

- b) Use the formula to predict the global investment in 2001.
- c) Find the year in which the global investment will reach \$250 billion.

87. The 2.4-meter rule. A 2.4-meter sailboat is a one-person boat that is about 13 feet in length, has a displacement of about 550 pounds, and a sail area of about 81 square feet. To compete in the 2.4-meter class, a boat must satisfy the formula

$$2.4 = \frac{L + 2D - F\sqrt{S}}{2.37},$$

where L = length, F = freeboard, D = girth, and S = sail area. Solve the formula for L .



PHOTO FOR EXERCISE 87

2.5

TRANSLATING VERBAL EXPRESSIONS INTO ALGEBRAIC EXPRESSIONS

In this section

- Writing Algebraic Expressions
- Pairs of Numbers
- Consecutive Integers
- Using Formulas
- Writing Equations

You translated some verbal expressions into algebraic expressions in Section 1.6; in this section you will study translating in more detail.

Writing Algebraic Expressions

The following box contains a list of some frequently occurring verbal expressions and their equivalent algebraic expressions.

Translating Words into Algebra

	Verbal Phrase	Algebraic Expression
Addition:	The sum of a number and 8	$x + 8$
	Five is added to a number	$x + 5$
	Two more than a number	$x + 2$
	A number increased by 3	$x + 3$
Subtraction:	Four is subtracted from a number	$x - 4$
	Three less than a number	$x - 3$
	The difference between 7 and a number	$7 - x$
	A number decreased by 2	$x - 2$
Multiplication:	The product of 5 and a number	$5x$
	Twice a number	$2x$
	One-half of a number	$\frac{1}{2}x$
	Five percent of a number	$0.05x$
Division:	The ratio of a number to 6	$\frac{x}{6}$
	The quotient of 5 and a number	$\frac{5}{x}$
	Three divided by some number	$\frac{3}{x}$

EXAMPLE 1**Writing algebraic expressions**

Translate each verbal expression into an algebraic expression.

- a) The sum of a number and 9 b) Eighty percent of a number
 c) A number divided by 4 d) The result of a number subtracted from 5
 e) Three less than a number

Solution

- a) If x is the number, then the sum of a number and 9 is $x + 9$.
 b) If w is the number, then eighty percent of the number is $0.80w$.
 c) If y is the number, then the number divided by 4 is $\frac{y}{4}$.
 d) If z is the number, then the result of subtracting z from 5 is $5 - z$.
 e) If a is the number, then 3 less than a is $a - 3$. ■

helpful hint

We know that x and $10 - x$ have a sum of 10 for any value of x . We can easily check that fact by adding:

$$x + 10 - x = 10$$

In general it is not true that x and $x - 10$ have a sum of 10, because

$$x + x - 10 = 2x - 10.$$

For what value of x is the sum of x and $x - 10$ equal to 10?

Pairs of Numbers

There is often more than one unknown quantity in a problem, but a relationship between the unknown quantities is given. For example, if one unknown number is 5 more than another unknown number, we can use

$$x \quad \text{and} \quad x + 5,$$

to represent them. Note that x and $x + 5$ can also be used to represent two unknown numbers that differ by 5, for if two numbers differ by 5, one of the numbers is 5 more than the other.

How would you represent two numbers that have a sum of 10? If one of the numbers is 2, the other is certainly $10 - 2$, or 8. Thus if x is one of the numbers, then $10 - x$ is the other. The expressions

$$x \quad \text{and} \quad 10 - x$$

have a sum of 10 for any value of x .

EXAMPLE 2**Algebraic expressions for pairs of numbers**

Write algebraic expressions for each pair of numbers.

- a) Two numbers that differ by 12 b) Two numbers with a sum of -8

Solution

- a) The expressions x and $x - 12$ represent two numbers that differ by 12. We can check by subtracting:

$$x - (x - 12) = x - x + 12 = 12$$

Of course, x and $x + 12$ also differ by 12 because $x + 12 - x = 12$.

- b) The expressions x and $-8 - x$ have a sum of -8 . We can check by addition:

$$x + (-8 - x) = x - 8 - x = -8 \quad \blacksquare$$

Pairs of numbers occur in geometry in discussing measures of angles. You will need the following facts about degree measures of angles.

