

Elementary Algebra Review
to accompany
Elementary and Intermediate Algebra
Dugopolski

Section EA.1 Real Numbers and Their Properties

In this section:

- The Real Numbers
- Fractions
- Operations on Real Numbers
- Algebraic Expressions
- Properties of the Real Numbers

The Real Numbers

A **set** is a collection of objects. We use braces $\{ \}$ to enclose the members of a set. The symbol \in is used to designate a member of a set. For example, consider the set $A = \{a, b, c\}$, we can say that $a \in A$ because a is a member of set A . $d \notin A$, d is not a member of set A . We classify all real numbers into sets according to characteristics such as positive or negative sign, representation as a terminating or non-terminating decimal, and so on.

The set of **Natural Numbers** is the set of counting numbers. Natural Numbers could occur in nature. For example, we count three trees, four dogs, two people. The set of Natural Numbers is designated with the letter N . To use set notation, $N = \{1, 2, 3, \dots\}$. The three dots inside the braces are the mathematical version of “et cetera (etc)”, called an ellipsis, and are understood to mean that “the pattern continues”.

Notice that the set of Natural Numbers does not include a symbol to represent a zero quantity. The set of **Whole Numbers** includes all Natural Numbers and 0. The set of Whole Numbers is designated with the letter W . To use set notation, $W = \{0, 1, 2, 3, \dots\}$.

Neither the set of Natural Numbers nor the set of Whole Numbers includes a representation of signed numbers. Signed numbers are used to represent an increase or deficit, or a change in direction. The set of **Integers** includes the Whole Numbers with both positive and negative value. The set of Integers is designated with the letter I . To use set notation, $I = \{\dots, -1, 0, 1, \dots\}$.

Sometimes we need a number to represent a fractional quantity. The set of **Rational Numbers** is defined as $Q = \left\{ \frac{a}{b}, \text{ such that } a \text{ and } b \text{ are both integers, with } b \neq 0 \right\}$. When a rational number is converted to decimal form, the decimal either terminates or repeats indefinitely. For example, 2 is a rational number, $\frac{1}{2} = 0.5$ is a rational number, $\frac{1}{3} = 0.333\dots$ is a rational number.

If a number is not rational, we say that the number is irrational. **Irrational Numbers** when converted to decimal form neither terminate nor repeat. For example π is an irrational

number. Any square root that is not an integer is an irrational number. For example, $\sqrt{2}$ is irrational.

The **absolute value** of a number, $|a|$, is the distance between that number and zero. For example, $|4| = 4$ and $|-4| = 4$.

Fractions

All rational numbers can be expressed as **fractions**. The **denominator** of a fraction shows the number of equivalent parts in the whole, and the **numerator** shows how many parts are being considered. For example, $\frac{3}{4}$ represents 3 of 4 equal parts. We classify fractions in several ways. A fraction is considered to be **proper** if the value of the numerator is less than the value of the denominator. A fraction is considered to be **improper** if the value of the numerator is larger than the value of the denominator. For example, $\frac{2}{3}$ is a proper fraction, $\frac{7}{3}$ is an improper fraction.

We can write improper fractions as **mixed numbers**. We convert an improper fraction to a mixed number by dividing the numerator by the denominator and writing the result as a whole number and a fraction. The numerator is the remainder when dividing. For example, the improper fraction $\frac{7}{3}$ becomes the mixed number $2\frac{1}{3}$. ($7 \div 3 = 2$ with a remainder of 1.)

Two fractions are considered to be **equivalent** if the fractions can be reduced to the same value. For example, $\frac{16}{10} = \frac{8}{5}$. Equivalent fractions can be obtained by multiplying or dividing the numerator and denominator by the same nonzero number.

The **rule for multiplying fractions** is $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$, where $b \neq 0$ and $d \neq 0$. For example, $\frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$. Be sure to reduce your answer.

The **rule for dividing fractions** is $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where $b \neq 0$ and $c \neq 0$. For example, $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$. Be sure to reduce your answer.

The **rule for adding two fractions** with equal denominators is $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$. Be sure to reduce your answer.

The **rule for subtracting two fractions** with equal denominators is $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$. Be sure to reduce your answer.

In order to add or subtract two fractions, the fractions must have a **common denominator**.

Operations on Real Numbers

Signed numbers are real numbers with either positive or negative value. If we are combining two numbers and both are positive or both are negative, we say that the numbers have **like signs**. If we are combining two numbers and one is positive and one is negative, we say that the numbers have **unlike signs**.

To add two numbers with like sign, we add their absolute values. The sum has the same sign as the given numbers. For example, $-2 + -5 = -7$. **To add two numbers with unlike sign**, we subtract their absolute values. The answer is assigned the sign of the larger number. For example, $-2 + 5 = 3$.

When we **multiply or divide two numbers with like signs**, the result is a positive number. For example, $-2 \cdot -5 = 10$. When we **multiply or divide two numbers with unlike signs**, the result is a negative number. For example, $-2 \cdot 5 = -10$.

The **order of operations** is a protocol that helps us to determine the order in which to perform mathematical operations. The order in which to perform mathematical operations is:

- Work from left to right
- Perform all operations within grouping symbols
- Perform all exponential operations
- Perform multiplication and division
- Perform addition and subtraction

You may find the following mnemonic device helpful to recall the order of operations: “Please Excuse My Dear Aunt Sally” (Parentheses, Exponents, Multiply, Divide, Add, Subtract).

Algebraic Expressions

An **algebraic expression** is a combination of numbers and variables with the basic operations of arithmetic (add, subtract, multiply, divide). An algebraic expression does not have an equal sign and may not be solved. An algebraic expression may be evaluated for a given value of a variable, for example: $x + y$ has a value of $3 + 4 = 7$ when $x = 3$ and $y = 4$.

Properties of the Real Numbers

Properties of the real numbers are attributes of those numbers. The commutative properties apply to the operations of addition and multiplication for real numbers. The commutative property relates to the order in which we perform these operations. The **Commutative Property for Addition of Real Numbers** is stated as follows: $a + b = b + a$. The **Commutative Property for Multiplication of Real Numbers** is stated as follows: $ab = ba$. The associative properties apply to the grouping of real numbers when performing addition or multiplication. The **Associative Property for Addition of Real Numbers** is stated as follows:

$a + (b + c) = (a + b) + c$. The **Associative Property for Multiplication of Real Numbers** is stated as follows: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. The **Distributive Property for Real Numbers** is stated as follows: $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. The **Additive Identity for Real Numbers** is 0. We state the property: $a + 0 = a$. The **Multiplicative Identity for Real Numbers** is 1. We state the property: $a \cdot 1 = a$. The **Additive Inverse of a Real Number** is also known as the **opposite** of that number. The **Additive Inverse Property** is stated as follows:

$a + -a = 0$. The **Multiplicative Inverse of a Real Number** is also known as the **reciprocal** of that number. The **Multiplicative Inverse Property for a Non-Zero Real Number** is stated as

follows: $a \cdot \frac{1}{a} = 1$

Exercises.

Which of the numbers $\sqrt{2}, -6, \frac{2}{3}, \pi, 12, 0,$ and 5 are

- | | |
|------------------------|----------------------|
| 1. natural numbers? | 2. whole numbers? |
| 3. integers? | 4. rational numbers? |
| 5. irrational numbers? | 6. real numbers? |

True or false? Explain your answer.

- Every whole number is a natural number.
- Every integer is a rational number.
- The decimal number 0.45 is an irrational number.
- Every integer greater than -1 is a whole number.
- The square root of the number 16 is irrational.

Evaluate the following.

- | | |
|--------------|------------|
| 12. $ -2 $ | 13. $ 34 $ |
| 14. $- -12 $ | 15. $- 5 $ |

Perform the indicated operations.

