

## 4.3

## EQUATIONS OF LINES IN SLOPE-INTERCEPT AND STANDARD FORM

## In this section

- Slope-Intercept Form
- Standard Form
- Using Slope-Intercept Form for Graphing
- Writing the Equation for a Line
- Applications

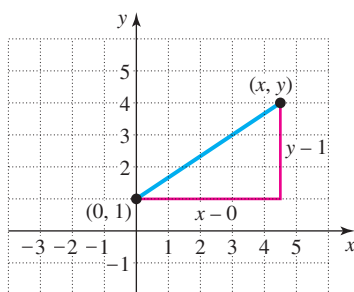


FIGURE 4.22

## study tip

Keep reviewing. After you have done your current assignment, go back a section or two and try a few problems. You will be amazed at how much your knowledge will improve with a regular review.

In Section 4.1 you learned that the graph of all solutions to a linear equation in two variables is a straight line. In this section we start with a line or a description of a line and write an equation for the line. The equation of a line in any form is called a **linear equation in two variables**.

## Slope-Intercept Form

Consider the line through  $(0, 1)$  with slope  $\frac{2}{3}$  shown in Fig. 4.22. If we use the points  $(x, y)$  and  $(0, 1)$  in the slope formula, we get an equation that is satisfied by every point on the line:

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= m && \text{Slope formula} \\ \frac{y - 1}{x - 0} &= \frac{2}{3} && \text{Let } (x_1, y_1) = (0, 1) \text{ and } (x_2, y_2) = (x, y). \\ \frac{y - 1}{x} &= \frac{2}{3}\end{aligned}$$

Now solve the equation for  $y$ :

$$\begin{aligned}x \cdot \frac{y - 1}{x} &= \frac{2}{3} \cdot x && \text{Multiply each side by } x. \\ y - 1 &= \frac{2}{3}x \\ y &= \frac{2}{3}x + 1 && \text{Add 1 to each side.}\end{aligned}$$

Because  $(0, 1)$  is on the  $y$ -axis, it is called the  **$y$ -intercept** of the line. Note how the slope  $\frac{2}{3}$  and the  $y$ -coordinate of the  $y$ -intercept  $(0, 1)$  appear in  $y = \frac{2}{3}x + 1$ . For this reason it is called the **slope-intercept form** of the equation of the line.

## Slope-Intercept Form

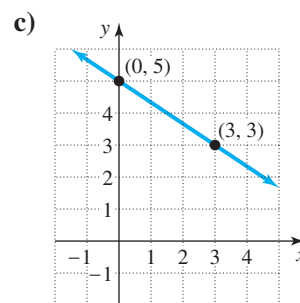
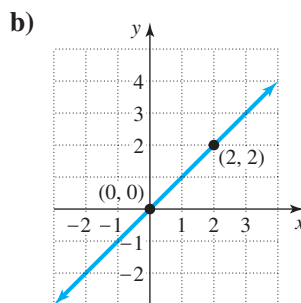
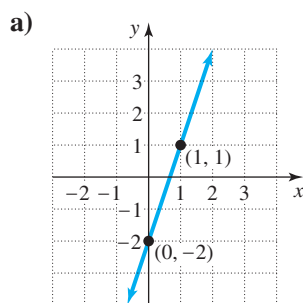
The equation of the line with  $y$ -intercept  $(0, b)$  and slope  $m$  is

$$y = mx + b.$$

## EXAMPLE 1

## Using slope-intercept form

Write the equation of each line in slope-intercept form.



## calculator



## close-up

Since a graphing calculator screen has a finite number of pixels, a graphing calculator plots only a finite number of ordered pairs that satisfy an equation. A graph is supposed to be a picture of all of the ordered pairs that satisfy an equation. Because a calculator plots many ordered pairs accurately and quickly, it can be a great help in drawing the graph of an equation.

