy Computations that consider the time value of money are performed on the cash flows of each alternative to obtain the measure of worth.

Alternative Selection The measure-of-worth values are compared, and an alternative is selected. This is the result of the engineering economy analysis. For example, the result of a rate-of-return analysis may be: Select alternative 1, where the rate of return is estimated at $18.4 \%$ per year, over alternative 2 with an expected $10 \%$ per year return. Some combination of economic criteria using the measure of worth, and the noneconomic and intangible factors, may be applied to help select one alternative.

If only one feasible alternative is defined, a second is often present in the form of the do-nothing alternative. This is the as-is or status quo alternative. Do nothing can be selected if no alternative has a favorable measure of worth.

Whether we are aware of it or not, we use criteria every day to choose between alternatives. For example, when you drive to campus, you decide to take the "best" route. But how did you define best? Was the best route the safest, shortest, fastest, cheapest, most scenic, or what? Obviously, depending upon which criterion or combination of criteria is used to identify the best, a different route might be selected each time. In economic analysis, financial units (dollars or other currency) are generally used as the tangible basis for evaluation. Thus, when there are several ways of accomplishing a stated objective, the alternative with the lowest overall cost or highest overall net income is selected.

An after-tax analysis is performed during project evaluation, usually with only significant effects for asset depreciation and income taxes accounted for. Taxes imposed by local, state, federal, and international governments usually take the form of an income tax on revenues, value-added tax (VAT), import taxes, sales taxes, real estate taxes, and others. Taxes affect alternative estimates for cash flows; they tend to improve cash flow estimates for expenses, cost savings, and asset depreciation, while they reduce cash flow estimates for revenue and after-tax net income. This text delays the details of after-tax analysis until the fundamental tools and techniques of engineering economy are covered. Until then, it is assumed that all alternatives are taxed equally by prevailing tax laws. (If the effects of taxes must be considered earlier, it is recommended that Chapters 16 and 17 be covered after Chapter 6,8 or 11.)

Now, we turn to some fundamentals of engineering economy that are applicable in the everyday life of engineering practice, as well as personal decision making.

### 1.4 INTEREST RATE AND RATE OF RETURN



Interest is the manifestation of the time value of money. Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is negative, there is no interest. There are always two perspectives to an amount of interest-interest paid and interest earned. Interest is paid when a person or organization borrowed money (obtained a loan) and repays a larger amount. Interest is earned when a person or organization saved, invested, or lent money and obtains a return of a larger amount. It is shown below that the computations and numerical values are essentially the same for both perspectives, but there are different interpretations.

Interest paid on borrowed funds (a loan) is determined by using the relation

$$
\begin{equation*}
\text { Interest }=\text { amount owed now }- \text { original amount } \tag{1.1}
\end{equation*}
$$

When interest paid over a specific time unit is expressed as a percentage of the original amount (principal), the result is called the interest rate.

$$
\begin{equation*}
\text { Interest rate }(\%)=\underline{\text { interest accrued per time unit }} \times 100 \% \tag{1.2}
\end{equation*}
$$

The time unit of the rate is called the interest period. By far the most common interest period used to state an interest rate is 1 year. Shorter time periods can be used, such as, $1 \%$ per month. Thus, the interest period of the interest rate should always be included. If only the rate is stated, for example, $8.5 \%$, a 1-year interest period is assumed.

## EXAMPLE 1.3

An employee at LaserKinetics.com borrows $\$ 10,000$ on May 1 and must repay a total of $\$ 10,700$ exactly 1 year later. Determine the interest amount and the interest rate paid.

## Solution

The perspective here is that of the borrower since $\$ 10,700$ repays a loan. Apply Equation [1.1] to determine the interest paid.

$$
\text { Interest }=\$ 10,700-10,000=\$ 700
$$

Equation [1.2] determines the interest rate paid for 1 year.

$$
\text { Percent interest rate }=\frac{\$ 700}{\$ 10,000} \times 100 \%=7 \% \text { per year }
$$

## EXAMPLE 1.4

Stereophonics, Inc., plans to borrow \$20,000 from a bank for 1 year at $9 \%$ interest for new recording equipment. (a) Compute the interest and the total amount due after 1 year. (b) Construct a bar graph that shows the original amount and total amount due after one year used to compute the loan interest rate of $9 \%$ per year.

## Solution

(a) Compute the total interest accrued by solving Equation [1.2] for interest accrued.

$$
\text { Interest }=\$ 20,000(0.09)=\$ 1800
$$

The total amount due is the sum of principal and interest.

$$
\text { Total due }=\$ 20,000+1800=\$ 21,800
$$

(b) Figure 1-2 shows the values used in Equation [1.2]: \$1800 interest, \$20,000 original loan principal, 1-year interest period.


Figure 1-2
Values used to compute an interest rate of $9 \%$ per year, Example 1.4.

## Comment

Note that in part (a), the total amount due may also be computed as

$$
\text { Total due }=\operatorname{principal}(1+\text { interest rate })=\$ 20,000(1.09)=\$ 21,800
$$

Later we will use this method to determine total amounts when the interest rate is in effect longer than one interest period.

From the perspective of a saver, a lender, or an investor, interest earned is the final amount minus the initial amount, or principal.

$$
\begin{equation*}
\text { Interest }=\text { total amount now }- \text { original amount } \tag{1.3}
\end{equation*}
$$

Interest paid over a specific period of time is expressed as a percentage of the original amount and is called rate or return ( $R O R$ ).

$$
\begin{equation*}
\text { Rate of return }(\%)=\frac{\text { interest accrued per time unit }}{\text { original amount }} \times 100 \% \tag{1.4}
\end{equation*}
$$

Investment rate of return



## EXAMPLE 1.4 CONTINUED

The time unit for rate of return is called the interest period, just as for the borrower's perspective. Again, the most common period is 1 year.

The term return on investment (ROI) is used equivalently with ROR in different industries and settings, especially where large capital funds are committed to engineeringoriented programs.

The numerical values in the two sets of equations above are the same, but the term interest rate paid is more appropriate for the borrower's perspective, and the rate of return earned is better for the investor's perspective.

## EXAMPLE 1.5

(a) Calculate the amount deposited 1 year ago to have $\$ 1000$ now at an interest rate of 5\% per year.
(b) Calculate the amount of interest earned during this time period.

## Solution

(a) The total amount accrued is the sum of the original deposit and the earned interest. If $X$ is the original deposit,

$$
\begin{aligned}
\text { Total accrued } & =\text { original }+ \text { original(interest rate }) \\
\$ 1000 & =X+X(0.05)=X(1+0.05)=1.05 X
\end{aligned}
$$

The original deposit is

$$
X=\frac{1000}{1.05}=\$ 952.38
$$

(b) Apply Equation [1.3] to determine interest earned.

$$
\text { Interest }=\$ 1000-952.38=\$ 47.62
$$

In Examples 1.3 to 1.5 the interest period was 1 year, and the interest amount was calculated at the end of one period. When more than one interest period is involved (e.g., if we wanted the amount of interest owed after 3 years in Example 1.4), it is necessary to state whether the interest is accrued on a simple or compound basis from one period to the next.

An additional economic consideration for any engineering economy study is inflation. Several comments about the fundamentals of inflation are warranted at this early stage. In truth, from the borrower's perspective, the rate of inflation is simply another interest rate tacked on to the stated interest rate. And, from the vantage point of the saver or investor, inflation reduces the real rate of return on an investment. Inflation means that cost and revenue cash flow estimates increase over time. This increase is due to the changing value of money that is forced upon a country's currency by inflation, thus making a unit of currency (one dollar) worth less relative to its value at a previous time. We see the effect of
inflation in that money purchases less now than it did at a previous time. Inflation contributes to

- A reduction in purchasing power.
- An increase in the CPI (consumer price index).
- An increase in the cost of equipment and its maintenance.
- An increase in the cost of salaried professionals and hourly employees.
- A reduction in the real rate of return on personal savings and corporate investments.

In other words, inflation can materially contribute to changes in corporate and personal economic analysis.

Commonly, engineering economy studies assume that inflation affects all estimated values equally. Accordingly, an interest rate or rate of return, such as $8 \%$ per year, is applied throughout the analysis without accounting for an additional inflation rate. However, if inflation were explicitly taken into account, and it was reducing the value of money at, say, an average of $4 \%$ per year, it would be necessary to perform the economic analysis using an inflated interest rate of $12.32 \%$ per year. On the other hand, if the stated ROR on an investment is $8 \%$ with inflation included, the same inflation rate of $4 \%$ per year results in a real rate of return of only $3.85 \%$ per year! (The relevant relations are derived in Chapter 14.)

