4.4 THE POINT-SLOPE FORM

In this section

- Point-Slope Form
- Parallel Lines
- Perpendicular Lines

In Section 4.3 we wrote the equation of a line given its slope and y-intercept. In this section you will learn to write the equation of a line given the slope and *any* other point on the line.

Point-Slope Form

Consider a line through the point (4, 1) with slope $\frac{2}{3}$ as shown in Fig. 4.25. Because the slope can be found by using any two points on the line, we use (4, 1) and an arbitrary point (x, y) in the formula for slope:

$$\frac{y_2 - y_1}{x_2 - x_1} = m \qquad \text{Slope formula}$$

$$\frac{y - 1}{x - 4} = \frac{2}{3} \qquad \text{Let } m = \tfrac{2}{3},\ (x_1, y_1) = (4, 1), \text{ and } (x_2, y_2) = (x, y).$$

$$y - 1 = \frac{2}{3}(x - 4) \qquad \text{Multiply each side by } x - 4.$$

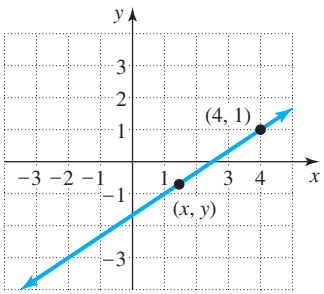


FIGURE 4.25

Note how the coordinates of the point (4, 1) and the slope $\frac{2}{3}$ appear in the above equation. We can use the same procedure to get the equation of any line given one point on the line and the slope. The resulting equation is called the **point-slope form** of the equation of the line.

helpful hint

If a point (x, y) is on a line with slope m through (x_1, y_1), then

$$\frac{y - y_1}{x - x_1} = m.$$

Multiplying each side of this equation by $x - x_1$ gives us the point-slope form.

Point-Slope Form

The equation of the line through the point (x_1, y_1) with slope m is

$$y - y_1 = m(x - x_1).$$

EXAMPLE 1

Writing an equation given a point and a slope

Find the equation of the line through $(-2, 3)$ with slope $\frac{1}{2}$, and write it in slope-intercept form.

Solution

Because we know a point and the slope, we can use the point-slope form:

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - 3 = \frac{1}{2}[x - (-2)] \qquad \text{Substitute } m = \tfrac{1}{2} \text{ and } (x_1, y_1) = (-2, 3).$$

$$y - 3 = \frac{1}{2}(x + 2) \qquad \text{Simplify.}$$

$$y - 3 = \frac{1}{2}x + 1 \qquad \text{Distributive property}$$

$$y = \frac{1}{2}x + 4 \qquad \text{Slope-intercept form}$$

Alternate Solution

Replace m by $\frac{1}{2}$, x by -2, and y by 3 in the slope-intercept form:

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ 3 &= \frac{1}{2}(-2) + b && \text{Substitute } m = \quad \text{and } (x, y) = (-2, 3). \\ 3 &= -1 + b && \text{Simplify.} \\ 4 &= b \end{aligned}$$

Since $b = 4$, we can write $y = \frac{1}{2}x + 4$. ■

CAUTION The point-slope form can be used to find the equation of a line for *any* given point and slope. However, if the given point is the y-intercept, then it is simpler to use the slope-intercept form.

EXAMPLE 2 Writing an equation given two points

Find the equation of the line that contains the points $(-3, -2)$ and $(4, -1)$, and write it in standard form.

