98. ***Height difference.*** A red ball and a green ball are simultaneously tossed into the air. The red ball is given an initial velocity of 96 feet per second, and its height t seconds after it is tossed is $-16t^2 + 96t$ feet. The green ball is given an initial velocity of 30 feet per second, and its height t seconds after it is tossed is $-16t^2 + 80t$ feet.

a) Find a polynomial that represents the difference in the heights of the two balls.

b) How much higher is the red ball 2 seconds after the balls are tossed?

c) In reality, when does the difference in the heights stop increasing?

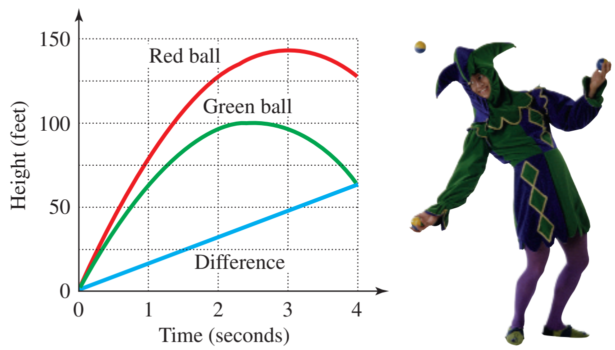

FIGURE FOR EXERCISE 98

99. ***Total interest.*** Donald received $0.08(x + 554)$ dollars interest on one investment and $0.09(x + 335)$ interest on another investment. Write a polynomial that represents the total interest he received. What is the total interest if $x = 1000$?

100. ***Total acid.*** Deborah figured that the amount of acid in one bottle of solution is $0.12x$ milliliters and the amount of acid in another bottle of solution is $0.22(75 - x)$ milliliters. Find a polynomial that represents the total amount of acid. What is the total amount of acid if $x = 50$?

101. ***Harris-Benedict for females.*** The Harris-Benedict polynomial

$$655.1 + 9.56w + 1.85h - 4.68a$$

represents the number of calories needed to maintain a female at rest for 24 hours, where w is her weight in kilograms, h is her height in centimeters, and a is her age. Find the number of calories needed by a 30-year-old 54-kilogram female who is 157 centimeters tall.

102. ***Harris-Benedict for males.*** The Harris-Benedict polynomial

$$66.5 + 13.75w + 5.0h - 6.78a$$

represents the number of calories needed to maintain a male at rest for 24 hours, where w is his weight in kilograms, h is his height in centimeters, and a is his age. Find the number of calories needed by a 40-year-old 90-kilogram male who is 185 centimeters tall.

GETTING MORE INVOLVED

103. ***Discussion.*** Is the sum of two natural numbers always a natural number? Is the sum of two integers always an integer? Is the sum of two polynomials always a polynomial? Explain.

104. ***Discussion.*** Is the difference of two natural numbers always a natural number? Is the difference of two rational numbers always a rational number? Is the difference of two polynomials always a polynomial? Explain.

105. ***Writing.*** Explain why the polynomial $2^4 - 7x^3 + 5x^2 - x$ has degree 3 and not degree 4.

106. ***Discussion.*** Which of the following polynomials does not have degree 2? Explain.

a) πr^2 **b)** $\pi^2 - 4$ **c)** $y^2 - 4$ **d)** $x^2 - x^4$
e) $a^2 - 3a + 9$

In this section

- Multiplying Monomials with the Product Rule
- Multiplying Polynomials
- The Opposite of a Polynomial
- Applications

5.2 MULTIPLICATION OF POLYNOMIALS

You learned to multiply some polynomials in Chapter 1. In this section you will learn how to multiply any two polynomials.

Multiplying Monomials with the Product Rule

To multiply two monomials, such as x^3 and x^5, recall that

$$x^3 = x \cdot x \cdot x \qquad \text{and} \qquad x^5 = x \cdot x \cdot x \cdot x \cdot x.$$

So

$$x^3 \cdot x^5 = \underbrace{\overbrace{(x \cdot x \cdot x)}^{3 \text{ factors}}\overbrace{(x \cdot x \cdot x \cdot x \cdot x)}^{5 \text{ factors}}}_{8 \text{ factors}} = x^8.$$

The exponent of the product of x^3 and x^5 is the sum of the exponents 3 and 5. This example illustrates the **product rule** for multiplying exponential expressions.