### 1.5 EQUIVALENCE

Equivalent terms are used very often in the transfer from one scale to another. Some common equivalencies or conversions are as follows:

Length: 100 centimeters $=1$ meter 1000 meters $=1$ kilometer
12 inches $=1$ foot $\quad 3$ feet $=1$ yard $\quad 39.370$ inches $=1$ meter
Pressure: 1 atmosphere $=1$ Newton/meter ${ }^{2}$
1 atmosphere $=10^{3}$ pascal $=1$ kilopascal
Many equivalent measures are a combination of two or more scales. For example, 110 kilometers per hour (kph) is equivalent to 68 miles per hour ( mph ), based on the equivalence that 1 mile $=1.6093$ kilometers. We can further conclude that driving at approximately 68 mph for 2 hours is equivalent to traveling a total of about 220 kilometers, or 136 miles. Three scales-time in hours, length in miles, and length in kilometers-are combined to develop equivalent statements. An additional use of these equivalencies is to estimate driving time in hours between two cities using two maps, one indicating distance in miles, a second showing kilometers, if the average of $110 \mathrm{kph}(68 \mathrm{mph})$ is assumed. Note that throughout these statements the fundamental relation 1 mile $=$ 1.6093 kilometers is used. If this relation changes, then the other equivalencies are in error.

When considered together, the time value of money and the interest rate help develop the concept of economic equivalence, which means that different sums of money at different times are equal in economic value. For example, if the

Figure 1-3
Equivalence of three amounts at a $6 \%$ per year interest rate.

6\% per year interest rate

interest rate is $6 \%$ per year, $\$ 100$ today (present time) is equivalent to $\$ 106$ one year from today.

$$
\text { Amount accrued }=100+100(0.06)=100(1+0.06)=\$ 106
$$

So, if someone offered you a gift of $\$ 100$ today or $\$ 106$ one year from today, it would make no difference which offer you accepted. In either case you have $\$ 106$ one year from today. However, the two sums of money are equivalent to each other only when the interest rate is $6 \%$ per year. At a higher or lower interest rate, $\$ 100$ today is not equivalent to $\$ 106$ one year from today.

In addition to future equivalence, we can apply the same logic to determine equivalence for previous years. A total of $\$ 100$ now is equivalent to $\$ 100 / 1.06=$ $\$ 94.34$ one year ago at an interest rate of $6 \%$ per year. From these illustrations, we can state the following: $\$ 94.34$ last year, $\$ 100$ now, and $\$ 106$ one year from now are equivalent at an interest rate of $6 \%$ per year. The fact that these sums are equivalent can be verified by computing the two interest rates for 1 -year interest periods.

$$
\frac{\$ 6}{\$ 100} \times 100 \%=6 \% \text { per year }
$$

and

$$
\frac{\$ 5.66}{\$ 94.34} \times 100 \%=6 \% \text { per year }
$$

Figure 1-3 indicates the amount of interest each year necessary to make these three different amounts equivalent at $6 \%$ per year.

## EXAMPLE 1.6

AC-Delco makes auto batteries available to General Motors dealers through privately owned distributorships. In general, batteries are stored throughout the year, and a 5\% cost increase is added each year to cover the inventory carrying charge for the distributorship owner. Assume you own the City Center Delco facility. Make the calculations necessary at an interest rate of 5\% per year to show which of the following statements
are true and which are false about battery costs.
(a) The amount of $\$ 98$ now is equivalent to a cost of $\$ 105.60$ one year from now.
(b) A truck battery cost of $\$ 200$ one year ago is equivalent to $\$ 205$ now.
(c) $\mathrm{A} \$ 38$ cost now is equivalent to $\$ 39.90$ one year from now.
(d) $\mathrm{A} \$ 3000$ cost now is equivalent to $\$ 2887.14$ one year ago.
(e) The carrying charge accumulated in 1 year on an investment of $\$ 2000$ worth of batteries is $\$ 100$.

## Solution

(a) Total amount accrued $=98(1.05)=\$ 102.90 \neq \$ 105.60$; therefore, it is false. Another way to solve this is as follows: Required original cost is $105.60 / 1.05=$ $\$ 100.57 \neq \$ 98$.
(b) Required old cost is $205.00 / 1.05=\$ 195.24 \neq \$ 200$; therefore, it is false.
(c) The cost 1 year from now is $\$ 38(1.05)=\$ 39.90$; true
(d) Cost now is $2887.14(1.05)=\$ 3031.50 \neq \$ 3000$; false.
(e) The charge is $5 \%$ per year interest, or $2000(0.05)=\$ 100$; true.

### 1.6 SIMPLE AND COMPOUND INTEREST

The terms interest, interest period, and interest rate (introduced in Section 1.4) are useful in calculating equivalent sums of money for one interest period in the past and one period in the future. However, for more than one interest period, the terms simple interest and compound interest become important.

Simple interest is calculated using the principal only, ignoring any interest accrued in preceding interest periods. The total simple interest over several periods is computed as

$$
\text { Interest }=(\text { principal })(\text { number of periods })(\text { interest rate })
$$

where the interest rate is expressed in decimal form.


## EXAMPLE 1.7

Pacific Telephone Credit Union loaned money to an engineering staff member for a radiocontrolled model airplane. The loan is for $\$ 1000$ for 3 years at $5 \%$ per year simple interest. How much money will the engineer repay at the end of 3 years? Tabulate the results.

## Solution

The interest for each of the 3 years is
Interest per year $=1000(0.05)=\$ 50$
Total interest for 3 years from Equation [1.5] is

$$
\text { Total interest }=1000(3)(0.05)=\$ 150
$$

The amount due after 3 years is

$$
\text { Total due }=\$ 1000+150=\$ 1150
$$

## EXAMPLE 1.7 CONTINUED

The $\$ 50$ interest accrued in the first year and the $\$ 50$ accrued in the second year do not earn interest. The interest due each year is calculated only on the $\$ 1000$ principal.

The details of this loan repayment are tabulated in Table 1-1 from the perspective of the borrower. The year zero represents the present, that is, when the money is borrowed. No payment is made until the end of year 3. The amount owed each year increases uniformly by $\$ 50$, since simple interest is figured on only the loan principal.

## table 1-1 Simple Interest Computations

| $(1)$ <br> End of <br> Year | (2) <br> Amount <br> Borrowed | (3) | (4) <br> Amount <br> Owed | (5) <br> Amount <br> Paid |
| :---: | :---: | :---: | :---: | ---: |
| 0 | $\$ 1000$ |  |  |  |
| 1 | - | $\$ 50$ | $\$ 1050$ | $\$$ |
| 2 | - | 50 | 1100 | 0 |
| 3 | - | 1150 | 1150 |  |

For compound interest, the interest accrued for each interest period is calculated on the principal plus the total amount of interest accumulated in all previous periods. Thus, compound interest means interest on top of interest. Compound interest reflects the effect of the time value of money on the interest also. Now the interest for one period is calculated as

$$
\begin{equation*}
\text { Interest }=(\text { principal }+ \text { all accrued interest)(interest rate }) \tag{1.6}
\end{equation*}
$$

## EXAMPLE 1.8

If an engineer borrows $\$ 1000$ from the company credit union at $5 \%$ per year compound interest, compute the total amount due after 3 years. Graph and compare the results of this and the previous example.

## Solution

The interest and total amount due each year are computed separately using Equation [1.6].

$$
\begin{aligned}
\text { Year } 1 \text { interest: } & \$ 1000(0.05)=\$ 50.00 \\
\text { Total amount due after year 1: } & \$ 1000+50.00=\$ 1050.00 \\
\text { Year 2 interest: } & \$ 1050(0.05)=\$ 52.50 \\
\text { Total amount due after year 2: } & \$ 1050+52.50=\$ 1102.50 \\
\text { Year 3 interest: } & \$ 1102.50(0.05)=\$ 55.13 \\
\text { Total amount due after year 3: } & \$ 1102.50+55.13=\$ 1157.63
\end{aligned}
$$

## table 1-2 Compound Interest Computations, Example 1.8

| (1) <br> End of <br> Year | $(2)$ <br> Amount <br> Borrowed | Interest | (3) <br> Amount <br> Owed | (5) <br> Amount <br> Paid |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\$ 1000$ |  |  |  |
| 1 | - | $\$ 50.00$ | $\$ 1050.00$ | $\$$ |
| 2 | - | 52.50 | 1102.50 | 0 |
| 3 | - | 55.13 | 1157.63 | 1157.63 |

The details are shown in Table 1-2. The repayment plan is the same as that for the simple interest example-no payment until the principal plus accrued interest is due at the end of year 3 .

Figure 1-4 shows the amount owed at the end of each year for 3 years. The difference due to the time value of money is recognized for the compound interest case. An extra $\$ 1157.63-\$ 1150=\$ 7.63$ of interest is paid compared with simple interest over the 3 -year period.


Figure 1-4
Comparison of simple and compound interest calculations, Examples 1.7 and 1.8.