- | | |
|-------------------------------------|--------------------------------------|
| 16. $\frac{1}{5} + \frac{3}{5}$ | 17. $\frac{3}{5} + \frac{4}{5}$ |
| 18. $\frac{1}{3} + \frac{1}{4}$ | 19. $\frac{2}{3} + \frac{3}{4}$ |
| 20. $\frac{3}{4} - \frac{1}{4}$ | 21. $\frac{4}{5} - \frac{3}{4}$ |
| 22. $6 \cdot \frac{2}{3}$ | 23. $\frac{2}{7} \cdot \frac{3}{14}$ |
| 24. $\frac{3}{4} \cdot \frac{8}{9}$ | 25. $6 \div \frac{2}{3}$ |
| 26. $\frac{2}{3} \div 6$ | 27. $\frac{3}{4} \div \frac{8}{9}$ |

Evaluate the following.

- | | |
|------------------------------|-------------------------------|
| 28. $-2 \cdot -27$ | 29. $-17 \cdot 3$ |
| 30. -3^2 | 31. $(-3)^2$ |
| 32. $(2 - 5) \cdot (12 - 9)$ | 33. $2^2 - 7 + 3^2$ |
| 34. $3 + 5(2 - 6)$ | 35. $6 \div 3 \cdot 4 \div 2$ |

Let $a = -3$, $b = 5$ and $c = -6$. Find the value of the following.

36. a^2

37. ab^2

38. $a^2 - c^2$

39. $12a + 3b$

40. $\frac{ab}{c}$

Name the property that justifies each statement.

41. $ax + ay = a(x + y)$

42. $(a + b) + c = c + (a + b)$

43. $ab = ba$

44. $a \cdot \frac{1}{a} = 1$

45. $(a + b) + c = a + (b + c)$

SMART CD Lessons:

Topic	SMART CD Location	SMART CD Lessons
The Real Numbers	1.1	<ol style="list-style-type: none"> 1. Comparing Numbers on the Number Line 2. Natural Numbers, Whole Numbers, and Integers 3. Types of Real Numbers 4. Absolute Value
Fractions	1.2	<ol style="list-style-type: none"> 1. Multiplying Fractions 2. Dividing Fractions 3. Adding Fractions 4. Subtracting Fractions
Operations on Real Numbers	1.3	<ol style="list-style-type: none"> 1. Addition of Signed Numbers 2. Subtraction of Signed Numbers
Operations on Real Numbers	1.4	<ol style="list-style-type: none"> 1. Multiplication of Signed Numbers 2. Division of Signed Numbers
Algebraic Expressions	1.5	<ol style="list-style-type: none"> 1. Evaluating Arithmetic Expressions 2. Evaluating Exponential Expressions 3. The Order of Operations
Algebraic Expressions	1.6	<ol style="list-style-type: none"> 1. Naming Expressions 2. Evaluating Algebraic Expressions 3. Solutions to Equations
Properties of the Real Numbers	1.7	<ol style="list-style-type: none"> 1. The Commutative and Associative Properties 2. The Distributive Property 3. More Properties
Properties of the Real Numbers	1.8	<ol style="list-style-type: none"> 1. Using the Properties in Computation 2. Combining Like Terms 3. Finding Products and Quotients 4. Removing Parentheses

Video Lesson Locations:

Topic	Video Location
The Real Numbers	Chapter One Tape 1:41
Fractions	Chapter One Tape 9:30
Operations on Real Numbers	Chapter One Tape 22:01
Operations on Real Numbers	Chapter One Tape 28:52
Algebraic Expressions	Chapter One Tape 33:38
Algebraic Expressions	Chapter One Tape 46:52
Properties of the Real Numbers	Chapter One Tape 59:11
Properties of the Real Numbers	Chapter One Tape 1:13:59

Midtext Diagnostic Review Exercises:

Topic	Text Location
Fractions	Page 391, Problems 3&4
Algebraic Expressions	Page 391, Problems 1&2
Properties of the Real Numbers	Page 391, Problems 5-12

Section EA.2 Linear Equations and Inequalities in One Variable

In this section:

- Solving Linear Equations
- Literal Equations
- Solving Linear Inequalities
- Compound Inequalities
- Absolute Value Equations and Inequalities
- Application Problems

Solving Linear Equations

A **linear equation** in one variable x is an equation that can be written in the form $ax + b = 0$, where a and b are real numbers and $a \neq 0$. To **solve** an equation means to find all of the solutions to the equation. The set of all solutions to an equation is the **solution set** to the equation. Equations that have the same solution set are **equivalent equations**.

In solving equations, our goal is to isolate the variable. We use either the **Addition Property of Equality** or the **Multiplication Property of Equality** to eliminate unwanted terms. The **Addition Property of Equality** states that adding the same number to both sides of an equation does not change the solution to the equation. In symbols, if $a = b$, then $a + c = b + c$. For example, to solve the equation $x + 1.2 = 4$, we subtract 1.2 from both sides to find the solution $x = 2.8$. The **Multiplication Property of Equality** states that multiplying both sides of an equation by the same nonzero number does not change the solution to the equation. In symbols, if $a = b$ and $c \neq 0$, then $ac = bc$. For example, to solve the equation $5x = 4.5$, we multiply both sides of the equation by $\frac{1}{5}$ to find the solution $x = 0.9$.

To solve a linear equation of the form $ax + b = cx + d$ we use a combination of the addition and the multiplication properties of equality. For example, to solve the equation $2x + 5 = 4x - 7$ the following steps are necessary: $2x + 5 - 5 = 4x - 7 - 5$; the result of this operation is the equation $2x = 4x - 12$, we then subtract $4x$ from both sides; $2x - 4x = 4x - 4x - 12$ for a result of $-2x = -12$. We then multiply both sides by $-\frac{1}{2}$ to find the solution $x = 6$.

An equation for which the solution is all real numbers is known as an **identity**. An equation for which a finite number of solutions exist is known as a **conditional equation**. An equation that has no solution is called an **inconsistent equation**.

Literal Equations

A **literal equation** is an equation involving two or more variables. **Formulas** are literal equations. The process of rewriting a formula for one variable in terms of another is called **solving for a certain variable**. To **solve for a certain variable**, we use the same techniques that we use in solving equations. For example, to rewrite the formula for the perimeter of a rectangle in terms of the length, the problem would be stated: “Solve $P = 2L + 2W$ for L ”. We would first subtract $2W$ from both sides of the equation: $P - 2W = 2L + 2W - 2W$. The result of this operation is the equation: $P - 2W = 2L$. To complete the solution for L , we divide both sides of the equation by 2: $\frac{P - 2W}{2} = L$. At each step of the solution process, we have been working with a literal equation.

Solving Linear Inequalities

Statements that express the inequality of algebraic expressions are called **inequalities**. We use the following symbols to represent inequality relationships:

<u>Symbol</u>	<u>Meaning</u>
$<$	is less than
\leq	is less than or equal to
$>$	is greater than
\geq	is greater than or equal to

A **linear inequality** in one variable x is any inequality that can be written in the form $ax + b < 0$, where a and b are real numbers, with $a \neq 0$. In place of $<$ we may also use \leq , $>$, or \geq .

To solve a linear inequality, we use the **Addition Property of Inequality** and the **Multiplication Property of Inequality**. The **Addition Property of Inequality** states that: if the same number is added to both sides of an inequality, then the solution set to the inequality is unchanged. For example, we can solve the linear inequality $x - 2.5 < 6$ by adding 2.5 to both sides of the relationship; $x - 2.5 + 2.5 < 6 + 2.5$. The result of adding 2.5 to both sides of the inequality is the solution, $x < 8.5$. The **Multiplication Property of Inequality** states that: if both sides of an inequality are multiplied by the same positive number, then the solution set to the inequality is unchanged. For example, to solve the linear inequality $2x < -12$, we multiply both sides of the inequality by $\frac{1}{2}$; $\frac{1}{2} \cdot 2x < \frac{1}{2} \cdot -12$. The result of multiplying both sides of the inequality by $\frac{1}{2}$ is the solution $x < -6$.

