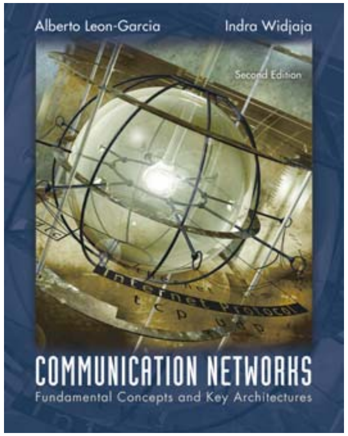
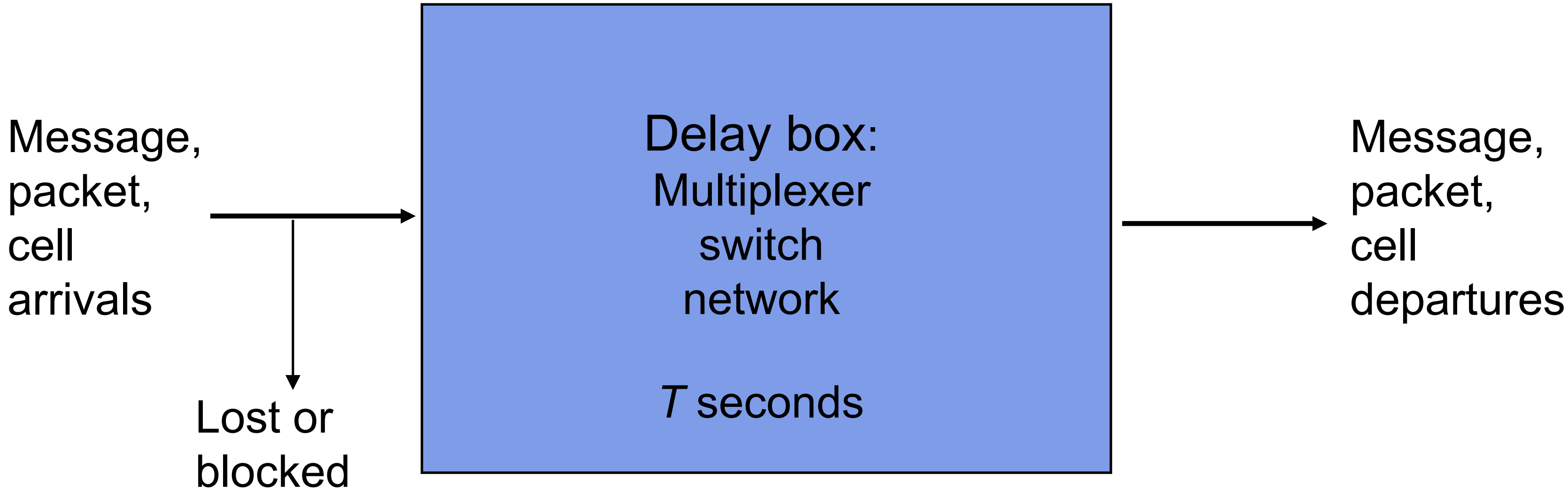
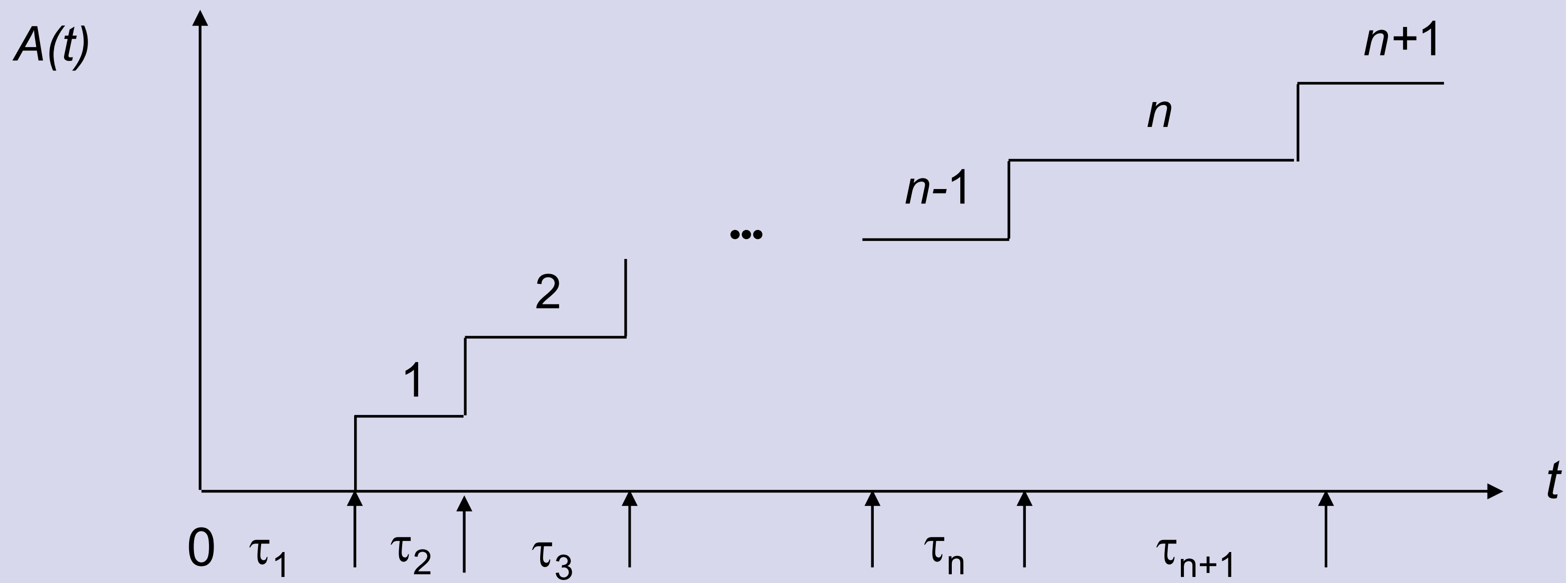
Appendix A
Delay and Loss Performance



