

## CHAPTER FIVE

## Capacity Planning

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## LEARNING OBJECTIVES

After completing this chapter, you should be able to:

1 Explain the importance of capacity planning.
2 Discuss ways of defining and measuring capacity.
3 Describe the factors that determine effective capacity alternatives

4 Discuss the major considerations related to developing capacity alternatives.
5 Briefly describe approaches that are useful for evaluating capacity alternatives.
capacity The upper limit or ceiling on the load that an operating unit can handle.

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Capacity planning encompasses many basic decisions with long-term consequences for the organization. In this chapter, you will learn about the importance of capacity decisions, the measurement of capacity, how capacity requirements are determined, and the development and evaluation of capacity alternatives.

## Introduction

Capacity issues are important for all organizations, and at all levels of an organization. Capacity refers to an upper limit or ceiling on the load that an operating unit can handle. The operating unit might be a plant, department, machine, store, or worker.

The capacity of an operating unit is an important piece of information for planning purposes: It enables managers to quantify production capability in terms of inputs or outputs, and thereby make other decisions or plans related to those quantities. The basic questions in capacity planning are the following:

1. What kind of capacity is needed?
2. How much is needed?
3. When is it needed?

The question of what kind of capacity is needed depends on the products and services that management intends to produce or provide. Hence, in a very real sense, capacity planning is governed by those choices.

The most fundamental decisions in any organization concern the products and/or services it will offer. Virtually all other decisions pertaining to capacity, facilities, location, and the like are governed by product and service choices.

In some instances, capacity choices are made very infrequently; in others, they are made regularly, as part of an ongoing process. Generally, the factors that influence this frequency are the stability of demand, the rate of technological change in equipment and product design, and competitive factors. Other factors relate to the type of product or service and whether style changes are important (e.g., automobiles and clothing). In any case, management must review product and service choices periodically to ensure that the company makes capacity changes when they are needed for cost, competitive effectiveness, or other reasons.

## NEWSCLIP

Less Trash Leaves Landfills in a Bind
www.gm.com
ot too long ago, dire predictions were made about the lack of landfill capacity to handle the growing amounts of trash companies and residences were generating. Now, some landfills around the country are not getting the trash (and the fees) they need to survive. What was once regarded as undercapacity has now turned into overcapacity.

The reasons for this turnaround can be found in strong efforts by the general public to recycle-stronger than most ex-
perts had predicted. Companies, too, are recycling more, a result of government regulations and cost-saving measures. They are also incorporating more recyclable and reusable parts and materials in their products, and they are reducing the amount of materials used to package their products.

But landfills, like many other kinds of operations, are designed to operate at a certain level. It is difficult (and in some states illegal) for them to operate above their design capacity, and it is inefficient to operate at levels much below design capacity. The shortfall that some landfills are experiencing underscores the risks involved in long-term capacity planning and the importance of good forecasts of future demand.

Source: Based on "Riga Landfill Strains to Survive," Michael Caputo, Rochester Democrat and Chronicle, July 28, 1997, p. 1A.

## Importance of Capacity Decisions

For a number of reasons, capacity decisions are among the most fundamental of all the design decisions that managers must make.

1. Capacity decisions have a real impact on the ability of the organization to meet future demands for products and services; capacity essentially limits the rate of output possible. Having capacity to satisfy demand can allow a company to take advantage of tremendous opportunities. When the Quigley (www.quigleyco.com) Corporation's zinc gluconate lozenges, sold under the name Cold-Eze ${ }^{\mathrm{TM}}$, attracted the public's interest during the height of the cold and flu season in 1997, drugstores and supermarkets quickly sold out. The product was so popular that the company couldn't keep up with demand. Because of this, the company was unable to take full advantage of the strong demand.
2. Capacity decisions affect operating costs. Ideally, capacity and demand requirements will be matched, which will tend to minimize operating costs. In practice, this is not always achieved because actual demand either differs from expected demand or tends to vary (e.g., cyclically). In such cases, a decision might be made to attempt to balance the costs of over- and undercapacity.
3. Capacity is usually a major determinant of initial cost. Typically, the greater the capacity of a productive unit, the greater its cost. This does not necessarily imply a one-for-one relationship; larger units tend to cost proportionately less than smaller units.
4. Capacity decisions often involve long-term commitment of resources and the fact that, once they are implemented, it may be difficult or impossible to modify those decisions without incurring major costs.
5. Capacity decisions can affect competitiveness. If a firm has excess capacity, or can quickly add capacity, that fact may serve as a barrier to entry by other firms. Then too, capacity can affect delivery speed, which can be a competitive advantage.
6. Capacity affects the ease of management; having appropriate capacity makes management easier than when capacity is mismatched.

## Defining and Measuring Capacity

Capacity often refers to an upper limit on the rate of output. Even though this seems simple enough, there are subtle difficulties in actually measuring capacity in certain cases. These difficulties arise because of different interpretations of the term capacity and problems with identifying suitable measures for a specific situation.

In selecting a measure of capacity, it is important to choose one that does not require updating. For example, dollar amounts are often a poor measure of capacity (e.g., capacity of $\$ 30$ million a year) because price changes necessitate updating of that measure.

Where only one product or service is involved, the capacity of the productive unit may be expressed in terms of that item. However, when multiple products or services are involved, as is often the case, using a simple measure of capacity based on units of output can be misleading. An appliance manufacturer may produce both refrigerators and freezers. If the output rates for these two products are different, it would not make sense to simply state capacity in units without reference to either refrigerators or freezers. The problem is compounded if the firm has other products. One possible solution is to state capacities in terms of each product. Thus, the firm may be able to produce 100 refrigerators per day or 80 freezers per day. Sometimes this approach is helpful, sometimes not. For instance, if an organization has many different products or services, it may not be practical to list all of the relevant capacities. This is especially true if there are frequent changes in the mix of output, because this would necessitate a frequently changing composite index of capacity. The preferred alternative in such cases is to use a measure of capacity that refers to availability of inputs. Thus, a hospital has a certain number of beds, a factory has a certain number of machine hours available, and a bus has a certain number of seats and a certain amount of standing room.

No single measure of capacity will be appropriate in every situation. Rather, the measure of capacity must be tailored to the situation. Table 5-1 provides some examples of commonly used measures of capacity.

## TABLE 5-1

Measures of capacity

## Example 1

## Solution

| Business | Inputs | Outputs |
| :---: | :---: | :---: |
| Auto manufacturing | Labor hours, machine hours | Number of cars per shift |
| Steel mill | Furnace size | Tons of steel per day |
| Oil refinery | Refinery size | Gallons of fuel per day |
| Farming | Number of acres, number of cows | Bushels of grain per acre per year, gallons of milk per day |
| Restaurant | Number of tables, seating capacity | Number of meals served per day |
| Theater | Number of seats | Number of tickets sold per performance |
| Retail sales | Square feet of floor space | Revenue generated per day |

Up to this point, we have been using a working definition of capacity. Although it is functional, it can be refined into two useful definitions of capacity:

1. Design capacity: the maximum output that can possibly be attained.
2. Effective capacity: the maximum possible output given a product mix, scheduling difficulties, machine maintenance, quality factors, and so on.

Design capacity is the maximum rate of output achieved under ideal conditions. Effective capacity is usually less than design capacity (it cannot exceed design capacity) owing to realities of changing product mix, the need for periodic maintenance of equipment, lunch breaks, coffee breaks, problems in scheduling and balancing operations, and similar circumstances. Actual output cannot exceed effective capacity and is often less because of machine breakdowns, absenteeism, shortages of materials, and quality problems, as well as factors that are outside the control of the operations managers.

These different measures of capacity are useful in defining two measures of system effectiveness: efficiency and utilization. Efficiency is the ratio of actual output to effective capacity. Utilization is the ratio of actual output to design capacity.

$$
\begin{align*}
& \text { Efficiency }=\frac{\text { Actual output }}{\text { Effective capacity }}  \tag{5-1}\\
& \text { Utilization }=\frac{\text { Actual output }}{\text { Design capacity }} \tag{5-2}
\end{align*}
$$

It is common for managers to focus exclusively on efficiency, but in many instances, this emphasis can be misleading. This happens when effective capacity is low compared with design capacity. In those cases, high efficiency would seem to indicate effective use of resources when it does not. The following example illustrates this point.

Given the information below, compute the efficiency and the utilization of the vehicle repair department:

$$
\begin{aligned}
\text { Design capacity } & =50 \text { trucks per day } \\
\text { Effective capacity } & =40 \text { trucks per day } \\
\text { Actual output } & =36 \text { trucks per day }
\end{aligned}
$$

Efficiency $=\frac{\text { Actual output }}{\text { Effective capacity }}=\frac{36 \text { trucks per day }}{40 \text { trucks per day }}=90 \%$
Utilization $=\frac{\text { Actual output }}{\text { Design capacity }}=\frac{36 \text { trucks per day }}{50 \text { trucks per day }}=72 \%$

Thus, compared with the effective capacity of 40 units per day, 36 units per day looks pretty good. However, compared with the design capacity of 50 units per day, 36 units per day is much less impressive although probably more meaningful.

Because effective capacity acts as a lid on actual output, the real key to improving capacity utilization is to increase effective capacity by correcting quality problems, maintaining equipment in good operating condition, fully training employees, and fully utilizing bottleneck equipment.

Hence, increasing utilization depends on being able to increase effective capacity, and this requires a knowledge of what is constraining effective capacity.

The following section explores some of the main determinants of effective capacity. It is important to recognize that the benefits of high utilization are realized only in instances where there is demand for the output. When demand is not there, focusing exclusively on utilization can be counterproductive, because the excess output not only results in additional variable costs; it also generates the costs of having to carry the output as inventory Another disadvantage of high utilization is that operating costs may increase because of increasing waiting time due to bottleneck conditions.

## Determinants of Effective Capacity

Many decisions about system design have an impact on capacity. The same is true for many operating decisions. This section briefly describes some of these factors, which are then elaborated on elsewhere in the book. The main factors relate to the following:

1. Facilities
2. Products or services
3. Processes
4. Human considerations
5. Operations
6. External forces

## FACILITIES FACTORS

The design of facilities, including size and provision for expansion, is key. Locational factors, such as transportation costs, distance to market, labor supply, energy sources, and


A mass producer of large-frame computers, the Sujitsu plant in Numazu, Japan, conducts product testing. Building efficiency into product testing increases their capacity.


Artisans making rugs by hand in a carpet work shop at the Grand Bazaar in Istanbul, Turkey, Capacity is very limited in production systems such as this where items are specialized and produced one at a time.
room for expansion, are also important. Likewise, layout of the work area often determines how smoothly work can be performed, and environmental factors such as heating, lighting, and ventilation also play a significant role in determining whether personnel can perform effectively or whether they must struggle to overcome poor design characteristics.

## PRODUCT/SERVICE FACTORS

Product or service design can have a tremendous influence on capacity. For example, when items are similar, the ability of the system to produce those items is generally much greater than when successive items differ. Thus, a restaurant that offers a limited menu can usually prepare and serve meals at a faster rate than a restaurant with an extensive menu. Generally speaking, the more uniform the output, the more opportunities there are for standardization of methods and materials, which leads to greater capacity. The particular mix of products or services rendered must also be considered since different items will have different rates of output.

## PROCESS FACTORS

The quantity capability of a process is an obvious determinant of capacity. A more subtle determinant is the influence of output quality. For instance, if quality of output does not meet standards, the rate of output will be slowed by the need for inspection and rework activities.

## HUMAN FACTORS

The tasks that make up a job, the variety of activities involved, and the training, skill, and experience required to perform a job all have an impact on the potential and actual output. In addition, employee motivation has a very basic relationship to capacity, as do absenteeism and labor turnover.

## OPERATIONAL FACTORS

Scheduling problems may occur when an organization has differences in equipment capabilities among alternative pieces of equipment or differences in job requirements. Inventory stocking decisions, late deliveries, acceptability of purchased materials and parts, and quality inspection and control procedures also can have an impact on effective capacity.

Inventory shortages of even one component of an assembled item (e.g., computers, refrigerators, automobiles) can cause a temporary halt to assembly operations until new components become available. This can have a major impact on effective capacity. Thus, insufficient capacity in one area can affect overall capacity.

## EXTERNAL FACTORS

Product standards, especially minimum quality and performance standards, can restrict management's options for increasing and using capacity. Thus, pollution standards on products and equipment often reduce effective capacity, as does paperwork required by government regulatory agencies by engaging employees in nonproductive activities. A similar effect occurs when a union contract limits the number of hours and type of work an employee may do.

