1.8 USING THE PROPERTIES TO SIMPLIFY EXPRESSIONS

In this section

- Using the Properties in Computation
- Like Terms
- Combining Like Terms
- Products and Quotients
- Removing Parentheses

The properties of the real numbers can be helpful when we are doing computations. In this section we will see how the properties can be applied in arithmetic and algebra.

Using the Properties in Computation

The properties of the real numbers can often be used to simplify computations. For example, to find the product of 26 and 200, we can write

$$\begin{aligned}(26)(200) &= (26)(2 \cdot 100)\\ &= (26 \cdot 2)(100)\\ &= 52 \cdot 100\\ &= 5200\end{aligned}$$

It is the associative property that allows us to multiply 26 by 2 to get 52, then multiply 52 by 100 to get 5200.

EXAMPLE 1 **Using the properties**

Use the appropriate property to aid you in evaluating each expression.

a) $347 + 35 + 65$ **b)** $3 \cdot 435 \cdot \frac{1}{3}$ **c)** $6 \cdot 28 + 4 \cdot 28$

Solution

a) Notice that the sum of 35 and 65 is 100. So apply the associative property as follows:

$$\begin{aligned}347 + (35 + 65) &= 347 + 100\\ &= 447\end{aligned}$$

b) Use the commutative and associative properties to rearrange this product. We can then do the multiplication quickly:

$$\begin{aligned}3 \cdot 435 \cdot \frac{1}{3} &= 435\left(3 \cdot \frac{1}{3}\right)\\ &= 435 \cdot 1\\ &= 435\end{aligned}$$

c) Use the distributive property to rewrite this expression.

$$\begin{aligned}6 \cdot 28 + 4 \cdot 28 &= (6 + 4)28\\ &= 10 \cdot 28\\ &= 280\end{aligned}$$

■

study tip

Being a full-time student is a full-time job. A successful student spends from two to four hours studying outside of class for every hour spent in the classroom. It is rare to find a person who can handle two full-time jobs and it is just as rare to find a successful full-time student who also works full time.

Like Terms

An expression containing a number or the product of a number and one or more variables raised to powers is called a **term.** For example,

$$-3, \quad 5x, \quad -3x^2y, \quad a, \quad \text{and} \quad -abc$$

are terms. The number preceding the variables in a term is called the **coefficient.** In the term $5x$, the coefficient of x is 5. In the term $-3x^2y$ the coefficient of x^2y is -3. In the term a, the coefficient of a is 1 because $a = 1 \cdot a$. In the term $-abc$ the coefficient of abc is -1 because $-abc = -1 \cdot abc$. If two terms contain the same variables with the same exponents, they are called **like terms.** For example, $3x^2$ and $-5x^2$ are like terms, but $3x^2$ and $-5x^3$ are not like terms.

Combining Like Terms

Using the distributive property on an expression involving the sum of like terms allows us to combine the like terms as shown in the next example.

EXAMPLE 2 **Combining like terms**

Use the distributive property to perform the indicated operations.

a) $3x + 5x$ **b)** $-5xy - (-4xy)$

study tip

When you get a test back, do not simply file it in your notebook or the waste basket. While the material is fresh in your mind, rework all problems that you missed. Ask questions about anything that you don't understand and save your test for future reference.

Solution

a) $3x + 5x = (3 + 5)x$
$= 8x$

Because the distributive property is valid for any real numbers, we have $3x + 5x = 8x$ no matter what number is used for x.

b) $-5xy - (-4xy) = [-5 - (-4)]xy$
$= -1xy$
$= -xy$

■

Of course, we do not want to write out all of the steps shown in Example 2 every time we combine like terms. We can combine like terms as easily as we can add or subtract their coefficients.