Degree Measures of Angles

Two angles are called **complementary** if the sum of their degree measures is 90° .

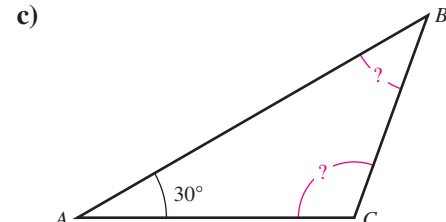
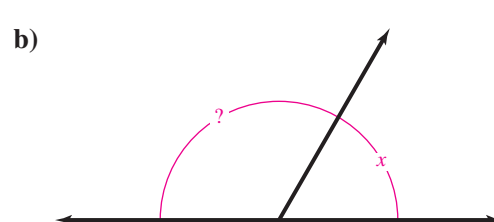
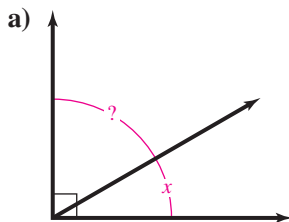
Two angles are called **supplementary** if the sum of their degree measures is 180° .

The sum of the degree measures of the three angles of any triangle is 180° .

For complementary angles, we use x and $90 - x$ for their degree measures. For supplementary angles, we use x and $180 - x$. Complementary angles that share a common side form a right angle. Supplementary angles that share a common side form a straight angle or straight line.

EXAMPLE 3 Degree measures

Write algebraic expressions for each pair of angles shown.



Solution

- Since the angles shown are complementary, we can use x to represent the degree measure of the smaller angle and $90 - x$ to represent the degree measure of the larger angle.
- Since the angles shown are supplementary, we can use x to represent the degree measure of the smaller angle and $180 - x$ to represent the degree measure of the larger angle.
- If we let x represent the degree measure of angle B , then $180 - x - 30$, or $150 - x$, represents the degree measure of angle C . ■

Consecutive Integers

To gain practice in problem solving, we will solve problems about consecutive integers in Section 2.6. Note that each integer is 1 larger than the previous integer, while consecutive even integers as well as consecutive odd integers differ by 2.

EXAMPLE 4 Expressions for integers

Write algebraic expressions for the following unknown integers.

- Three consecutive integers, the smallest of which is w
- Three consecutive even integers, the smallest of which is z

Solution

- Since each integer is 1 larger than the preceding integer, we can use w , $w + 1$, and $w + 2$ to represent them.
- Since consecutive even integers differ by 2, these integers can be represented by z , $z + 2$, and $z + 4$. ■

helpful hint

If x is even, then x , $x + 2$, and $x + 4$ represent three consecutive even integers. If x is odd, then x , $x + 2$, and $x + 4$ represent three consecutive odd integers. Is it possible for x , $x + 1$, and $x + 3$ to represent three consecutive even integers or three consecutive odd integers?

Using Formulas

In writing expressions for unknown quantities, we often use standard formulas such as those given at the back of the book.

EXAMPLE 5

Writing algebraic expressions using standard formulas

Find an algebraic expression for

- the distance if the rate is 30 miles per hour and the time is T hours.
- the discount if the rate is 40% and the original price is p dollars.

Solution

- Using the formula $D = RT$, we have $D = 30T$. So $30T$ is an expression that represents the distance in miles.
- Since the discount is the rate times the original price, an algebraic expression for the discount is $0.40p$ dollars. ■

Writing Equations

To solve a problem using algebra, we describe or **model** the problem with an equation. In this section we write the equations only, and in Section 2.6 we write and solve them. Sometimes we must write an equation from the information given in the problem and sometimes we use a standard model to get the equation. Some standard models are shown in the following box.

study tip

It is a good idea to work with others, but don't be misled. Working a problem with help is not the same as working a problem on your own. In the end, mathematics is personal. Make sure that you can do it.