If both sides of an inequality are multiplied by the same negative number, the inequality symbol must be reversed. For example, to solve the linear inequality $\frac{x}{-3} > 5$, we multiply both sides by -3 and change the direction of the inequality symbol: $-3 \cdot \frac{x}{-3} < -3 \cdot 5$. The solution is $x < -15$.

Compound Inequalities

A **compound inequality** is a way of combining two inequality relationships. For example, the relationships $x > 3$ and $x < 6$ can be written $3 < x < 6$. To solve a compound inequality, we apply the Addition Property of Inequality or the Multiplication Property of Inequality as necessary. For example, to solve the relationship $5 < 2x + 3 < 12$, we must first subtract 3 from all parts of the relationship; $5 - 3 < 2x + 3 - 3 < 12 - 3$. The result of subtracting 3 from all parts of the relationship is the compound inequality $2 < 2x < 9$. We now multiply each part of the relationship by $\frac{1}{2}$ to complete the solution process; $\frac{1}{2} \cdot 2 < \frac{1}{2} \cdot 2x < \frac{1}{2} \cdot 9$. The result is the solution $1 < x < \frac{9}{2}$.

Absolute Value Equations and Inequalities

To solve an **absolute value** equation or inequality, we must consider the nature of an absolute value relationship. For example, both the absolute value of 4 and the absolute value of -4 have the same value. $|4| = 4$ and $|-4| = 4$. If $|x| = 4$, there are two values, 4 and -4 that satisfy this equation. Both numbers are 4 units from 0.

To **solve an absolute value equation**, we use the Addition and Multiplication Properties of Equality and we consider the absolute value relationship. For example, to solve the relationship $|x + 3| = 5$, we must solve two equations: $x + 3 = 5$ and $x + 3 = -5$. We solve both of these equations using the addition property of equality, the solutions are $x = 2$ and $x = -8$.

To **solve an absolute value inequality**, we use the Addition and Multiplication Properties of Equality. We must also consider the relationship that is being described. For example, the solution for $|x| < 5$ is the set of numbers that are less than 5 and greater than -5 . The numbers that solve the absolute value inequality $|x| < 5$ are the solution set for two inequality relationships, $x < 5$ and $x > -5$. It is possible for a real number to satisfy both conditions, so the solution may be written as one set of numbers, the numbers that solve the relationship $-5 < x < 5$. The numbers that solve the absolute value inequality $|x| > 5$ are the solution set for two inequality relationships: $x > 5$ and $x < -5$. A real number cannot meet both conditions, $x > 5$ and $x < -5$ so the solution for this inequality is two sets of numbers, $x > 5$ or $x < -5$.

Application Problems

To solve a problem using algebra, we describe or **model** the problem with an equation. When solving application problems, it is helpful to find “clue words” within the problem. Look for words that describe the basic operations of addition, subtraction, multiplication, and division. For example:

Operation

Addition
 Subtraction
 Multiplication
 Division

Clue Words

sum, added to, more than, increased by
 difference, subtracted from, less than, decreased by
 product, “twice”, “one-half”, percent of
 quotient, ratio, divided by

Apply the following strategies to solve application problems successfully:

1. Read the problem as many times as necessary. Guessing the answer and checking it will help you understand the problem.
2. If possible, draw a diagram to illustrate the problem.
3. Choose a variable and write what it represents.
4. Write algebraic expressions for any other unknowns in terms of that variable.
5. Write an equation that describes the situation.
6. Solve the equation.
7. Answer the original question.
8. Check your answer by using it to solve the original problem (not the equation).

Exercises.

Solve each equation and check your answer.

- | | |
|--------------------------------|----------------------------------|
| 1. $x - 3.4 = 6$ | 2. $12 = x + 2.8$ |
| 3. $\frac{3}{4}x = 6$ | 4. $-2y = 34$ |
| 5. $3x - 5 = 12$ | 6. $5y - 4 = 6y$ |
| 7. $3(x - 7) + 5 = -(16 - 3x)$ | 8. $4(x - 7) - 5 = 5 - (3 - 4x)$ |

Solve each equation for the given variable.

- | | |
|----------------------------|---------------------------------|
| 9. $P = a + b + c$ for b | 10. $A = \frac{1}{2}bh$ for h |
| 11. $2x + 3y = 6$ for y | 12. $ax + b = 0$ for x |

Solve each inequality and check your answer.

- | | |
|--|----------------------------|
| 13. $2 - 3x < 12$ | 14. $-3(4 - 2x) > 15$ |
| 15. $\frac{1}{2}x + 5 \leq \frac{2}{3}x - 7$ | 16. $5(x + 2) > -2(x - 1)$ |
| 17. $ x < 9$ | 18. $ x \geq 12$ |
| 19. $ 2x + 3 < 6$ | 20. $ x - 4 > 11$ |

21. $6 < 2x < 16$

22. $-2 \leq 3x + 7 < 16$

23. $5 < 4 - x < 12$

24. $\frac{2}{3}x + 4 < x + 5 < 6$

Solve each of the following problems. Write an equation to solve the problem, then solve the problem.

25. The area of a rectangle is 24 square centimeters. The length of the rectangle is 6 centimeters. Find the width of the rectangle.
26. If a banner in the shape of a triangle has an area of 12 square feet with a base of 4 feet, then what is the height of the banner?
27. Jose paid \$525 simple interest on a loan of \$3500. If the annual interest rate was 5%, then what was the time?
28. Chelsea paid \$420 in simple interest on a loan of \$1000 for 6 years. What was the rate?
29. Diana drove for 3 hours on the freeway, then decreased her speed by 20 miles per hour and drove for 2 more hours on a country road. If the total trip was 300 miles, what was her speed on the freeway?
30. On Monday, Sam drove to work in 45 minutes. On Tuesday he averaged 12 miles per hour more, and it took him 9 minutes less to get to work. How far does he travel to work?
31. Find three consecutive integers whose sum is 33.
32. Find two consecutive odd integers whose sum is 36.
33. At a 20% off sale, Kim saved \$60 on a bicycle. What was the original price of the bicycle?
34. Isaac bought a new car. The selling price plus the 6% sales tax was \$19610. What was the selling price of the car?
35. How many gallons of milk containing 1% butterfat must be mixed with 40 gallons of milk containing 5% butterfat to obtain a mixture containing 2% butterfat?
36. Nikolai needs 20 quarts of 50% antifreeze solution in his radiator. He plans to obtain this by mixing some pure antifreeze with an appropriate amount of a 20% antifreeze solution. How many quarts of each should he use?

SMART CD Lessons:

Topic	SMART CD Location	SMART CD Lessons
Solving Linear Equations	2.1	1. The Addition Property of Equality 2. The Multiplication Property of Equality 3. Variables on Both Sides
Solving Linear Equations	2.2	1. Equations of the Form $ax + b = 0$ 2. Equations of the Form $ax + b = cx + d$ 3. Equations with Parentheses
Solving Linear Equations	2.3	1. Identities, Conditional Equations, and Inconsistent Equations 2. Equations Involving Fractions 3. Equations Involving Decimals
Literal Equations	2.4	1. Solving for a Variable 2. Finding the Value of a Variable
Solving Linear Inequalities	3.1	1. Verifying Inequalities 2. Solving Inequalities
Compound Inequalities	3.2	1. Checking Compound Inequalities 2. Solving Compound Inequalities
Absolute Value Equations and Inequalities	3.3	1. Absolute Value Equations 2. Solving Absolute Value Inequalities
Application Problems	2.5	1. Writing Algebraic Expressions 2. Pairs of Numbers 3. Writing Equations
Application Problems	2.6	1. Number Problems 2. Geometric Problems 3. Uniform Motion Problems
Application Problems	2.7	1. Discount Problems 2. Investment Problems 3. Mixture Problems

Video Lesson Locations:

Topic	Video Location
Solving Linear Equations	Chapter Two Tape 1:38
Solving Linear Equations	Chapter Two Tape 10:09
Solving Linear Equations	Chapter Two Tape 18:27
Literal Equations	Chapter Two Tape 26:10
Solving Linear Inequalities	Chapter Three Tape 1:38
Compound Inequalities	Chapter Three Tape 13:30
Absolute Value Equations and Inequalities	Chapter Three Tape 23:44
Application Problems	Chapter Two Tape 33:00
Application Problems	Chapter Two Tape 41:06
Application Problems	Chapter Two Tape 49:31