EXAMPLE 3 **Combining like terms**

Perform the indicated operations.

a) $w + 2w$ **b)** $-3a + (-7a)$ **c)** $-9x + 5x$
d) $7xy - (-12xy)$ **e)** $2x^2 + 4x^2$

Solution

a) $w + 2w = 1w + 2w = 3w$ **b)** $-3a + (-7a) = -10a$
c) $-9x + 5x = -4x$ **d)** $7xy - (-12xy) = 19xy$
e) $2x^2 + 4x^2 = 6x^2$

■

CAUTION There are no like terms in expressions such as

$$2 + 5x, \quad 3xy + 5y, \quad 3w + 5a, \quad \text{and} \quad 3z^2 + 5z$$

The terms in these expressions cannot be combined.

Products and Quotients

In the next example we use the associative property of multiplication to simplify the product of two expressions.

EXAMPLE 4

Finding products

Simplify.

a) $3(5x)$ **b)** $2\left(\frac{x}{2}\right)$ **c)** $(4x)(6x)$ **d)** $(-2a)(4b)$

Solution

a) $3(5x) = (3 \cdot 5)x$
$= (15)x$
$= 15x$

b) $2\left(\frac{x}{2}\right) = 2\left(\frac{1}{2} \cdot x\right)$
$= \left(2 \cdot \frac{1}{2}\right)x$
$= 1 \cdot x$
$= x$

c) $(4x)(6x) = 4 \cdot 6 \cdot x \cdot x$
$= 24x^2$

d) $(-2a)(4b) = -2 \cdot 4 \cdot a \cdot b = -8ab$ ■

study tip

Note how the exercises are keyed to the examples. This serves two purposes. If you have missed class and are studying on your own, you should study an example and then immediately try to work the corresponding exercises. If you have seen an explanation in class, then you can start the exercises and refer back to the examples as necessary.

CAUTION Be careful with expressions such as $3(5x)$ and $3(5 + x)$. In $3(5x)$ we multiply 5 by 3 to get $3(5x) = 15x$. In $3(5 + x)$, both 5 and x are multiplied by the 3 to get $3(5 + x) = 15 + 3x$.

In Example 4 we showed how the properties are used to simplify products. However, in practice we usually do not write out any steps for these problems—we can write just the answer.

EXAMPLE 5

Finding products quickly

Find each product.

a) $(-3)(4x)$ **b)** $(-4a)(-7a)$ **c)** $(-3a)\left(\frac{b}{3}\right)$ **d)** $6 \cdot \frac{x}{2}$

Solution

a) $-12x$ **b)** $28a^2$ **c)** $-ab$ **d)** $3x$ ■

In Section 1.1 we found the quotient of two numbers by inverting the divisor and then multiplying. Since $a \div b = a \cdot \frac{1}{b}$, any quotient can be written as a product.

EXAMPLE 6

Simplifying quotients

Simplify.

a) $\frac{10x}{5}$ **b)** $\frac{4x + 8}{2}$

Solution

a) Since dividing by 5 is equivalent to multiplying by $\frac{1}{5}$, we have

$$\frac{10x}{5} = \frac{1}{5}(10x) = \left(\frac{1}{5} \cdot 10\right)x = (2)x = 2x.$$

Note that you can simply divide 10 by 5 to get 2.

b) Since dividing by 2 is equivalent to multiplying by $\frac{1}{2}$, we have

$$\frac{4x + 8}{2} = \frac{1}{2}(4x + 8) = 2x + 4.$$

Note that both 4 and 8 are divided by 2.

CAUTION It is not correct to divide only one term in the numerator by the denominator. For example,

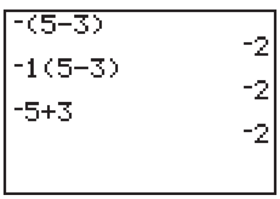
$$\frac{4 + 7}{2} \neq 2 + 7$$

because $\frac{4 + 7}{2} = \frac{11}{2}$ and $2 + 7 = 9$.

Removing Parentheses

calculator close-up

A negative sign in front of parentheses changes the sign of every term inside the parentheses.