Uniform Motion Model

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = R \cdot T$$

Selling Price and Discount Model

$$\text{Discount} = \text{Rate of discount} \cdot \text{Original price}$$

$$\text{Selling Price} = \text{Original price} - \text{Discount}$$

Real Estate Commission Model

$$\text{Commission} = \text{Rate of commission} \cdot \text{Selling price}$$

$$\text{Amount for owner} = \text{Selling price} - \text{Commission}$$

Percentage Models

$$\text{What number is } 5\% \text{ of } 40?$$

$$x = 0.05 \cdot 40$$

$$\text{Ten is what percent of } 80?$$

$$10 = x \cdot 80$$

$$\text{Twenty is } 4\% \text{ of what number?}$$

$$20 = 0.04 \cdot x$$

Geometric Models for Area

$$\text{Rectangle: } A = LW$$

$$\text{Square: } A = s^2$$

$$\text{Parallelogram: } A = bh$$

$$\text{Triangle: } A = \frac{1}{2}bh$$

Geometric Models for Perimeter

Perimeter of any figure = the sum of the lengths of the sides

$$\text{Rectangle: } P = 2L + 2W$$

$$\text{Square: } P = 4s$$

More geometric formulas can be found in Appendix A.

EXAMPLE 6 Writing equations

Identify the variable and write an equation that describes each situation.

- Find two numbers that have a sum of 14 and a product of 45.
- A coat is on sale for 25% off the list price. If the sale price is \$87, then what is the list price?
- What percent of 8 is 2?
- The value of x dimes and $x - 3$ quarters is \$2.05.

Solution

- a) Let $x =$ one of the numbers and $14 - x =$ the other number. Since their product is 45, we have

$$x(14 - x) = 45.$$

- b) Let $x =$ the list price and $0.25x =$ the amount of discount. We can write an equation expressing the fact that the selling price is the list price minus the discount:

$$\text{List price} - \text{discount} = \text{selling price}$$

$$x - 0.25x = 87$$

- c) If we let x represent the percentage, then the equation is $x \cdot 8 = 2$, or $8x = 2$.
- d) The value of x dimes at 10 cents each is $10x$ cents. The value of $x - 3$ quarters at 25 cents each is $25(x - 3)$ cents. We can write an equation expressing the fact that the total value of the coins is 205 cents:

$$\text{Value of dimes} + \text{value of quarters} = \text{total value}$$

$$10x + 25(x - 3) = 205$$

CAUTION The value of the coins in Example 6(d) is either 205 cents or 2.05 dollars. If the total value is expressed in dollars, then all of the values must be expressed in dollars. So we could also write the equation as

$$0.10x + 0.25(x - 3) = 2.05.$$

helpful hint

At this point we are simply learning to write equations that model certain situations. Don't worry about solving these equations now. In Section 2.6 we will solve problems by writing an equation and solving it.

WARM - UPS**True or false? Explain your answer.**

- For any value of x , the numbers x and $x + 6$ differ by 6.
- For any value of a , a and $10 - a$ have a sum of 10.
- If Jack ran at x miles per hour for 3 hours, he ran $3x$ miles.
- If Jill ran at x miles per hour for 10 miles, she ran for $10x$ hours.
- If the realtor gets 6% of the selling price and the house sells for x dollars, the owner gets $x - 0.06x$ dollars.
- If the owner got \$50,000 and the realtor got 10% of the selling price, the house sold for \$55,000.
- Three consecutive odd integers can be represented by x , $x + 1$, and $x + 3$.
- The value in cents of n nickels and d dimes is $0.05n + 0.10d$.
- If the sales tax rate is 5% and x represents the price of the goods purchased, then the total bill is $1.05x$.
- If the length of a rectangle is 4 feet more than the width w , then the perimeter is $w + (w + 4)$ feet.

2.5 EXERCISES

Reading and Writing After reading this section, write out the answers to these questions. Use complete sentences.

- What are the different ways of verbally expressing the operation of addition?
- How can you algebraically express two numbers using only one variable?
- What are complementary angles?
- What are supplementary angles?
- What is the relationship between distance, rate, and time?
- What is the difference between expressing consecutive even integers and consecutive odd integers algebraically?
- Two numbers with a sum of -8
- Two numbers such that one is 3 larger than the other
- Two numbers such that one is 8 smaller than the other
- Two numbers such that one is 5% of the other
- Two numbers such that one is 40% of the other
- Two numbers such that one is 30% more than the other
- Two numbers such that one is 20% smaller than the other

Each of the following figures shows a pair of angles. Write algebraic expressions for the degree measures of each pair of angles. See Example 3.

31.