Table 5-2 summarizes these factors. In general, inadequate planning is a major limiting determinant of effective capacity.

## Determining Capacity Requirements

Capacity planning decisions involve both long-term and short-term considerations. Longterm considerations relate to overall level of capacity, such as facility size; short-term considerations relate to probable variations in capacity requirements created by such things as seasonal, random, and irregular fluctuations in demand. Because the time intervals

| A. Facilities | 5. Compensation |
| :--- | :--- |
| 1. Design | 6. Learning rates |
| 2. Location | 7. Absenteeism and labor |
| 3. Larnover |  |
| 4. Environment | E. Operational |
| B. Product/Service | 1. Scheduling |
| 1. Design | 2. Materials management |
| 2. Product or service mix | 3. Quality assurance |
| C. Process | 4. Maintenance policies |
| 1. Quantity capabilities | 5. Equipment breakdowns |
| 2. Quality capabilities | F. External factors |
| D. Human factors | 1. Product standards |
| 1. Job content | 2. Safety regulations |
| 2. Job design | 3. Unions |
| 3. Training and experience | 4. Pollution control |
| 4. Motivation | standards |
|  |  |


covered by each of these categories can vary significantly from industry to industry, it would be misleading to put times on the intervals. However, the distinction will serve as a framework within which to discuss capacity planning.

We determine long-term capacity needs by forecasting demand over a time horizon and then converting those forecasts into capacity requirements. Figure 5-1 illustrates some basic demand patterns that might be identified by a forecast. In addition to basic patterns there are more complex patterns, such as a combination of cycles and trends.

When trends are identified, the fundamental issues are (1) how long the trend might persist, because few things last forever, and (2) the slope of the trend. If cycles are identified, interest focuses on (1) the approximate length of the cycles, and (2) the amplitude of the cycles (i.e., deviation from average).

Short-term capacity needs are less concerned with cycles or trends than with seasonal variations and other variations from average. These deviations are particularly important because they can place a severe strain on a system's ability to satisfy demand at some times and yet result in idle capacity at other times.

An organization can identify seasonal patterns using standard forecasting techniques. Although commonly thought of as annual fluctuations, seasonal variations are also

## TABLE 5-2

Factors that determine effective capacity

## FICURE 5-1

## TABL: 5-3 <br> Examples of seasonal demand

 patterns| Period | Ifems |
| :---: | :---: |
| Year | Beer sales, toy sales, airline traffic, clothing, vacations, tourism, power usage, gasoline consumption, sports and recreation, education |
| Month | Welfare and social security checks, bank transactions |
| Week | Retail sales, restaurant meals, automobile traffic, automotive rentals, hotel registrations |
| Day | Telephone calls, power usage, automobile traffic, public transportation, classroom utilization, retail sales, restaurant meals |

reflected in monthly, weekly, and even daily capacity requirements. Table 5-3 provides some examples of items that tend to exhibit seasonal demand patterns.

When time intervals are too short to have seasonal variations in demand, the analysis can often describe the variations by probability distributions such as a normal, uniform, or Poisson distribution. For example, we might describe the amount of coffee served during the midday meal at a luncheonette by a normal distribution with a certain mean and standard deviation. The number of customers who enter a bank branch on Monday mornings might be described by a Poisson distribution with a certain mean. It does not follow, however, that every instance of random variability will lend itself to description by a standard statistical distribution. Service systems in particular may experience a considerable amount of variability in capacity requirements unless requests for service can be scheduled. Manufacturing systems, because of their typical isolation from customers and the more uniform nature of production, are less likely to experience variations. Waiting-line models and simulation models can be useful when analyzing service systems. These models are described in Chapter 19.

Irregular variations are perhaps the most troublesome: They are virtually impossible to predict. They are created by such diverse forces as major equipment breakdowns, freak storms that disrupt normal routines, foreign political turmoil that causes oil shortages, discovery of health hazards (nuclear accidents, unsafe chemical dumping grounds, carcinogens in food and drink), and so on.

The link between marketing and operations is crucial to realistic determination of capacity requirements. Through customer contracts, demographic analyses, and forecasts, marketing can supply vital information to operations for ascertaining capacity needs for both the long term and the short term.

## Developing Capacity Alternatives

Aside from the general considerations about the development of alternatives (i.e., conduct a reasonable search for possible alternatives, consider doing nothing, take care not to overlook nonquantitative factors), there are other things that can be done to enhance capacity management:

1. Design flexibility into systems. The long-term nature of many capacity decisions and the risks inherent in long-term forecasts suggest potential benefits from designing flexible systems. For example, provision for future expansion in the original design of a structure frequently can be obtained at a small price compared to what it would cost to remodel an existing structure that did not have such a provision. Hence, if future expansion of a restaurant seems likely, water lines, power hookups, and waste disposal lines can be put in place initially so that if expansion becomes a reality, modification to the existing structure can be minimized. Similarly, a new golf course may start as a nine-hole operation, but if provision is made for future expansion by obtaining options on adjacent land, it may progress to a larger (18-hole) course. Other considerations in flexible design involve layout of equipment, location, equipment selection, production planning, scheduling, and inventory policies, which will be discussed in later chapters.
2. Differentiate between new and mature products or services. Mature products or services tend to be more predictable in terms of capacity requirements, and they may have
limited life spans. The predictable demand pattern means less risk of choosing an incorrect capacity, but the possible limited life span of the product or service may necessitate finding an alternative use for the additional capacity at the end of the life span. New products tend to carry higher risk because of the uncertainty often associated with predicting the quantity and duration of demand. That makes flexibility appealing to managers.
3. Take a "big picture" approach to capacity changes. When developing capacity alternatives, it is important to consider how parts of the system interrelate. For example, when making a decision to increase the number of rooms in a motel, one should also take into account probable increased demands for parking, entertainment and food, and housekeeping. This is a "big picture" approach.
4. Prepare to deal with capacity "chunks." Capacity increases are often acquired in fairly large chunks rather than smooth increments, making it difficult to achieve a match between desired capacity and feasible capacity. For instance, the desired capacity of a certain operation may be 55 units per hour; but suppose that machines used for this operation are able to produce 40 units per hour each. One machine by itself would cause capacity to be 15 units per hour short of what is needed, but two machines would result in an excess capacity of 25 units per hour. The illustration becomes even more extreme if we shift the topic-to open-hearth furnaces or to the number of airplanes needed to provide a desired level of capacity.
5. Attempt to smooth out capacity requirements. Unevenness in capacity requirements also can create certain problems. For instance, during periods of inclement weather, public transportation ridership tends to increase substantially relative to periods of pleasant weather. Consequently, the system tends to alternate between underutilization and overutilization. Increasing the number of buses or subway cars will reduce the burden during periods of heavy demand, but this will aggravate the problem of overcapacity at other times and certainly add to the cost of operating the system.

We can trace the unevenness in demand for products and services to a variety of sources. The bus ridership problem is weather related to a certain extent, but demand could be considered to be partly random (i.e., varying because of chance factors). Still another source of varying demand is seasonality. Seasonal variations are generally easier to cope with than random variations because they are predictable. Consequently, management can make allowances in planning and scheduling activities and inventories. However, seasonal variations can still pose problems because of their uneven demands on the system: At certain times the system will tend to be overloaded, while at other times it will tend to be underloaded. One possible approach to this problem is to identify products or services that have complementary demand patterns, that is, patterns that tend to offset each other. For instance, demand for snow skis and demand for water skis might complement each other: Demand for water skis is greater in the spring and summer months, and demand for snow skis is greater in the fall and winter months. The same might apply to heating and air-conditioning equipment. The ideal case is one in which products or services with complementary demand patterns involve the use of the same resources but at different times, so that overall capacity requirements remain fairly stable. Figure 5-2 illustrates complementary demand patterns.

Variability in demand can pose a problem for managers. Simply adding capacity by increasing the size of the operation (e.g., increasing the size of the facility, the workforce, or the amount of processing equipment) is not always the best approach, because that reduces flexibility and adds to fixed costs. Consequently, managers often choose to respond to higher than normal demand in other ways. One way is through the use of overtime work. Another way is to subcontract some of the work. A third way is to draw down finished goods inventories during periods of high demand and replenish them during periods of slow demand. These options and others are discussed in detail in the chapter on aggregate planning.
6. Identify the optimal operating level. Production units typically have an ideal or optimal level of operation in terms of unit cost of output. At the ideal level, cost per unit is the lowest for that production unit; larger or smaller rates of output will result in a

## FGURE 5-2

$A$ and $B$ have complementary demand patterns


## FIGURE 5-3

Production units have an optimal rate of output for minimum cost

higher unit cost. Figure 5-3 illustrates this concept. Notice how unit costs rise as the rate of output varies from the optimal level.

The explanation for the shape of the cost curve is that at low levels of output, the costs of facilities and equipment must be absorbed (paid for) by very few units. Hence, the cost per unit is high. As output is increased, there are more units to absorb the "fixed" cost of facilities and equipment, so unit costs decrease. However, beyond a certain point, unit costs will start to rise. To be sure, the fixed costs are spread over even more units, so that does not account for the increase, but other factors now become important: worker fatigue; equipment breakdowns; the loss of flexibility, which leaves less of a margin for error; and, generally, greater difficulty in coordinating operations.

Both optimal operating rate and the amount of the minimum cost tend to be a function of the general capacity of the operating unit. For example, as the general capacity of a plant increases, the optimal output rate increases and the minimum cost for the optimal rate decreases. Thus, larger plants tend to have higher optimal output rates and lower minimum costs than smaller plants. Figure 5-4 illustrates these points.

In choosing the capacity of an operating unit, management must take these relationships into account along with the availability of financial and other resources and forecasts of expected demand. To do this, it is necessary to determine enough points for each size facility to be able to make a comparison among different sizes. In some instances, facility sizes are givens, whereas in others, facility size is a continuous variable (i.e., any size can be selected). In the latter case, an ideal facility size can be selected. Usually, management must make a choice from given sizes, and none may have a minimum at the desired rate of output.


## Planning Service Capacity

Three very important factors in planning service capacity are (1) the need to be near customers, (2) the inability to store services, and (3) the degree of volatility of demand.

Convenience for customers is often an important aspect of service. Generally, a service must be located near customers. For example hotel rooms must be where customers want to stay; having a vacant room in another city won't help. Thus, capacity and location are closely tied.

Capacity must also be matched with the timing of demand. Unlike goods, services cannot be produced in one period and stored for use in a later period. Thus, an unsold seat on an airplane, train, or bus cannot be stored for use on a later trip. Similarly, inventories of goods allow customers to immediately satisfy wants, whereas a customer who wants a service may have to wait. This can result in a variety of negatives for an organization that provides the service. Thus, speed of delivery, or customer waiting time, becomes a major concern in service capacity planning. For example, deciding on the number of police officers and fire trucks to have on duty at any given time affects the speed of response and brings into issue the cost of maintaining that capacity. Some of these issues are addressed in the chapter on waiting lines.

Demand volatility presents problems for capacity planners. Demand volatility tends to be higher for services than for goods, not only in timing of demand, but also in the time

## FICURE 5-4 <br> Minimum cost and optimal

 operating rate are functions of size of a production unitVol. 8, Tape 2, Seg. 1 McDonald's


For routine service needs that come in frequently and in high volume, employees at a call center may provide customer service most efficiently. For more specialized customer service needs, individualized service may provide the best result.
$\square$
Vol. 3, Seg. 4 Toyota
required to service individual customers. For example, banks tend to experience higher volumes of demand on certain days of the week, and the number and nature of transactions tend to vary substantially for different individuals. Then, too, a wide range of social, cultural, and even weather factors can cause major peaks and valleys in demand. The fact that services can't be stored means service systems cannot turn to inventory to smooth demand requirements on the system the way goods-producing systems are able to. Instead, service planners have to devise other methods of coping with demand volatility. For example, to cope with peak demand periods, planners might consider hiring extra workers, outsourcing some or all of a service, or using pricing and promotion to shift some demand to slower periods.

In some instances, demand management is a strategy that can be used to offset capacity limitations. Pricing, promotions, discounts, and similar tactics can help to shift some demand away from peak periods and into slow periods, allowing organizations to achieve a closer match in supply and demand.

## Evaluating Alternatives

An organization needs to examine alternatives for future capacity from a number of different perspectives. Most obvious are economic considerations: Will an alternative be economically feasible? How much will it cost? How soon can we have it? What will operating and maintenance costs be? What will its useful life be? Will it be compatible with present personnel and present operations?