Product Rule

If a is any real number and m and n are any positive integers, then

$$a^m \cdot a^n = a^{m+n}.$$

EXAMPLE 1

Multiplying monomials

Find the indicated products.

a) $x^2 \cdot x^4 \cdot x$ **b)** $(-2ab)(-3ab)$ **c)** $-4x^2y^2 \cdot 3xy^5$ **d)** $(3a)^2$

Solution

a) $x^2 \cdot x^4 \cdot x = x^2 \cdot x^4 \cdot x^1$

$= x^7$ Product rule

b) $(-2ab)(-3ab) = (-2)(-3) \cdot a \cdot a \cdot b \cdot b$

$= 6a^2b^2$ Product rule

c) $(-4x^2y^2)(3xy^5) = (-4)(3)x^2 \cdot x \cdot y^2 \cdot y^5$

$= -12x^3y^7$ Product rule

d) $(3a)^2 = 3a \cdot 3a$

$= 9a^2$

study tip

As soon as possible after class, find a quiet place and work on your homework. The longer you wait the harder it is to remember what happened in class.

CAUTION Be sure to distinguish between adding and multiplying monomials. You can add like terms to get $3x^4 + 2x^4 = 5x^4$, but you cannot combine the terms in $3w^5 + 6w^2$. However, you can multiply any two monomials: $3x^4 \cdot 2x^4 = 6x^8$ and $3w^5 \cdot 6w^2 = 18w^7$.

Multiplying Polynomials

To multiply a monomial and a polynomial, we use the distributive property.

EXAMPLE 2

Multiplying monomials and polynomials

Find each product.

a) $3x^2(x^3 - 4x)$ **b)** $(y^2 - 3y + 4)(-2y)$ **c)** $-a(b - c)$

Solution

a) $3x^2(x^3 - 4x) = 3x^2 \cdot x^3 - 3x^2 \cdot 4x$ Distributive property

$= 3x^5 - 12x^3$

b) $(y^2 - 3y + 4)(-2y) = y^2(-2y) - 3y(-2y) + 4(-2y)$ Distributive property

$= -2y^3 - (-6y^2) + (-8y)$

$= -2y^3 + 6y^2 - 8y$

c) $-a(b - c) = (-a)b - (-a)c$ Distributive property

$= -ab + ac$

$= ac - ab$

Note in part (c) that either of the last two binomials is the correct answer. The last one is just a little simpler to read. ■

Just as we use the distributive property to find the product of a monomial and a polynomial, we can use the distributive property to find the product of two binomials and the product of a binomial and a trinomial. Before simplifying, the product of two binomials has four terms and the product of a binomial and a trinomial has six terms.

EXAMPLE 3 **Multiplying polynomials**

Use the distributive property to find each product.

a) $(x + 2)(x + 5)$ **b)** $(x + 3)(x^2 + 2x - 7)$

Solution

a) First multiply each term of $x + 5$ by $x + 2$:

$$(x + 2)(x + 5) = (x + 2)x + (x + 2)5 \quad \text{Distributive property}$$
$$= x^2 + 2x + 5x + 10 \quad \text{Distributive property}$$
$$= x^2 + 7x + 10 \quad \text{Combine like terms.}$$

b) First multiply each term of the trinomial by $x + 3$:

$$(x + 3)(x^2 + 2x - 7) = (x + 3)x^2 + (x + 3)2x + (x + 3)(-7) \quad \text{Distributive property}$$
$$= x^3 + 3x^2 + 2x^2 + 6x - 7x - 21 \quad \text{Distributive property}$$
$$= x^3 + 5x^2 - x - 21 \quad \text{Combine like terms.}$$ ■

Products of polynomials can also be found by arranging the multiplication vertically like multiplication of whole numbers.

EXAMPLE 4 **Multiplying vertically**

Find each product.

a) $(x - 2)(3x + 7)$ **b)** $(x + 2)(x^2 - x + 1)$

Solution

a)

	$3x + 7$	
	$x - 2$	
	$-6x - 14$	← -2 times $3x + 7$
$3x^2 + 7x$		← x times $3x + 7$
$3x^2 + x - 14$		Add.

b)

	$x^2 - x + 1$
	$x + 2$
	$2x^2 - 2x + 2$
$x^3 - x^2 + x$	
$x^3 + x^2 - x + 2$	

■

helpful hint

Many students find vertical multiplication easier than applying the distributive property twice horizontally. However, you should learn both methods because horizontal multiplication will help you with factoring by grouping in Section 6.2.

These examples illustrate the following rule.

Multiplication of Polynomials

To multiply polynomials, multiply each term of one polynomial by every term of the other polynomial, then combine like terms.

The Opposite of a Polynomial

The opposite or additive inverse of y is $-y$ because $y + (-y) = 0$. *To find the opposite of a polynomial we change the sign of every term of the polynomial.* For example, the opposite of $x^2 - 3x + 1$ is $-x^2 + 3x - 1$ because

$$(x^2 - 3x + 1) + (-x^2 + 3x - 1) = 0.$$

We also write $-(x^2 - 3x + 1) = -x^2 + 3x - 1$. As another example, we find the opposite of $a - b$:

$$-(a - b) = -a + b = b - a$$

So $-(a - b) = b - a$. Since $(a - b) + (b - a) = 0$, we can be sure that $a - b$ and $b - a$ are opposites or additive inverses of each other.