Solution

First find the slope using the two given points:

$$m = \frac{-2 - (-1)}{-3 - 4} = \frac{-1}{-7} = \frac{1}{7}$$

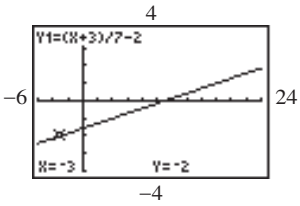
Now use one of the points, say $(-3, -2)$, and slope $\frac{1}{7}$ in the point-slope form:

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-slope form} \\ y - (-2) &= \frac{1}{7}[x - (-3)] && \text{Substitute.} \\ y + 2 &= \frac{1}{7}(x + 3) && \text{Simplify.} \\ 7(y + 2) &= 7 \cdot \frac{1}{7}(x + 3) && \text{Multiply each side by 7.} \\ 7y + 14 &= x + 3 \\ 7y &= x - 11 && \text{Subtract 14 from each side.} \\ -x + 7y &= -11 && \text{Subtract } x \text{ from each side.} \\ x - 7y &= 11 && \text{Multiply each side by } -1. \end{aligned}$$

The equation in standard form is $x - 7y = 11$. Using the other given point, $(4, -1)$, would give the same final equation in standard form. Try it. ■

calculator close-up

Graph $y = (x + 3)/7 - 2$ to see that the line goes through $(-3, -2)$ and $(4, -1)$.



Note that the form of the equation does not matter on the calculator as long as it is solved for y.

Parallel Lines

In Section 4.2 you learned that parallel lines have the same slope. For example, the lines $y = 6x - 4$ and $y = 6x + 7$ are parallel because each has slope 6. In the next example we write the equation of a line that is parallel to a given line and contains a given point.

EXAMPLE 3 **Writing an equation given a point and a parallel line**

Write the equation of the line that is parallel to the line $3x + y = 9$ and contains the point $(2, -1)$. Give the answer in slope-intercept form.

Solution

We want the equation of the line through $(2, -1)$ that is parallel to $3x + y = 9$, as shown in Fig. 4.26. First write $3x + y = 9$ in slope-intercept form to determine its slope:

$$3x + y = 9$$
$$y = -3x + 9$$

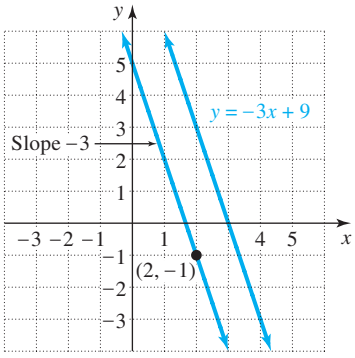


FIGURE 4.26

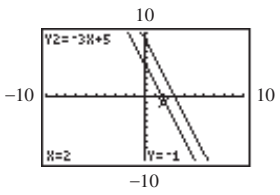
The slope of $3x + y = 9$ and any line parallel to it is -3. So we want the equation of the line that has slope -3 and contains $(2, -1)$. Use the point-slope form:

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - (-1) = -3(x - 2) \quad \text{Substitute.}$$
$$y + 1 = -3x + 6 \quad \text{Simplify.}$$
$$y = -3x + 5 \quad \text{Slope-intercept form}$$

The line $y = -3x + 5$ has slope -3 and contains the point $(2, -1)$. Check that $(2, -1)$ satisfies $y = -3x + 5$. ■

calculator close-up

Graph $y = -3x + 9$ and $y = -3x + 5$ as follows:



Because the lines look parallel and $y = -3x + 5$ goes through $(2, -1)$, this graph supports the answer to Example 3.

Perpendicular Lines

In Section 4.2 you learned that lines with slopes m and $-\frac{1}{m}$ (for $m \neq 0$) are perpendicular to each other. For example, the lines

$$y = -2x + 7 \quad \text{and} \quad y = \frac{1}{2}x - 8$$

are perpendicular to each other. In the next example we will write the equation of a line that is perpendicular to a given line and contains a given point.

EXAMPLE 4 **Writing an equation given a point and a perpendicular line**

Write the equation of the line that is perpendicular to $3x + 2y = 8$ and contains the point $(1, -3)$. Write the answer in slope-intercept form.