Multiplying a number by -1 merely changes the sign of the number. For example,

$$(-1)(7) = -7 \qquad \text{and} \qquad (-1)(-8) = 8.$$

So -1 times a number is the *opposite* of the number. Using variables, we write

$$(-1)x = -x \qquad \text{or} \qquad -1(y + 5) = -(y + 5).$$

When a minus sign appears in front of a sum, we can change the minus sign to -1 and use the distributive property. For example,

$$\begin{aligned} -(w + 4) &= -1(w + 4) \\ &= (-1)w + (-1)4 \\ &= -w + (-4) \\ &= -w - 4 \end{aligned}$$

Note how the minus sign in front of the parentheses caused all of the signs to change: $-(w + 4) = -w - 4$. As another example, consider the following:

$$\begin{aligned} -(x - 3) &= -1(x - 3) \\ &= (-1)x - (-1)3 \\ &= -x + 3 \end{aligned}$$

CAUTION When removing parentheses preceded by a minus sign, you must change the sign of *every* term within the parentheses.

EXAMPLE 7 **Removing parentheses**

Simplify each expression.

a) $5 - (x + 3)$ **b)** $3x - 6 - (2x - 4)$ **c)** $-6x - (-x + 2)$

Solution

a)
$$\begin{aligned} 5 - (x + 3) &= 5 - x - 3 \\ &= 5 - 3 - x \\ &= 2 - x \end{aligned}$$

Solve each problem.

107. ***Married filing jointly.*** The expression

$$0.15(43{,}850) + 0.28(x - 43{,}850)$$

gives the 2000 federal income tax for a married couple filing jointly with a taxable income of x dollars, where x is over \$43,850 but not over \$105,950 (Internal Revenue Service, www.irs.gov).

a) Simplify the expression. $0.28x - 5700.5$

b) Use the expression to find the amount of tax for a couple with a taxable income of \$80,000. \$16,700

c) Use the graph shown here to estimate the 2000 federal income tax for a couple with a taxable income of \$150,000. \$43,000

d) Use the graph to find the approximate taxable income for a couple who paid \$70,000 in federal income tax. \$240,000

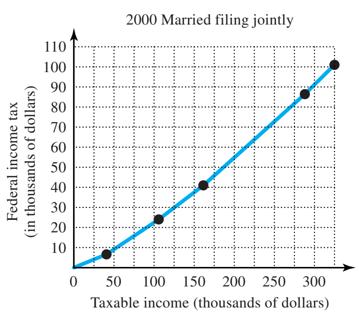


FIGURE FOR EXERCISE 107

108. ***Marriage penalty.*** The expression

$$0.15(26{,}250) + 0.28(x - 26{,}250)$$

gives the 2000 federal income tax for a single taxpayer with taxable income of x dollars, where x is over \$26,250 but not over \$63,550.

a) Simplify the expression. $0.28x - 3412.5$

b) Find the amount of tax for a single taxpayer with taxable income of \$40,000. \$7788

c) Who pays more, two single taxpayers with taxable incomes of \$40,000 each or one married couple with taxable income of \$80,000 together? See Exercise 107. Married couple pays \$1124 more.

109. ***Perimeter of a corral.*** The perimeter of a rectangular corral that has width x feet and length $x + 40$ feet is $2(x) + 2(x + 40)$. Simplify the expression for the perimeter. Find the perimeter if $x = 30$ feet.
$4x + 80$, 200 feet

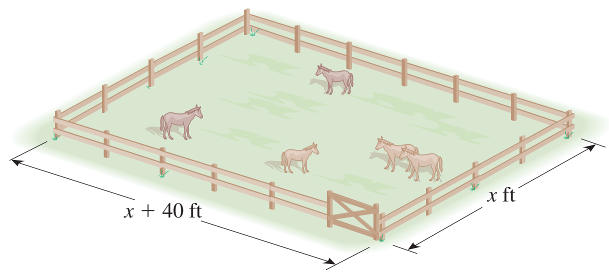


FIGURE FOR EXERCISE 109

GETTING MORE INVOLVED

110. ***Discussion.*** What is wrong with the way in which each of the following expressions is simplified?