## EXAMPLE 1.8 CONTINUED

Comment
The difference between simple and compound interest grows each year. If the computations are continued for more years, for example, 10 years, the difference is $\$ 128.90$; after 20 years compound interest is $\$ 653.30$ more than simple interest.

If $\$ 7.63$ does not seem like a significant difference in only 3 years, remember that the beginning amount here is $\$ 1000$. If we make these same calculations for an initial amount of $\$ 100,000$ or $\$ 1$ million, we are talking real money. This indicates that the power of compounding is vitally important in all economics-based analyses.

Another and shorter way to calculate the total amount due after 3 years in Example 1.8 is to combine calculations rather then perform them on a year-byyear basis. The total due each year is as follows:

$$
\begin{array}{ll}
\text { Year 1: } & \$ 1000(1.05)^{1}=\$ 1050.00 \\
\text { Year 2: } & \$ 1000(1.05)^{2}=\$ 1102.50 \\
\text { Year 3: } & \$ 1000(1.05)^{3}=\$ 1157.63
\end{array}
$$

The year 3 total is calculated directly; it does not require the year 2 total. In general formula form

Total due after a number of years $=\operatorname{principal}(1+\text { interest rate })^{\text {number of years }}$
This fundamental relation is used many times in upcoming chapters.
We combine the concepts of interest rate, simple interest, compound interest, and equivalence to demonstrate that different loan repayment plans may be equivalent, but differ substantially in monetary amounts from one year to another. This also shows that there are many ways to take into account the time value of money. The following example illustrates equivalence for five different loan repayment plans.

## EXAMPLE 1.9

(a) Demonstrate the concept of equivalence using the different loan repayment plans described below. Each plan repays a $\$ 5000$ loan in 5 years at $8 \%$ interest per year.

- Plan 1: Simple interest, pay all at end. No interest or principal is paid until the end of year 5. Interest accumulates each year on the principal only.
- Plan 2: Compound interest, pay all at end. No interest or principal is paid until the end of year 5. Interest accumulates each year on the total of principal and all accrued interest.
- Plan 3: Simple interest paid annually, principal repaid at end. The accrued interest is paid each year, and the entire principal is repaid at the end of year 5.
- Plan 4: Compound interest and portion of principal repaid annually. The accrued interest and one-fifth of the principal (or \$1000) is repaid each
year. The outstanding loan balance decreases each year, so the interest for each year decreases.
- Plan 5: Equal payments of compound interest and principal made annually. Equal payments are made each year with a portion going toward principal repayment and the remainder covering the accrued interest. Since the loan balance decreases at a rate slower than that in plan 4 due to the equal end-of-year payments, the interest decreases, but at a slower rate.
(b) Make a statement about the equivalence of each plan at $8 \%$ simple or compound interest, as appropriate.


## Solution

(a) Table 1-3 presents the interest, payment amount, total owed at the end of each year, and total amount paid over the 5-year period (column 4 totals).

| TABLE 1-3 | 3 Different Repayment Schedules Over 5 Years for \$5000 at 8\% Per Year Interest |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (1) | (2) | (3) | (4) | (5) |
| End of Year | Interest Owed for Year | Total Owed at End of Year | End-of-Year Payment | Total Owed after Payment |
| Plan 1: Simple Interest, Pay All at End |  |  |  |  |
| 0 |  |  |  | \$5000.00 |
| 1 | \$400.00 | \$5400.00 | - | 5400.00 |
| 2 | 400.00 | 5800.00 | - | 5800.00 |
| 3 | 400.00 | 6200.00 | - | 6200.00 |
| 4 | 400.00 | 6600.00 | - | 6600.00 |
| 5 | 400.00 | 7000.00 | \$7000.00 |  |
| Totals |  |  | \$7000.00 |  |
| Plan 2: Compound Interest, Pay All at End |  |  |  |  |
| 0 |  |  |  | \$5000.00 |
| 1 | \$400.00 | \$5400.00 | - | 5400.00 |
| 2 | 432.00 | 5832.00 | - | 5832.00 |
| 3 | 466.56 | 6298.56 | - | 6298.56 |
| 4 | 503.88 | 6802.44 | - | 6802.44 |
| 5 | 544.20 | 7346.64 | \$7346.64 |  |
| Totals |  |  | \$7346.64 |  |
| Plan 3: Simple Interest Paid Annually; Principal Repaid at End |  |  |  |  |
| 0 |  |  |  | \$5000.00 |
| 1 | \$400.00 | \$5400.00 | \$400.00 | 5000.00 |
| 2 | 400.00 | 5400.00 | 400.00 | 5000.00 |
| 3 | 400.00 | 5400.00 | 400.00 | 5000.00 |
| 4 | 400.00 | 5400.00 | 400.00 | 5000.00 |
| 5 | 400.00 | 5400.00 | 5400.00 |  |
| Totals |  |  | \$7000.00 |  |

## EXAMPLE 1.9 CONTINUED

| TABLE $1-3$ | (Continued) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (1) | (2) | (3) | (4) | (5) |
| End of <br> Year | Interest Owed <br> for Year | Total Owed at <br> End of Year | End-of-Year <br> Payment | Total Owed <br> after Payment |

Plan 4: Compound Interest and Portion of Principal Repaid Annually

| 0 |  |  |  | $\$ 5000.00$ |
| :---: | ---: | ---: | ---: | ---: |
| 1 | $\$ 400.00$ | $\$ 5400.00$ | $\$ 1400.00$ | 4000.00 |
| 2 | 320.00 | 4320.00 | 1320.00 | 3000.00 |
| 3 | 240.00 | 3240.00 | 1240.00 | 2000.00 |
| 4 | 160.00 | 2160.00 | 1160.00 | 1000.00 |
| 5 | 80.00 | 1080.00 | 1080.00 |  |
| Totals |  |  | $\$ 6200.00$ |  |

Plan 5: Equal Annual Payments of Compound Interest and Principal

| 0 |  |  |  | $\$ 5000.00$ |
| :---: | ---: | ---: | ---: | ---: |
| 1 | $\$ 400.00$ | $\$ 5400.00$ | $\$ 1252.28$ | 4147.72 |
| 2 | 331.82 | 4479.54 | 1252.28 | 3227.25 |
| 3 | 258.18 | 3485.43 | 1252.28 | 2233.15 |
| 4 | 178.65 | 2411.80 | 1252.28 | 1159.52 |
| 5 | 92.76 | 1252.28 | $\underline{1252.28}$ |  |
| Totals |  |  | $\$ 6261.41$ |  |

The amounts of interest (column 2) are determined as follows:
Plan 1 Simple interest $=($ original principal $)(0.08)$
Plan 2 Compound interest = (total owed previous year) $(0.08)$
Plan 3 Simple interest $=($ original principal $)(0.08)$
Plan 4 Compound interest $=($ total owed previous year $)(0.08)$
Plan 5 Compound interest = (total owed previous year)(0.08)
Note that the amounts of the annual payments are different for each repayment schedule and that the total amounts repaid for most plans are different, even though each repayment plan requires exactly 5 years. The difference in the total amounts repaid can be explained (1) by the time value of money, (2) by simple or compound interest, and (3) by the partial repayment of principal prior to year 5 .
(b) Table $1-3$ shows that $\$ 5000$ at time 0 is equivalent to each of the following:

Plan $1 \$ 7000$ at the end of year 5 at $8 \%$ simple interest.
Plan $2 \$ 7346.64$ at the end of year 5 at $8 \%$ compound interest.
Plan $3 \$ 400$ per year for 4 years and $\$ 5400$ at the end of year 5 at $8 \%$ simple interest.
Plan 4 Decreasing payments of interest and partial principal in years 1 (\$1400) through 5 (\$1080) at $8 \%$ compound interest.
Plan $5 \quad \$ 1252.28$ per year for 5 years at $8 \%$ compound interest.
An engineering economy study uses plan 5; interest is compounded, and a constant amount is paid each period. This amount covers accrued interest and a partial amount of principal repayment.

### 1.7 TERMINOLOGY AND SYMBOLS

The equations and procedures of engineering economy utilize the following terms and symbols. Sample units are indicated.
$P=$ value or amount of money at a time designated as the present or time 0 . Also $P$ is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC); dollars
$F=$ value or amount of money at some future time. Also $F$ is called future worth (FW) and future value (FV); dollars
$A=$ series of consecutive, equal, end-of-period amounts of money.
Also $A$ is called the annual worth (AW) and equivalent uniform annual worth (EUAW); dollars per year, dollars per month
$n=$ number of interest periods; years, months, days
$i=$ interest rate or rate of return per time period; percent per year, percent per month
$t=$ time, stated in periods; years, months, days
The symbols $P$ and $F$ represent one-time occurrences: $A$ occurs with the same value once each interest period for a specified number of periods. It should be clear that a present value $P$ represents a single sum of money at some time prior to a future value $F$ or prior to the first occurrence of an equivalent series amount $A$.