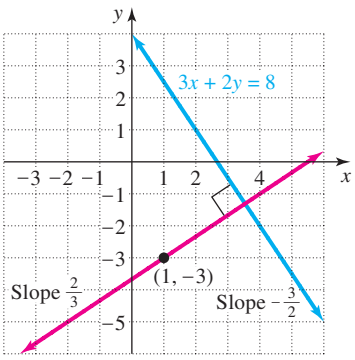
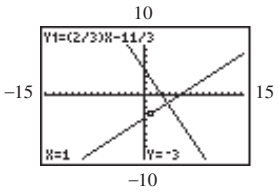


FIGURE 4.27

calculator close-up

Graph $y_1 = (2/3)x - 11/3$ and $y_2 = (-3/2)x + 4$ as shown:



Because the lines look perpendicular and y_1 goes through $(1, -3)$, the graph supports the answer to Example 4.

Solution

First graph $3x + 2y = 8$ and a line through $(1, -3)$ that is perpendicular to $3x + 2y = 8$ as shown in Fig. 4.27. The right angle symbol is used in the figure to indicate that the lines are perpendicular. Now write $3x + 2y = 8$ in slope-intercept form to determine its slope:

$$\begin{aligned} 3x + 2y &= 8 \\ 2y &= -3x + 8 \\ y &= -\frac{3}{2}x + 4 \end{aligned}$$ Slope-intercept form

The slope of the given line is $-\frac{3}{2}$. The slope of any line perpendicular to it is $\frac{2}{3}$. Now we use the point-slope form with the point $(1, -3)$ and the slope $\frac{2}{3}$:

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$\begin{aligned} y - (-3) &= \frac{2}{3}(x - 1) \\ y + 3 &= \frac{2}{3}x - \frac{2}{3} \\ y &= \frac{2}{3}x - \frac{2}{3} - 3 \end{aligned}$$ Subtract 3 from each side.

$$y = \frac{2}{3}x - \frac{11}{3}$$ Slope-intercept form

So $y = \frac{2}{3}x - \frac{11}{3}$ is the equation of the line that contains $(1, -3)$ and is perpendicular to $3x + 2y = 8$. Check that $(1, -3)$ satisfies $y = \frac{2}{3}x - \frac{11}{3}$. ■

WARM-UPS

True or false? Explain your answer.

1. The formula $y = m(x - x_1)$ is the point-slope form for a line.
2. It is impossible to find the equation of a line through $(2, 5)$ and $(-3, 1)$.
3. The point-slope form will not work for the line through $(3, 4)$ and $(3, 6)$.
4. The equation of the line through the origin with slope 1 is $y = x$.
5. The slope of the line $5x + y = 4$ is 5.
6. The slope of any line perpendicular to the line $y = 4x - 3$ is $-\frac{1}{4}$.
7. The slope of any line parallel to the line $x + y = 1$ is -1.
8. The line $2x - y = -1$ goes through the point $(-2, -3)$.
9. The line is $2x + y = 4$ and $y = -2x + 7$ are parallel.
10. The equation of the line through $(0, 0)$ perpendicular to $y = x$ is $y = -x$.

4.4 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is the point-slope form for the equation of a line?

2. For what is the point-slope form used?

3. What is the procedure for finding the equation of a line when given two points on the line?

4. How can you find the slope of a line when given the equation of the line?

5. What is the relationship between the slopes of parallel lines?

6. What is the relationship between the slopes of perpendicular lines?

Write each equation in slope-intercept form. See Example 1.

7. $y - 1 = 5(x + 2)$ **8.** $y + 3 = -3(x - 6)$

9. $3x - 4y = 80$ **10.** $2x + 3y = 90$

11. $y - \frac{1}{2} = \frac{2}{3}\left(x - \frac{1}{4}\right)$ **12.** $y + \frac{2}{3} = -\frac{1}{2}\left(x - \frac{2}{5}\right)$

Find the equation of each line. Write each answer in slope-intercept form. See Example 1.