Chapter Figures

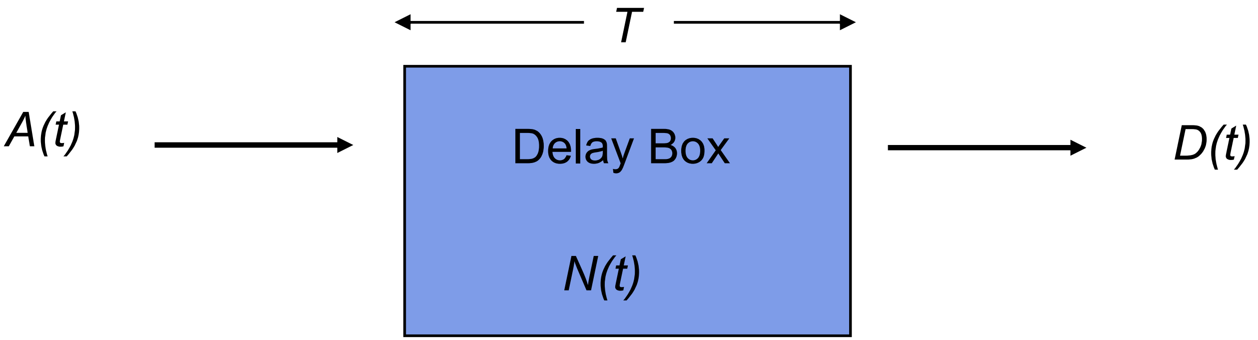

Message,
packet,
cell
arrivals
Lost or
blocked
Delay box:
Multiplexer
switch
network
T seconds
Message,
packet,
cell
departures