CAUTION The opposite $a + b$ is not $a - b$ because $(a + b) + (a - b) = 2a$. The opposite of $a + b$ is $-a - b$ because $(a + b) + (-a - b) = 0$.

EXAMPLE 5 **Opposite of a polynomial**

Find the opposite of each polynomial.

a) $x - 2$ **b)** $9 - y^2$ **c)** $a + 4$ **d)** $-x^2 + 6x - 3$

Solution

a) $-(x - 2) = 2 - x$

b) $-(9 - y^2) = y^2 - 9$

c) $-(a + 4) = -a - 4$

d) $-(-x^2 + 6x - 3) = x^2 - 6x + 3$ ■

Applications

EXAMPLE 6 **Multiplying polynomials**

A parking lot is 20 yards wide and 30 yards long. If the college increases the length and width by the same amount to handle an increasing number of cars, then what polynomial represents the area of the new lot? What is the new area if the increase is 15 yards?

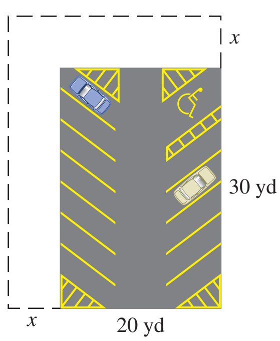

FIGURE 5.1

Solution

If x is the amount of increase, then the new lot will be $x + 20$ yards wide and $x + 30$ yards long as shown in Fig. 5.1. Multiply the length and width to get the area:

$$\begin{aligned}(x + 20)(x + 30) &= (x + 20)x + (x + 20)30 \\ &= x^2 + 20x + 30x + 600 \\ &= x^2 + 50x + 600\end{aligned}$$

The polynomial $x^2 + 50x + 600$ represents the area of the new lot. If $x = 15$, then

$$x^2 + 50x + 600 = (15)^2 + 50(15) + 600 = 1575.$$

If the increase is 15 yards, then the area of the lot will be 1575 square yards. ■

WARM-UPS

True or false? Explain your answer.

1. $3x^3 \cdot 5x^4 = 15x^{12}$ for any value of x.
2. $3x^2 \cdot 2x^7 = 5x^9$ for any value of x.
3. $(3y^3)^2 = 9y^6$ for any value of y.
4. $-3x(5x - 7x^2) = -15x^3 + 21x^2$ for any value of x.
5. $2x(x^2 - 3x + 4) = 2x^3 - 6x^2 + 8x$ for any number x.
6. $-2(3 - x) = 2x - 6$ for any number x.
7. $(a + b)(c + d) = ac + ad + bc + bd$ for any values of a, b, c, and d.
8. $-(x - 7) = 7 - x$ for any value of x.
9. $83 - 37 = -(37 - 83)$
10. The opposite of $x + 3$ is $x - 3$ for any number x.

5.2 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is the product rule for exponents?
2. Why is the sum of two monomials not necessarily a monomial?
3. What property of the real numbers is used when multiplying a monomial and a polynomial?
4. What property of the real numbers is used when multiplying two binomials?
5. How do we multiply any two polynomials?
6. How do we find the opposite of a polynomial?

Find each product. See Example 1.

7. $3x^2 \cdot 9x^3$
8. $5x^7 \cdot 3x^5$
9. $2a^3 \cdot 7a^8$
10. $3y^{12} \cdot 5y^{15}$
11. $-6x^2 \cdot 5x^2$
12. $-2x^2 \cdot 8x^5$
13. $(-9x^{10})(-3x^7)$
14. $(-2x^2)(-8x^9)$
15. $-6st \cdot 9st$
16. $-12sq \cdot 3s$
17. $3wt \cdot 8w^7t^6$
18. $h^8k^3 \cdot 5h$
19. $(5y)^2$
20. $(6x)^2$
21. $(2x^3)^2$
22. $(3y^5)^2$

Find each product. See Example 2.

23. $4y^2(y^5 - 2y)$
24. $6t^3(t^5 + 3t^2)$
25. $-3y(6y - 4)$
26. $-9y(y^2 - 1)$
27. $(y^2 - 5y + 6)(-3y)$
28. $(x^3 - 5x^2 - 1)7x^2$
29. $-x(y^2 - x^2)$

30. $-ab(a^2 - b^2)$

31. $(3ab^3 - a^2b^2 - 2a^3b)5a^3$

32. $(3c^2d - d^3 + 1)8cd^2$

33. $-\frac{1}{2}t^2v(4t^3v^2 - 6tv - 4v)$

34. $-\frac{1}{3}m^2n^3(-6mn^2 + 3mn - 12)$

Use the distributive property to find each product. See Example 3.