Less obvious, but nonetheless important, is possible negative public opinion. For instance, the decision to build a new power plant is almost sure to stir up reaction, whether the plant is coal-fired, hydroelectric, or nuclear. Any option that could disrupt lives and property is bound to generate hostile reactions. Construction of new facilities may necessitate moving personnel to a new location. Embracing a new technology may mean retraining some people and terminating some jobs. Relocation can cause unfavorable reactions, particularly if a town is about to lose a major employer. Conversely, community pressure in a new location may arise if the presence of the company is viewed unfavorably (noise, traffic, pollution).

A number of techniques are useful for evaluating capacity alternatives from an economic standpoint. Some of the more common are cost-volume analysis, financial analysis, decision theory, and waiting-line analysis. Cost-volume analysis is described in this chapter. Financial analysis is mentioned briefly; decision analysis is described in the chapter supplement and waiting-line analysis are described in chapter 19.

## CALCULATING PROCESSING REQUIREMENTS

When evaluating capacity alternatives, a necessary piece of information is the capacity requirements of products that will be processed with a given alternative. To get this information, one must have reasonably accurate demand forecasts for each product, and know the standard processing time per unit for each product on each alternative machine, the number of workdays per year, and the number of shifts that will be used.

A department works one eight-hour shift, 250 days a year, and has these figures for usage of a machine that is currently being considered:

| Product | Annual <br> Demand | Standard Processing <br> Time per Unit (Hr) | Processing Time <br> Needed (Hr) |
| :---: | :---: | :---: | :---: |
| \#1 | 400 | 5.0 | 2,000 |
| $\# 2$ | 300 | 8.0 | 2,400 |
| $\# 3$ | 700 | 2.0 | $\underline{1,400}$ |
|  |  |  | 5,800 |

Working one eight-hour shift, 250 days a year provides an annual capacity of $8 \times 250$ $=2,000$ hours per year. We can see that three of these machines would be needed to handle the required volume:

$$
\frac{5,800 \text { hours }}{2,000 \text { hour } / \text { machine }}=2.90 \text { machines }
$$

## COST-VOLUME ANALYSIS

Cost-volume analysis focuses on relationships between cost, revenue, and volume of output. The purpose of cost-volume analysis is to estimate the income of an organization under different operating conditions. It is particularly useful as a tool for comparing capacity alternatives.

Use of the technique requires identification of all costs related to the production of a given product. These costs are then designated as fixed costs or variable costs. Fixed costs tend to remain constant regardless of volume of output. Examples include rental costs, property taxes, equipment costs, heating and cooling expenses, and certain administrative costs. Variable costs vary directly with volume of output. The major components of variable costs are generally materials and labor costs. We will assume that variable cost per unit remains the same regardless of volume of output.

Table 5-4 summarizes the symbols used in the cost-volume formulas.
The total cost associated with a given volume of output is equal to the sum of the fixed cost and the variable cost per unit times volume:

$$
\begin{align*}
\mathrm{TC} & =\mathrm{FC}+\mathrm{VC}  \tag{5-3}\\
\mathrm{VC} & =Q \times v
\end{align*}
$$

where $v=$ variable cost per unit. Figure 5-5A shows the relationship between volume of output and fixed costs, total variable costs, and total (fixed plus variable) costs.

Revenue per unit, like variable cost per unit, is assumed to be the same regardless of quantity of output. Total revenue will have a linear relationship to output, as illustrated in Figure 5-5B. Assume that all output can be sold. The total revenue associated with a given quantity of output, $Q$, is:

$$
\begin{equation*}
\mathrm{TR}=R \times Q \tag{5-5}
\end{equation*}
$$

Figure 5-5C describes the relationship between profit-which is the difference between total revenue and total (i.e., fixed plus variable) cost-and volume of output. The volume at which total cost and total revenue are equal is referred to as the break-even point (BEP). When volume is less than the break-even point, there is a loss; when volume is greater than the break-even point, there is a profit. The greater the deviation from this point, the greater the profit or loss. Total profit can be computed using the formula:

$$
P=\mathrm{TR}-\mathrm{TC}=R \times Q-(\mathrm{FC}+v \times Q)
$$

$$
\begin{aligned}
& \text { FC }=\text { Fixed cost } \\
& \text { VC }=\text { Total variable cost } \\
& v=\text { Variable cost per unit } \\
& T C=\text { Total cost } \\
& T R=\text { Total revenue } \\
& R=\text { Revenue per unit } \\
& Q=\text { Quantity or volume of output } \\
& Q_{\text {BEP }}=\text { Break-even quantity } \\
& P=\text { Profit }
\end{aligned}
$$

break-even point (BEP) The
volume of output at which total cost and total revenue are equal.

## TABLE 5-4

## Cost-volume symbols


A. Fixed, variable, and total costs.

B. Total revenue increases linearly with output.

C. Profit $=T R-T C$.

## FIGURE 5-5

## Excmple 3

## Solution

Rearranging terms, we have

$$
\begin{equation*}
P=Q(R-v)-\mathrm{FC} \tag{5-6}
\end{equation*}
$$

The required volume, $Q$, needed to generate a specified profit is:

$$
\begin{equation*}
Q=\frac{P+\mathrm{FC}}{R-v} Q \tag{5-7}
\end{equation*}
$$

A special case of this is the volume of output needed for total revenue to equal total cost. This is the break-even point, computed using the formula:

$$
\begin{equation*}
Q_{\mathrm{BEP}}=\frac{\mathrm{FC}}{R-v} \tag{5-8}
\end{equation*}
$$

The owner of Old-Fashioned Berry Pies, S. Simon, is contemplating adding a new line of pies, which will require leasing new equipment for a monthly payment of $\$ 6,000$. Variable costs would be $\$ 2.00$ per pie, and pies would retail for $\$ 7.00$ each.
a. How many pies must be sold in order to break even?
b. What would the profit (loss) be if 1,000 pies are made and sold in a month?
c. How many pies must be sold to realize a profit of $\$ 4,000$ ?

$$
\mathrm{FC}=\$ 6,000, \quad \mathrm{VC}=\$ 2 \text { per pie }, \quad \operatorname{Rev}=\$ 7 \text { per pie }
$$

a. $Q_{\mathrm{BEP}}=\frac{\mathrm{FC}}{\mathrm{Rev}-\mathrm{VC}}=\frac{\$ 6,000}{\$ 7-\$ 2}=1,200$ pies $/ \mathrm{month}$
b. For $Q=1,000, P=Q(R-v)-\mathrm{FC}=1,000(\$ 7-\$ 2)-\$ 6,000=-\$ 1,000$
c. $P=\$ 4,000$; solve for $Q$ using equation 5-7:

$$
Q=\frac{\$ 4,000+\$ 6,000}{\$ 7-\$ 2}=2,000 \mathrm{pies}
$$

Capacity alternatives may involve step costs, which are costs that increase stepwise as potential volume increases. For example, a firm may have the option of purchasing one, two, or three machines, with each additional machine increasing the fixed cost, although perhaps not linearly. (See Figure 5-6A.) Then fixed costs and potential volume would

A. Step fixed costs and variable costs.

B. Multiple break-even points.
depend on the number of machines purchased. The implication is that multiple break-even quantities may occur, possibly one for each range. Note, however, that the total revenue line might not intersect the fixed-cost line in a particular range, meaning that there would be no break-even point in that range. This possibility is illustrated in Figure 5-6B, where there is no break-even point in the first range. In order to decide how many machines to purchase, a manager must consider projected annual demand (volume) relative to the multiple break-even points and choose the most appropriate number of machines, as Example 4 shows.

A manager has the option of purchasing one, two, or three machines. Fixed costs and potential volumes are as follows:

| Number of <br> Machines | Total Annual <br> Fixed Costs | Corresponding <br> Range of Output |
| :---: | :---: | :---: |
| 1 | $\$ 9,600$ | 0 to 300 |
| 2 | 15,000 | 301 to 600 |
| 3 | $\$ 20,000$ | 601 to 900 |

Variable cost is $\$ 10$ per unit, and revenue is $\$ 40$ per unit.
a. Determine the break-even point for each range.
b. If projected annual demand is between 580 and 660 units, how many machines should the manager purchase?
a. Compute the break-even point for each range using the formula $Q_{\mathrm{BEP}}=F C / R-v$.

For one machine
$Q_{\text {BEP }}=\frac{\$ 9,600}{\$ 40 / \text { unit }-\$ 10 / \text { unit }}=320$ units [not in range, so there is no BEP]
For two machines $Q_{\text {BEP }}=\frac{\$ 15,000}{\$ 40 / \mathrm{unit}-\$ 10 / \mathrm{unit}}=500$ units
For three machines $Q_{\text {BEP }}=\frac{\$ 20,000}{\$ 40 / \text { unit }-\$ 10 / \text { unit }}=666.67$ units
b. Comparing the projected range of demand to the two ranges for which a break-even point occurs, you can see that the break-even point is 500 , which is in the range 301 to 600. This means that even if demand is at the low end of the range, it would be above the break-even point and thus yield a profit. That is not true of range 601 to 900 . At the top

## FICURE 5-6

Break-even problem with step fixed costs

## Example 4

## Solution

cash flow Difference between cash received from sales and other sources, and cash outflow for labor, material, overhead, and taxes.
present value The sum, in current value, of all future cash flows of an investment proposal.
end of projected demand, the volume would still be less than the break-even point for that range, so there would be no profit. Hence, the manager should choose two machines.

Cost-volume analysis can be a valuable tool for comparing capacity alternatives if certain assumptions are satisfied:

1. One product is involved.
2. Everything produced can be sold.
3. The variable cost per unit is the same regardless of the volume.
4. Fixed costs do not change with volume changes, or they are step changes.
5. The revenue per unit is the same regardless of volume.
6. Revenue per unit exceeds variable cost per unit.

As with any quantitative tool, it is important to verify that the assumptions on which the technique is based are reasonably satisfied for a particular situation. For example, revenue per unit or variable cost per unit are not always constant. In addition, fixed costs may not be constant over the range of possible output. If demand is subject to random variations, one must take that into account in the analysis. Also, cost-volume analysis requires that fixed and variable costs can be separated, and this is sometimes exceedingly difficult to accomplish.

Cost-volume analysis works best with one product or a few products that have the same cost characteristics. Nevertheless, a notable benefit of cost-volume considerations is the conceptual framework it provides for integrating cost, revenue, and profit estimates into capacity decisions. If a proposal looks attractive using cost-volume analysis, the next step would be to develop cash flow models to see how it fares with the addition of time and more flexible cost functions.

## FINANCIAL ANALYSIS

A problem that is universally encountered by managers is how to allocate scarce funds. A common approach is to use financial analysis to rank investment proposals.

Two important terms in financial analysis are cash flow and present value:
Cash flow refers to the difference between the cash received from sales (of goods or services) and other sources (e.g., sale of old equipment) and the cash outflow for labor, materials, overhead, and taxes.
Present value expresses in current value the sum of all future cash flows of an investment proposal.

The three most commonly used methods of financial analysis are payback, present value, and internal rate of return.

Payback is a crude but widely used method that focuses on the length of time it will take for an investment to return its original cost. For example, an investment with an original cost of $\$ 6,000$ and a monthly net cash flow of $\$ 1,000$ has a payback period of six months. Payback ignores the time value of money. Its use is easier to rationalize for shortterm than for long-term projects.

The present value ( $P V$ ) method summarizes the initial cost of an investment, its estimated annual cash flows, and any expected salvage value in a single value called the equivalent current value, taking into account the time value of money (i.e., interest rates).

The internal rate of return (IRR) summarizes the initial cost, expected annual cash flows, and estimated future salvage value of an investment proposal in an equivalent interest rate. In other words, this method identifies the rate of return that equates the estimated future returns and the initial cost.

These techniques are appropriate when there is a high degree of certainty associated with estimates of future cash flows. In many instances, however, operations managers and
other managers must deal with situations better described as risky or uncertain. When conditions of risk or uncertainty are present, decision theory is often applied.

## DECISION THEORY

Decision theory is a helpful tool for financial comparison of alternatives under conditions of risk or uncertainty. It is suited to capacity decisions and to a wide range of other decisions managers must make. Decision theory is described in the supplement to this chapter.

## WAITING-LINE ANALYSIS

Analysis of lines is often useful for designing service systems. Waiting lines have a tendency to form in a wide variety of service systems (e.g., airport ticket counters, telephone calls to a cable television company, hospital emergency rooms). The lines are symptoms of bottleneck operations. Analysis is useful in helping managers choose a capacity level that will be cost-effective through balancing the cost of having customers wait with the cost of providing additional capacity. It can aid in the determination of expected costs for various levels of service capacity.

This topic is described in Chapter 19.