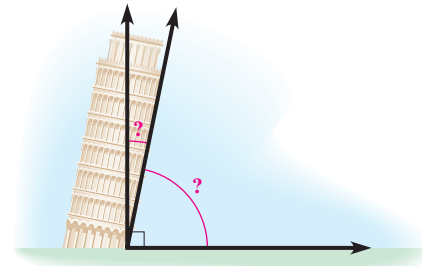


FIGURE FOR EXERCISE 31

32.

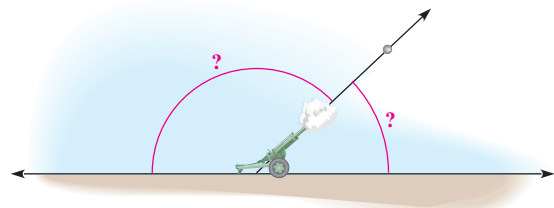


FIGURE FOR EXERCISE 32

33.

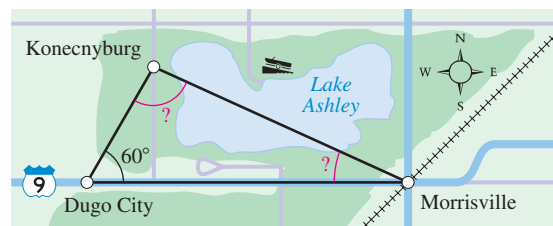


FIGURE FOR EXERCISE 33

Translate each verbal expression into an algebraic expression. See Example 1.

- The sum of a number and 3
- Two more than a number
- Three less than a number
- Four subtracted from a number
- The product of a number and 5
- Five divided by some number
- Ten percent of a number
- Eight percent of a number
- The ratio of a number and 3
- The quotient of 12 and a number
- One-third of a number
- Three-fourths of a number

Write algebraic expressions for each pair of numbers. See Example 2.

- Two numbers with a difference of 15
- Two numbers that differ by 9
- Two numbers with a sum of 6
- Two numbers with a sum of 5
- Two numbers with a sum of -4

34.

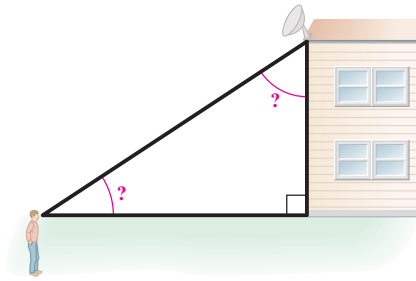


FIGURE FOR EXERCISE 34

Write algebraic expressions for the following unknown integers. See Example 4.

35. Two consecutive even integers, the smallest of which is n
36. Two consecutive odd integers, the smallest of which is x
37. Two consecutive integers
38. Three consecutive even integers
39. Three consecutive odd integers
40. Three consecutive integers
41. Four consecutive even integers
42. Four consecutive odd integers
- Find an algebraic expression for the quantity in italics using the given information. See Example 5.
43. The *distance*, given that the rate is x miles per hour and the time is 3 hours
44. The *distance*, given that the rate is $x + 10$ miles per hour and the time is 5 hours
45. The *discount*, given that the rate is 25% and the original price is q dollars
46. The *discount*, given that the rate is 10% and the original price is t yen
47. The *time*, given that the distance is x miles and the rate is 20 miles per hour
48. The *time*, given that the distance is 300 kilometers and the rate is $x + 30$ kilometers per hour
49. The *rate*, given that the distance is $x - 100$ meters and the time is 12 seconds
50. The *rate*, given that the distance is 200 feet and the time is $x + 3$ seconds
51. The *area* of a rectangle with length x meters and width 5 meters
52. The *area* of a rectangle with sides b yards and $b - 6$ yards
53. The *perimeter* of a rectangle with length $w + 3$ inches and width w inches
54. The *perimeter* of a rectangle with length r centimeters and width $r - 1$ centimeters
55. The *width* of a rectangle with perimeter 300 feet and length x feet
56. The *length* of a rectangle with area 200 square feet and width w feet
57. The *length* of a rectangle, given that its width is x feet and its length is 1 foot longer than twice the width
58. The *length* of a rectangle, given that its width is w feet and its length is 3 feet shorter than twice the width
59. The *area* of a rectangle, given that the width is x meters and the length is 5 meters longer than the width
60. The *perimeter* of a rectangle, given that the length is x yards and the width is 10 yards shorter
61. The *simple interest*, given that the principal is $x + 1000$, the rate is 18%, and the time is 1 year
62. The *simple interest*, given that the principal is $3x$, the rate is 6%, and the time is 1 year
63. The *price per pound* of peaches, given that x pounds sold for \$16.50
64. The *rate per hour* of a mechanic who gets \$480 for working x hours
65. The *degree measure* of an angle, given that its complementary angle has measure x degrees
66. The *degree measure* of an angle, given that its supplementary angle has measure x degrees
- Identify the variable and write an equation that describes each situation. Do not solve the equation. See Example 6.
67. Two numbers differ by 5 and have a product of 8
68. Two numbers differ by 6 and have a product of -9
69. Herman's house sold for x dollars. The real estate agent received 7% of the selling price, and Herman received \$84,532.
70. Gwen sold her car on consignment for x dollars. The saleswoman's commission was 10% of the selling price, and Gwen received \$6570.
71. What percent of 500 is 100?
72. What percent of 40 is 120?
73. The value of x nickels and $x + 2$ dimes is \$3.80.
74. The value of d dimes and $d - 3$ quarters is \$6.75.
75. The sum of a number and 5 is 13.
76. Twelve subtracted from a number is -6 .