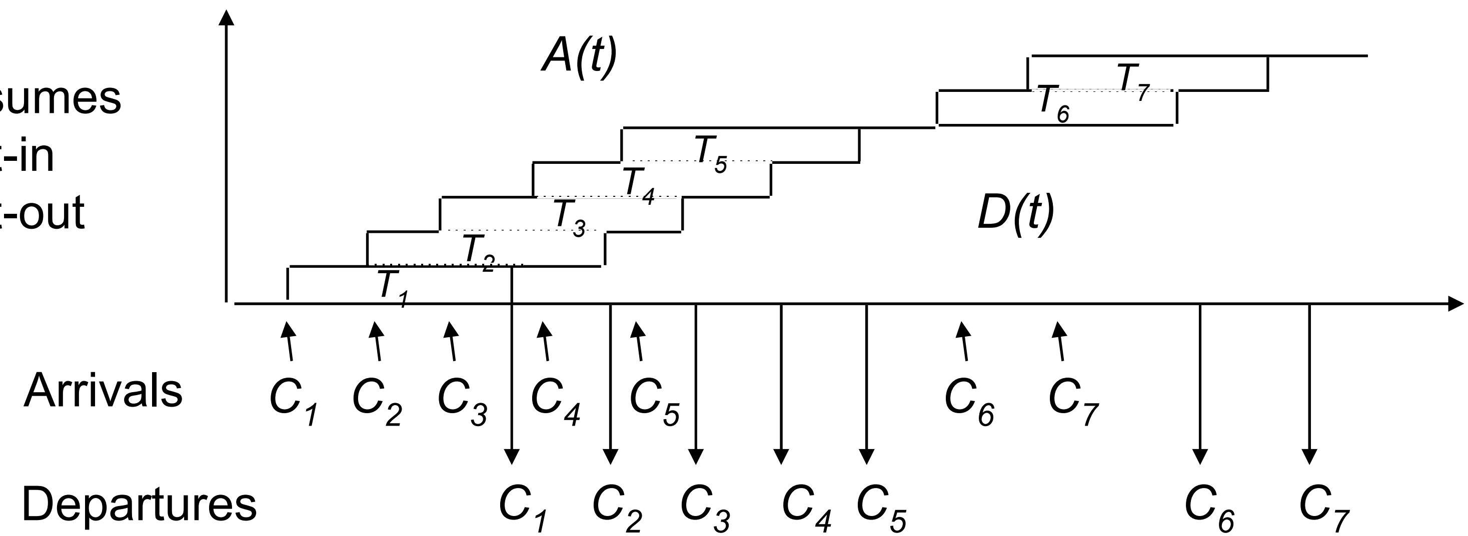
Time of nth arrival $= \tau_1 + \tau_2 + \ldots + \tau_n$

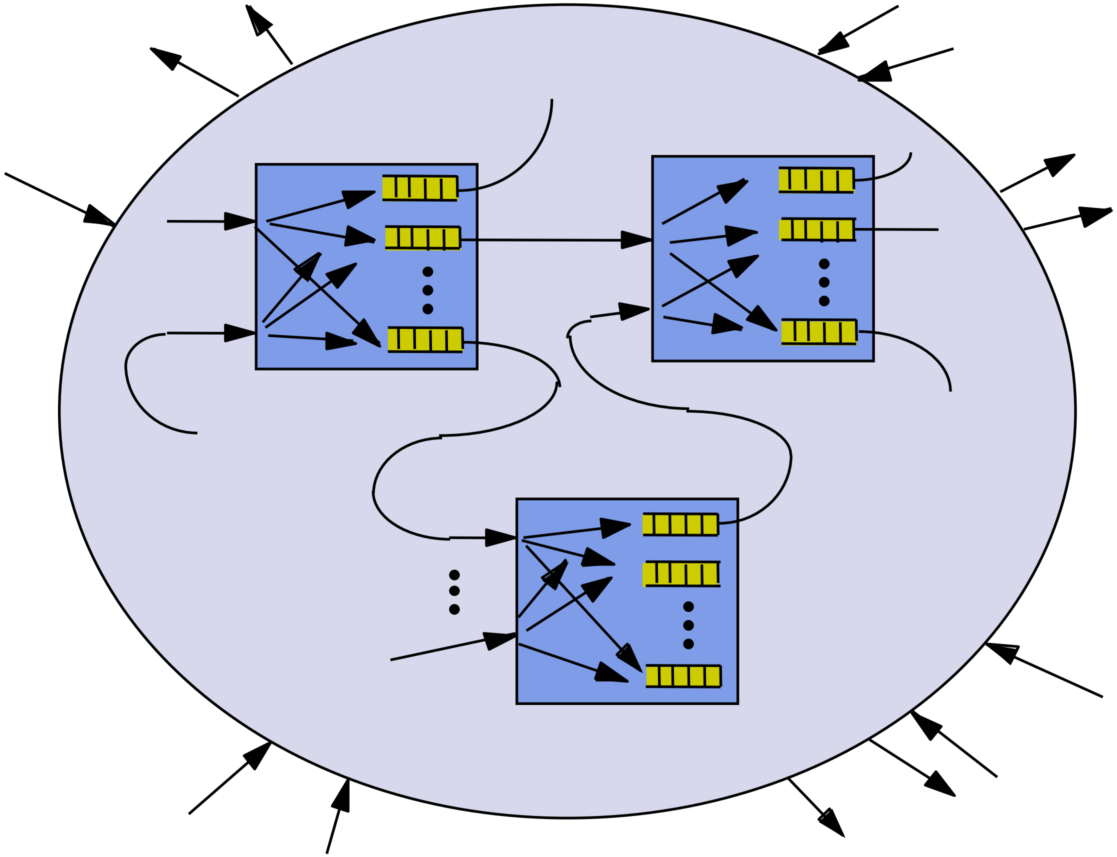
$$\text{Arrival Rate} = \frac{n \text{ arrivals}}{\tau_1 + \tau_2 + \ldots + \tau_n \text{ seconds}} = \frac{1}{(\tau_1+\tau_2+\ldots+\tau_n)/n} \longrightarrow \frac{1}{E[\tau]}$$