It is important to note that the symbol $A$ always represents a uniform amount (i.e., the same amount each period) that extends through consecutive interest periods. Both conditions must exist before the series can be represented by $A$.

The interest rate $i$ is assumed to be a compound rate, unless specifically stated as simple interest. The rate $i$ is expressed in percent per interest period, for example, $12 \%$ per year. Unless stated otherwise, assume that the rate applies throughout the entire $n$ years or interest periods. The decimal equivalent for $i$ is always used in engineering economy computations.

All engineering economy problems involve the element of time $t$. Of the other five, every problem will involve at least four of the symbols, $P, F, A, n$, and $i$, with at least three of them estimated or known.

## EXAMPLE 1.10

A new college graduate has a job with Boeing Aerospace. She plans to borrow $\$ 10,000$ now to help in buying a car. She has arranged to repay the entire principal plus $8 \%$ per year interest after 5 years. Identify the engineering economy symbols involved and their values for the total owed after 5 years.

## EXAMPLE 1.10 CONTINUED

Solution
In this case only $P$ and $F$ are involved, since all amounts are single payments. Time is expressed in years.

$$
P=\$ 10,000 \quad i=8 \% \text { per year } \quad n=5 \text { years } \quad F=?
$$

The future amount $F$ is unknown.

## EXAMPLE 1.11

Assume you borrow $\$ 2000$ now at $7 \%$ per year for 10 years and must repay the loan in equal yearly payments. Determine the symbols involved and their values.

## Solution

Time is in years.

$$
\begin{aligned}
P & =\$ 2000 \\
A & =? \text { per year for } 5 \text { years } \\
i & =7 \% \text { per year } \\
n & =10 \text { years }
\end{aligned}
$$

In Examples 1.10 and 1.11, the $P$ value is a receipt to the borrower, and $F$ or $A$ is a disbursement from the borrower. It is equally correct to use these symbols in the reverse roles.

## EXAMPLE 1.12

On July 1, 2002, your new employer Ford Motor Company deposits $\$ 5000$ into your money market account, as part of your employment bonus, the account pays interest at $5 \%$ per year. You expect to withdraw an equal annual amount for the following 10 years. Identify the symbols and their values.

## Solution

Time is in years.

$$
\begin{aligned}
P & =\$ 5000 \\
A & =? \text { per year } \\
i & =5 \% \text { per year } \\
n & =10 \text { years }
\end{aligned}
$$

## EXAMPLE 1.13

You plan to make a lump-sum deposit of $\$ 5000$ now into an investment account that pays $6 \%$ per year, and you plan to withdraw an equal end-of-year amount of \$1000 for 5 years, starting next year. At the end of the sixth year, you plan to close your account by withdrawing the remaining money. Define the engineering economy symbols involved.

## Solution

Time is expressed in years.

$$
\begin{aligned}
P & =\$ 5000 \\
A & =\$ 1000 \text { per year for } 5 \text { years } \\
F & =? \text { at end of year } 6 \\
i & =6 \% \text { per year } \\
n & =5 \text { years for the } A \text { series and } 6 \text { for the } F \text { value }
\end{aligned}
$$

## EXAMPLE 1.14

Last year Jane's grandmother offered to put enough money into a savings account to generate $\$ 1000$ this year to help pay Jane's expenses at college. (a) Identify the symbols, and (b) calculate the amount that had to be deposited exactly 1 year ago to earn $\$ 1000$ in interest now, if the rate of return is $6 \%$ per year.

## Solution

(a) Time is in years.

$$
\begin{aligned}
P & =? \\
i & =6 \% \text { per year } \\
n & =1 \text { year } \\
F & =P+\text { interest } \\
& =?+\$ 1000
\end{aligned}
$$

(b) Refer to Equations [1.3] and [1.4]. Let $F=$ total amount now and $P=$ original amount. We know that $F-P=\$ 1000$ is the accrued interest. Now we can determine $P$ for Jane and her grandmother.

$$
F=P+P(\text { interest rate })
$$

The $\$ 1000$ interest can be expressed as

$$
\begin{aligned}
\text { Interest } & =F-P=[P+P(\text { interest rate })]-P \\
& =P(\text { interest rate }) \\
\$ 1000 & =P(0.06) \\
P & =\frac{1000}{0.06}=\$ 16,666.67
\end{aligned}
$$

### 1.8 INTRODUCTION TO SOLUTION BY COMPUTER

The functions on a computer spreadsheet can greatly reduce the amount of hand and calculator work for equivalency computations involving compound interest and the terms $P, F, A, i$, and $n$. The power of the electronic spreadsheet often makes it possible to enter a predefined spreadsheet function into one cell and obtain the final answer immediately. Any spreadsheet system can be used-one off the shelf, such as Microsoft Excel ${ }^{\oplus}$, or one specially developed with built-in financial functions and operators. Excel is used throughout this book because it is readily available and easy to use.

Appendix A is a primer on using spreadsheets and Excel. The functions used in engineering economy are described there in detail, with explanations of all the parameters (also called arguments) placed between parentheses after the function identifier. The Excel online help function provides similar information. Appendix A also includes a section on spreadsheet layout that is useful when the economic analysis is presented to someone else-a coworker, a boss, or a professor.

A total of six Excel functions can perform most of the fundamental engineering economy calculations. However, these functions are no substitute for knowing how the time value of money and compound interest work. The functions are great supplemental tools, but they do not replace the understanding of engineering economy relations, assumptions, and techniques.

Using the symbols $P, F, A, i$, and $n$ exactly as defined in the previous section, the Excel functions most used in engineering economic analysis are formulated as follows.

To find the present value $P: \operatorname{PV}(i \%, n, A, F)$
To find the future value $F$ : $\mathrm{FV}(i \%, n, A, P)$
To find the equal, periodic value $A$ : $\operatorname{PMT}(i \%, n, P, F)$
To find the number of periods $n$ : $\operatorname{NPER}(i \%, A, P, F)$
To find the compound interest rate $i$ : $\operatorname{RATE}(n, A, P, F)$
To find the compound interest rate $i$ : IRR(first_cell:last_cell)
To find the present value $P$ of any series: NPV( $i \%$, second_cell:last_cell) + first_cell

If some of the parameters don't apply to a particular problem, they can be omitted and zero is assumed. If the parameter omitted is an interior one, the comma must be entered. The last two functions require that a series of numbers be entered into contiguous spreadsheet cells, but the first five can be used with no supporting data. In all cases, the function must be preceded by an equals sign ( $=$ ) in the cell where the answer is to be displayed.

Each of these functions will be introduced and illustrated at the point in this text where they are most useful. However, to get an idea of how they work, look back at Examples 1.10 and 1.11. In Example 1.10, the future amount $F$ is unknown, as indicated by $F=$ ? in the solution. In the next chapter, we will learn how the time value of money is used to find $F$, given $P, i$, and $n$. To find $F$ in this example using a spreadsheet, simply enter the FV function preceded by an equals


Figure 1-5
Excel spreadsheet func-
tions for (a) Example
1.10, and (b) Example
1.11 .
sign into any cell. The format is $\mathrm{FV}(i \%, n, P)$ or $\mathrm{FV}(8 \%, 5,, 10000)$. The comma is entered because there is no $A$ involved. Figure $1-5 a$ is a screen image of the Excel spreadsheet with the FV function entered into cell B2. The answer of $\$-14,693.28$ is displayed. The answer is in red on the actual Excel screen to indicate a negative amount from the borrower's perspective to repay the loan after 5 years. The FV function is shown in the formula bar above the worksheet itself. Also, we have added a cell tag to show the format of the FV function.

In Example 1.11, the uniform annual amount $A$ is sought, and $P, i$, and $n$ are known. Find $A$, using the function $\operatorname{PMT}(i \%, n, P)$ or, in this example, PMT $(7 \%, 10,2000)$. Figure $1-5 b$ shows the result in cell C4. The format of the FV function is shown in the formula bar and the cell tag.


Because these functions can be used so easily and rapidly, we will detail them in many of the examples throughout the book. A special checkered-flag icon, with $Q-\operatorname{solv}$ (for quick solution) printed on it, is placed in the margin when just one function provides an answer. In the introductory chapters of Level One, the entire spreadsheet and detailed function are shown. In succeeding chapters, the Q-solv icon is shown in the margin, and the spreadsheet function is contained within the solution of the example.

When the power of the computer is used to solve a more complex problem utilizing several functions and possibly an Excel chart (graph), the icon in the margin is a lightening bolt with the term $e$-solve printed. These spreadsheets are more complex and contain much more information and computation, especially when sensitivity analysis is performed. The Solution by Computer answer to an example is always presented after the Solution by Hand. As mentioned earlier, the spreadsheet function is not a replacement for the correct understanding and application of the engineering economy relations. Therefore, the hand and computer solutions complement each other.