## Operations Strategy

The strategic implications of capacity decisions can be enormous for an organization, impacting all areas of the organization. From an operations management standpoint, capacity decisions establish a set of conditions within which operations will be required to function. Hence, it is extremely important to include input from operations management people in making capacity decisions.

Flexibility can be a key issue in capacity decisions, although flexibility is not always an option, particularly in capital-intensive industries. However, where possible, flexibility allows an organization to be responsive (agile) to changes in the marketplace. Also, it reduces to a certain extent the dependence on long-range forecasts to accurately predict demand. And flexibility makes it easier for organizations to take advantage of technological and other innovations.

Bottleneck management can be a way to increase effective capacity, by scheduling non-bottleneck operations to achieve maximum utilization of bottleneck operations.

Capacity refers to a system's potential for producing goods or delivering services over a specified time interval. Capacity decisions are important because capacity is a ceiling on output and a major determinant of operating costs.

The capacity planning decision is one of the most important decisions that managers make. The capacity decision is strategic and long-term in nature, often involving a significant initial investment of capital. Capacity planning is particularly difficult in cases where returns will accrue over a lengthy period and risk is a major consideration.

A variety of factors can interfere with capacity utilization, so effective capacity is usually somewhat less than design capacity. These factors include facilities design and layout, human factors, product/service design, equipment failures, scheduling problems, and quality considerations.

Capacity planning involves long-term and short-term considerations. Long-term considerations relate to the overall level of capacity; short-term considerations relate to variations in capacity requirements due to seasonal, random, and irregular fluctuations in demand. Ideally, capacity will match demand. Thus, there is a close link between forecasting and capacity planning, particularly in the long term. In the short term, emphasis shifts to describing and coping with variations in demand.

Development of capacity alternatives is enhanced by taking a systems approach to planning, by recognizing that capacity increments are often acquired in chunks, by designing flexible systems, and by considering product/service complements as a way of dealing with various patterns of demand.


Special service help when needed by transfers at the airport can help keep lines moving at the ticket counter.
c

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## Summary

## Solved Problems

## Problem 1

## Solution

## Problem 2

## Solution

In evaluating capacity alternatives. a manager must consider both quantitative and qualitative aspects. Quantitative analysis usually reflects economic factors, and qualitative considerations include intangibles such as public opinion and personal preferences of managers. Cost-volume analysis can be useful for analyzing alternatives.
break-even point (BEP), 000
capacity, 000

A firm's manager must decide whether to make or buy a certain item used in the production of vending machines. Cost and volume estimates are as follows:

|  | Make | Buy |
| :--- | :--- | :--- |
| Annual fixed cost | $\$ 150,000$ | None |
| Variable cost/unit | $\$ 60$ | $\$ 80$ |
| Annual volume (units) | 12,000 | 12,000 |

a. Given these numbers, should the firm buy or make this item?
$b$. There is a possibility that volume could change in the future. At what volume would the manager be indifferent between making and buying?
a. Determine the annual cost of each alternative:

Total cost $=$ Fixed cost + Volume $\times$ Variable cost
Make: $\quad \$ 150,000+12,000(\$ 60)=\$ 870,000$
Buy: $\quad 0+12,000(\$ 80)=\$ 960,000$
Because the annual cost of making the item is less than the annual cost of buying it, the manager would reasonably choose to make the item.
$b$. To determine the volume at which the two choices would be equivalent, set the two total costs equal to each other, and solve for volume: $\mathrm{TC}_{\text {make }}=\mathrm{TC}_{\text {buy }}$. Thus, $\$ 150,000+Q(\$ 60)$ $=0+Q(\$ 80)$. Solving, $Q=7,500$ units. Therefore, at a volume of 7,500 units a year, the manager would be indifferent between making and buying. For lower volumes, the choice would be to buy, and for higher volumes, the choice would be to make.


A small firm produces and sells novelty items in a five-state area. The firm expects to consolidate assembly of its electric turtle line at a single location. Currently, operations are in three widely scattered locations. The leading candidate for location will have a monthly fixed cost of $\$ 42,000$ and variable costs of $\$ 3$ per turtle. Turtles sell for $\$ 7$ each. Prepare a table that shows total profits, fixed costs, variable costs, and revenues for monthly volumes of $10,000,12,000$, and 15,000 units. What is the break-even point?

$$
\text { Revenue }=\$ 7 \text { per unit }
$$

Variable cost $=\$ 3$ per unit

$$
\begin{aligned}
\text { Fixed cost } & =\$ 42,000 \text { per month } \\
\text { Profit } & =Q(R-v)-\mathrm{FC} \\
\text { Total cost } & =\mathrm{FC}+v \times Q
\end{aligned}
$$

| Volume | Total <br> Revenue | Total <br> VC | Fixed <br> Cost | Total <br> Cost | Total <br> Profit |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $10,000 \ldots \ldots \ldots$ | $\$ 70,000$ | $\$ 30,000$ | $\$ 42,000$ | $\$ 72,000$ | $\$(2,000)$ |
| $12,000 \ldots \ldots$ | 84,000 | 36,000 | 42,000 | 78,000 | 6,000 |
| $15,000 \ldots \ldots$ | 105,000 | 45,000 | 42,000 | 87,000 | 18,000 |
| $Q_{\mathrm{BEP}}=\frac{\mathrm{FC}}{R-\mathrm{v}}=\frac{\$ 42,000}{\$ 7-\$ 3}=10,500$ units per month |  |  |  |  |  |

Refer to Problem 2. Develop an equation that can be used to compute profit for any volume. Use that equation to determine profit when volume equals 22,000 units.

$$
\begin{aligned}
& \text { Profit }=Q(R-v)-\mathrm{FC}=Q(\$ 7-\$ 3)-\$ 42,000=\$ 4 Q-\$ 42,000 \\
& \text { For } Q=22,000 \text {, profit is } \\
& \$ 4(22,000)-\$ 42,000=\$ 46,000
\end{aligned}
$$

A manager must decide which type of equipment to buy, Type A or Type B. Type A equipment costs $\$ 15,000$ each, and Type B costs $\$ 11,000$ each. The equipment can be operated 8 hours a day, 250 days a year.

Either machine can be used to perform two types of chemical analysis, C 1 and C 2 . Annual service requirements and processing times are shown in the following table. Which type of equipment should be purchased, and how many of that type will be needed? The goal is to minimize total purchase cost.

PROCESSING TIME PER ANALYSIS (HR)

| Analysis Type | Annual Volume | C1 | C2 |
| :---: | :---: | :---: | :---: |
| C1 | 1,200 | 1 | 2 |
| C2 | 900 | 3 | 2 |

Total processing time (annual volume $\times$ processing time per analysis) needed by type of equipment:

| Analysis Type | A | B |
| :---: | :---: | :---: |
| C1 | 1,200 | 2,400 |
| C2 | $\underline{2,700}$ | $\underline{1,800}$ |
|  | Total | 3,900 |

Total processing time available per piece of equipment is 8 hours/day $\times 250$ days/year $=2,000$. Hence, one piece can handle 2,000 hours of analysis, two pieces of equipment can handle 4,000 hours, etc.

Given the total processing requirements, two of Type A would be needed, for a total cost of $2 \times \$ 15,000=\$ 30,000$, or three of Type B, for a total cost of $3 \times \$ 11,000=\$ 33,000$. Thus, two pieces of Type A would have sufficient capacity to handle the load at a lower cost than three of Type B.

## Problem 3

Solution

Problem 4

## Solution

1. Contrast design capacity and effective capacity.
2. List and briefly explain three factors that may inhibit capacity utilization.
3. How do long-term and short-term capacity considerations differ?
4. Give an example of a good and a service that exhibit these seasonal demand patterns:
a. Annual
b. Monthly

## Discussion and <br> Review Questions

## Memo Writing Exercises

## Problems

## a. 46,000 units

b. (1) $\$ 3,000$
(2) $\$ 8,200$
c. 126,000 units
d. 25,556 units
e. Graph in IM
a. (A) 8,000 units
(B) 7,500 units
b. 10,000 units
c. $P_{A}=\$ 20,000$
$P_{B}=\$ 18,000$
a. 39,683 units
b. $\$ 1.71$
a. (A) $\$ 82$
(B) $\$ 92$
(C) $\$ 100$
b. Graph in IM
c. Weekly
d. Daily
5. Give some examples of building flexibility into system design.
6. Why is it important to adopt a "big picture" approach to capacity planning?
7. What is meant by "capacity in chunks," and why is that a factor in capacity planning?
8. What kinds of capacity problems do many elementary and secondary schools periodically experience? What are some alternatives to deal with those problems?
9. How can a systems approach to capacity planning be useful?
10. How do capacity decisions influence productivity?
11. Why is it important to match process capabilities with product requirements?
12. Briefly discuss how uncertainty affects capacity decisions.
13. Discuss the importance of capacity planning in deciding on the number of police officers or fire trucks to have on duty at a given time.

1. Write a short memo to your boss, Al Thomas, outlining the general impact on break-even quantities of an increase in the proportion of automation in a process.
2. Write a one-page memo to Don Jones, a production supervisor. Don has questioned your practice of sometimes scheduling production that is below capacity, resulting in less than full utilization of personnel and equipment.
3. A producer of pottery is considering the addition of a new plant to absorb the backlog of demand that now exists. The primary location being considered will have fixed costs of $\$ 9,200$ per month and variable costs of 70 cents per unit produced. Each item is sold to retailers at a price that averages 90 cents.
a. What volume per month is required in order to break even?
b. What profit would be realized on a monthly volume of 61,000 units? 87,000 units?
c. What volume is needed to obtain a profit of $\$ 16,000$ per month?
d. What volume is needed to provide a revenue of $\$ 23,000$ per month?
$e$. Plot the total cost and total revenue lines.
4. A small firm intends to increase the capacity of a bottleneck operation by adding a new machine. Two alternatives, $A$ and $B$, have been identified, and the associated costs and revenues have been estimated. Annual fixed costs would be $\$ 40,000$ for A and $\$ 30,000$ for B; variable costs per unit would be $\$ 10$ for $A$ and $\$ 12$ for $B$; and revenue per unit would be $\$ 15$ for $A$ and \$16 for B.
a. Determine each alternative's break-even point in units.
b. At what volume of output would the two alternatives yield the same profit?
c. If expected annual demand is 12,000 units, which alternative would yield the higher profit?
5. A producer of felt-tip pens has received a forecast of demand of 30,000 pens for the coming month from its marketing department. Fixed costs of $\$ 25,000$ per month are allocated to the felt-tip operation, and variable costs are 37 cents per pen.
$a$. Find the break-even quantity if pens sell for $\$ 1$ each.
b. At what price must pens be sold to obtain a monthly profit of $\$ 15,000$, assuming that estimated demand materializes?
6. A real estate agent is considering installing a cellular telephone in her car. There are three billing plans to choose from, all of which involve a weekly charge of $\$ 20$. Plan A has a cost of $\$ 0.45$ a minute for daytime calls and $\$ 0.20$ a minute for evening calls. Plan B has a charge of $\$ 0.55$ a minute for daytime calls and a charge of $\$ 0.15$ a minute for evening calls. Plan C has a flat rate of $\$ 80$ with 200 minutes of calls allowed per week and a cost of $\$ 0.40$ per minute beyond that, day or evening.
a. Determine the total charge under each plan for this case: 120 minutes of day calls and 40 minutes of evening calls in a week.
b. Prepare a graph that shows total weekly cost for each plan versus daytime call minutes.
$c$. If the agent will use the service for daytime calls, over what range of call minutes will each plan be optimal?
7. Refer to Problem 4. Suppose that the agent expects both daytime and evening calls. At what point (i.e., percentage of call minutes for daytime calls) would she be indifferent between plans A and B ?
8. A firm plans to begin production of a new small appliance. The manager must decide whether to purchase the motors for the appliance from a vendor at $\$ 7$ each or to produce them in-house. Either of two processes could be used for in-house production; one would have an annual fixed cost of $\$ 160,000$ and a variable cost of $\$ 5$ per unit, and the other would have an annual fixed cost of $\$ 190,000$ and a variable cost of $\$ 4$ per unit. Determine the range of annual volume for which each of the alternatives would be best.
9. A manager is trying to decide whether to purchase a certain part or to have it produced internally. Internal production could use either of two processes. One would entail a variable cost of $\$ 17$ per unit and an annual fixed cost of $\$ 200,000$; the other would entail a variable cost of $\$ 14$ per unit and an annual fixed cost of $\$ 240,000$. Three vendors are willing to provide the part. Vendor A has a price of $\$ 20$ per unit for any volume up to 30,000 units. Vendor B has a price of $\$ 22$ per unit for demand of 1,000 units or less, and $\$ 18$ per unit for larger quantities. Vendor C offers a price of $\$ 21$ per unit for the first 1,000 units, and $\$ 19$ per unit for additional units.
$a$. If the manager anticipates an annual volume of 10,000 units, which alternative would be best from a cost standpoint? For 20,000 units, which alternative would be best?
$b$. Determine the range for which each alternative is best. Are there any alternatives that are never best? Which?
10. A company manufactures a product using two machine cells. Each cell has a design capacity of 250 units per day and an effective capacity of 230 units per day. At present, actual output averages 200 units per cell, but the manager estimates that productivity improvements soon will increase output to 225 units per day. Annual demand is currently 50,000 units. It is forecasted that within two years, annual demand will triple. The company could produce at the rate of 400 per day using available capacity. How many cells should the company plan to produce to satisfy predicted demand under these conditions? Assume 240 workdays per year.
11. A manager must decide which type of machine to buy, A, B, or C. Machine costs are:

| Machine | Cost |
| :---: | :---: |
| A | $\$ 40,000$ |
| B | $\$ 30,000$ |
| C | $\$ 80,000$ |