Midtext Diagnostic Review Exercises:

Topic	Text Location
Solving Linear Equations	Page 391, Problems 13-16
Literal Equations	Page 391, Problems 17-20
Solving Linear Inequalities	Page 391, Problems 25, 26, 33, 35
Compound Inequalities	Page 391, problems 29-32
Absolute Value Equations and Inequalities	Page 391, problems 27, 34, 36
Applications	Page 391, problems 21-24

Section EA.3 Linear Equations in Two Variables and Their Graphs

In this section:

- Graphing Lines in the Coordinate Plane
- Slope
- Equations of Lines

Graphing Lines in the Coordinate Plane

The equation $x + y = 5$ is an example of a **linear equation in two variables**. A **solution** for this equation must include a value for x and a value for y . For example, if $x = 2, y = 3$ because $2 + 3 = 5$. If $x = 4, y = 1$ is also a solution for the equation $x + y = 5$. There are an infinite number of solutions for this type of equation. Each solution must be written to include both the x and y values for that solution. We write the solutions as **ordered pairs**. The first number of an ordered pair is known as the **abscissa**, the second number of an ordered pair is known as the **ordinate**. We call these values **coordinates**. For example, the ordered pair $(2,3)$ has an abscissa or x -coordinate of 2 and an ordinate or y -coordinate of 3.

We graph equations in two variables using the **rectangular (Cartesian) coordinate system**. We draw a horizontal line, the **x-axis** and a vertical line, the **y-axis**. The point where the x - and y -axes intersect is known as the **origin** and is labeled $(0,0)$. The x -axis and the y -axis divide the coordinate plane into four sections known as **quadrants**. The quadrants are labeled with Roman numerals as shown in figure EA.3.1.

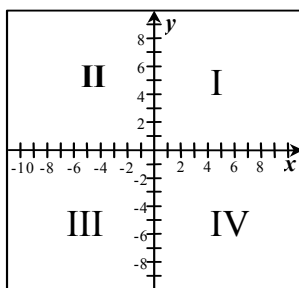


Figure EA 3.1

A linear equation in two variables can be graphed using one of three methods:

- The method of plotting points
- The method of using x - and y -intercepts
- The method of using the slope-intercept form of the equation of the line

To **graph a linear equation using the method of plotting points**, we find three points that are solutions of the equation, plot those points, then draw the line. For example, to graph the equation $x + y = 5$, we might choose the values -1, 0, and 1 for x . We can represent the results using a table:

x	y
-1	6
0	5
1	4

Figure EA.3.2 shows the graph of the equation $x + y = 5$:

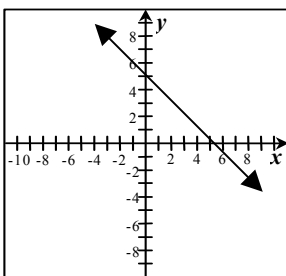


Figure EA 3.2

To **graph a linear equation using x- and y-intercepts**, we find the x- and y-intercepts for the equation, plot those points, and then draw the line. At the x-intercept, the graph is crossing the x-axis, therefore y has a value of 0. At the y-intercept, the graph is crossing the y-axis, therefore x has a value of 0. We find the x- and y-intercepts by setting x and y equal to zero and finding the corresponding value to complete the ordered pair. For the equation $x + y = 5$, the x-intercept is at $(5, 0)$ and the y-intercept is at $(0, 5)$. We can observe that in Figure EA.3.2, the graph of $x + y = 5$, the line crosses the x-axis at 5 and the y-axis at 5.

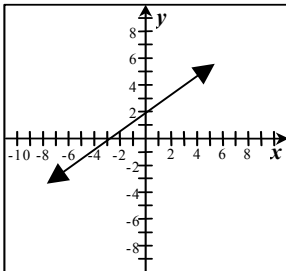
When we solve a linear equation for y , the equation is in **slope-intercept form**. The slope-intercept form of the equation of a line is $y = mx + b$. When the equation of a line is in slope intercept form, the coefficient (m) of x is the slope and the constant (b) is the y-intercept for that line. For the equation $x + y = 5$, the slope-intercept form is $y = -x + 5$. The slope of this line is -1 and the y-intercept is $(0, 5)$. To use this information to graph the line, we plot the y-intercept and then locate other points on the line by using the slope.

Slope

The slope of a line describes the “slant” of the line. The slope of a line is defined as the ratio of the rise (vertical change) of the line to the run (horizontal change) of the line. We use the

letter m to represent the slope of a line. In Figure EA.3.3, lines with positive, negative, zero, and undefined slope are shown.

positive slope



negative slope

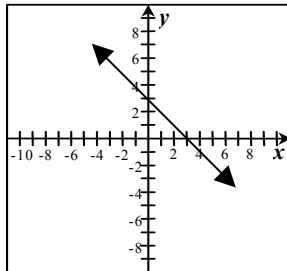
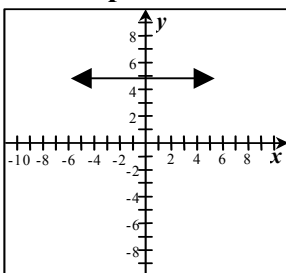
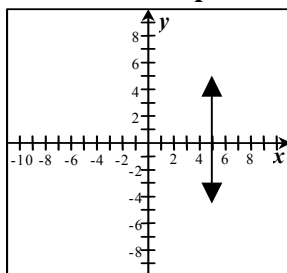


Figure EA 3.3

zero slope



undefined slope



Given two points, we can calculate the slope of a line by finding the difference between the x-values and the difference between the y-values. The **formula for slope** is: $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

where (x_1, y_1) and (x_2, y_2) are the coordinates of two points on the line. For example, to find the slope of the line connecting the points $(-2, 1)$ and $(3, 5)$, we substitute the coordinate values

into the formula for slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{3 - (-2)} = \frac{4}{5}$.

The **slope of a vertical line** is undefined. The **slope of a horizontal line** is 0. Two lines that are **parallel** have the same slope. Two lines that are **perpendicular** have slopes that are negative reciprocals.

Equations of Lines

There are three forms of the equation of a line. The **standard form of the equation of a line** is $Ax + By = C$, where $A > 0$; A , B , and C are integer values; and A and B are not both 0.

The **slope-intercept form of the equation of a line** is $y = mx + b$. The **point-slope form of the equation of a line** is $y - y_1 = m(x - x_1)$.

The **standard form** of the equation of a line is the most formal version of the equation for a line. Be sure to clear all fractions and that the x-term has a positive coefficient. For example, to change the equation $y = \frac{2}{3}x + 1$ to standard form, we first clear the fraction by multiplying by

the Least Common Denominator, 3: $3 \cdot y = 3 \cdot \frac{2}{3}x + 3 \cdot 1$. This results in the equation

$3y = 2x + 3$. We now subtract the x-term from both sides of the equation: $3y - 2x = 2x - 2x + 3$ with the result $-2x + 3y = 3$. Because the x-coefficient must be positive, we multiply both sides of the equation by -1. The result is $2x - 3y = -3$, the standard form of the equation $y = \frac{2}{3}x + 1$.

When the equation of a line is in **slope-intercept** form, the coefficient of x is the slope and the constant term is the y-intercept for the line. For example, the line with the equation

$y = \frac{2}{3}x + 1$ has a slope of $\frac{2}{3}$ and a y-intercept at (0,1). If we know the slope of a line and the y-intercept for that line, we can write the equation of the line. For example, if the slope of a line is $m = \frac{-1}{2}$ and the y-intercept for the line is 1, then the equation for this line is $y = \frac{-1}{2}x + 1$.