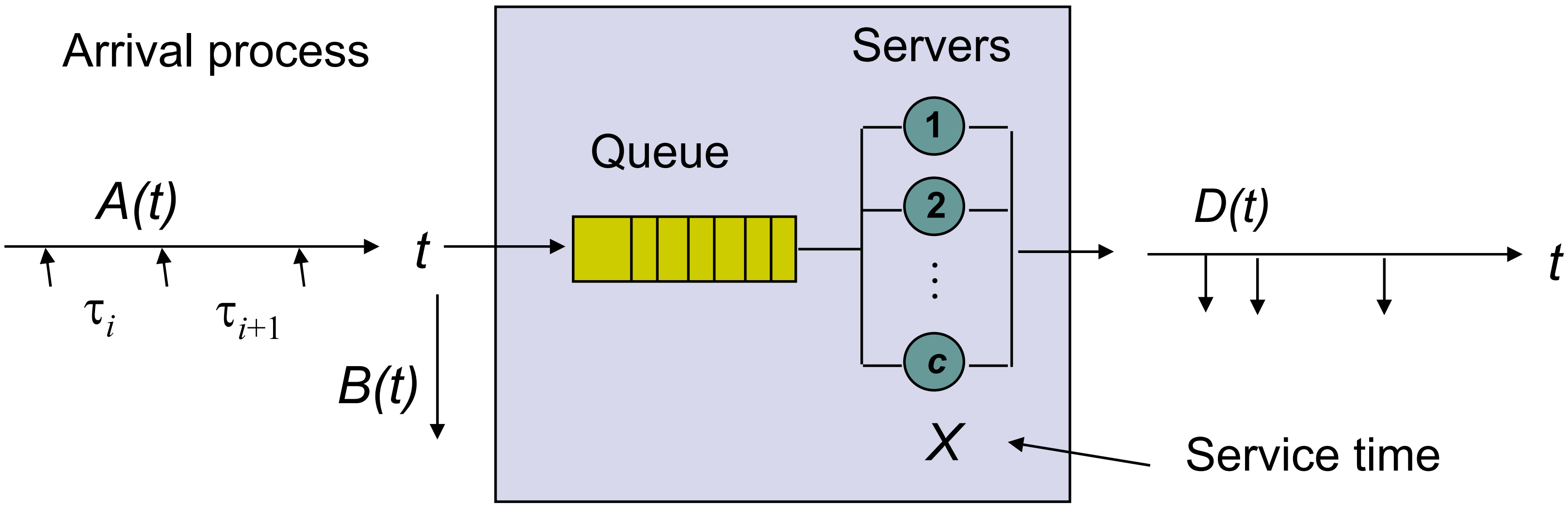
Arrival Rate = 1 / mean interarrival time


T
A(t)
Delay Box
N(t)
D(t)

Assumes first-in first-out






Arrival process
A(t)
τ_i
τ_{i+1}
t
B(t)
Queue
Servers
1
2
c
X
Service time
D(t)
t