77. The sum of three consecutive integers is 42.
78. The sum of three consecutive odd integers is 27.
79. The product of two consecutive integers is 182.
80. The product of two consecutive even integers is 168.
81. Twelve percent of Harriet's income is \$3000.
82. If 9% of the members buy tickets, then we will sell 252 tickets to this group.
83. Thirteen is 5% of what number?
84. Three hundred is 8% of what number?
85. The length of a rectangle is 5 feet longer than the width, and the area is 126 square feet.
86. The length of a rectangle is 1 yard shorter than twice the width, and the perimeter is 298 yards.
87. The value of n nickels and $n - 1$ dimes is 95 cents.
88. The value of q quarters, $q + 1$ dimes, and $2q$ nickels is 90 cents.
89. The measure of an angle is 38° smaller than the measure of its supplementary angle.
90. The measure of an angle is 16° larger than the measure of its complementary angle.
91. **Target heart rate.** For a cardiovascular work out, fitness experts recommend that you reach your target heart rate and stay at that rate for at least 20 minutes (*Cycling, Burkett and Darst*). To find your target heart rate, find the sum of your age and your resting heart rate, then subtract that sum from 220. Find 60% of that result and add it to your resting heart rate.

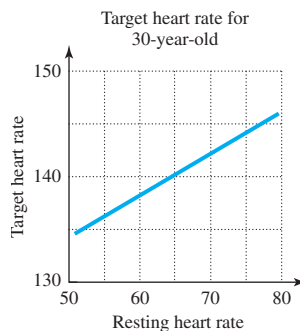


FIGURE FOR EXERCISE 91

- b) Judging from the accompanying graph, does the target heart rate for a 30-year-old increase or decrease as the resting heart rate increases.

92. **Adjusting the saddle.** The saddle height on a bicycle should be 109% of the rider's inside leg measurement L (*Cycling, Burkett and Darst*). See the figure. Write an equation expressing the fact that the saddle height for Brenda is 36 in.

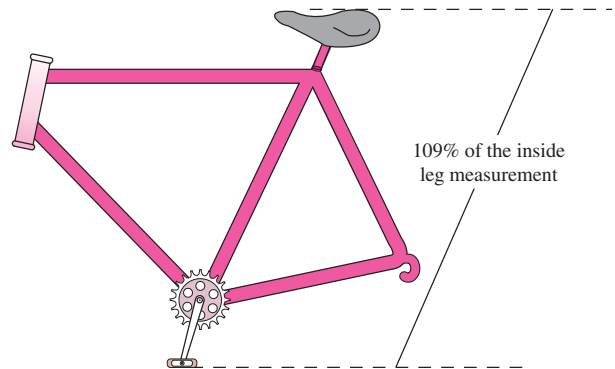


FIGURE FOR EXERCISE 92

Given that the area of each figure is 24 square feet, use the dimensions shown to write an equation expressing this fact. Do not solve the equation.