Product forecasts and processing times on the machines are as follows:

|  |  | PROCESSING <br> TIME PER UNIT <br> (MINUTES) |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Demand | A |  |  |
|  | Annual | B | C |  |
| 1 | 16,000 | 3 | 4 | 2 |
| 2 | 12,000 | 4 | 4 | 3 |
| 3 | 6,000 | 5 | 6 | 4 |
| 4 | 30,000 | 2 | 2 | 1 |

Assume that only purchasing costs are being considered. Which machine would have the lowest total cost, and how many of that machine would be needed? Machines operate 10 hours a day, 250 days a year.
10. Refer to Problem 9. Consider this additional information: The machines differ in terms of hourly operating costs: The A machines have an hourly operating cost of \$10 each, B machines have an hourly operating cost of $\$ 11$ each, and $C$ machines have an hourly operating cost of $\$ 12$ each. Which alternative would be selected, and how many machines, in order to minimize total cost while satisfying capacity processing requirements?
11. A manager must decide how many machines of a certain type to purchase. Each machine can process 100 customers per hour. One machine will result in a fixed cost of $\$ 2,000$ per day,

minutes. C is optimal for 178 or
more.
33 percent daytime minutes
For $Q$ less than 63,333 , vendor is
best. For larger quantities, best to
produce in-house at $\$ 4$ per unit.
a. For 10,000 units:
Int. 1: \$370,000
Int. 2: \$380,000
Vend. A: \$200,000
Vend. B: $\$ 180,000$
Vend. C: \$192,000
For 20,000 units:
Int. 1: \$540,000
Int. 2: \$520,000
Vend. A: \$400,000
Vend. B: \$360,000
Vend. C: S382,000
b. Range Best Alt.
1,000 to $59,999 \quad$ B
60,000 or more Int. 2
Total processing time per machine:
A: 186,000
B: 208,000
C: 122,000
Number of each machine needed
and total cost:
A: 2, \$80,000
B: 2, \$60,000
C: 1, \$80,000
Buy 2 Bs
A. 111,000
B. 98,133
C. 104,400

Buy 2 machine B
b. 2


The High Acres Landfill is located on a 70-acre site outside Fairport, New York. Opened in 1971, it is licensed to handle residential, commercial, and industrial nonhazardous waste. The landfill has 27 employees, and it receives approximately 900 tons of waste per day.

The public often has certain preconceived notions about a landfill, chief among them that landfills are dirty and unpleasant. However, a visit to the landfill dispelled some of those misconceptions. The entrance is nicely landscaped. Most of the site is planted with grass and a few trees. Although unpleasant odors can emanate from arriving trucks or at the dump site, the remainder of the landfill is relatively free of odors.

A major portion of the landfill consists of a large hill within which the waste is buried. Initially, the landfill began not as a hill but as a large hole in the ground. After a number of years of depositing waste, the hole eventually was filled. From that point on, as additional layers were added, the landfill began to take the shape of a flattop hill. Each layer is a little narrower than the preceding one, giving the hill a slope. The sides of the
hill are planted with grass. Only the "working face" along the top remains unplanted. When the designated capacity is exhausted (this may take another 10 years), the landfill will be closed to further waste disposal. The site will be converted into a public park with hiking trails and picnic and recreation areas, and given to the town.

The construction and operation of landfills are subject to numerous state and federal regulations. For example, nonpermeable liners must be placed on the bottom and sides of the landfill to prevent leakage of liquids into the groundwater. (Independent firms monitor groundwater to determine if there is any leakage into wells placed around the perimeter of the hill.) Mindful of public opinion, every effort is made to minimize the amount of time that waste is left exposed. At the end of each day, the waste that has been deposited in the landfill is compacted and covered with six inches of soil.

The primary source of income for the landfill is the fees it charges users. The landfill also generates income from methane gas, a by-product of organic waste decomposition, that accumulates within the landfill. A collection system is in place to capture and extract the gas from the landfill, and it is then sold to the local power company. Also, the landfill has a composting operation in which leaves and other yard wastes are converted into mulch.


Part of the liner construction of a new landfill at the High Acres Landfill and Recycling Center in Fairport, New York. The hill in the background is a "closed" landfill, which has been through final cover.

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## Selected

Bibliography and
further Reciding

## SUPPLEMENT TO CHAPTER 5 Decision Theory

## SUPPLEMENT OUTLINE

Introduction, 000
Causes of Poor Decisions, 000
Decision Environments, 000
Decision Making under
Certainty, 000
Decision Making under
Uncertainty, 000
Decision Making under Risk, 000
Decision Trees, 000
Expected Value of Perfect
Information, 000

Sensitivity Analysis, 000
Summary, 000
Key Terms, 000
Solved Problems, 000
Discussion and Review
Questions, 000
Problems, 000
Selected Bibliography and Further Reading, 000

## LEARNING OBJECTIVES

After completing this supplement, you should be able to:

1 Describe the different environments under which operations decisions are made.
2 Describe and use techniques that apply to decision making under uncertainty.
3 Describe and use the expectedvalue approach.
4 Construct a decision tree and use it to analyze a problem.
5 Compute the expected value of perfect information.
6 Conduct sensitivity analysis on a simple decision problem.

## Introduction

Decision theory represents a general approach to decision making. It is suitable for a wide range of operations management decisions. Among them are capacity planning, product and service design, equipment selection, and location planning. Decisions that lend themselves to a decision theory approach tend to be characterized by these elements:

1. A set of possible future conditions exists that will have a bearing on the results of the decision.
2. A list of alternatives for the manager to choose from.
3. A known payoff for each alternative under each possible future condition.

To use this approach, a decision maker would employ this process:

1. Identify the possible future conditions (e.g., demand will be low, medium, or high; the number of contracts awarded will be one, two, or three; the competitor will or will not introduce a new product). These are called states of nature.
2. Develop a list of possible alternatives, one of which may be to do nothing.
3. Determine or estimate the payoff associated with each alternative for every possible future condition.
4. If possible, estimate the likelihood of each possible future condition.
5. Evaluate alternatives according to some decision criterion (e.g., maximize expected profit), and select the best alternative.
The information for a decision is often summarized in a payoff table, which shows the expected payoffs for each alternative under the various possible states of nature. These tables are helpful in choosing among alternatives because they facilitate comparison of alternatives. Consider the following payoff table, which illustrates a capacity planning problem.
payoff table Table showing the expected payoffs for each alternative in every possible state of nature.

The payoffs are shown in the body of the table. In this instance, the payoffs are in terms of present values, which represent equivalent current dollar values of expected future income less costs. This is a convenient measure because it places all alternatives on a comparable basis. If a small facility is built, the payoff will be the same for all three possible states of nature. For a medium facility, low demand will have a present value of $\$ 7 \mathrm{mil}$ lion, whereas both moderate and high demand will have present values of $\$ 12$ million. A large facility will have a loss of $\$ 4$ million if demand is low, a present value of $\$ 2$ million if demand is moderate, and a present value of $\$ 16$ million if demand is high.

The problem for the decision maker is to select one of the alternatives, taking the present value into account.

Evaluation of the alternatives differs according to the degree of certainty associated with the possible future conditions.

## Causes of Poor Decisions

Despite the best efforts of a manager, a decision occasionally turns out poorly due to unforeseeable circumstances. Luckily, such occurrences are not common. Often, failures can be traced to some combination of mistakes in the decision process, bounded rationality, or suboptimization.
bounded rationality The limitations on decision making caused by costs, human abilities, time, technology, and availability of information.
suboptimization The result of different departments each attempting to reach a solution that is optimum for that department.
certainty Environment in which relevant parameters have known values.
risk Environment in which certain future events have probable outcomes.
uncertainty Environment in which it is impossible to assess the likelihood of various future events.

In many cases, managers fail to appreciate the importance of each step in the decisionmaking process. They may skip a step or not devote enough effort to completing it before jumping to the next step. Sometimes this happens owing to a manager's style of making quick decisions or a failure to recognize the consequences of a poor decision. The manager's ego can be a factor. This sometimes happens when the manager has experienced a series of successes-important decisions that turned out right. Some managers then get the impression that they can do no wrong. But they soon run into trouble, which is usually enough to bring them back down to earth. Other managers seem oblivious to negative results and continue the process they associate with their previous successes, not recognizing that some of that success may have been due more to luck than to any special abilities of their own. A part of the problem may be the manager's unwillingness to admit a mistake. Yet other managers demonstrate an inability to make a decision; they stall long past the time when the decision should have been rendered.

Of course, not all managers fall into these traps-it seems safe to say that the majority do not. Even so, this does not necessarily mean that every decision works out as expected. Another factor with which managers must contend is bounded rationality, or the limits imposed on decision making by costs, human abilities, time, technology, and the availability of information. Because of these limitations, managers cannot always expect to reach decisions that are optimal in the sense of providing the best possible outcome (e.g., highest profit, least cost). Instead, they must often resort to achieving a satisfactory solution.

Still another cause of poor decisions is that organizations typically departmentalize decisions. Naturally, there is a great deal of justification for the use of departments in terms of overcoming span-of-control problems and human limitations. However, suboptimization can occur. This is a result of different departments attempting to reach a solution that is optimum for each. Unfortunately, what is optimal for one department may not be optimal for the organization as a whole.

## Decision Environments

Operations management decision environments are classified according to the degree of certainty present. There are three basic categories: certainty, risk, and uncertainty.
Certainty means that relevant parameters such as costs, capacity, and demand have known values.
Risk means that certain parameters have probabilistic outcomes.
Uncertainty means that it is impossible to assess the likelihood of various possible future events.

Consider these situations:

1. Profit per unit is $\$ 5$. You have an order for 200 units. How much profit will you make? (This is an example of certainty since unit profits and total demand are known.)
2. Profit is $\$ 5$ per unit. Based on previous experience, there is a 50 percent chance of an order for 100 units and a 50 percent chance of an order for 200 units. What is expected profit? (This is an example of risk since demand outcomes are probabilistic.)
3. Profit is $\$ 5$ per unit. The probabilities of potential demands are unknown. (This is an example of uncertainty.)

The importance of these different decision environments is that they require different analysis techniques. Some techniques are better suited for one category than for others. You should make note of the environments for which each technique is appropriate.

## Decision Making under Certainty

When it is known for certain which of the possible future conditions will actually happen, the decision is usually relatively straightforward: Simply choose the alternative that has the best payoff under that state of nature. Example S-1 illustrates this.

Determine the best alternative in the preceding payoff table for each of the cases: It is known with certainty that demand will be: (a) low, (b) moderate, (c) high.

Choose the alternative with the highest payoff. Thus, if we know demand will be low, we would elect to build the small facility and realize a payoff of $\$ 10$ million. If we know demand will be moderate, a medium factory would yield the highest payoff ( $\$ 12$ million versus either $\$ 10$ or $\$ 2$ million). For high demand, a large facility would provide the highest payoff.

Although complete certainty is rare in such situations, this kind of exercise provides some perspective on the analysis. Moreover, in some instances, there may be an opportunity to consider allocation of funds to research efforts, which may reduce or remove some of the uncertainty surrounding the states of nature.