a) $4(2 + x) = 8 + x$ $\quad 4(2 + x) = 8 + 4x$

b) $4(2x) = 8 \cdot 4x = 32x$ $\quad 4(2x) = (4 \cdot 2)x = 8x$

c) $\dfrac{4 + x}{2} = 2 + x$ $\quad \dfrac{4 + x}{2} = \dfrac{1}{2}(4 + x) = 2 + \dfrac{1}{2}x$

d) $5 - (x - 3) = 5 - x - 3 = 2 - x$
$5 - (x - 3) = 5 - x + 3 = 8 - x$

111. ***Discussion.*** An instructor asked his class to evaluate the expression $1/2x$ for $x = 5$. Some students got 0.1; others got 2.5. Which answer is correct and why?
If $x = 5$, then $1/2 \cdot 5 = \frac{1}{2} \cdot 5 = 2.5$ because we do division and multiplication from left to right.

COLLABORATIVE ACTIVITIES

Remembering the Rules

This chapter reviews different types of numbers used in algebra. This activity will review the rules for the basic operations: addition, subtraction, multiplication, and division for fractions, decimals, and real numbers.

Part I: Remembering the rules. Have each member of your group choose an operation: addition, subtraction, multiplication, or division.

1. Fractions:
   a. Write the rules for working a fraction problem using the operation you have chosen. Use your book as a reference and consider the following sample problems:

$$\frac{1}{2} + \frac{2}{5}, \quad \frac{1}{3} \cdot \frac{6}{7}, \quad \frac{3}{5} - \frac{1}{3}, \quad \frac{1}{3} \div \frac{2}{3}$$

   b. Starting with addition, each of you will share what he or she has written with the other members of the group. Make additions or corrections if needed.

Grouping: 4 students
Topic: Fractions, decimals, and signed numbers

Switch operations. Each member of the group now takes the operation of the person to his or her right.

2. Decimals: Repeat parts 1(a) and 1(b) for the following sample problems:

$$0.012 + 3,\ 2.1 - 0.25,\ 3.2 \cdot 0.23,\ 5.4 \div 1.2$$

Switch operations. Each member of the group now takes the operation of the person to his or her right.

3. Signed numbers: Repeat parts 1(a) and 1(b) for the following sample problems:

$$-3 + 5,\ 3 - (-2),\ -2 \cdot 3,\ -6 \div -2$$

Part II: Testing your rules. As a group, work through the following problems together, using the rules you have written from Part I. Add any new rules that come up while you work.

1. $\frac{1}{3} + \frac{2}{5}$
2. $0.076 + 7 + 2.005$
3. $-8 + 17$
4. $2\frac{9}{14} - 1\frac{5}{7}$
5. $8 - 3.024$
6. $-5 - (-19)$
7. $3\frac{1}{3} \cdot 2\frac{2}{5}$
8. $0.0723 \cdot 100$
9. $12(-3)(-2)$
10. $1\frac{2}{3} \div 5\frac{1}{2}$
11. $1.024 \div 3.2$
12. $-405 \div 15$
13. $-2\left(\frac{2}{3} - \frac{1}{4}\right) \div 2\frac{1}{2} + (3.052 - (-0.948))$

Extension: Before each exam, form a study group to review material. Write any new rules or definitions used in each chapter. Save these to study for your final exam.

WRAP-UP CHAPTER 1

SUMMARY

The Real Numbers		Examples
Counting or natural numbers	$\{1, 2, 3, \ldots\}$	
Whole numbers	$\{0, 1, 2, 3, \ldots\}$	
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$	
Rational numbers	$\left\{\frac{a}{b} \middle\vert\ a \text{ and } b \text{ are integers with } b \neq 0\right\}$	$\frac{3}{2}, 5, -6, 0$
Irrational numbers	$\{x \mid x \text{ is a real number that is not rational}\}$	$\sqrt{2}, \sqrt{3}, \pi$
Real numbers	The set of real numbers consists of all rational numbers together with all irrational numbers.	

Fractions		Examples
Reducing fractions	$\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$	$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3}$
Building up fractions	$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$	$\frac{3}{8} = \frac{3 \cdot 5}{8 \cdot 5} = \frac{15}{40}$
Multiplying fractions	$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$
Dividing fractions	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$