Arrival Process / Service Time / Servers / Max Occupancy

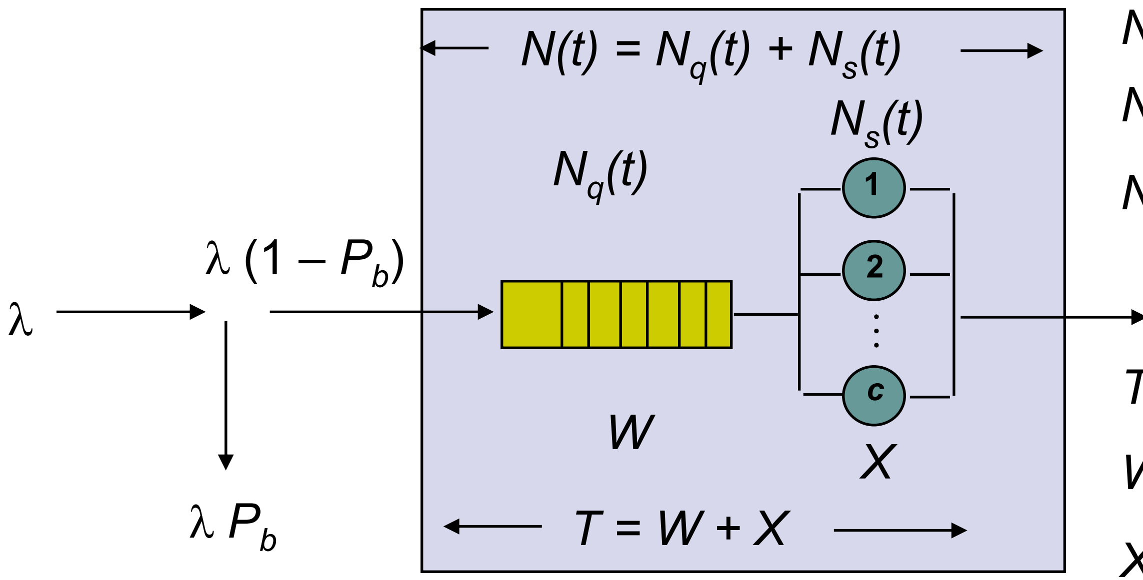
Interarrival times τ
M = exponential
D = deterministic
G = general
Arrival Rate:
$\lambda = 1/E[\tau]$

Service times X
M = exponential
D = deterministic
G = general
Service Rate:
$\mu = 1/E[X]$

1 server
c servers
infinite

K customers
unspecified if
unlimited

Multiplexer Models: M/M/1/K, M/M/1, M/G/1, M/D/1
Trunking Models: M/M/c/c, M/G/c/c
User Activity: M/M/∞, M/G/∞