We use the **point-slope form** of the equation of a line when we know a point on the line and the slope of the line, or when we know two points on the line. For example, given that the point (2,3) is on the line with a slope of $\frac{1}{2}$, we write the equation of the line as follows:

$y - 3 = \frac{1}{2}(x - 2)$. We can change this equation to slope-intercept form by distributing the slope

value (in this case, $\frac{1}{2}$) and then isolating the y-value. $y - 3 = \frac{1}{2}x - \frac{1}{2} \cdot 2$; $y - 3 = \frac{1}{2}x - 1$;

$y = \frac{1}{2}x + 2$. To change this equation to standard form, we clear the fraction and isolate the

constant term. $y = \frac{1}{2}x + 2$; $2y = 2 \cdot \frac{1}{2}x + 2 \cdot 2$; $2y = x + 4$; $-x + 2y = 4$; $x - 2y = -4$.

A **vertical line** has a constant x-value, thus the equation of a vertical line has the form $x = a$, where a is the x-coordinate for all points on the line. The equation of the vertical line through the point (-2, 3) is $x = -2$.

A **horizontal line** has a constant y-value, thus the equation of a horizontal line has the form $y = b$, where b is the y-coordinate for all points on the line. The equation of the horizontal line through the point (-2,3) is $y = 3$.

Exercises.

Sketch the graph of each equation by finding three ordered pairs that satisfy the equation.

1. $y = x + 4$

2. $x - 2y = 6$

3. $y = \frac{1}{2}x + 3$

4. $x + y = 4$

Sketch the graph of each equation by using the method of finding the x- and y-intercepts.

5. $2x - 5y = 10$

6. $3x + 2y = 12$

7. $y = \frac{3}{2}x - 1$

8. $y = x + 1$

Sketch the graph of each equation by using the slope and the y-intercept for the given line.

9. $y = x - 2$

10. $y = \frac{2}{3}x + 1$

11. $x + y = 3$

12. $3x - 2y = 6$

Sketch the graph of each of the following lines.

13. $x = 4$

14. $y = -3$

Determine the slope for each of the following lines.

15. The line through the points (1,3) and (4,5)

16. The line with the equation $y = \frac{-1}{2}x + 3$

17. The line through the points (2,4) and (2, 7)

18. The line parallel to the line $y = \frac{3}{5}x - 2$

19. The line perpendicular to the line $x = 5$

Write the equation for each of the following lines in standard form.

20. $y = \frac{2}{5}x - 3$

21. $y = x - 5$

Write the equation for each of the following lines in slope-intercept form.

22. $4x - 3y = 24$

23. $x - 2y = 4$

Write the equation for each of the following lines in point-slope form.

24. The line through (1,4) with slope -2

25. The line through (3,2) with slope $\frac{2}{3}$

Write the equation for each of the following lines. Write the answer in standard form.

26. The line through the points (-2,4) and (3,5)

27. The line through the point (0,3) with slope -1

28. The line through (1,2) with slope $\frac{5}{2}$

29. The line through the points (2,4) and (2,5)

30. The line through the points (2,4) and (-1,4)

31. The line parallel to $y = \frac{2}{3}x + 1$ with y-intercept 5

32. The line perpendicular to $y = 3x - 4$ through the point (6,-1)

33. The line parallel to $y = -2$ through the point (3,4)

34. The line perpendicular to $y = -2$ through the point (3,4)

SMART CD Lessons:

Topic	SMART CD Location	SMART CD Lessons
Graphing Lines in the Coordinate Plane	4.1	1. Finding Solutions to Equations 2. Graphing Lines Using Intercepts
Slope	4.2	1. Finding the Slope of a Line
Equations of Lines	4.3	1. Slope-Intercept Form 2. Writing the Equation for a Line
Equations of Lines	4.4	1. Writing an Equation Given a Point and a Slope 2. Writing an Equation Given Two Points

Video Lesson Locations:

Topic	Video Location
Graphing Lines in the Coordinate Plane	Chapter Four Tape 1:37
Slope	Chapter Four Tape 12:17
Equations of Lines	Chapter Four Tape 24:16
Equations of Lines	Chapter Four Tape 35:27

Midtext Diagnostic Review Exercises:

Topic	Text Location
Graphing Lines in the Coordinate Plane	Pages 391-392, Problems 46-48
Slope	Page 391, Problem 37
Equations of Lines	Page 391, Problems 38-45

Section EA.4 Polynomials and Exponents

In this section:

- Addition and Subtraction of Polynomials
- Multiplication of Polynomials
- Division of Polynomials
- Rules for Integral Exponents

Addition and Subtraction of Polynomials

A **constant** is a number. A **variable** is a letter or a symbol used to represent a numerical value. An algebraic expression involving only the operations of addition, subtraction, and multiplication on variables and constants is called a **polynomial**. For example, $3x^2y^4$, $5x - 4$, and $x^2 + 2x - 3$ are all polynomials. When a constant is multiplied by a variable or a set of variables, it is called a **coefficient**. For example, in the expression

$x^2 + 2x - 3$, -3 is a constant and 2 is a coefficient. The coefficient on the x^2 term is 1 . We call a polynomial with one term a **monomial**. The polynomial $3x^2y^4$ is a monomial. A polynomial with two terms is a **binomial**. The polynomial $5x - 4$ is a binomial. A polynomial with three terms is a **trinomial**. The polynomial $x^2 + 2x - 3$ is a trinomial. Any expression with more than three terms is known as a **polynomial**.

To add or subtract two polynomials, we combine the like terms. Two terms are called **like terms** if they have exactly the same variable factors and each factor is to the same power. For example, $3x^2$ and $5x$ are not like terms because the power on each respective x -term is different. $3x^2$ and $5x^2$ are like terms because they have the same variable factor to the same power. Like terms are combined by adding their numerical coefficients. For example, $(3x^2 - 2xy + 4) + (5x^2 + 6xy - 2) = 8x^2 + 4xy + 2$.

Multiplication of Polynomials

The expression x^3 represents $x \cdot x \cdot x$ and the expression x^5 represents $x \cdot x \cdot x \cdot x \cdot x$. When we multiply these two expressions, the result is $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^8$. The exponent of the product of the two expressions is the sum of their respective exponents. We can apply the product rule when multiplying polynomials. The **Product Rule** for multiplying exponential expressions states that: If a is any real number and m and n are any positive integers, then $a^m \cdot a^n = a^{m+n}$. To **multiply polynomials**, multiply each term of one polynomial by every term of the other polynomial, then combine like terms. Consider the following three examples:

An example of a monomial times a monomial: $3xy \cdot 2x^2y = 6x^3y^2$.

An example of a monomial times a binomial: $3xy(2x + 3y) = 3x^2y + 9xy^2$.

An example of a binomial times a binomial:

$$(3x + 2y)(5x - 4y) = 15x^2 - 12xy + 10xy - 8y^2 = 15x^2 - 2xy - 8y^2.$$

Division of Polynomials

To divide two polynomial expressions, we apply the quotient rule. The **Quotient Rule** states that: If $a \neq 0$ and m and n are positive integers, then $\frac{a^m}{a^n} = a^{m-n}$. To divide two monomials, we divide the coefficients and apply the quotient rule to the variables. For example, $\frac{6x^4}{3x^3} = 2x$. To divide other types of polynomials, we apply the division algorithm that we use with real numbers. For example, $x^2 - 3x - 10 \div x + 2 = x - 5$.

Rules for Integral Exponents

An integer is a positive or negative whole number or zero. The word “integral” is used to describe a value that is an integer. **Integral exponents** are exponents that have integer values. There are rules associated with various operations involving integral exponents.

Rule	Example
Zero Exponent Rule: $a^0 = 1$	$(2x)^0 = 1$
Product Rule: $a^m \cdot a^n = a^{m+n}$	$x^2 \cdot x^3 = x^5$
Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^5}{x^2} = x^3$
Power Rule: $(a^m)^n = a^{mn}$	$(x^3)^2 = x^6$
Power of a Product Rule: $(ab)^n = a^n b^n$	$(2x^4)^3 = 8x^{12}$
Power of a Quotient Rule: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{x}\right)^4 = \frac{16}{x^4}$
Negative Exponent Rule: $a^{-n} = \frac{1}{a^n}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Exercises.