**Solution**

- a) The  $y$ -intercept is  $(0, -2)$ , and the slope is 3. Use the form  $y = mx + b$  with  $b = -2$  and  $m = 3$ . The equation in slope-intercept form is

$$y = 3x - 2.$$

- b) The  $y$ -intercept is  $(0, 0)$ , and the slope is 1. So the equation is

$$y = x.$$

- c) The  $y$ -intercept is  $(0, 5)$ , and the slope is  $-\frac{2}{3}$ . So the equation is

$$y = -\frac{2}{3}x + 5.$$

The equation of a line may take many different forms. The easiest way to find the slope and  $y$ -intercept for a line is to rewrite the equation in slope-intercept form.

**EXAMPLE 2****Finding slope and  $y$ -intercept**

Determine the slope and  $y$ -intercept of the line  $3x - 2y = 6$ .

**Solution**

Solve for  $y$  to get slope-intercept form:

$$\begin{aligned} 3x - 2y &= 6 \\ -2y &= -3x + 6 \\ y &= \frac{3}{2}x - 3 \end{aligned}$$

The slope is  $\frac{3}{2}$ , and the  $y$ -intercept is  $(0, -3)$ .

**Standard Form**

The graph of the equation  $x = 3$  is a vertical line. Because slope is not defined for vertical lines, this line does not have an equation in slope-intercept form. Only non-vertical lines have equations in slope-intercept form. However, there is a form that includes all lines. It is called **standard form**.

**helpful hint**

In geometry we learn that two points determine a line. However, if you locate two points that are close together and draw a line through them, your line can have a lot of error in it at locations far from the two chosen points. Locating five points on the graph will improve your accuracy. If you draw a straight line with two points, they should be chosen as far as possible from each other.

**Standard Form**

Every line has an equation in the form

$$Ax + By = C$$

where  $A$ ,  $B$ , and  $C$  are real numbers with  $A$  and  $B$  not both zero.

To write the equation  $x = 3$  in this form, let  $A = 1$ ,  $B = 0$ , and  $C = 3$ . We get

$$1 \cdot x + 0 \cdot y = 3,$$

which is equivalent to

$$x = 3.$$

In Example 2 we converted an equation in standard form to slope-intercept form. Any linear equation in standard form with  $B \neq 0$  can be written in slope-intercept

form by solving for  $y$ . In the next example we convert an equation in slope-intercept form to standard form.

### EXAMPLE 3

#### Converting to standard form

Write the equation of the line  $y = \frac{2}{5}x + 3$  in standard form using only integers.

#### Solution

To get standard form, first subtract  $\frac{2}{5}x$  from each side:

$$\begin{aligned} y &= \frac{2}{5}x + 3 \\ -\frac{2}{5}x + y &= 3 \\ -5\left(-\frac{2}{5}x + y\right) &= -5 \cdot 3 && \text{Multiply each side by } -5 \text{ to eliminate the fraction} \\ &&& \text{and get positive } 2x. \\ 2x - 5y &= -15 \end{aligned}$$

The answer  $2x - 5y = -15$  in Example 3 is not the only answer using only integers. Equations such as  $-2x + 5y = 15$  and  $4x - 10y = -30$  are equivalent equations in standard form. We prefer to write  $2x - 5y = -15$  because the greatest common factor of 2, 5, and 15 is 1 and the coefficient of  $x$  is positive.

### Using Slope-Intercept Form for Graphing

One way to graph a linear equation is to find several points that satisfy the equation and then draw a straight line through them. We can also graph a linear equation by using the  $y$ -intercept and the slope.

#### Strategy for Graphing a Line Using Slope and $y$ -Intercept

1. Write the equation in slope-intercept form if necessary.
2. Plot the  $y$ -intercept.
3. Starting from the  $y$ -intercept, use the rise and run to locate a second point.
4. Draw a line through the two points.

### EXAMPLE 4

#### Graphing a line using $y$ -intercept and slope

Graph the line  $2x - 3y = 3$ .

#### Solution

First write the equation in slope-intercept form:

$$\begin{aligned} 2x - 3y &= 3 \\ -3y &= -2x + 3 && \text{Subtract } 2x \text{ from each side.} \\ y &= \frac{2}{3}x - 1 && \text{Divide each side by } -3. \end{aligned}$$

The slope is  $\frac{2}{3}$ , and the  $y$ -intercept is  $(0, -1)$ . A slope of  $\frac{2}{3}$  means a rise of 2 and a run of 3. Start at  $(0, -1)$  and go up two units and to the right three units to locate a second point on the line. Now draw a line through the two points. See Fig. 4.23 for the graph of  $2x - 3y = 3$ .