35. $(x + 1)(x + 2)$

36. $(x + 6)(x + 3)$

37. $(x - 3)(x + 5)$

38. $(y - 2)(y + 4)$

39. $(t - 4)(t - 9)$

40. $(w - 3)(w - 5)$

41. $(x + 1)(x^2 + 2x + 2)$

42. $(x - 1)(x^2 + x + 1)$

43. $(3y + 2)(2y^2 - y + 3)$

44. $(4y + 3)(y^2 + 3y + 1)$

45. $(y^2z - 2y^4)(y^2z + 3z^2 - y^4)$

46. $(m^3 - 4mn^2)(6m^4n^2 - 3m^6 + m^2n^4)$

Find each product vertically. See Example 4.

47. $\begin{array}{r} 2a - 3 \\ \underline{a + 5} \end{array}$

48. $\begin{array}{r} 2w - 6 \\ \underline{w + 5} \end{array}$

49. $\begin{array}{r} 7x + 30 \\ \underline{2x + 5} \end{array}$

50. $\begin{array}{r} 5x + 7 \\ \underline{3x + 6} \end{array}$

51. $\begin{array}{r} 5x + 2 \\ \underline{4x - 3} \end{array}$

52. $\begin{array}{r} 4x + 3 \\ \underline{2x - 6} \end{array}$

53. $\begin{array}{r} m - 3n \\ \underline{2a + b} \end{array}$

54. $\begin{array}{r} 3x + 7 \\ \underline{a - 2b} \end{array}$

55. $\begin{array}{r} x^2 + 3x - 2 \\ \underline{x + 6} \end{array}$

56. $\begin{array}{r} -x^2 + 3x - 5 \\ \underline{x - 7} \end{array}$

57. $\begin{array}{r} 2a^3 - 3a^2 + 4 \\ \underline{-2a - 3} \end{array}$

58. $\begin{array}{r} -3x^2 + 5x - 2 \\ \underline{-5x - 6} \end{array}$

59. $\begin{array}{r} x - y \\ \underline{x + y} \end{array}$

60. $\begin{array}{r} a^2 + b^2 \\ \underline{a^2 - b^2} \end{array}$

61. $\begin{array}{r} x^2 - xy + y^2 \\ \underline{x + y} \end{array}$

62. $\begin{array}{r} 4w^2 + 2wv + v^2 \\ \underline{2w - v} \end{array}$

Find the opposite of each polynomial. See Example 5.

63. $3t - u$

64. $-3t - u$

65. $3x + y$

66. $x - 3y$

67. $-3a^2 - a + 6$

68. $3b^2 - b - 6$

69. $3v^2 + v - 6$

70. $-3t^2 + t - 6$

Perform the indicated operation.

71. $-3x(2x - 9)$

72. $-1(2 - 3x)$

73. $2 - 3x(2x - 9)$

74. $6 - 3(4x - 8)$

75. $(2 - 3x) + (2x - 9)$

76. $(2 - 3x) - (2x - 9)$

77. $(6x^6)^2$

78. $(-3a^3b)^2$

79. $3ab^3(-2a^2b^7)$

80. $-4xst \cdot 8xs$

81. $(5x + 6)(5x + 6)$

82. $(5x - 6)(5x - 6)$

83. $(5x - 6)(5x + 6)$

84. $(2x - 9)(2x + 9)$

85. $2x^2(3x^5 - 4x^2)$

86. $4a^3(3ab^3 - 2ab^3)$

87. $(m - 1)(m^2 + m + 1)$

88. $(a + b)(a^2 - ab + b^2)$

89. $(3x - 2)(x^2 - x - 9)$

90. $(5 - 6y)(3y^2 - y - 7)$

Solve each problem. See Example 6.

91. ***Office space.*** The length of a professor's office is x feet, and the width is $x + 4$ feet. Write a polynomial that represents the area. Find the area if $x = 10$ ft.

92. ***Swimming space.*** The length of a rectangular swimming pool is $2x - 1$ meters, and the width is $x + 2$ meters. Write a polynomial that represents the area. Find the area if x is 5 meters.

93. ***Area of a truss.*** A roof truss is in the shape of a triangle with a height of x feet and a base of $2x + 1$ feet. Write a polynomial that represents the area of the triangle. What is the area if x is 5 feet?

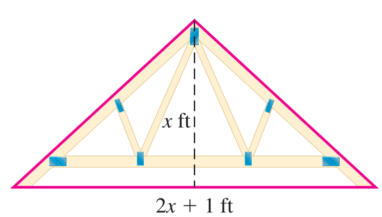

FIGURE FOR EXERCISE 93