13. The line through (2, 3) with slope $\frac{1}{3}$

14. The line through (1, 4) with slope $\frac{1}{4}$

15. The line through $(-2, 5)$ with slope $-\frac{1}{2}$

16. The line through $(-3, 1)$ with slope $-\frac{1}{3}$

17. The line with slope -6 that goes through $(-1, -7)$

18. The line with slope -8 that goes through $(-1, -5)$

Write each equation in standard form using only integers. See Example 2.

19. $y - 3 = 2(x - 5)$ **20.** $y + 2 = -3(x - 1)$

21. $y = \frac{1}{2}x - 3$ **22.** $y = \frac{1}{3}x + 5$

23. $y - 2 = \frac{2}{3}(x - 4)$ **24.** $y + 1 = \frac{3}{2}(x + 4)$

Find the equation of each line. Write each answer in standard form using only integers. See Example 2.

25. The line through the points (1, 2) and (5, 8)

26. The line through the points (3, 5) and (8, 15)

27. The line through the points $(-2, -1)$ and $(3, -4)$

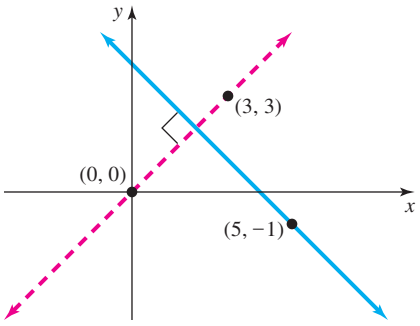
28. The line through the points $(-1, -3)$ and $(2, -1)$

29. The line through the points $(-2, 0)$ and $(0, 2)$

30. The line through the points (0, 3) and (5, 0)

The lines in each figure are perpendicular. Find the equation (in slope-intercept form) for the solid line.

31.



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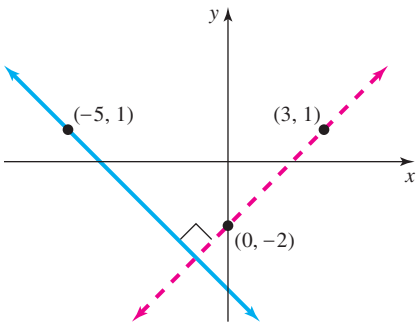

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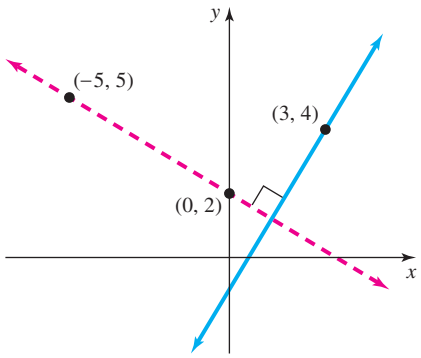

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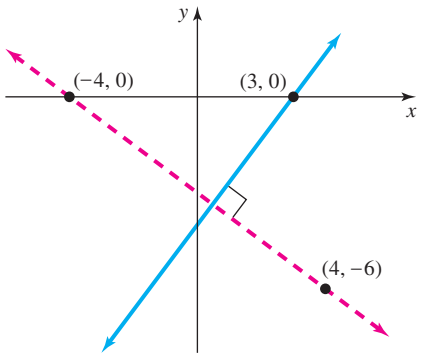

Find the equation of each line. Write each answer in slope-intercept form. See Examples 3 and 4.

35. The line contains the point (3, 4) and is perpendicular to $y = 3x - 1$.

36. The line contains the point $(-2, 3)$ and is perpendicular to $y = 2x + 7$.

37. The line is parallel to $y = x - 9$ and goes through the point (7, 10).

38. The line is parallel to $y = -x + 5$ and goes through the point $(-3, 6)$.

39. The line is perpendicular to $3x - 2y = 10$ and passes through the point (1, 1).

40. The line is perpendicular to $x - 5y = 4$ and passes through the point $(-1, 1)$.

41. The line is parallel to $2x + y = 8$ and contains the point $(-1, -3)$.

42. The line is parallel to $-3x + 2y = 9$ and contains the point $(-2, 1)$.

43. The line goes through $(-1, 2)$ and is perpendicular to $3x + y = 5$.

44. The line goes through (1, 2) and is perpendicular to $y = \frac{1}{2}x - 3$.

45. The line goes through (2, 3) and is parallel to $-2x + y = 6$.

46. The line goes through (1, 4) and is parallel to $x - 2y = 6$.

Solve each problem.