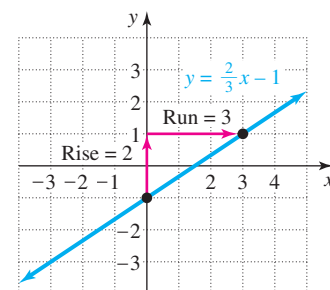
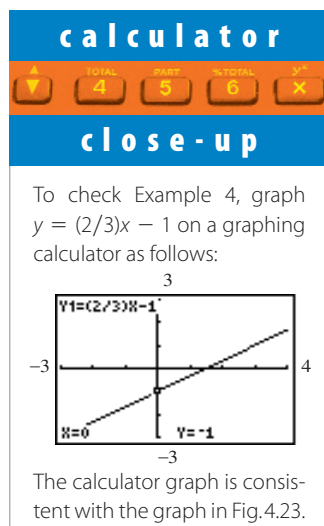


FIGURE 4.23

**CAUTION** When using the slope to find a second point on the line, be sure to start at the  $y$ -intercept, not at the origin.

### EXAMPLE 5 Graphing a line using $y$ -intercept and slope

Graph the line  $y = -3x + 4$ .

#### Solution

The slope is  $-3$ , and the  $y$ -intercept is  $(0, 4)$ . Because  $-3 = \frac{-3}{1}$ , we use a rise of  $-3$  and a run of  $1$ . To locate a second point on the line, start at  $(0, 4)$  and go down three units and to the right one unit. Draw a line through the two points. See Fig. 4.24.

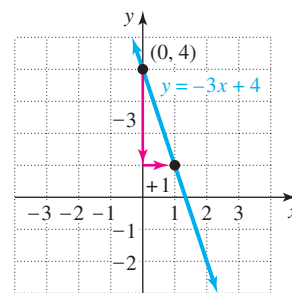


FIGURE 4.24

### Writing the Equation for a Line

In Example 1 we wrote the equation of a line by finding its slope and  $y$ -intercept from a graph. In the next example we write the equation of a line from a description of the line.

### EXAMPLE 6 Writing an equation

Write the equation in slope-intercept form for the line through  $(0, 4)$  that is perpendicular to the line  $2x - 4y = 1$ .

#### Solution

First find the slope of  $2x - 4y = 1$ :

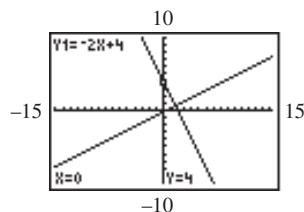
$$\begin{aligned} 2x - 4y &= 1 \\ -4y &= -2x + 1 \\ y &= \frac{1}{2}x - \frac{1}{4} \end{aligned} \quad \text{The slope of this line is } \frac{1}{2}.$$

The slope of the line that we are interested in is the opposite of the reciprocal of  $\frac{1}{2}$ . So the line has slope  $-2$  and  $y$ -intercept  $(0, 4)$ . Its equation is  $y = -2x + 4$ .

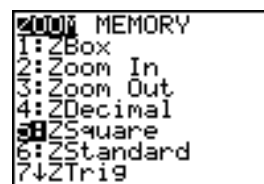
### calculator close-up

If you use the same minimum and maximum window values for  $x$  and  $y$ , then the length of one unit on the  $x$ -axis is larger than on the  $y$ -axis because the screen is longer in the  $x$ -direction. In this case, perpendicular lines will not look perpendicular. The viewing window chosen here for the lines in Example 6 makes them look perpendicular.

Any viewing window proportional to this one will also produce approximately the same unit length on each axis. Some



calculators have a square feature that automatically makes the unit length the same on both axes.



## Applications

The slope-intercept and standard forms are both important in applications.

### EXAMPLE 7

#### Changing forms

A landscaper has a total of \$800 to spend on bushes at \$20 each and trees at \$50 each. So if  $x$  is the number of bushes and  $y$  is the number of trees he can buy, then  $20x + 50y = 800$ . Write this equation in slope-intercept form. Find and interpret the  $y$ -intercept and the slope.