$N(t)$ = number in system

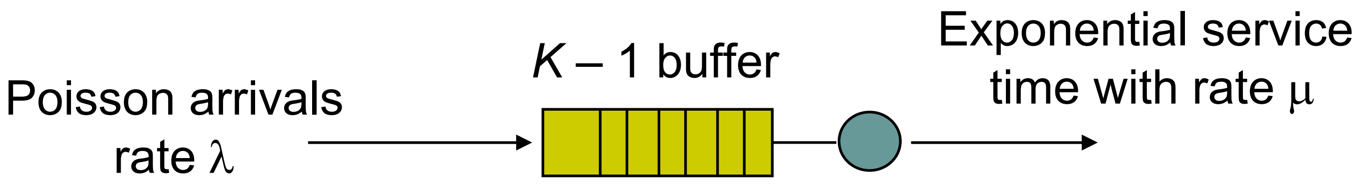
$N_q(t)$ = number in queue

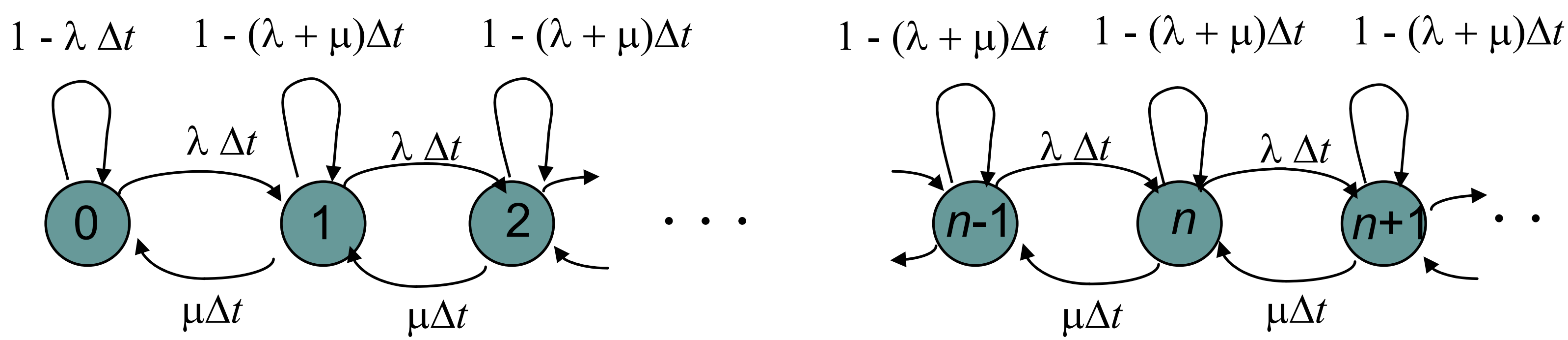
$N_s(t)$ = number in service

T = total delay

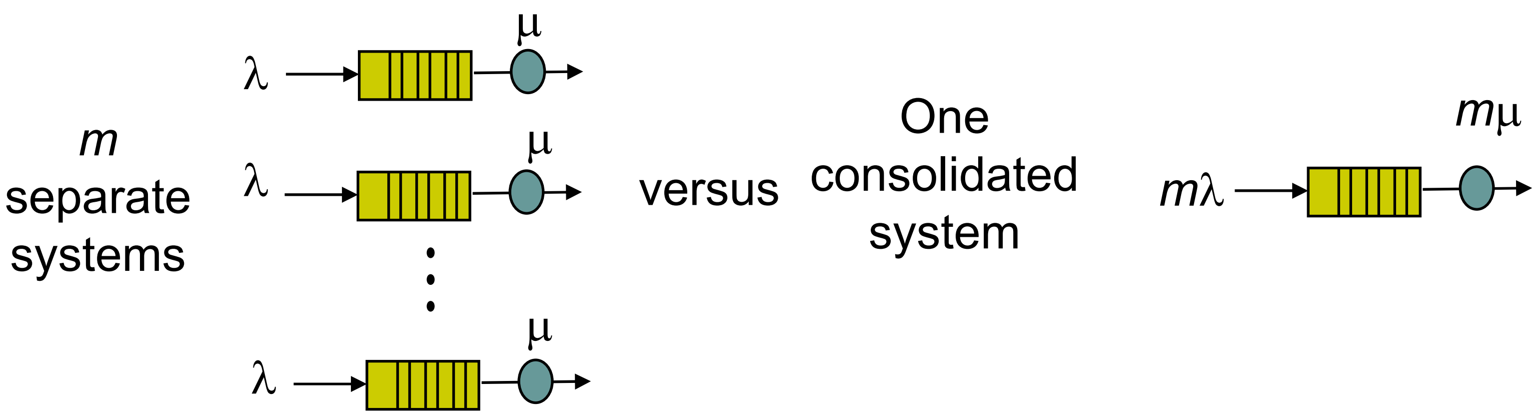
W = waiting time

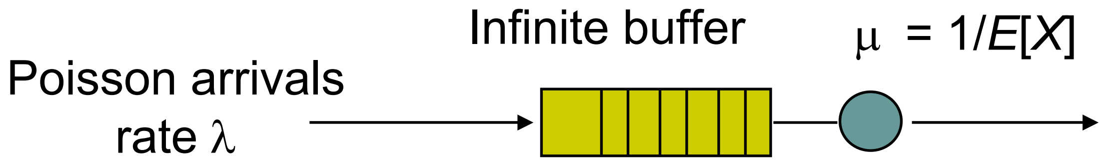
X = service time


K – 1 buffer
Exponential service time with rate μ
Poisson arrivals rate λ

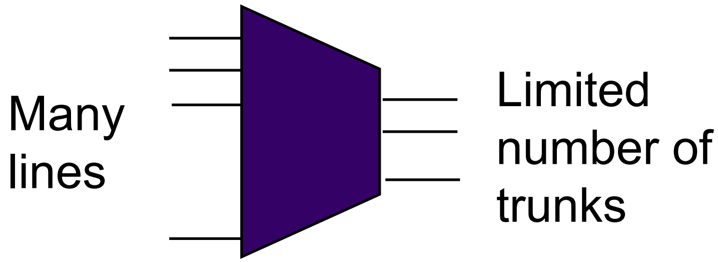


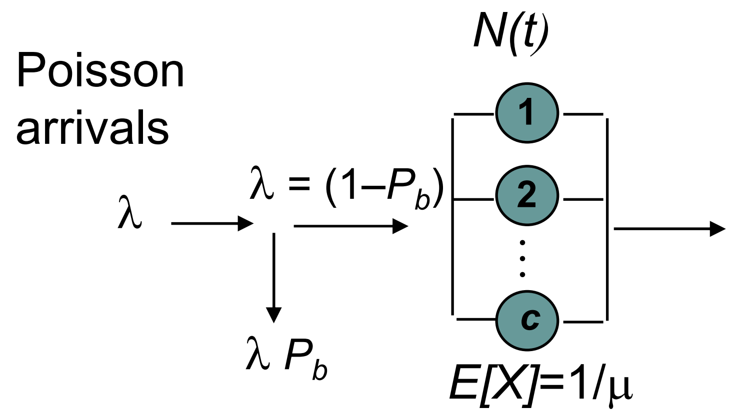
1 - λ Δt
1 - (λ + μ)Δt
1 - (λ + μ)Δt
1 - (λ + μ)Δt
1 - (λ + μ)Δt
1 - (λ + μ)Δt
λ Δt
λ Δt
λ Δt
λ Δt
0
1
2
. . .
n-1
n
n+1
. . .
μΔt
μΔt
μΔt
μΔt


m
separate
systems
λ
μ
λ
μ
λ
μ
versus
One
consolidated
system
mλ
mμ


Infinite buffer