## Decision Making under Uncertainty

At the opposite extreme is complete uncertainty: no information is available on how likely the various states of nature are. Under those conditions, four possible decision criteria are maximin, maximax, Laplace, and minimax regret. These approaches can be defined as follows:
Maximin-Determine the worst possible payoff for each alternative, and choose the alternative that has the "best worst." The maximin approach is essentially a pessimistic one because it takes into account only the worst possible outcome for each alternative. The actual outcome may not be as bad as that, but this approach establishes a "guaranteed minimum."
Maximax-Determine the best possible payoff, and choose the alternative with that payoff. The maximax approach is an optimistic, "go for it" strategy; it does not take into account any payoff other than the best.
Laplace-Determine the average payoff for each alternative, and choose the alternative with the best average. The Laplace approach treats the states of nature as equally likely.
Minimax regret-Determine the worst regret for each alternative, and choose the alternative with the "best worst." This approach seeks to minimize the difference between the payoff that is realized and the best payoff for each state of nature.

The next two examples illustrate these decision criteria.

Referring to the preceding payoff table, determine which alternative would be chosen under each of these strategies:
a. Maximin
b. Maximax
c. Laplace
a. Using maximin, the worst payoffs for the alternatives are:

Small facility: $\quad \$ 10$ million
Medium facility: 7 million
Large facility: $\quad-4$ million
Hence, since $\$ 10$ million is the best, choose to build the small facility using the maximin strategy.
b. Using maximax, the best payoffs are:
maximin Choose the alternative with the best of the worst possible payoffs.
maximax Choose the alternative with the best possible payoff.

Laplace Choose the alternative with the best average payoff of any of the alternatives.
minimax regret Choose the alternative that has the least of the worst regrets.

## Example S-2

## Solution

## Example S-1

## Solution

$\square$

## Fxample S-3

## Solution

regret The difference between a given payoff and the best payoff for a state of nature.

| Small facility: | $\$ 10$ million |
| :--- | ---: |
| Medium facility: | 12 million |
| Large facility: | 16 million |

The best overall payoff is the $\$ 16$ million in the third row. Hence, the maximax criterion leads to building a large facility.
c. For the Laplace criterion, first find the row totals, and then divide each of those amounts by the number of states of nature (three in this case). Thus, we have:

|  | Row Total <br> (in \$ millions) | Row Average <br> (in \$ millions) |
| :--- | :---: | :---: |
| Small facility | $\$ 30$ | $\$ 10.00$ |
| Medium facility | 31 | 10.33 |
| Large facility | 14 | 4.67 |

Because the medium facility has the highest average, it would be chosen under the Laplace criterion.

Determine which alternative would be chosen using a minimax regret approach to the capacity planning program.

The first step in this approach is to prepare a table of opportunity losses, or regrets. To do this, subtract every payoff in each column from the best payoff in that column. For instance, in the first column, the best payoff is 10 , so each of the three numbers in that column must be subtracted from 10 . Going down the column, the regrets will be $10-10$ $=0,10-7=3$, and $10-(-4)=14$. In the second column, the best payoff is 12 . Subtracting each payoff from 12 yields 2,0 , and 10 . In the third column, 16 is the best payoff. The regrets are 6,4 , and 0 . These results are summarized in a regret table:

|  | REGRETS (in \$ millions) |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Alternatives | Low | Moderate | High |  |
|  | Worst |  |  |  |
| Small facility | $\$ 0$ | $\$ 2$ | $\$ 6$ | $\$ 6$ |
| Medium facility | 3 | 0 | 4 | 4 |
| Large facility | 14 | 10 | 0 | 14 |

The second step is to identify the worst regret for each alternative. For the first alternative, the worst is 6 ; for the second, the worst is 4 ; and for the third, the worst is 14 .

The best of these worst regrets would be chosen using minimax regret. The lowest regret is 4 , which is for a medium facility. Hence, that alternative would be chosen.

Solved Problem 6 at the end of this supplement illustrates decision making under uncertainty when the payoffs represent costs.

The main weakness of these approaches (except for Laplace) is that they do not take into account all of the payoffs. Instead, they focus on the worst or best, and so they lose some information. Still, for a given set of circumstances, each has certain merits that can be helpful to a decision maker.

## Decision Making under Risk

Between the two extremes of certainty and uncertainty lies the case of risk: The probability of occurrence for each state of nature is known. (Note that because the states are mutually exclusive and collectively exhaustive, these probabilities must add to 1.00.) A widely used approach under such circumstances is the expected monetary value criterion.

The expected value is computed for each alternative, and the one with the highest expected value is selected. The expected value is the sum of the payoffs for an alternative where each payoff is weighted by the probability for the relevant state of nature. Thus, the approach is:
Expected monetary value (EMV) criterion-Determine the expected payoff of each alternative, and choose the alternative that has the best expected payoff.

Using the expected monetary value criterion, identify the best alternative for the previous payoff table for these probabilities: low $=.30$, moderate $=.50$, and high $=.20$.

Find the expected value of each alternative by multiplying the probability of occurrence for each state of nature by the payoff for that state of nature and summing them:

$$
\begin{aligned}
\mathrm{EV}_{\text {small }} & =.30(\$ 10)+.50(\$ 10)+.20(\$ 10)=\$ 10 \\
\mathrm{EV}_{\text {medium }} & =.30(\$ 7)+.50(\$ 12)+.20(\$ 12)=\$ 10.5 \\
\mathrm{EV}_{\text {large }} & =.30(-4)+.50(\$ 2)+.20(\$ 16)=\$ 3
\end{aligned}
$$

Hence, choose the medium facility because it has the highest expected value.

The expected monetary value approach is most appropriate when a decision maker is neither risk averse nor risk seeking, but is risk neutral. Typically, well-established organizations with numerous decisions of this nature tend to use expected value because it provides an indication of the long-run, average payoff. That is, the expected-value amount (e.g., $\$ 10.5$ million in the last example) is not an actual payoff but an expected or average amount that would be approximated if a large number of identical decisions were to be made. Hence, if a decision maker applies this criterion to a large number of similar decisions, the expected payoff for the total will approximate the sum of the individual expected payoffs.

## Decision Trees

A decision tree is a schematic representation of the alternatives available to a decision maker and their possible consequences. The term gets its name from the treelike appearance of the diagram (see Figure 5S-1). Although tree diagrams can be used in place of a payoff table, they are particularly useful for analyzing situations that involve sequential decisions.


## expected monetary value (EMV) criterion The best

 expected value among the alternatives.
## Example S-4

## Solution

decision tree A schematic representation of the available alternatives and their possible consequences.

## FICURE 5S-1

Format of a decision tree

## Example S-5

## Solution

For instance, a manager may initially decide to build a small facility only to discover that demand is much higher than anticipated. In this case, the manager may then be called upon to make a subsequent decision on whether to expand or build an additional facility.

A decision tree is composed of a number of nodes that have branches emanating from them (see Figure 5S-1). Square nodes denote decision points, and circular nodes denote chance events. Read the tree from left to right. Branches leaving square nodes represent alternatives; branches leaving circular nodes represent chance events (i.e., the possible states of nature).

After the tree has been drawn, it is analyzed from right to left; that is, starting with the last decision that might be made. For each decision, choose the alternative that will yield the greatest return (or the lowest cost). If chance events follow a decision, choose the alternative that has the highest expected monetary value (or lowest expected cost).

A manager must decide on the size of a video arcade to construct. The manager has narrowed the choices to two: large or small. Information has been collected on payoffs, and a decision tree has been constructed. Analyze the decision tree and determine which initial alternative (build small or build large) should be chosen in order to maximize expected monetary value.


The dollar amounts at the branch ends indicate the estimated payoffs if the sequence of chance events and decisions that is traced back to the initial decision occurs. For example, if the initial decision is to build a small facility and it turns out that demand is low, the payoff will be $\$ 40$ (thousand). Similarly, if a small facility is built, demand turns out high, and a later decision is made to expand, the payoff will be $\$ 55$. The figures in parentheses on branches leaving the chance nodes indicate the probabilities of those states of nature. Hence, the probability of low demand is .4 , and the probability of high demand is .6. Payoffs in parentheses indicate losses.

Analyze the decisions from right to left:

1. Determine which alternative would be selected for each possible second decision. For a small facility with high demand, there are three choices: do nothing, work overtime, and expand. Because expand has the highest payoff, you would choose it. Indicate this by placing a double slash through each of the other alternatives. Similarly, for a large facility with low demand, there are two choices: do nothing and reduce prices. You would choose reduce prices because it has the higher expected value, so a double slash is placed on the other branch.
2. Determine the product of the chance probabilities and their respective payoffs for the remaining branches:

Build small
Low demand $.4(\$ 40)=\$ 16$
High demand $.6(\$ 55)=33$
Build large
Low demand $.4(\$ 50)=20$
High demand $\quad 6(\$ 70)=42$
3. Determine the expected value of each initial alternative:
$\begin{array}{ll}\text { Build small } & \$ 16+\$ 33=\$ 49 \\ \text { Build large } & \$ 20+\$ 42=\$ 62\end{array}$
Hence, the choice should be to build the large facility because it has a larger expected value than the small facility.

## Expected Value of Perfect Information

In certain situations, it is possible to ascertain which state of nature will actually occur in the future. For instance, the choice of location for a restaurant may weigh heavily on whether a new highway will be constructed or whether a zoning permit will be issued. A decision maker may have probabilities for these states of nature; however, it may be possible to delay a decision until it is clear which state of nature will occur. This might involve taking an option to buy the land. If the state of nature is favorable, the option can be exercised; if it is unfavorable, the option can be allowed to expire. The question to consider is whether the cost of the option will be less than the expected gain due to delaying the decision (i.e., the expected payoff above the expected value). The expected gain is the expected value of perfect information, or EVPI.

Expected value of perfect information (EVPI)-the difference between the expected payoff with perfect information and the expected payoff under risk.
Other possible ways of obtaining perfect information depend somewhat on the nature of the decision being made. Information about consumer preferences might come from market research, additional information about a product could come from product testing, or legal experts might be called on.

There are two ways to determine the EVPI. One is to compute the expected payoff under certainty and subtract the expected payoff under risk. That is,

$$
\begin{align*}
& \text { Expected value of } \\
& \text { perfect information }
\end{aligned}=\begin{aligned}
& \text { Expected payoff }  \tag{5S-1}\\
& \text { under certainty }
\end{aligned}-\begin{aligned}
& \text { Expected payoff } \\
& \text { under risk }
\end{align*}
$$

Using the information from Example S-4, determine the expected value of perfect information using Formula 5S-1.
First, compute the expected payoff under certainty. To do this, identify the best payoff under each state of nature. Then combine these by weighting each payoff by the probability of that state of nature and adding the amounts. Thus, the best payoff under low demand is $\$ 10$, the best under moderate demand is $\$ 12$, and the best under high demand is $\$ 16$. The expected payoff under certainty is, then:

$$
.30(\$ 10)+.50(\$ 12)+.20(\$ 16)=\$ 12.2
$$

expected value of perfect information (EVPI) The difference between the expected payoff with perfect information and the expected payoff under risk.

## Example S-6

## Solution

| $\square$ | Th <br> fe <br> Th |
| :--- | :--- |
|  |  |
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The expected payoff under risk, as computed in Example 4, is $\$ 10.5$. The EVPI is the difference between these:

$$
\mathrm{EVPI}=\$ 12.2-\$ 10.5=\$ 1.7
$$

This figure indicates the upper limit on the amount the decision maker should be willing to spend to obtain perfect information in this case. Thus, if the cost equals or exceeds this amount, the decision maker would be better off not spending additional money and simply going with the alternative that has the highest expected payoff.

A second approach is to use the regret table to compute the EVPI. To do this, find the expected regret for each alternative. The minimum expected regret is equal to the EVPI.

## Solution

Determine the expected value of perfect information for the capacity-planning problem using the expected regret approach.

Using information from Examples 2, 3, and 4, we can compute the expected regret for each alternative. Thus:

Small facility $\quad .30(0)+.50(2)+.20(6)=2.2$
Medium facility $\quad .30(3)+.50(0)+.20(4)=1.7$ [minimum]
Large facility $\quad .30(14)+.50(10)+.20(0)=9.2$
The lowest expected regret is 1.7 , which is associated with the second alternative. Hence, the EVPI is $\$ 1.7$ million, which agrees with the previous example using the other approach.
sensitivity analysis
Determining the range of probability for which an alternative has the best expected payoff.

## Sensitivity Analysis

Generally speaking, both the payoffs and the probabilities in this kind of a decision problem are estimated values. Consequently, it can be useful for the decision maker to have some indication of how sensitive the choice of an alternative is to changes in one or more of these values. Unfortunately, it is impossible to consider all possible combinations of every variable in a typical problem. Nevertheless, there are certain things a decision maker can do to judge the sensitivity of probability estimates.