#### Solution

Write in slope-intercept form:

$$\begin{aligned} 20x + 50y &= 800 \\ 50y &= -20x + 800 \\ y &= -\frac{2}{5}x + 16 \end{aligned}$$

The slope is  $-\frac{2}{5}$  and the intercept is  $(0, 16)$ . So he can get 16 trees if he buys no bushes and he loses  $\frac{2}{5}$  of a tree for each additional bush that he purchases. ■

## WARM-UPS

#### True or false? Explain your answer.

1. There is only one line with  $y$ -intercept  $(0, 3)$  and slope  $-\frac{4}{3}$ .
2. The equation of the line through  $(1, 2)$  with slope 3 is  $y = 3x + 2$ .
3. The vertical line  $x = -2$  has no  $y$ -intercept.
4. The equation  $x = 5$  has a graph that is a vertical line.
5. The line  $y = x - 3$  is perpendicular to the line  $y = 5 - x$ .
6. The line  $y = 2x - 3$  is parallel to the line  $y = 4x - 3$ .
7. The line  $2y = 3x - 8$  has a slope of 3.
8. Every straight line in the coordinate plane has an equation in standard form.
9. The line  $x = 2$  is perpendicular to the line  $y = 5$ .
10. The line  $y = x$  has no  $y$ -intercept.

## 4.3 EXERCISES

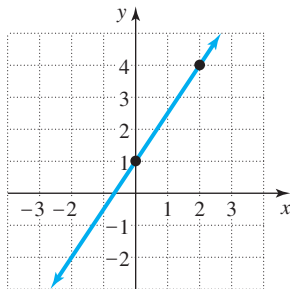
**Reading and Writing** After reading this section, write out the answers to these questions. Use complete sentences.

1. What is the slope-intercept form for the equation of a line?
2. How can you determine the slope and  $y$ -intercept from the slope-intercept form?
3. What is the standard form for the equation of a line?
4. How can you graph a line when the equation is in slope-intercept form?

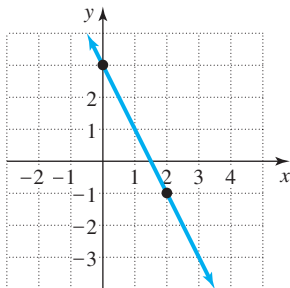
5. What form is used in this section to write an equation of a line from a description of the line?
6. What makes lines look perpendicular on a graph?

Write an equation for each line. Use slope-intercept form if possible. See Example 1.