Perform the indicated operations.

- $(2x + 3y) - (4x - 5y)$
- $(3x^2y + 5xy^2) + (10xy - 6xy^2)$
- $(x^2 - 5x - 6) + (x^2 + 2x + 10)$
- $(x^2 + 4x + 12) - (x^2 - 5x + 8)$
- $5xy(2xy)$
- $-5y(6xy)$
- $5xy(2xy + 4)$
- $-2x(3x^2 - 5)$
- $(x + 2)(x - 3)$
- $(x + 3y)(x - 3y)$
- $(2x - 5y)(3x + 4y)$
- $(x + 3y)(x^2 + 1)$
- $5xy(x^2 + 2x + 1)$
- $-2x(x^2 - 3x - 4)$
- $(x + 2y)(x^2 - 4x + 3)$
- $(x + 1)(x^2 - x + 1)$
- $(a + b + c)(a + b + c)$
- $(a^2 + 3a - 4)(a^2 + a + 5)$

19. $\frac{6xy}{2y}$

21. $(x^2 + 5x + 6) \div (x + 2)$

23. $(x^2 - 9) \div (x + 3)$

25. $(x^2 + 4) \div (x - 2)$

20. $8x^2y \div 4xy$

22. $(x^2 - x - 6) \div (x + 3)$

24. $(x^3 + 8) \div (x + 2)$

26. $(x^3 - 1) \div (x + 3)$

Use the rules for integral exponents to simplify each of the following expressions. Use only positive exponents in answers.

27. $x^5 \cdot x^3$

29. $(x^2)^3$

31. $\frac{x^4}{x^2}$

33. $\frac{8x^2y^5}{2xy^6}$

35. $2x^{-1}$

37. $(2x + 3)^0$

28. $3x^2 \cdot 5x^4$

30. $(3xy^2)^3$

32. $\frac{12x^6}{4x}$

34. $(2x)^{-1}$

36. $(5x)^0$

38. $(2x^3)^2(5x)^0$

SMART CD Lessons:

Topic	SMART CD Location	SMART CD Lessons
Addition and Subtraction of Polynomials	5.1	1. Evaluating Polynomials 2. Adding and Subtracting Polynomials
Multiplication of Polynomials	5.2	1. Multiplying Monomials 2. Multiplying Polynomials
Multiplication of Polynomials	5.3	1. The FOIL Method
Multiplication of Polynomials	5.4	1. The Square of a Binomial 2. The Product of a Sum and a Difference
Division of Polynomials	5.5	1. Dividing Monomials Using the Quotient Rule 2. Dividing a Polynomial by a Monomial 3. Dividing a Polynomial by a Binomial
Rules for Integral Exponents	5.6	1. The Power Rule for Exponents 2. Power of a Product and Power of a Quotient Rules
Rules for Integral Exponents	5.7	1. Negative Integral Exponents 2. Scientific Notation

Video Lesson Locations:

Topic	Video Location
Addition and Subtraction of Polynomials	Chapter Five Tape 1:49
Multiplication of Polynomials	Chapter Five Tape 16:04
Multiplication of Polynomials	Chapter Five Tape 24:08
Multiplication of Polynomials	Chapter Five Tape 31:12
Division of Polynomials	Chapter Five Tape 40:39
Rules for Integral Exponents	Chapter Five Tape 52:42
Rules for Integral Exponents	Chapter Five Tape 1:02:02

Midtext Diagnostic Review Exercises:

Topic	Text Location
Addition and Subtraction of Polynomials	Page 392, Problem 49
Multiplication of Polynomials	Page 392, problems 50, 51, 53
Division of Polynomials	Page 392, Problem 52
Rules for Integral Exponents	Page 392, Problems 54-60

Section EA.5 Factoring Polynomials

In this section:

- Factoring Out Common Factors
- Factoring the Special Products and Factoring by Grouping
- Factoring $ax^2 + bx + c$
- Solving Quadratic Equations by Factoring

Factoring Out Common Factors

A **product** is the result of multiplying two values. The values that are multiplied are known as **factors**. For example, $2 \cdot 3 = 6$, 2 and 3 are factors and 6 is the product of multiplying these factors. For $(x+2)(x+3) = x^2 + 5x + 6$ ($x+2$) and ($x+3$) are factors and the product is $x^2 + 5x + 6$.

When working with polynomials, it is sometimes useful to factor out a **common factor**. A common factor is a shared factor. For example, $8 = 2 \cdot 4$ and $6 = 2 \cdot 3$. A common factor for 6 and 8 is the number 2. It is sometimes important to know the greatest common factor for two numbers. For example, $12 = 2 \cdot 2 \cdot 3$ and $8 = 2 \cdot 2 \cdot 2$. The greatest common factor for 12 and 8 is $2 \cdot 2 = 4$. For polynomial expressions, the **greatest common factor** may be a number, a variable, or a combination of numbers and variables. For the polynomial expression $6x + 9$, 3 is the greatest common factor; $6x + 9 = 3(2x + 3)$. For the polynomial expression $x^2 + xy$, x is the greatest common factor; $x^2 + xy = x(x + y)$. For the polynomial expression $6x^2 + 9xy$ the greatest common factor is $3x$; $6x^2 + 9xy = 3x(2x + 3y)$.

Factoring the Special Products and Factoring by Grouping

When multiplying polynomials, we observe that there are “special cases” where the result becomes predictable. For example, if we multiply $(x+3)(x-3)$, the result is $x^2 - 9$. This result is known as the **difference of two squares** and is the result of multiplying two binomials where each binomial has the same terms with a sign change in one binomial. We designate the **difference of two squares** as a **special product**, noting that $(a+b)(a-b) = a^2 - b^2$. The factorization of $a^2 - b^2$ is $(a+b)(a-b)$. For example, $x^2 - 49 = (x+7)(x-7)$ and $4x^2 - 9y^2 = (2x+3y)(2x-3y)$. Because multiplication is commutative, the order of the factors does not matter. $x^2 - 49 = (x+7)(x-7) = (x-7)(x+7)$ and $4x^2 - 9y^2 = (2x+3y)(2x-3y) = (2x-3y)(2x+3y)$.

Another “special case” of polynomial multiplication is the square of a binomial. For example, if we multiply $(x+3)^2$, the result is $x^2 + 6x + 9$. This is an example of a perfect square trinomial. We designate the **perfect square trinomial** as a **special product**, noting that $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$. The factorization of $a^2 + 2ab + b^2$

is $(a+b)(a+b) = (a+b)^2$. The factorization of $a^2 - 2ab + b^2$ is $(a-b)(a-b) = (a-b)^2$. For example, $x^2 + 6x + 9 = (x+3)(x+3) = (x+3)^2$ and $x^2 - 6x + 9 = (x-3)(x-3) = (x-3)^2$.

It is sometimes simpler to factor a polynomial by “grouping” the terms of the polynomial. This technique is known as **factoring by grouping**. For example, $ax + 2a + bx + 2b$ does not have any factors that are common to all terms. If we group the first two terms and the last two terms, we can identify a common factor. $ax + 2a + bx + 2b = a(x+2) + b(x+2) = (x+2)(a+b)$.

Factoring $ax^2 + bx + c$

There are two methods that we use to factor the trinomial $ax^2 + bx + c$. These methods are the method of trial and error and the ac method. A polynomial is considered to be a **prime polynomial** if it cannot be factored using real numbers.

To factor a polynomial using **trial-and-error**, we make a reasonable guess for the factors of the polynomial, check the guess by multiplying, and try again until we have factored correctly or until we have determined that the polynomial is a prime polynomial. For example, it would be reasonable to assume that the factorization of $x^2 + 5x + 6$ might include the factors of 6, so we would try either $1 \cdot 6$ or $2 \cdot 3$. Because $2 + 3 = 5$, 2 and 3 are the most reasonable first guess. We try $(x+2)(x+3) = x^2 + 5x + 6$ and we find that we have factored the polynomial correctly.