47. ***Automated tellers.*** ATM volume reached 10.6 billion transactions in 1996 (Electronic Commerce Data Base). The accompanying graph shows the steady growth of automatic tellers.

a) Write the equation of the line through (92, 7.0) and (96, 10.6).

b) Use the equation to predict the number of transactions at automated teller machines in the year 2005?

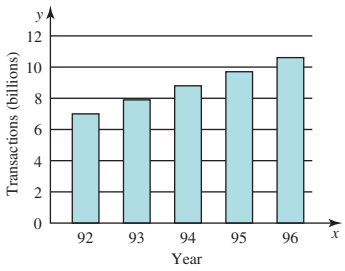

FIGURE FOR EXERCISE 47

48. ***Direct deposit.*** In 1994, one-third of all workers participated in direct deposit of their paychecks and this number is expected to reach three-fourths by the year 2000. (New York Automated Clearing House, www.nyach.org).

a) Write the equation of the line through (1993, 1/3) and (2000, 3/4).

b) Use the accompanying graph to predict the year in which 100% of all workers will participate in direct deposit of their paychecks.

c) Use the equation from part (a) to predict the year in which 100% of all workers will participate in direct deposit of their paychecks.

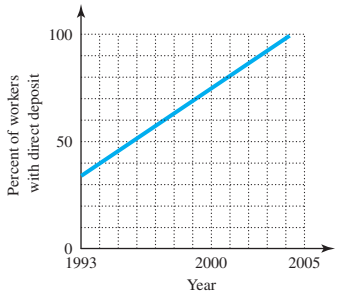

FIGURE FOR EXERCISE 48

49. ***Gross domestic product.*** The U.S. per capita gross domestic product went from $14,000 in 1970 to $18,000 in 1992 (*World Resources,* 1997).

a) Write the equation of the line through the points (1970, 14,000) and (1992, 18,000).

b) What do x and y represent in your equation?

c) Graph the equation

d) By how much is the gross domestic product increasing per year?

e) Use your equation to predict the per capita gross domestic product in the year 2000.

50. ***Body-mass index.*** The body mass index BMI is used to assess the level of fat in a person's body. When Tim weighed 147 pounds his BMI was 23.4. When his weight went to 185, his BMI was 29.5.

a) Find the equation of the line through (147, 23.4) and (185, 29.5).

b) What do x and y represent in your equation?

c) Graph the equation.

d) Interpret the slope of this line.

e) What is his BMI when his weight is 160?

GETTING MORE INVOLVED

51. ***Exploration.*** What is the slope of the line $2x + 3y = 9$? What is the slope of $4x - 5y = 6$? Write a formula for the slope of $Ax + By = C$, where $B \neq 0$.

GRAPHING CALCULATOR EXERCISES

52. Graph each equation on a graphing calculator. Choose a viewing window that includes both the x- and y-intercepts. Use the calculator output to help you draw the graph on paper.

a) $y = 20x - 300$

b) $y = -30x + 500$

c) $2x - 3y = 6000$

53. Graph $y = 2x + 1$ and $y = 1.99x - 1$ on a graphing calculator. Are these lines parallel? Explain your answer.

54. Graph $y = 0.5x + 0.8$ and $y = 0.5x + 0.7$ on a graphing calculator. Find a viewing window in which the two lines are separate.

55. Graph $y = 3x + 1$ and $y = -\frac{1}{3}x + 2$ on a graphing calculator. Do the lines look perpendicular? Explain.