### 1.9 MINIMUM ATTRACTIVE RATE OF RETURN

For any investment to be profitable, the investor (corporate or individual) expects to receive more money than the amount invested. In other words, a fair rate of return, or return on investment, must be realizable. The definition of ROR in Equation [1.4] is used in this discussion, that is, amount earned divided by the original amount.

Engineering alternatives are evaluated upon the prognosis that a reasonable ROR can be expected. Therefore, some reasonable rate must be established for the selection criteria phase of the engineering economy study (Figure 1-1). The reasonable rate is called the Minimum Attractive Rate of Return (MARR) and is higher than the rate expected from a bank or some safe investment that involves minimal investment risk. Figure 1-6 indicates the relations between different rate of return values. In the United States, the current U.S. Treasury bill return is sometimes used as the benchmark safe rate.

The MARR is also referred to as the hurdle rate for projects; that is, to be considered financially viable the expected ROR must meet or exceed the MARR or hurdle rate. Note that the MARR is not a rate that is calculated like an ROR. The MARR is established by (financial) management and used as a criterion against which an alternative's ROR is measured, when making the accept/reject decision.

To develop a foundation-level understanding of how a MARR value is established and used, we must return to the term capital introduced in Section 1.1. Capital is also referred to as capital funds and capital investment money. It always costs money in the form of interest to raise capital. The interest, stated as a percentage rate, is called the cost of capital. For example, if you want to purchase a new music system, but don't have sufficient money (capital), you could obtain a credit union loan at some interest rate, say, $9 \%$ per year, and pay cash to the merchant now. Or, you could use your (newly acquired) credit card and pay

off the balance on a monthly basis. This approach will probably cost you at least $18 \%$ per year. Or, you could use funds from your savings account that earns $5 \%$ per year and pay cash. The $9 \%, 18 \%$, and $5 \%$ rates are your cost of capital estimates to raise the capital for the system by different methods of capital financing. In analogous ways, corporations estimate the cost of capital from different sources to raise funds for engineering projects and other types of projects.

In general, capital is developed in two ways-equity financing and debt financing. A combination of these two is very common for most projects. Chapter 10 covers these in greater detail, but a snapshot description follows.

Equity financing. The corporation uses it own funds from cash on hand, stock sales, or retained earnings. Individuals can use their own cash, savings, or investments. In the example above, using money from the 5\% savings account is equity financing.
Debt financing. The corporation borrows from outside sources and repays the principal and interest according to some schedule, much like the plans in Table 1-3. Sources of debt capital may be bonds, loans, mortgages, venture capital pools, and many others. Individuals, too, can utilize debt sources, such as the credit card and credit union options described in the music system example.

Figure 1-6
Size of MARR relative to other rate of return values.


Combinations of debt-equity financing mean that a weighted average cost of capital (WACC) results. If the music system is purchased with $40 \%$ credit card money at $18 \%$ per year and $60 \%$ savings account funds earning $5 \%$ per year, the weighted average cost of capital is $0.4(18)+0.6(5)=10.2 \%$ per year.

For a corporation, the established MARR used as a criterion to accept or reject an alternative will always be higher than the weighted average cost of capital that the corporation must bear to obtain the necessary capital funds. So the inequality

$$
\begin{equation*}
\text { ROR } \geq \text { MARR }>\text { cost of capital } \tag{1.7}
\end{equation*}
$$

must be correct for an accepted project. Exceptions may be government-regulated requirements (safety, security, environmental, legal, etc.), economically lucrative ventures expected to lead to other opportunities, etc. Value-added engineering projects usually follow Equation [1.7].

Even though there are many alternatives that may yield a ROR which exceeds the MARR as indicated in Figure 1-6, there will be likely not be sufficient capital available for all, or the project's risk may be estimated as too high to take the investment chance. New projects that are undertaken because they have an expected ROR > MARR are usually those projects which have an expected return that is at least as great as the return on another alternative not yet funded. Such a selected new project would be proposal represented by the top ROR arrow in Figure 1-6. For example, assume MARR $=12 \%$ and proposal 1 with an expected ROR $=13 \%$ cannot be funded due to a lack of capital funds. Meanwhile, proposal 2 has a $\mathrm{ROR}=14.5 \%$ and is funded from available capital. Since proposal 1 is not pursued due to the lack of capital, its estimated ROR of $13 \%$ is referred to as the opportunity cost; that is, the opportunity to make an additional $13 \%$ return is forgone.

### 1.10 CASH FLOWS: THEIR ESTIMATION AND DIAGRAMMING

In Section 1.3 cash flows are described as the inflows and outflows of money. These cash flows may be estimates or observed values. Every person or company has cash receipts-revenue and income (inflows); and cash disbursementsexpenses, and costs (outflows). These receipts and disbursements are the cash flows, with a plus sign representing cash inflows and a minus sign representing cash outflows. Cash flows occur during specified periods of time, such as 1 month or 1 year.

Of all the elements of the engineering economy study approach (Figure 1-1), cash flow estimation is likely the most difficult and inexact. Cash flow estimates are just that-estimates about an uncertain future. Once estimated, the techniques of this book guide the decision making process. But the time-proven accuracy of an alternative's estimated cash inflows and outflows clearly dictates the quality of the economic analysis and conclusion.

Cash inflows, or receipts, may be comprised of the following, depending upon the nature of the proposed activity and the type of business involved.

Samples of Cash Inflow Estimates
Revenues (usually incremental resulting from an alternative).
Operating cost reductions (resulting from an alternative).
Asset salvage value.
Receipt of loan principal.
Income tax savings.
Receipts from stock and bond sales.
Construction and facility cost savings.
Saving or return of corporate capital funds.
Cash outflows, or disbursements, may be comprised of the following, again depending upon the nature of the activity and type of business.

Samples of Cash Outflow Estimates
First cost of assets.
Engineering design costs.
Operating costs (annual and incremental).
Periodic maintenance and rebuild costs.
Loan interest and principal, payments.
Major, expected upgrade costs.
Income taxes.
Expenditure of corporate capital funds.
Background information for estimates may be available in departments such as accounting, finance, marketing, sales, engineering, design, manufacturing, production, field services, and computer services. The accuracy of estimates is largely dependent upon the experiences of the person making the estimate with similar situations. Usually point estimates are made; that is, a single-value estimate is developed for each economic element of an alternative. If a statistical approach to the engineering economy study is undertaken, a range estimate or distribution estimate may be developed. Though more involved computationally, a statistical study provides more complete results when key estimates are expected to vary widely. We will use point estimates throughout most of this book. Final chapters discuss decision making under risk.

Once the cash inflow and outflow estimates are developed, the net cash flow can be determined.

$$
\begin{align*}
\text { Net cash flow } & =\text { receipts }- \text { disbursements } \\
& =\text { cash inflows }- \text { cash outflows } \tag{1.8}
\end{align*}
$$

Since cash flows normally take place at varying time points within an interest period, a simplifying assumption is made.

The end-of-period convention means that all cash flows are assumed to occur at the end of an interest period. When several receipts and disbursements occur within a given interest period, the net cash flow is assumed to occur at the end of the interest period.


However, it should be understood that, although $F$ or $A$ amounts are located at the end of the interest period by convention, the end of the period is not necessarily December 31. In Example 1.12 the deposit took place on July 1, 2002, and the withdrawals will take place on July 1 of each succeeding year for 10 years. Thus, end of the period generally means one time period from the date of the transaction.

The cash flow diagram is a very important tool in an economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on a time scale. The diagram includes what is known, what is estimated, and what is needed. That is, once the cash flow diagram is complete, another person should be able to work the problem by looking at the diagram.

Cash flow diagram time $t=0$ is the present, and $t=1$ is the end of time period 1. We assume that the periods are in years for now. The time scale of Figure $1-7$ is set up for 5 years. Since the end-of-year convention places cash flows at the end of years, the " 1 " marks the end of year 1.

While it is not necessary to use an exact scale on the cash flow diagram, you will probably avoid errors if you make a neat diagram to approximate scale for both time and relative cash flow magnitude.

The direction of the arrows on the cash flow diagram is important. A vertical arrow pointing up indicates a positive cash flow. Conversely, an arrow pointing down indicates a negative cash flow. Figure 1-8 illustrates a receipt (cash inflow) at the end of year 1 and equal disbursements (cash outflows) at the end of years 2 and 3 .

The perspective or vantage point must be determined prior to placing a sign on each cash flow and diagramming it. As an illustration, if you borrow $\$ 2500$ to buy a $\$ 2000$ used Harley-Davidson for cash, and you use the remaining $\$ 500$ for a new paint job, there may be several different perspectives taken. Possible

Figure 1-7 A typical cash flow time scale for 5 years.