To factor $ax^2 + bx + c$ using the **ac method**, we separate the b term into two terms, thus allowing us to factor by grouping. For example, $2x^2 - 7x - 4 = 2x^2 - 8x + x - 4 = x(2x - 4) + 1(x - 4) = (x - 4)(2x + 1)$. In order to find the correct values to use for the separation of the b term, we rely on the product of the a term and the c term.

To factor $ax^2 + bx + c$ using the ac method:

1. Find two numbers that have a product equal to ac and a sum equal to b .
2. Replace bx by two terms using the two new numbers as coefficients.
3. Factor the resulting four-term polynomial by grouping.

We can apply the following strategy to factor polynomials completely:

1. If there are any common factors, factor them out first.
2. When factoring a binomial, check to see whether it is a difference of two squares, a difference of two cubes, or a sum of two cubes.
3. When factoring a trinomial, check to see whether it is a perfect square trinomial.
4. When factoring a trinomial that is not a perfect square, use the ac method or the trial-and-error method.
5. If the polynomial has four terms, try factoring by grouping.
6. Check to see whether any of the factors can be factored again.

Solving Quadratic Equations by Factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where a, b , and c are real numbers and $a \neq 0$. If a quadratic equation can be factored, we can apply the Zero Factor Property to solve the equation. The **Zero Factor Property** states that: If $a \cdot b = 0$, then either $a = 0$, $b = 0$, or both a and b are equal to zero.

To solve an equation by factoring:

1. Rewrite the equation with 0 on the right-hand side.
2. Factor the left-hand side of the equation completely.
3. Use the zero factor property to get simple linear equations.
4. Solve the linear equations.
5. Check the answers in the original equation.
6. State the solution(s) to the original equation.

For example, to solve the equation $x^2 - x - 6 = 0$, we factor the equation, then set each factor equal to zero and solve. $x^2 - x - 6 = 0$; $x^2 - x - 6 = (x - 3)(x + 2)$; $(x - 3) = 0$; $x = 3$; $(x + 2) = 0$; $x = -2$. The solutions for the equation $x^2 - x - 6 = 0$ are -2 and 3.

Exercises.

Factor the following polynomials completely, if possible.

- | | |
|-------------------------|---------------------|
| 1. $3x - 12y$ | 2. $6x^2 + 18xy$ |
| 3. $x^2 - 16$ | 4. $9x^2 - 16y^2$ |
| 5. $x^2 + 12x + 36$ | 6. $x^2 - 18x + 81$ |
| 7. $4x^2 + 12x + 9$ | 8. $2x^2 + 4x - 6$ |
| 9. $x^2 - 3x - 10$ | 10. $x^2 + 4x - 12$ |
| 11. $x^2 + 16$ | 12. $2x^2 - x - 15$ |
| 13. $x^2 + 5xy - 14y^2$ | 14. $4x^2 - 9y^2$ |
| 15. $6x^2 + 17x + 12$ | 16. $12x^2 - x - 1$ |

Solve the following equations.

- | | |
|--------------------------|-------------------------|
| 17. $(x + 2)(x + 3) = 0$ | 18. $3x(x - 5) = 0$ |
| 19. $x^2 - 36 = 0$ | 20. $x^2 + 8x + 16 = 0$ |
| 21. $x^2 - x - 12 = 0$ | 22. $x^2 + 6x - 16 = 0$ |
| 23. $x^2 + 12x + 32 = 0$ | 24. $x^2 - 7x + 10 = 0$ |
| 25. $x^2 + 5x = -6$ | 26. $x^2 = 25$ |
| 27. $x^2 = 2x + 15$ | 28. $x^2 - 4x = 32$ |
| 29. $x(x - 4) = 12$ | 30. $x(x + 3) = 10$ |

SMART CD Lessons:

Topic	SMART CD Location	SMART CD Lessons
Factoring Out Common Factors	6.1	1. The Greatest Common Factor 2. Factoring Out the Greatest Common Factor
Factoring the Special Products and Factoring by Grouping	6.2	1. Factoring a Difference of Two Squares 2. Factoring a Perfect Square Trinomial 3. Factoring by Grouping
Factoring $ax^2 + bx + c$	6.3	1. Factoring $ax^2 + bx + c$ with $a = 1$ 2. Prime Polynomials
Factoring $ax^2 + bx + c$	6.4	1. Factoring $ax^2 + bx + c$ with $a \neq 1$ 2. Factoring by Trial and Error
Factoring $ax^2 + bx + c$	6.5	1. Factoring a Difference or Sum of Two Cubes 2. Factoring Polynomials Completely
Solving Quadratic Equations by Factoring	6.6	1. Solving Quadratic Equations by Factoring

Video Lesson Locations:

Topic	Video Location
Factoring Out Common Factors	Chapter Six Tape 1:37
Factoring the Special Products and Factoring by Grouping	Chapter Six Tape 13:34
Factoring $ax^2 + bx + c$	Chapter Six Tape 26:13
Factoring $ax^2 + bx + c$	Chapter Six Tape 35:07
Factoring $ax^2 + bx + c$	Chapter Six Tape 46:51
Solving Quadratic Equations by Factoring	Chapter Six Tape 56:19

Midtext Diagnostic Review Exercises:

Topic	Text Location
Factoring Out Common Factors	Page 392, Problem 61
Factoring the Special Products and Factoring by Grouping	Page 392, Problem 63
Factoring $ax^2 + bx + c$	Page 392, Problems 62&64
Solving Quadratic Equations by Factoring	Page 392, Problems 65-70

Section EA.6 Rational Expressions

In this section:

- Reducing Rational Expressions
- Multiplication and Division of Rational Expressions
- Addition and Subtraction of Rational Expressions
- Complex Fractions
- Solving Equations with Rational Expressions
- Ratio and Proportion

Reducing Rational Expressions

A **rational expression** is the ratio of two polynomials with the denominator not equal to 0.

Apply the following strategy for reducing rational expressions:

1. Reducing is done by dividing out all common factors.
2. Factor the numerator and denominator completely to see the common factors.
3. Use the quotient rule $\left(\frac{a^m}{a^n} = a^{m-n}\right)$ to reduce a ratio of two monomials.
4. You may have to factor out a common factor with a negative sign to get identical factors in the numerator and denominator.
5. The quotient of $a - b$ and $b - a$ is -1 .

For example:

$$\frac{x^6}{x^3} = x^3 \quad \frac{x^2 + 5x + 6}{x + 3} = \frac{(x + 2)(x + 3)}{x + 3} = x + 2 \quad \frac{x^2 - 9}{3 - x} = \frac{(x + 3)(x - 3)}{-(x - 3)} = -(x + 3)$$

Multiplication and Division of Rational Expressions

Two rational expressions are **multiplied** by multiplying their numerators and multiplying their denominators. We say that: if $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. For example,

$$\frac{8x}{15y} \cdot \frac{5y}{4xy} = \frac{2}{3y} \quad \text{and} \quad \frac{x + 3}{x^2 - 4} \cdot \frac{x^2 + 5x + 6}{x + 2} = \frac{x + 3}{(x + 2)(x - 2)} \cdot \frac{(x + 2)(x + 3)}{x + 2} = \frac{(x + 3)^2}{x - 2}$$

To **divide** two rational expressions, we multiply by the reciprocal of the divisor. We say that: if $b \neq 0$, $c \neq 0$, and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. For example, $\frac{9x}{4y} \div \frac{3xy}{2} = \frac{9x}{4y} \cdot \frac{2}{3xy} = \frac{3}{2y^2}$.

Addition and Subtraction of Rational Expressions

In order to add two rational expressions, we must first find a common denominator. Apply the following strategy to find the **Least Common Denominator (LCD)** for polynomials:

1. Factor each denominator completely. Use exponent notation for repeated factors.

- Write the product of all of the different factors that appear in the denominators.
- On each factor, use the highest power that appears on that factor in any of the denominators. For example, $x^2 - 9 = (x + 3)(x - 3)$ and $x^2 + 4x + 3 = (x + 1)(x + 3)$ the LCD for $x^2 - 9$ and $x^2 + 4x + 3$ is $(x + 3)(x - 3)(x + 1)$.