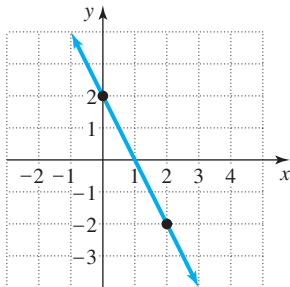
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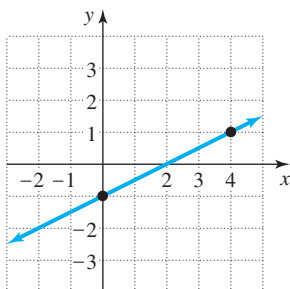
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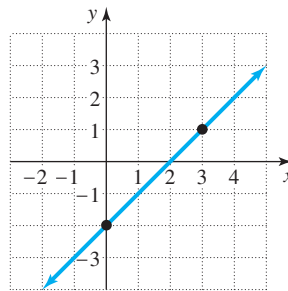
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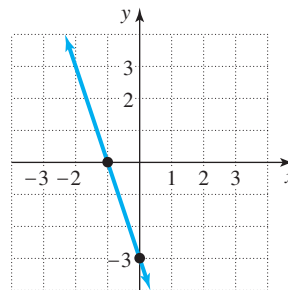
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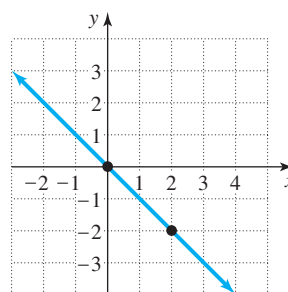
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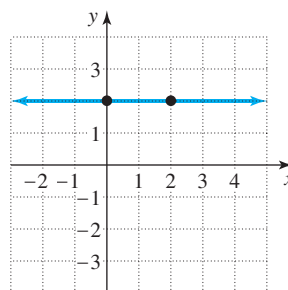
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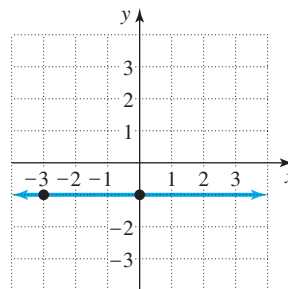
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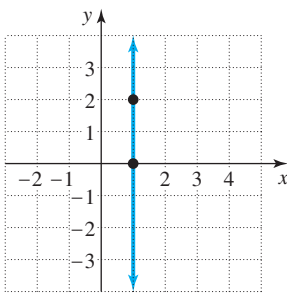
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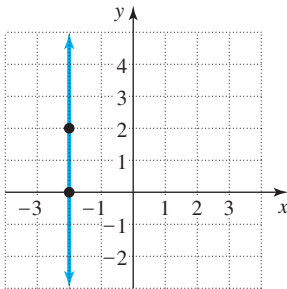
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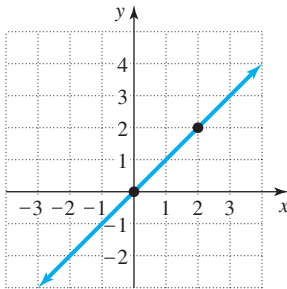
16.



17.



18.



Find the slope and y-intercept for each line that has a slope and y-intercept. See Example 2.

19.  $y = 3x - 9$     20.  $y = -5x + 4$     21.  $y = 4$

22.  $y = -5$     23.  $y = -3x$     24.  $y = 2x$

25.  $x + y = 5$     26.  $x - y = 4$     27.  $x - 2y = 4$

28.  $x + 2y = 3$     29.  $2x - 5y = 10$     30.  $2x + 3y = 9$

31.  $2x - y + 3 = 0$     32.  $3x - 4y - 8 = 0$

33.  $x = -3$

34.  $\frac{2}{3}x = 4$

Write each equation in standard form using only integers. See Example 3.

35.  $y = -x + 2$

36.  $y = 3x - 5$

37.  $y = \frac{1}{2}x + 3$

38.  $y = \frac{2}{3}x - 4$

39.  $y = \frac{3}{2}x - \frac{1}{3}$

40.  $y = \frac{4}{5}x + \frac{2}{3}$

41.  $y = -\frac{3}{5}x + \frac{7}{10}$

42.  $y = -\frac{2}{3}x - \frac{5}{6}$

43.  $\frac{3}{5}x + 6 = 0$

44.  $\frac{1}{2}x - 9 = 0$

45.  $\frac{3}{4}y = \frac{5}{2}$

46.  $\frac{2}{3}y = \frac{1}{9}$

47.  $\frac{x}{2} = \frac{3y}{5}$

48.  $\frac{x}{8} = -\frac{4y}{5}$

49.  $y = 0.02x + 0.5$

50.  $0.2x = 0.03y - 0.1$

Draw the graph of each line using its y-intercept and its slope. See Examples 4 and 5.

51.  $y = 2x - 1$

52.  $y = 3x - 2$

53.  $y = -3x + 5$

54.  $y = -4x + 1$

55.  $y = \frac{3}{4}x - 2$

56.  $y = \frac{3}{2}x - 4$

57.  $2y + x = 0$

62.  $y + 5 = 0$

58.  $2x + y = 0$

Write an equation in slope-intercept form, if possible, for each line. See Example 6. In each case, make a sketch.

63. The line through  $(0, 6)$  that is perpendicular to the line  $y = 3x - 5$

64. The line through  $(0, -1)$  that is perpendicular to the line  $y = x$

65. The line with  $y$ -intercept  $(0, 3)$  that is parallel to the line  $2x + y = 5$

66. The line through the origin that is parallel to the line  $2x - 5y = 8$

59.  $3x - 2y = 10$

67. The line through  $(2, 3)$  that runs parallel to the  $x$ -axis

68. The line through  $(-3, 5)$  that runs parallel to the  $y$ -axis

69. The line through  $(0, 4)$  and  $(5, 0)$

70. The line through  $(0, -3)$  and  $(4, 0)$

Solve each problem. See Example 7.