μ = 1/E[X]
Poisson arrivals
rate λ

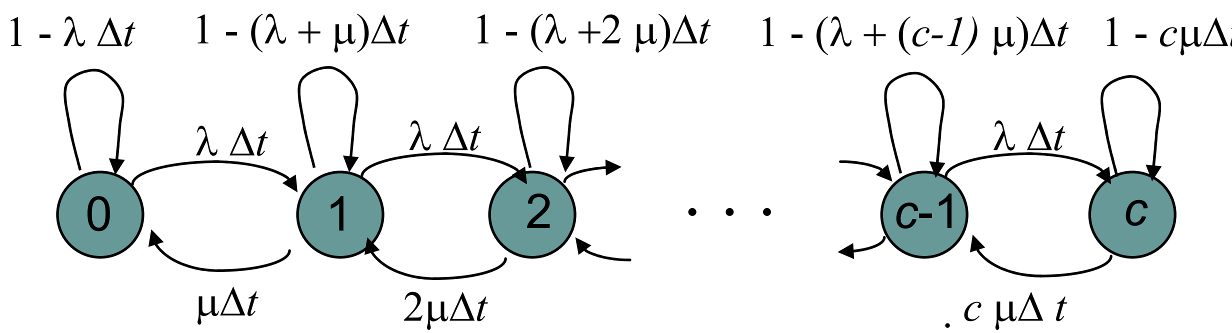
	M/D/1	M/ Er/1	M/M/1	M/H/1
Interarrivals	Constant	Erlang	Exponential	Hyperexponential
C_X^2	0	<1	1	>1
$E[W]/E[W_{M/M/1}]$	1/2	1/2< , <1	1	>1





- Blocked calls are cleared from the system; no waiting allowed.
- Performance parameter: P_b = fraction of arrivals that are blocked
- $P_b = P[N(t)=c] = B(c,a)$ where $a=\lambda/\mu$
- *B(c,a)* is the Erlang B formula which is valid for ***any*** service time distribution

$$B(c,a) = \frac{\frac{a^c}{c!}}{\sum_{j=0}^{c} \frac{a^j}{j!}}$$


1 - λ Δt
1 - (λ + μ)Δt
1 - (λ +2 μ)Δt
1 - (λ + (c-1) μ)Δt
1 - cμΔt
λ Δt
λ Δt
λ Δt
0
1
2
. . .
c-1
c
μΔt
2μΔt
c μΔ t