We can **add or subtract rational expressions** with a common denominator according to the following definition: If $b \neq 0$ then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ and $\frac{a}{b} - \frac{c}{d} = \frac{a-c}{d}$. For example,

$$\frac{x}{x+3} + \frac{3}{x+3} = \frac{x+3}{x+3} = 1 \quad \text{and} \quad \frac{2xy}{x+3} + \frac{6y}{x+3} = \frac{2xy+6y}{x+3} = \frac{2y(x+3)}{x+3} = 2y.$$

Complex Fractions

A **complex fraction** is a fraction having rational expressions in the numerator, denominator,

or both. For example, $\frac{1 + \frac{2}{x}}{\frac{1}{3} - x}$ is a complex fraction. We simplify a complex fraction by

multiplying the numerator and the denominator of the complex fraction by the LCD for all denominators in the fraction. For example, to simplify the previous complex fraction, we

multiply the numerator and the denominator by the LCD, $3x$.
$$\frac{1 + \frac{2}{x}}{\frac{1}{3} - x} = \frac{3x \cdot \left(1 + \frac{2}{x}\right)}{3x \cdot \left(\frac{1}{3} - x\right)} = \frac{3x + 6}{x - 3x^2}.$$

Apply the following strategy for simplifying a complex fraction:

- Find the LCD for all the denominators in the complex fraction.
- Multiply both the numerator and the denominator of the complex fraction by the LCD. Use the distributive property if necessary.
- Combine like terms if possible.
- Reduce to lowest terms when possible.

Solving Equations with Rational Expressions

To **solve an equation with rational expressions**, we use the strategy of “clearing” the fractions. To **clear the fractions** in an equation, we multiply all terms of the equation by the LCD. For example, to solve the equation $\frac{1}{6} - \frac{1}{8x} = \frac{1}{72}$, we multiply each term by the LCD of

$72x$. $72x \cdot \frac{1}{6} - 72x \cdot \frac{1}{8x} = 72x \cdot \frac{1}{72}$. The resulting equation is a much simpler version of the original equation. We use the addition and multiplication properties of equality to solve the equation. $12x - 9 = x$. $-9 = -11x$. $x = \frac{9}{11}$. In a rational expression we can replace the

variable only by real numbers that do not cause the denominator to be 0. If a number causes a denominator to be 0, then it cannot be a solution to the equation.

Ratio and Proportion

A **ratio** is a comparison of two numbers. If a and b are any real numbers, with $b \neq 0$, then the ratio of a to b can be written $\frac{a}{b} = a : b = a \text{ to } b$. Two ratios are **equivalent** if they can be reduced to the same value. For example, $\frac{6}{9} = \frac{2}{3}$, so we can say that $\frac{6}{9}$ and $\frac{2}{3}$ are equivalent ratios.

A **proportion** is any statement expressing the equality of two ratios. $\frac{6}{9} = \frac{2}{3}$ is considered to be a proportion. Observe that if we cross-multiply, the same value is obtained.

$6 \cdot 3 = 2 \cdot 9, 18 = 18$. In the proportion $\frac{a}{b} = \frac{c}{d}$, a and d are called the **extremes**. b and c are called the **means**. If $b \neq 0$ and $d \neq 0$ then $ad = bc$. We can use this rule to solve equations such as $\frac{x}{5} = \frac{2}{3}$. We cross-multiply to obtain the equation: $3x = 10$. We then solve this equation using the multiplication property of equality: $x = \frac{10}{3}$.

Exercises.

Reduce each rational expression to lowest terms.

1. $\frac{2xy}{18x^2y}$

2. $\frac{xy^2}{5xy}$

3. $\frac{x-4}{3x-12}$

4. $\frac{x^2-36}{x+6}$

5. $\frac{x^2-x-12}{x^2-16}$

6. $\frac{x^2+2x-15}{x^2+7x+10}$

Perform the indicated operation.

7. $\frac{2}{x} + \frac{y}{x}$

8. $\frac{x}{x+1} + \frac{1}{x+1}$

9. $\frac{2xy}{x+5} + \frac{10y}{x+5}$

10. $\frac{2}{3} - \frac{5}{x}$

11. $\frac{x}{x+3} - \frac{2}{x}$

12. $\frac{x}{5} \cdot \frac{5x+10}{x^2-4}$

13. $\frac{x^2-x-6}{x+5} \cdot \frac{x^2-25}{x^2-8x+15}$

14. $\frac{x^2+9}{x+3} \cdot \frac{x^2+7x+12}{4x^2+36}$

15. $\frac{2}{x} \div \frac{x}{4}$

17. $\frac{x+7}{3y} \div \frac{3x+21}{6xy}$

16. $\frac{5}{x} \div \frac{35}{x}$

18. $\frac{x^2 - x - 30}{x+6} \div \frac{x^2 - 36}{x^2 + 7x + 6}$

Simplify each complex fraction.

19. $\frac{1 + \frac{1}{x}}{2 + \frac{1}{y}}$

21. $\frac{\frac{x}{3} + 2}{\frac{x+1}{x+1} + x}$

20. $\frac{\frac{2}{a} - \frac{3}{b}}{\frac{1}{a} + \frac{5}{b}}$

22. $\frac{\frac{1}{x+3} - \frac{2}{x-1}}{\frac{x+3}{3} + \frac{x-1}{4}}$

Solve each equation.

23. $\frac{1}{x} + \frac{1}{4} = \frac{1}{5}$

25. $x + \frac{3x}{2-x} = \frac{6}{2-x}$

24. $\frac{2}{x} + \frac{4}{3x} = 1$

26. $\frac{5}{a^2 - 4} + \frac{1}{a+2} = \frac{3}{a-2}$

Solve each proportion.

27. $\frac{2}{x} = \frac{3}{7}$

29. $\frac{4}{1-x} = \frac{1}{x}$

28. $\frac{5}{x} = \frac{x}{5}$

30. $\frac{2}{x+1} = \frac{3}{x-1}$

SMART CD Lessons:

Topic	SMART CD Location	SMART CD Lessons
Reducing Rational Expressions	7.1	1. Evaluating a Rational Expression 2. Reducing Rational Expressions
Multiplication and Division of Rational Expressions	7.2	1. Multiplying Rational Expressions 2. Dividing Rational Expressions
Addition and Subtraction of Rational Expressions	7.3	1. Building Up the Denominator 2. Finding the Least Common Denominator
Addition and Subtraction of Rational Expressions	7.4	1. Addition and Subtraction of Rational Numbers 2. Addition and Subtraction of Rational Expressions
Complex Fractions	7.5	1. Complex Fractions
Solving Equations with Rational Expressions	7.6	1. Equations with Integers in the Denominators 2. Equations with Variables in the Denominators
Ratio and Proportion	7.7	1. Ratios 2. Proportions

Video Lesson Locations:

Topic	Video Location
Reducing Rational Expressions	Chapter Seven Tape 1:49
Multiplication and Division of Rational Expressions	Chapter Seven Tape 19:53
Addition and Subtraction of Rational Expressions	Chapter Seven Tape 32:01
Addition and Subtraction of Rational Expressions	Chapter Seven Tape 44:02
Complex Fractions	Chapter Seven Tape 53:48
Solving Equations with Rational Expressions	Chapter Seven Tape 57:20
Ratio and Proportion	Chapter Seven Tape 1:05:30

Midtext Diagnostic Review Exercises

Topic	Text Location
Multiplication and Division of Rational Expressions	Page 392, Problems 75&76
Addition and Subtraction of Rational Expressions	Page 392, Problems 71-74
Solving Equations with Rational Expressions	Page 392, Problems 77,79, 80
Ratio and Proportion	Page 392, Problem 78