60.  $4x + 3y = 9$

71. **Marginal cost.** A manufacturer plans to spend \$150,000 on research and development for a new lawn mower and then \$200 to manufacture each mower. The formula  $C = 200n + 150,000$  gives the cost in dollars of  $n$  mowers. What is the cost of 5000 mowers? What is the cost of 5001 mowers? By how much did the one extra lawn mower increase the cost? (The increase in cost is called the *marginal cost* of the 5001st lawn mower.)

72. **Marginal revenue.** A defense attorney charges her client \$4000 plus \$120 per hour. The formula  $R = 120n + 4000$  gives her revenue in dollars for

61.  $y - 2 = 0$

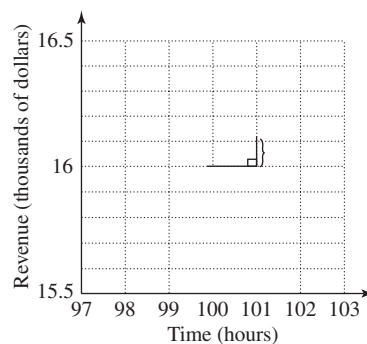


FIGURE FOR EXERCISE 72

$n$  hours of work. What is her revenue for 100 hours of work? What is her revenue for 101 hours of work? By how much did the one extra hour of work increase the revenue? (The increase in revenue is called the *marginal revenue* for the 101st hour.)

73. **In-house training.** The accompanying graph shows the percentage of U.S. workers receiving training by their employers. The percentage went from 5% in year 0 (1981) to 16% in year 14 (1995).
- Find the slope of this line.
  - Write the equation of the line in slope-intercept form.
  - Use your equation to predict the percentage that will be receiving training in the year 2000.

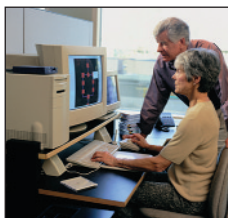
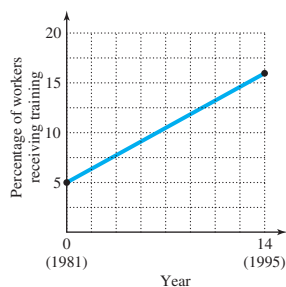


FIGURE FOR EXERCISE 73

74. **Women and marriage.** The percentage of women in the 20 to 24 age group who have never married went from 64% in year 0 (1970) to 33% in year 26 (1996) (Census Bureau, [www.census.gov](http://www.census.gov)).
- Find the equation of the line through the two points  $(0, 0.64)$  and  $(26, 0.33)$  in slope-intercept form.
  - Use your equation to predict what the percentage will be in the year 2000.
75. **Pansies and snapdragons.** A nursery manager plans to spend \$100 on 6-packs of pansies at 50 cents per pack and snapdragons at 25 cents per pack. The equation  $0.50x + 0.25y = 100$  can be used to model this situation.
- What do  $x$  and  $y$  represent?

b) Graph the equation.

c) Write the equation in slope-intercept form.

- What is the slope of the line?
- What does the slope tell you?

76. **Pens and pencils.** A bookstore manager plans to spend \$60 on pens at 30 cents each and pencils at 10 cents each. The equation  $0.10x + 0.30y = 60$  can be used to model this situation.

a) What do  $x$  and  $y$  represent?

b) Graph the equation.

c) Write the equation in slope-intercept form.

d) What is the slope of the line?

e) What does the slope tell you?



### GRAPHING CALCULATOR EXERCISES

Graph each pair of straight lines on your graphing calculator using a viewing window that makes the lines look perpendicular. Answers may vary.

77.  $y = 12x - 100$ ,  $y = -\frac{1}{12}x + 50$

78.  $2x - 3y = 300$ ,  $3x + 2y = -60$