Figure 1-8
Examples of positive and negative cash flows.

perspectives, cash flow signs, and amounts are as follows.

| Perspective | Cash Flow, \$ |
| :--- | :---: |
| Credit union | -2500 |
| You as borrower | +2500 |
| You as purchaser, | -2000 |
| $\quad$ and as paint customer | -500 |
| Used cycle dealer | +2000 |
| Paint shop owner | +500 |

## EXAMPLE 1.15

Reread Example 1.10, where $P=\$ 10,000$ is borrowed at $8 \%$ per year and $F$ is sought after 5 years. Construct the cash flow diagram.

## Solution

Figure 1-9 presents the cash flow diagram from the vantage point of the borrower. The present sum $P$ is a cash inflow of the loan principal at year 0 , and the future sum $F$ is the cash outflow of the repayment at the end of year 5 . The interest rate should be indicated on the diagram.


Figure 1-9
Cash flow diagram, Example 1.15.

## EXAMPLE 1.16

Each year Exxon-Mobil expends large amounts of funds for mechanical safety features throughout its worldwide operations. Carla Ramos, a lead engineer for Mexico and Central American operations, plans expenditures of $\$ 1$ million for this year and each of the next 4 years just for the improvement of field-based pressure-release valves. Construct the cash flow diagram to find the equivalent value of these expenditures at the end of year 4 , using a cost of capital estimate for safety-related funds of $12 \%$ per year.

## Solution

Figure 1-10 indicates the uniform and negative cash flow series (expenditures) for five periods, and the unknown $F$ value (positive cash flow equivalent) at exactly the same

## EXAMPLE 1.16 CONTINUED

time as the fifth expenditure. Since the expenditures start immediately, the first $\$ 1$ million is shown at time 0 , not time 1 . Therefore, the last negative cash flow occurs at the end of the fourth year, when $F$ also occurs. To make this diagram appear similar to that of Figure 1-9 with a full 5 years on the time scale, the addition of the year -1 prior to year 0 completes the diagram for a full 5 years. This addition demonstrates that year 0 is the end-of-period point for the year -1 .


Figure 1-10
Cash flow diagram, Example 1.16.

## EXAMPLE 1.17

A father wants to deposit an unknown lump-sum amount into an investment opportunity 2 years from now that is large enough to withdraw $\$ 4000$ per year for state university tuition for 5 years starting 3 years from now. If the rate of return is estimated to be $15.5 \%$ per year, construct the cash flow diagram.

## Solution

Figure 1-11 presents the cash flows from the father's perspective. The present value $P$ is a cash outflow 2 years hence and is to be determined ( $P=$ ?). Note that this present value does not occur at time $t=0$, but it does occur one period prior to the first $A$ value of $\$ 4000$, which is the cash inflow to the father.


Figure 1-11
Cash flow diagram, Example 1.17.

## Additional Examples 1.19 and 1.20.

### 1.11 RULE OF 72: ESTIMATING DOUBLING TIME AND INTEREST RATE

Sometimes it is helpful to estimate the number of years $n$ or the rate of return $i$ required for a single cash flow amount to double in size. The rule of 72 for compound interest rates can be used to estimate $i$ or $n$, given the other value. The estimation is simple; the time required for an initial single amount to double in size with compound interest is approximately equal to 72 divided by the rate of return in percent.

$$
\begin{equation*}
\text { Estimated } n=\frac{72}{i} \tag{1.9}
\end{equation*}
$$

For example, at a rate of $5 \%$ per year, it would take approximately $72 / 5=$ 14.4 years for a current amount to double. (The actual time required is 14.3 years, as will be shown in Chapter 2.) Table 1-4 compares the times estimated from the rule of 72 to the actual times required for doubling at several compounded rates. As you can see, very good estimates are obtained.

Alternatively, the compound rate $i$ in percent required for money to double in a specified period of time $n$ can be estimated by dividing 72 by the specified $n$ value.

$$
\begin{equation*}
\text { Estimated } i=\frac{72}{n} \tag{1.10}
\end{equation*}
$$

In order for money to double in a time period of 12 years, for example, a compound rate of return of approximately $72 / 12=6 \%$ per year would be required. The exact answer is $5.946 \%$ per year.

If the interest is simple, a rule of 100 may be used in the same way. In this case the answers obtained will always be exactly correct. As illustrations, money doubles in exactly 12 years at $100 / 12=8.33 \%$ simple interest. Or, at $5 \%$ simple interest it takes exactly $100 / 5=20$ years to double.

## table 1-4 Doubling Time Estimates Using the Rule of 72 and the Actual Time Using Compound Interest Calculations

|  | Doubling Time, Years |  |
| :---: | :---: | :---: |
| Rate of Return, <br> \% per Year | Rule-of-72 <br> Estimate | Actual <br> Years |
| 1 | 72 | 70 |
| 2 | 36 | 35.3 |
| 5 | 14.4 | 14.3 |
| 10 | 7.2 | 7.5 |
| 20 | 3.6 | 3.9 |
| 40 | 1.8 | 2.0 |

### 1.12 SPREADSHEET APPLICATION—SIMPLE AND COMPOUND INTEREST, AND CHANGING CASH FLOW ESTIMATES

The example below demonstrates how an Excel spreadsheet can be used to obtain equivalent future values. A key feature is the use of mathematical relations developed in the cells to perform sensitivity analysis for changing cash flow estimates and the interest rate. To answer these basic questions using hand solution can be time-consuming; the spreadsheet makes it much easier.

## EXAMPLE 1.18

A Japan-based architectural firm has asked a United States-based software engineering group to infuse GIS (geographical information system) sensing capability via satellite into monitoring software for high-rise structures in order to detect greater-than-expected horizontal movements. This software could be very beneficial as an advance warning of serious tremors in earthquake-prone areas in Japan and the United States. The inclusion of accurate GIS data is estimated to increase annual revenue over that for the current software system by $\$ 200,000$ for each of the next 2 years, and by $\$ 300,000$ for each of years 3 and 4 . The planning horizon is only 4 years due to the rapid advances made internationally in building-monitoring software. Develop spreadsheets to answer the questions below.
(a) Determine the equivalent future value in year 4 of the increased cash flows, using an $8 \%$ per year rate of return. Obtain answers for both simple and compound interest.
(b) Rework part (a) if the cash flow estimates in years 3 and 4 increase from \$300,000 to $\$ 600,000$.
(c) The financial manager of the U.S. company wants to consider the effects of $4 \%$ per year inflation in the analysis of part (a). As mentioned in Section 1.4, inflation reduces the real rate of return. For the $8 \%$ rate of return, an inflation rate of $4 \%$ per year compounded each year reduces the return to $3.85 \%$ per year.

## Solution by Computer

Refer to Figure $1-12 a$ to $c$ for the solutions. All three spreadsheets contain the same information, but the cell values are altered as required by the question. (Actually, all the questions posed here can be answered on one spreadsheet by simply changing the numbers. Three spreadsheets are shown here for explanation purposes only.)

The Excel functions are constructed with reference to the cells, not the values themselves, so that sensitivity analysis can be performed without function changes. This approach treats the value in a cell as a global variable for the spreadsheet. For example, the $8 \%$ (simple or compound interest) rate in cell B4 will be referenced in all functions as B4, not $8 \%$. Thus, a change in the rate requires only one alteration in the cell B4 entry, not in every spreadsheet relation and function where $8 \%$ is used. Key Excel relations are detailed in the cell tags.
(a) $8 \%$ simple interest. Refer to Figure 1-12a, columns C and D, for the answers. Simple interest earned each year (column C) incorporates Equation [1.5] one year at

(a)


## EXAMPLE 1.18 CONTINUED


(c)

Figure 1-12
Spreadsheet solution including sensitivity analysis, Example 1.18(a)-(c).
a time into the interest relation by using only the end-of-year (EOY) cash flow amounts $(\$ 200,000$ or $\$ 300,000)$ to determine interest for the next year. This interest is added to the interest from all previous years. In $\$ 1000$ units,

Year 2: $\quad \mathrm{C} 13=\mathrm{B} 12 * \mathrm{~B} 4=\$ 200(0.08)=\$ 16$ (see the cell tag)
Year 3: $\quad \mathrm{C} 14=\mathrm{C} 13+\mathrm{B} 13 * \mathrm{~B} 4=\$ 16+200(.08)=\$ 32$
Year 4: $\mathrm{C} 15=\mathrm{C} 14+\mathrm{B} 14 * \mathrm{~B} 4=\$ 32+300(.08)=\$ 56 \quad$ (see the cell tag)
Remember, an $=$ sign must precede each relation in the spreadsheet. Cell C16 contains the function SUM(C12:C15) to display the total simple interest of \$104,000 over the 4 years. The future value is in D15. It is $F=\$ 1,104,000$ which includes the cumulative amount of all cash flows and all simple interest. In $\$ 1000$ units, example functions are

Year 2: $\quad \mathrm{D} 13=\operatorname{SUM}(\mathrm{B} 13: \mathrm{C} 13)+\mathrm{D} 12=(\$ 200+16)+200=\$ 416$
Year 4: $\quad$ D15 $=\operatorname{SUM}(B 15: C 15)+$ D14 $=(\$ 300+56)+748=\$ 1104$
$8 \%$ compound interest. See Figure $1-12 a$, columns E and F. The spreadsheet structure is the same, except that Equation [1.6] is incorporated into the compound interest values in column E, thus adding interest on top of earned interest.