Sensitivity analysis provides a range of probability over which the choice of alternative would remain the same. The approach illustrated here is useful when there are two states of nature. It involves constructing a graph and then using algebra to determine a range of probabilities for which a given solution is best. In effect, the graph provides a visual indication of the range of probability over which the various alternatives are optimal, and the algebra provides exact values of the endpoints of the ranges. Example 8 illustrates the procedure.

Given the following table, determine the range of probability for state of nature \#2, that is, $P(2)$, for which each alternative is optimal under the expected-value approach.

|  |  | STATE OF <br> NATURE |  |
| :---: | :---: | :---: | ---: |
|  |  | \#1 | \#2 |
| Alternative | A | 4 | 12 |
|  | C | 16 | 2 |
|  |  | 12 | 8 |

First, plot each alternative relative to $P(2)$. To do this, plot the \#1 value on the left side of the graph and the \#2 value on the right side. For instance, for alternative A, plot 4 on the
left side of the graph and 12 on the right side. Then connect these two points with a straight line. The three alternatives are plotted on the graph as shown below.


The graph shows the range of values of $P(2)$ over which each alternative is optimal. Thus, for low values of $P(2)$ [and thus high values of $P(1)$, since $P(1)+P(2)=1.0$ ], alternative B will have the highest expected value; for intermediate values of $P(2)$, alternative C is best; and for higher values of $P(2)$, alternative A is best.

To find exact values of the ranges, determine where the upper parts of the lines intersect. Note that at the intersections, the two alternatives represented by the lines would be equivalent in terms of expected value. Hence, the decision maker would be indifferent between the two at that point. To determine the intersections, you must obtain the equation of each line. This is relatively simple to do. Because these are straight lines, they have the form $y$ $=a+b x$, where $a$ is the $y$-intercept value at the left axis, $b$ is the slope of the line, and $x$ is $P(2)$. Slope is defined as the change in $y$ for a one-unit change in $x$. In this type of problem, the distance between the two vertical axes is 1.0. Consequently, the slope of each line is equal to the right-hand value minus the left-hand value. The slopes and equations are:

|  | \#1 | \#2 | Slope | Equation |
| ---: | ---: | ---: | ---: | ---: |
| A | 4 | 12 | $12-4=+8$ | $4+8 P(2)$ |
| B | 16 | 2 | $2-16=-14$ | $16-14 P(2)$ |
| C | 12 | 8 | $8-12=-4$ | $12-4 P(2)$ |

From the graph, we can see that alternative B is best from $P(2)=0$ to the point where that straight line intersects the straight line of alternative C , and that begins the region where C is better. To find that point, solve for the value of $P(2)$ at their intersection. This requires setting the two equations equal to each other and solving for $P(2)$. Thus,

$$
16-14 P(2)=12-4 P(2)
$$

Rearranging terms yields

$$
4=10 P(2)
$$

Solving yields $P(2)=.40$. Thus, alternative B is best from $P(2)=0$ up to $P(2)=.40$. B and C are equivalent at $P(2)=.40$.

Alternative C is best from that point until its line intersects alternative A's line. To find that intersection, set those two equations equal and solve for $P(2)$. Thus,

$$
4+8 P(2)=12-4 P(2)
$$

Rearranging terms results in

Solving yields $P(2)=.67$. Thus, alternative C is best from $P(2)>.40$ up to $P(2)=.67$, where A and C are equivalent. For values of $P(2)$ greater than .67 up to $P(2)=1.0$, A is best.

Note: If a problem calls for ranges with respect to $P(1)$, find the $P(2)$ ranges as above, and then subtract each $P(2)$ from 1.00 (e.g., . 40 becomes .60 , and .67 becomes .33 ).

## Summary

## Key Terms

## Solved Problems

## Problem 1

## Solution

Problem 2

Decision making is an integral part of operations management.
Decision theory is a general approach to decision making that is useful in many different aspects of operations management. Decision theory provides a framework for the analysis of decisions. It includes a number of techniques that can be classified according to the degree of uncertainty associated with a particular decision problem. Two visual tools useful for analyzing some decision problems are decision trees and graphical sensitivity analysis.
bounded rationality, 000
certainty, 000
decision tree, 000
expected monetary value (EMV) criterion, 000
expected value of perfect information
(EVPI), 000
Laplace, 000
maximax, 000
maximin, 000
minimax regret, 000
opportunity losses, 000
payoff table, 000
regret, 000
risk, 000
sensitivity analysis, 000
suboptimization, 000
uncertainty, 000

The following solved problems refer to this payoff table:

|  | A | New <br> Bridge <br> Built | No <br> New <br> Bridge |
| :--- | :--- | :---: | :---: |
| Alternative capacity | B | 2 | 14 |
| for new store | C | 4 | 10 |
|  | C |  | 6 |

where $\mathrm{A}=$ small, $\mathrm{B}=$ medium, and $\mathrm{C}=$ large.
Assume the payoffs represent profits. Determine the alternative that would be chosen under each of these decision criteria:
a. Maximin.
b. Maximax.
c. Laplace.

| New <br> Bridge | No New <br> Bridge | Maximin <br> (worst) | Maximax <br> (best) | Laplace <br> (average) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 14 | 1 | 14 [best] | $15 \div 2=7.5$ [best] |
| B | 2 | 10 | 2 | 10 | $12 \div 2=6$ |
| C | 4 | 6 | 4 [best] | 6 | $10 \div 2=5$ |

Thus, the alternatives chosen would be C under maximin, A under maximax, and A under Laplace.
Using graphical sensitivity analysis, determine the probability for no new bridge for which each alternative would be optimal.

Plot a straight line for each alternative. Do this by plotting the payoff for new bridge on the left axis and the payoff for no new bridge on the right axis and then connecting the two points. Each line represents the expected profit for an alternative for the entire range of probability of no new bridge. Because the lines represent expected profit, the line that is highest for a given value of $P$ (no new bridge) is optimal. Thus, from the graph, you can see that for low values of this probability, alternative $C$ is best, and for higher values, alternative $A$ is best ( $B$ is never the highest line, so it is never optimal).


The dividing line between the ranges where C and A are optimal occurs where the two lines intersect. To find that probability, first formulate the equation for each line. To do this, let the intersection with the left axis be the $y$ intercept; the slope equals the right-side payoff minus the left-side payoff. Thus, for C you have $4+(6-4) P$, which is $4+2 P$. For A, $1+(14-1) P$, which is $1+13 P$. Setting these two equal to each other, you can solve for P :

$$
4+2 P=1+13 P
$$

Solving, $P=.27$. Therefore, the ranges for $P$ (no new bridge) are:
A: $.27<P \leq 1.00$
B: never optimal
C: $0 \leq P<.27$

Using the information in the payoff table, develop a table of regrets, and then:

To obtain the regrets, subtract all payoffs in each column from the best payoff in the column. The regrets are:
$\left.\begin{array}{ccc}\text { New } \\ \text { Bridge }\end{array} \quad \begin{array}{c}\text { No New } \\ \text { Bridge }\end{array}\right]$
a. Minimax regret involves finding the worst regret for each alternative and then choosing the alternative that has the "best" worst. Thus, you would choose A:

## Solution

$a$. Determine the alternative that would be chosen under minimax regret.
$b$. Determine the expected value of perfect information using the regret table, assuming that the probability of a new bridge being built is .60 .

|  | Worst |
| :--- | :--- |
| A | 3 [best] |
| B | 4 |
| C | 8 |

b. Once the regret table has been developed, you can compute the EVPI as the smallest expected regret. Since the probability of a new bridge is given as 60 , we can deduce that the probability of no new bridge is $1.00-.60=.40$. The expected regrets are:
A: $.60(3)+.40(0)=1.80$
B: $.60(2)+.40(4)=2.80$
C: $\quad .60(0)+.40(8)=3.20$
Hence, the EVPI is 1.80 .

Problem 4

## Solution

Problem 5

Solution

Using the probabilities of .60 for a new bridge and .40 for no new bridge, compute the expected value of each alternative in the payoff table, and identify the alternative that would be selected under the expected-value approach.

A: $.60(1)+.40(14)=6.20$ [best]
B: $\quad .60(2)+.40(10)=5.20$
C: $.60(4)+.40(6)=4.80$

Compute the EVPI using the information from the previous problem.
Using formula (5S-1), the EVPI is the expected payoff under certainty minus the maximum expected value. The expected payoff under certainty involves multiplying the best payoff in each column by the column probability and then summing those amounts. The best payoff in the first column is 4 , and the best in the second is 14 . Thus,

Expected payoff under certainty $=.60(4)+.40(14)=8.00$
Then

$$
\mathrm{EVPI}=8.00-6.20=1.80
$$

(This agrees with the result obtained in Solved Problem 3b.)
Excel solution:


Placing the problem data in the cell positions shown, the expected monetary value (EMV) for each alternative is shown in column J.

Then, the overall EMV is obtained in column J as the maximum of the values in $\mathrm{J} 6, \mathrm{~J} 7$, and J 8 .
The EVPI is obtained using the Opportunity Loss Table by summing the product of the maximum in column C 4 and the probability in C 4 , and the product of the maximum in column D and the probability in D4.

Suppose that the values in the payoff table represent costs instead of profits.
a. Determine the choice that you would make under each of these strategies: maximin, minimin, and Laplace.
b. Develop the regret table, and identify the alternative chosen using minimax regret. Then find the EVPI if $P$ (new bridge) $=.60$.
c. Using sensitivity analysis, determine the range of $P$ (no new bridge) for which each alternative would be optimal.
d. If $P($ new bridge $)=.60$ and $P($ no new bridge $)=.40$, find the alternative chosen to minimize expected cost.
a. Note: Minimin is the reverse of maximax; for costs minimin identifies the lowest (best) cost.

|  | New <br> Bridge | No New <br> Bridge | Maximin <br> (worst) | Minimin <br> (best) | Laplace <br> (average) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 14 | 14 | 1 [best] | $15 \div 2=7.5$ |
| B | 2 | 10 | 10 | 2 | $12 \div 2=6$ |
| C | 4 | 6 | 6 [best] | 4 | $10 \div 2=5$ [best] |

b. Develop the regret table by subtracting the lowest cost in each column from each of the values in the column. (Note that none of the values is negative.)

|  | New <br> Bridge | No New <br> Bridge | Worst |
| :--- | :---: | :---: | :--- |
| A | 0 | 8 | 8 |
| B | 1 | 4 | 4 |
| C | 3 | 0 | 3 [best] |
|  | EVPI $=.60(3)+.40(0)=1.80$ |  |  |

c. The graph is identical to that shown in Solved Problem 2. However, the lines now represent expected costs, so the best alternative for a given value of $P$ (no new bridge) is the lowest line. Hence, for very low values of $P$ (no new bridge), A is best; for intermediate values, B is best; and for high values, C is best. You can set the equations of A and B , and B and C , equal to each other in order to determine the values of $P$ (no new bridge) at their intersections. Thus,
$\mathrm{A}=\mathrm{B}: \quad 1+13 P=2+8 P$; solving, $P=.20$
$\mathrm{B}=\mathrm{C}: \quad 2+8 P=4+2 P$; solving, $P=.33$
Hence, the ranges are:
A best: $0 \leq P<.20$
B best: $.20<P<.33$
C best: $.33<P \leq 1.00$
d. Expected-value computations are the same whether the values represent costs or profits. Hence, the expected payoffs for costs are the same as the expected payoffs for profits that were computed in Solved Problem 4. However, now you want the alternative that has the lowest expected payoff rather than the one with the highest payoff. Consequently, alternative C is the best because its expected payoff is the lowest of the three.


1. What is the chief role of the operations manager?

## Discussion and <br> Review Cuestions

2. List the steps in the decision-making process.
3. Explain the term bounded rationality.
4. Explain the term suboptimization.
5. What are some of the reasons for poor decisions?

6 . What information is contained in a payoff table?
7. What is sensitivity analysis, and how can it be useful to a decision maker?
8. Contrast maximax and maximin decision strategies. Under what circumstances is each appropriate?
9. Under what circumstances is expected monetary value appropriate as a decision criterion? When isn't it appropriate?
10. Explain or define each of these terms:
a. Laplace criterion.
b. Minimax regret.
c. Expected value.
d. Expected value of perfect information.
11. What information does a decision maker need in order to perform an expected-value analysis of a problem? What options are available to the decision maker if the probabilities of the states of nature are unknown? Can you think of a way you might use sensitivity analysis in such a case?
12. Suppose a manager is using maximum EMV as a basis for making a capacity decision and, in the process, obtains a result in which there is a virtual tie between two of the seven alternatives. How is the manager to make a decision?