Interest at $8 \%$ is based on the accumulated cash flow at the end of the previous year. In $\$ 1000$ units,

Year 2 interest:

$$
\mathrm{E} 13=\mathrm{F} 12 * \mathrm{~B} 4=\$ 200(0.08)=\$ 16
$$

$$
\text { Cumulative cash flow: } \mathrm{F} 13=\mathrm{B} 13+\mathrm{E} 13+\mathrm{F} 12=\$ 200+16+200=\$ 416
$$

Year 4 interest: $\quad$ E15 $=$ F14*B4 $=\$ 749.28(0.08)=\$ 59.942$
(see the cell tag)
Cumulative cash flow: $\mathrm{F} 15=\mathrm{B} 15+\mathrm{E} 15+\mathrm{F} 14$

$$
=\$ 300+59.942+749.280=\$ 1109.222
$$

The equivalent future value is in cell F 15 , where $F=\$ 1,109,222$ is shown.
The cash flows are equivalent to $\$ 1,104,000$ at a simple $8 \%$ interest rate, and $\$ 1,109,222$ at a compound $8 \%$ interest rate. Using a compound interest rate increases the $F$ value by $\$ 5222$.

Note that it is not possible to use the FV function in this case because the $A$ values are not the same for all 4 years. We will learn how to use all the basic functions in more versatile ways in the next few chapters.
(b) Refer to Figure 1-12b. In order to initialize the spreadsheet with the two increased cash flow estimates, replace the $\$ 300,000$ values in B14 and B15 with $\$ 600,000$. All spreadsheet relations are identical, and the new interest and accumulated cash flow values are shown immediately. The equivalent fourth-year $F$ values have increased for both the $8 \%$ simple and compound interest (D15 and F15, respectively).
(c) Figure $1-12 c$ is identical to the spreadsheet in Figure 1-12a, except the cell B4 now contains the rate of $3.85 \%$. The corresponding $F$ value for compound interest in F15 has decreased to $\$ 1,051,247$ from the $\$ 1,109,222$ at $8 \%$. This represents an effect of inflation of $\$ 57,975$ in only four years. It is no surprise that governments, corporations, engineers, and all individuals are concerned when inflation rises and the currency is worth less over time.

## Comment

When working with an Excel spreadsheet, it is possible to display all of the entries and functions on the screen by simultaneously touching the $<$ Ctrl $>$ and $<{ }^{\prime}>$ keys, which may be in the upper left of the keyboard on the key with $\langle\sim\rangle$. Additionally, it may be necessary to widen some columns in order to display the entire function statement.

## ADDITIONAL EXAMPLES

## EXAMPLE 1.19

## CASH FLOW DIAGRAMS

A rental company spent $\$ 2500$ on a new air compressor 7 years ago. The annual rental income from the compressor has been $\$ 750$. Additionally, the $\$ 100$ spent on maintenance during the first year has increased each year by $\$ 25$. The company plans to sell the compressor at the end of next year for $\$ 150$. Construct the cash flow diagram from the company's perspective.

## EXAMPLE 1.19 CONTINUED

## Solution

Use time now as $t=0$. The incomes and costs for years -7 through 1 (next year) are tabulated below with net cash flow computed using Equation [1.8]. The net cash flows (one negative, eight positive) are diagrammed in Figure 1-13.

| End of <br> year | Income | Cost | Net <br> Cash Flow |
| :---: | :---: | ---: | :---: |
| -7 | $\$$ | 0 | $\$ 2500$ |
| -6 | 750 | 100 | $\$-2500$ |
| -5 | 750 | 125 | 650 |
| -4 | 750 | 150 | 625 |
| -3 | 750 | 175 | 600 |
| -2 | 750 | 200 | 575 |
| -1 | 750 | 225 | 550 |
| 0 | 750 | 250 | 525 |
| 1 | $750+150$ | 275 | 500 |



Figure 1.13
Cash flow diagram, Example 1.19.

## EXAMPLE 1.20

## CASH FLOW DIAGRAMS

Claudia wants to deposit an amount $P$ now such that she can withdraw an equal annual amount of $A_{1}=\$ 2000$ per year for the first 5 years starting 1 year after the deposit, and a different annual withdrawal of $A_{2}=\$ 3000$ per year for the following 3 years. How would the cash flow diagram appear if $i=8.5 \%$ per year?

## Solution

The cash flows are shown in Figure 1-14. The negative cash outflow $P$ occurs now. The first withdrawal (positive cash inflow) for the $A_{1}$ series occurs at the end of year 1, and $A_{2}$ occurs in years 6 through 8 .


Figure 1-14
Cash flow diagram with two different $A$ series, Example 1.20.

## CHAPTER SUMMARY

Engineering economy is the application of economic factors and criteria to evaluate alternatives, considering the time value of money. The engineering economy study involves computing a specific economic measure of worth for estimated cash flows over a specific period of time.

The concept of equivalence helps in understanding how different sums of money at different times are equal in economic terms. The differences between simple interest (based on principal only) and compound interest (based on principal and interest upon interest) have been described in formulas, tables, and graphs. This power of compounding is very noticeable, especially over long periods of time, as is the effect of inflation, introduced here.

The MARR is a reasonable rate of return established as a hurdle rate to determine if an alternative is economically viable. The MARR is always higher than the return from a safe investment.

Also, we learned about cash flows:
Difficulties with their estimation.
Difference between estimated and actual value.
End-of-year convention for cash flow location.
Net cash flow computation.
Different perspectives in determining the cash flow sign.
Construction of a cash flow diagram.

## PROBLEMS

1.1 What is the difference between a tangible and an intangible factor?
1.2 List three essential estimates that must be made in conducting an engineering economic analysis.
1.3 List at least four of the seven steps in the decision-making process.
1.4 List at least three measures of worth.
1.5 Identify at least three noneconomic attributes that may be used as evaluation criteria in the decision-making process.
1.6 What is meant by the do-nothing alternative?

## Interest Rate and Rate of Return

1.7 A broadband service company borrowed $\$ 2$ million for new equipment and repaid the loan in the amount of $\$ 2.25$ million after 1 year. What was the interest rate on the loan?
1.8 A design-build engineering firm completed a pipeline project wherein the company realized a rate of return of $32 \%$ per year. If the amount of money the company had invested was $\$ 6$ million, what was the amount of the profit the company made in the first year?
1.9 A publicly traded construction company reported that it just paid off a loan that it received 1 year earlier. If the total amount of money the company paid was $\$ 1.6$ million and the interest rate on the loan was $12 \%$ per year, how much money had the company borrowed 1 year ago?
1.10 A start-up chemical company has established a goal of making at least a $25 \%$ per year rate of return on its investment. If the company acquired $\$ 40$ million in venture capital, how much did it have to earn in the first year?

## Equivalence

1.11 At an interest rate of $10 \%$ per year, $\$ 10,000$ today is equivalent to how much (a) 1 year from now and (b) 1 year ago?
1.12 A medium-size consulting engineering firm is trying to decide whether it should replace its office furniture now or wait and do it 1 year from now. If the firm does it now, the cost will be $\$ 14,500$. If it waits 1 year, the cost is expected to be $\$ 16,000$. At an interest rate of $12 \%$ per year, should the company replace the furniture now or 1 year from now?
1.13 An investment of $\$ 40,000$ one year ago and $\$ 50,000$ one year from now are equivalent at what interest rate?

## Simple and Compound Interest

1.14 A local bank is offering to pay compound interest of 5\% per year on new savings accounts. An e-bank is offering 6\% per year simple interest on a 3-year certificate of deposit. Which offer is more attractive to a company which wants to set aside money now for a plant expansion 3 years from now?
1.15 Badger Pump Company invested \$500,000 five years ago in a new product line which is now worth $\$ 900,000$. What rate of return did the company earn (a) on a simple
interest basis and (b) on a compound interest basis?
1.16 How long will it take for an investment of $\$ 100,000$ to accumulate to $\$ 200,000$ at an interest rate of $10 \%$ per year simple interest?
1.17 An investment that was made 16 years ago is now worth $\$ 300,000$. How much was the initial investment at an interest rate of $10 \%$ per year (a) simple interest and (b) compound interest?
1.18 Companies frequently borrow money under an arrangement that requires them to make periodic payments of interest only and then pay the principal of the loan all at once. A company that manufactures odor control chemicals borrows $\$ 400,000$ for 3 years at $10 \%$ per year simple interest under such an arrangement. What is the difference in total payments for this arrangement and one in which the company makes no interest payments at the end of each year?
1.19 A company that manufactures in-line mixers for bulk manufacturing is considering borrowing $\$ 1.75$ million to update a production line. If it borrows the money now, it can do so at an interest rate of $10 \%$ per year simple interest for 5 years. If it borrows next year, the interest rate will be only $8 \%$ per year, but the interest will be compound interest for 4 years. (a) How much interest (total) will be paid under each scenario, and (b) should the company borrow now or 1 year from now? Assume the total amount due will be paid when the loan is due in either case.