1. A small building contractor has recently experienced two successive years in which work opportunities exceeded the firm's capacity. The contractor must now make a decision on capacity for next year. Estimated profits under each of the two possible states of nature are as shown in the table below. Which alternative should be selected if the decision criterion is:
a. Maximax?
b. Maximin?
c. Laplace?
d. Minimax regret?

NEXT YEAR'S
DEMAND

| Alternative | Low | High |
| :--- | :---: | :---: |
| Do nothing | $\$ 50^{*}$ | $\$ 60$ |
| Expand | 20 | 80 |
| Subcontract | 40 | 70 |

*Profit in \$ thousands.
2. Refer to Problem 1. Suppose after a certain amount of discussion, the contractor is able to subjectively assess the probabilities of low and high demand: $P($ low $)=.3$ and $P($ high $)=.7$.
$a$. Determine the expected profit of each alternative. Which alternative is best? Why?
$b$. Analyze the problem using a decision tree. Show the expected profit of each alternative on the tree.
c. Compute the expected value of perfect information. How could the contractor use this knowledge?
3. Refer to Problems 1 and 2. Construct a graph that will enable you to perform sensitivity analysis on the problem. Over what range of $P$ (high) would the alternative of doing nothing be best? Expand? Subcontract?
4. A firm that plans to expand its product line must decide whether to build a small or a large facility to produce the new products. If it builds a small facility and demand is low, the net present value after deducting for building costs will be $\$ 400,000$. If demand is high, the firm can either maintain the small facility or expand it. Expansion would have a net present value of $\$ 450,000$, and maintaining the small facility would have a net present value of $\$ 50,000$.

If a large facility is built and demand is high, the estimated net present value is $\$ 800,000$. If demand turns out to be low, the net present value will be $-\$ 10,000$.

The probability that demand will be high is estimated to be .60 , and the probability of low demand is estimated to be .40 .
a. Analyze using a tree diagram.
b. Compute the EVPI. How could this information be used?
c. Determine the range over which each alternative would be best in terms of the value of $P$ (demand low).
5. Determine the course of action that has the highest expected payoff for the following decision tree.
6. The lease of Theme Park, Inc., is about to expire. Management must decide whether to renew the lease for another 10 years or to relocate near the site of a proposed motel. The town planning board is currently debating the merits of granting approval to the motel. A consultant has estimated the net present value of Theme Park's two alternatives under each state of nature as shown below. What course of action would you recommend using:
a. Maximax?
b. Maximin?
c. Laplace?
d. Minimax regret?

| Options | Motel <br> Approved | Motel <br> Rejected |
| :--- | ---: | ---: |
| Renew | $\$ 500,000$ | $\$ 4,000,000$ |
| Relocate | $5,000,000$ | 100,000 |



a. Renew
\$2,775,000
Relocate $\quad \$ 1,815,000$
Decision: Renew lease
b. Graph in $I M$
c. $\mathrm{EVPI}=\$ 1,575,000$

Sign lease

[^0]
7. Refer to Problem 6. Suppose that the management of Theme Park, Inc., has decided that there is a .35 probability that the motel's application will be approved.
$a$. If management uses maximum expected monetary value as the decision criterion, which alternative should it choose?
$b$. Represent this problem in the form of a decision tree.
c. If management has been offered the option of a temporary lease while the town planning board considers the motel's application, would you advise management to sign the lease? The lease will cost $\$ 24,000$.
8. Construct a graph that can be used for sensitivity analysis for the preceding problem.
a. How sensitive is the solution to the problem in terms of the probability estimate of .35 ?
$b$. Suppose that, after consulting with a member of the town planning board, management decides that an estimate of approval is approximately .45. How sensitive is the solution to this revised estimate? Explain.
c. Suppose the management is confident of all the estimated payoffs except for $\$ 4$ million. If the probability of approval is .35 , for what range of payoff for renew/rejected will the alternative selected using maximum expected value remain the same?
9. A firm must decide whether to construct a small, medium, or large stamping plant. A consultant's report indicates a .20 probability that demand will be low and an .80 probability that demand will be high.

If the firm builds a small facility and demand turns out to be low, the net present value will be $\$ 42$ million. If demand turns out to be high, the firm can either subcontract and realize the net present value of $\$ 42$ million or expand greatly for a net present value of $\$ 48$ million.

The firm could build a medium-size facility as a hedge: If demand turns out to be low, its net present value is estimated at $\$ 22$ million; if demand turns out to be high, the firm could do nothing and realize a net present value of $\$ 46$ million, or it could expand and realize a net present value of $\$ 50$ million.

If the firm builds a large facility and demand is low, the net present value will be $-\$ 20$ million, whereas high demand will result in a net present value of $\$ 72$ million.
a. Analyze this problem using a decision tree.
b. What is the maximin alternative?
c. Compute the EVPI and interpret it.
d. Perform sensitivity analysis on $P$ (high).
10. A manager must decide how many machines of a certain type to buy. The machines will be used to manufacture a new gear for which there is increased demand. The manager has narrowed the decision to two alternatives: buy one machine or buy two. If only one machine is purchased and demand is more than it can handle, a second machine can be purchased at a later time. However, the cost per machine would be lower if the two machines were purchased at the same time.

The estimated probability of low demand is .30 , and the estimated probability of high demand is .70 .

The net present value associated with the purchase of two machines initially is $\$ 75,000$ if demand is low and $\$ 130,000$ if demand is high.

The net present value for one machine and low demand is $\$ 90,000$. If demand is high, there are three options. One option is to do nothing, which would have a net present value of $\$ 90,000$. A second option is to subcontract; that would have a net present value of $\$ 110,000$. The third option is to purchase a second machine. This option would have a net present value of $\$ 100,000$.

How many machines should the manager purchase initially? Use a decision tree to analyze this problem.
11. Determine the course of action that has the highest EMV for the accompanying tree diagram.

a. Figure in IM
b. Build large: $\quad \$ 53.6$ million
Build small: $\quad \$ 42$ million

c. | EPC $=\$ 66.0$ |  |
| :--- | :--- |
|  | $\mathrm{EVPI}=\$ 12.4$ |
| d. | P(high) $=x$ |
| Small: | $42+6 x$ |
| Medium: $\quad 22+28 x$ |  |
| Large: $\quad-20+92 x$ |  |
| Small and large EV |  |
| Same for $x=0.7209$ |  |

Graph in IM

Buy 2 machines
\$113,500

Alt. A: 22
Alt. B: 25
Choose Alt. B

| Graph in IM |  |
| :--- | :--- |
| Large | $\$ 476,000$ |
| Small | $\$ 430,000$ |


|  | Worst | Best | Avg. |
| :--- | :---: | :---: | :---: |
| Reassign | 85 | 50 | 65 |
| New staff | 60 | 60 | 60 |
| Redesign | 90 | 40 | 60 |
| a. New staff |  |  |  |
| b. Redesign |  |  |  |
| c. Table in IM |  |  |  |
| New staff |  |  |  |
| d. (tie) New staff or redesign |  |  |  |


| a. Reassign | $\$ 74$ |
| :--- | ---: |
| New staff | $\$ 60$ |
| Redesign | $\$ 73$ |
| b. Figure in IM |  |
| c. |  |
| Reassign | 19 |
| New staff | 5 |
| Redesign | 18 |

Figure in $I M$
12. A firm that plans to expand its product line must decide whether to build a small or a large facility to produce the new products. If it builds a small facility and demand is low, the net present value after deducting for building costs will be $\$ 400,000$. If demand is high, the firm can either maintain the small facility or expand it. Expansion would have a net present value of $\$ 450,000$, and maintaining the small facility would have a net present value of $\$ 50,000$.

If a large facility is built and demand is high, the estimated net present value is $\$ 800,000$. If demand turns out to be low, the net present value will be $-\$ 10,000$.

The probability that demand will be high is estimated to be .60 , and the probability of low demand is estimated to be .40 .

Analyze using a tree diagram.
13. The director of social services of a county has learned that the state has mandated additional information requirements. This will place an additional burden on the agency. The director has identified three acceptable alternatives to handle the increased workload. One alternative is to reassign present staff members, the second is to hire and train two new workers, and the third is to redesign current practice so that workers can readily collect the information with little additional effort. An unknown factor is the caseload for the coming year when the new data will be collected on a trial basis. The estimated costs for various options and caseloads are shown in the following table:

|  | CASELOAD |  |  |
| :--- | :---: | :---: | :---: |
|  | Moderate | High | Very High |
| Reassign staff | $\$ 50^{*}$ | 60 | 85 |
| New staff | 60 | 60 | 60 |
| Redesign collection | 40 | 50 | 90 |
| *Cost in \$ thousands. |  |  |  |

Assuming that past experience has shown the probabilities of various caseloads to be unreliable, what decision would be appropriate using each of the following criteria?
a. Maximin.
b. Maximax.
c. Minimax regret.
d. Laplace.
14. After contemplating the caseload question (see previous problem), the director of social services has decided that reasonable caseload probabilities are .10 for moderate, .30 for high, and .60 for very high.
$a$. Which alternative will yield the minimum expected cost?
b. Construct a decision tree for this problem. Indicate the expected costs for the three decision branches.
c. Determine the expected value of perfect information using an opportunity loss table.
15. Suppose the director of social services has the option of hiring an additional staff member if one staff member is hired initially and the caseload turns out to be high or very high. Under that plan, the first entry in row 2 of the cost table (see Problem 13) will be 40 instead of 60, the second entry will be 75 , and the last entry will be 80 . Assume the caseload probabilities are as noted in Problem 14. Construct a decision tree that shows the sequential nature of this decision, and determine which alternative will minimize expected cost.
16. A manager has compiled estimated profits for various capacity alternatives but is reluctant to assign probabilities to the states of nature. The payoff table is:

|  |  | STATE OF |  |
| :---: | :---: | :---: | ---: |
|  |  | \#ATURE |  |
|  | \#1 | \#2 |  |
| Alternative | A | $\$ 20^{*}$ | 140 |
|  | B | 120 | 80 |
|  | C | 100 | 40 |

* Cost in \$ thousands.
a. Plot the expected-value lines on a graph.
b. Is there any alternative that would never be appropriate in terms of maximizing expected profit? Explain on the basis of your graph.
c. For what range of $P(2)$ would alternative A be the best choice if the goal is to maximize expected profit?
d. For what range of $P(1)$ would alternative A be the best choice if the goal is to maximize expected profit?

17. Repeat all parts of Problem 16, assuming the values in the payoff table are estimated costs and the goal is to minimize expected costs.
18. The research staff of a marketing agency has assembled the following payoff table of estimated profits:

|  |  | Receive <br> Contract | Not Receive <br> Contract |
| :---: | :---: | :---: | :---: |
| Proposal | $\# 1$ | $\$ 10^{*}$ | -2 |
|  | $\# 2$ | 8 | 3 |
|  | $\# 3$ | 5 | 5 |
|  | $\# 4$ | 0 | 7 |

*Cost in \$ thousands.
Relative to the probability of not receiving the contract, determine the range of probability for which each of the proposals would maximize expected profit.
19. Given this payoff table:

|  |  | STATE OF <br> NATURE |  |
| :---: | :---: | :---: | ---: |
|  |  | \#1 | \#2 |
| Alternative | A | $\$ 120^{*}$ | 20 |
|  | B | 60 | 40 |
|  | $C$ | 10 | 110 |
|  | D | 90 | 90 |

*Cost in \$ thousands.
a. Determine the range of $P(1)$ for which each alternative would be best, treating the payoffs as profits.
$b$. Answer part $a$ treating the payoffs as costs.

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## a. Figure in $I M$

b. $C$ is always lower than $B$

C is never appropriate.
c. $P(2)>.625$
d. $P(1)<.375$
b. B is not appropriate
c. $P(2)<.444$
d. $P(1)>.556$
$E V_{1}=10-12 P$
$\mathrm{EV}_{2}=8-5 P$
$\mathrm{EV}_{3}=5-0 P$
$\mathrm{EV}_{4}=0+7 P$
1: $0 \leq P(2)<.286$
2: $.286<P(2)<.60$
3: $.60<P(2)<.714$
4: . $714<P(2) \leq 1.0$
a. A: $.70<P(1) \leq 1.00$

B: never
C: $0 \leq P(1)<.20$
D: $.20<P(1)<.70$
b. A: $0 \leq P(1)<.25$

B: $.25<P(1)<.583$
C: $.583<P(1) \leq 1.00$
D: never

## Selected <br> Blbliography and <br> Further Recidine


[^0]:    $a, b$ : Break-even probability of 46\%
    c. Range is $\$ 2,523,077$ or more