## Symbols and Spreadsheets

1.20 Define the symbols involved to determine the interest rate required for money to double in value in 5 years.
1.21 Define the symbols involved when a construction company wants to know how much money it can spend now in lieu of spending $\$ 50,000$ three years from now to purchase a new truck when the compound interest rate is $15 \%$ per year.
1.22 State the purpose for each of the following built-in Excel functions:
(a) $\mathrm{FV}(i \%, n, A, P)$
(b) IRR(first_cell:last_cell)
(c) $\operatorname{PMT}(i \%, n, P, F)$
(d) $\operatorname{PV}(i \%, n, A, F)$
1.23 What are the values of the engineering economy symbols $P, F, A, i$, and $n$ in the following Excel functions? Use a? for the symbol that is to be determined.
(a) $\mathrm{FV}(8 \%, 10,2000,10000)$
(b) $\operatorname{PMT}(12 \%, 30,16000)$
(c) $\operatorname{PV}(9 \%, 15,1000,700)$
1.24 Write the engineering economy symbol that corresponds to each of the following Excel functions.
(a) FV
(b) PMT
(c) NPER
(d) IRR
(e) PV
1.25 In a built-in Excel function, if a certain parameter does not apply, under what circumstances can it be left blank and when must a comma be entered in its place?

## MARR and Cost of Capital

1.26 Provide at least one example of a safe investment and one of a risky investment.
1.27 List three possible sources of (a) equity financing and $(b)$ debt financing.
1.28 Rank the following from lowest to highest interest rate: cost of capital, acceptable rate of return on an investment, minimum attractive rate of return, rate of return on a safe investment.
1.29 Three separate projects have calculated rates of return of 11,14 , and $19 \%$ per year. An engineer wants to know which projects to accept on the basis of rate of return. She learns from the Finance Department that company funds, which have a cost of capital of $16 \%$ per year, are commonly used to fund $35 \%$ of all capital projects. Later, she is told that borrowed money is currently costing $9 \%$ per year. If the MARR is established at exactly the weighted average cost of capital, which projects should she accept?

## Cash Flows

1.30 What is meant by the end-of-period convention?
1.31 The difference between cash inflows and cash outflows is known as what?
1.32 Calculate the net cash flow for each month for the following cash flows on a loan from two perspectives: (a) the borrower and (b) the bank.

| Month | Amount of <br> Money | Purpose |
| :--- | :---: | :---: |
| January | $\$ 1000$ | Borrowed $\$ 1000$ for <br> 6 months from bank |
| February | 100 | Paid interest on loan <br> balance to bank |
| April | 100 | Paid interest on loan <br> balance to bank |
| April | 500 | Paid interest on loan <br> balance to bank <br> Borrowed an additional <br> \$500 for 3 months |
| May | 150 | Paid interest on both loan <br> balances to bank |


| Month | Amount of <br> Money | Purpose |
| :--- | :---: | :---: |
| June | 150 | Paid interest on both loan <br> balances to bank |
| July | 150 | Paid interest on both loan <br> balances to bank |
| July | 1500 | Repaid principal on both <br> loans to bank |

1.33 Construct a cash flow diagram for the following cash flows: $\$ 10,000$ outflow at time zero, $\$ 3000$ per year inflow in years 1 through 5 at an interest rate of $10 \%$ per year, and an unknown future amount in year 5.
1.34 Construct a cash flow diagram to find the present worth for the following situation at an interest rate of $20 \%$ per year.

| Year | Cash Flow |
| :---: | ---: |
| 0 | $\$-50,000$ |
| $1-7$ | $-8,000$ |

1.35 Construct a cash flow diagram that represents the amount of money that would be accumulated in 15 years from an investment of $\$ 20,000$ now at an interest rate of $8 \%$ per year.

## Doubling the Value

1.36 Use the rule of 72 to estimate the time it would take for an initial investment of $\$ 10,000$ to accumulate to $\$ 20,000$ at a compound rate of $9 \%$ per year.
1.37 Estimate the time it would take (according to the rule of 72) for money to quadruple in value at a compound interest rate of $8 \%$ per year.
1.38 Use the rule of 72 to estimate the interest that would be required for $\$ 5000$ to accumulate to $\$ 10,000$ in 5 years.
1.39 If you now have $\$ 250,000$ in your retirement account and you want to retire when the account is worth $\$ 1$ million, estimate the rate of return the account must
earn if you want to retire in 20 years without adding any more money to the account.
(c) $\$ 11,000$
(d) $\$ 12,100$
1.43 An investment of $\$ 10,000$ six years ago has accumulated to $\$ 20,000$ now. The compound rate of return earned on the investment is closest to:
(a) $6 \%$
(b) $8 \%$
(c) $10 \%$
(d) $12 \%$
1.44 In most engineering economy studies, the best alternative is the one which:
(a) Will last the longest time
(b) Is easiest to implement
(c) Costs the least
(d) Is most politically attractive

## EXTENDED EXERCISE

## EFFECTS OF COMPOUND INTEREST

In an effort to maintain compliance with noise emission standards on the processing floor, National Semiconductors requires the use of noise-measuring instruments. The company plans to purchase new portable systems at the end of next year at a cost of $\$ 9000$ each. National estimates the maintenance cost to be $\$ 500$ per year for 3 years, after which they will be salvaged for $\$ 2000$ each.

## Questions

1. Construct the cash flow diagram. For a compound interest rate of $8 \%$ per year, find the equivalent $F$ value after 4 years, using calculations by hand.
2. Find the $F$ value in question 1 , using a spreadsheet.
3. Find the $F$ value if the maintenance costs are $\$ 300, \$ 500$, and $\$ 1000$ for each of the 3 years. By how much has the $F$ value changed?
4. Find the $F$ value in question 1 in terms of dollars needed in the future with an adjustment for inflation of $4 \%$ per year. This increases the interest rate from $8 \%$ to $12.32 \%$ per year.

## CASE STUDY

## DESCRIBING ALTERNATIVES FOR PRODUCING REFRIGERATOR SHELLS

## Background

Large refrigerator manufacturers like Whirlpool, General Electric, Frigidaire, and others may subcontract the molding of their plastic liners and door panels. One prime national subcontractor is Innovations Plastics. It is expected that in about 2 years improvements in mechanical properties will allow the molded plastic to sustain increased vertical and horizontal loading, thus significantly reducing the need for attached metal anchors for some shelving. However, improved molding equipment will be needed to enter this market. The company president wants a recommendation on whether Innovations should plan on offering the new technology to the major manufacturers and an estimate of the necessary capital investment to enter this market early.

You work as an engineer for Innovations. At this stage, you are not expected to perform a complete engineering economic analysis, for not enough information is available. You are asked to formulate reasonable alternatives, determine what data and estimates are needed for each one, and ascertain what criteria (economic and noneconomic) should be utilized to make the final decision.

## Information

Some information useful at this time is as follows:

- The technology and equipment are expected to last about 10 years before new methods are developed.
- Inflation and income taxes will not be considered in the analysis.
- The expected returns on capital investment used for the last three new technology projects were compound rates of 15,5 , and $18 \%$. The $5 \%$ rate was the criterion for enhancing an employeesafety system on an existing chemical-mixing process.
- Equity capital financing beyond $\$ 5$ million is not possible. The amount of debt financing and its cost are unknown.
- Annual operating costs have been averaging $8 \%$ of first cost for major equipment.
- Increased annual training costs and salary requirements for handling the new plastics and operating new equipment can range from $\$ 800,000$ to $\$ 1.2$ million.

There are two manufacturers working on the new-generation equipment, which answered your calls. You label these options as alternatives A and B.

## Case Study Exercises

1. Use the first four steps of the decision-making process to generally describe the alternatives and identify what economic-related estimates you will need to complete an engineering economy analysis for the president.
2. Identify any noneconomic factors and criteria to be considered in making the alternative selection.
3. During your inquiries about alternative B from its manufacturer, you learn that this company has
already produced a prototype molding machine and has sold it to a company in Germany for $\$ 3$ million (U.S. dollars). Upon inquiry, you further discover that the German company already has unused capacity on the equipment for manufacturing plastic shells. The company is willing to sell time on the equipment to Innovations
immediately to produce its own shells for U.S. delivery. This could allow an earlier market entry into the United States. Consider this as alternative C, and develop the estimates necessary to evaluate C at the same time as alternatives A and B.
