MECHANICAL ENGINEERING DESIGN
TUTORIAL 6, PART A: INTRODUCTION TO STATIC FAILURE THEORIES

APPROACH

Theories have been developed for the static failure of metals based upon the two classes of material failure; ductile metals yield while brittle metals fracture. Thus separate failure theories exist for ductile and brittle metals:

Failure Theories for Ductile Materials

1. Maximum Shear Stress (MSS)
2. Distortion Energy (DE)
3. Ductile Coulomb-Mohr (DCM)

### *Failure Theories for Brittle Materials*

1. Maximum Normal Stress (MNS)
2. Brittle Coulomb-Mohr (BCM)

These theories have grown out of hypotheses and experimental data in the following manner.

1. Experimental failure data is first collected through tensile tests.
2. The state of stress is correlated to the experimental data using Mohr's circle plots.
3. A failure theory is developed from a concept of the responsible failure mechanism.
4. A design envelope is established based upon the theoretical and empirical design equations.

In light of the extensive dependence of failure theories on experimental data, we will first review the acquisition and correlation of tensile test data to failure theory. Subsequently, the criteria and application of specific failure theories will be discussed.
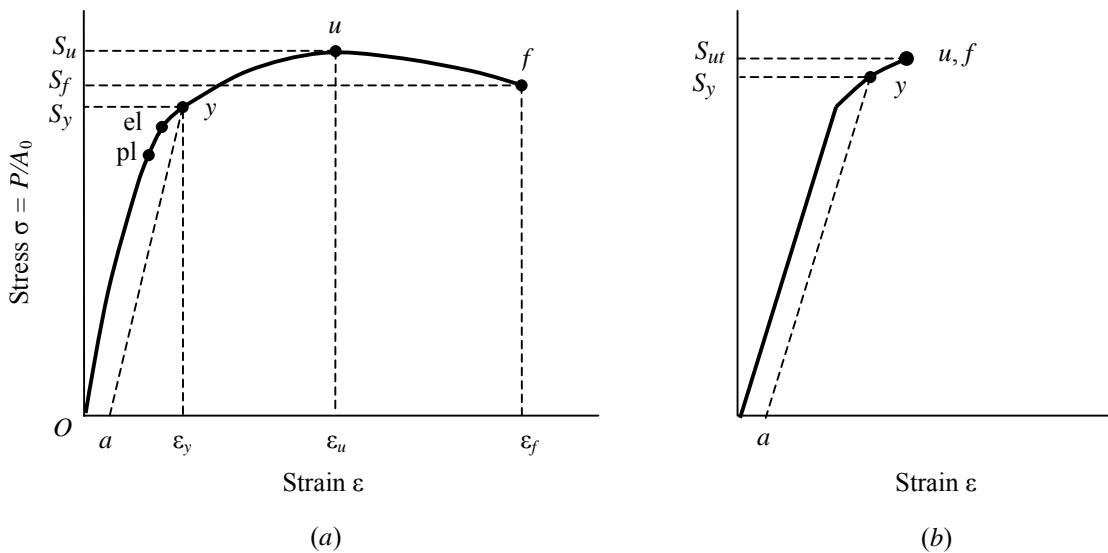
TENSILE TEST
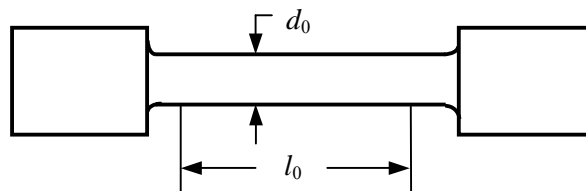
TEXT FIGURE 3-1: A typical tension-test specimen. Some of the standard dimensions used for d_0 are 2.5, 6.25, and 12.5 mm and 0.505 in, but other sections and sizes are in use. Common gauge lengths l_0 used are 10, 25, and 50 mm and 1 and 2 in.

[†] *Text* refers to *Mechanical Engineering Design,* 7th edition text by Joseph Edward Shigley, Charles R. Mischke and Richard G. Budynas; equations and examples with the prefix *T* refer to the present tutorial.

The tensile test is a standardized test (ASTM Standard E8 or E8m) and thus allows for the sharing of experimental data amongst researchers, typically in the form of stress-strain curves. Standard dimensions for the test-specimen are provided in Text Figure 3-1 while a comparison of characteristic stress-strain curves for ductile and brittle materials are shown in Text Figure 3-2. These *engineering* stress-strain diagrams graphically demonstrate the difference in the failure behavior of ductile and brittle metals, and the need for separate failure criteria. However, the curves do not represent true values of stress and strain; rather, they are calculated based upon the original specimen cross-sectional area, prior to loading.

Engineering Stress $\qquad \sigma = \dfrac{P}{A_0} = \dfrac{4P}{\pi d_0^2}$ $\qquad\qquad$ (Text Eq. 3-1)

Engineering Strain $\qquad \varepsilon = \dfrac{l - l_0}{l_0}$ $\qquad\qquad\qquad$ (Text Eq. 3-2)

TEXT FIGURE 3-2: Stress-strain diagram obtained from the standard tensile test (*a*) Ductile material; (*b*) brittle material. pl marks the proportional limit; el, the elastic limit; y, the offset yield strength as defined by offset strain *Oa*; u, the maximum or ultimate strength; and f, the fracture strength.

Referring to Text Fig. 3-2 (*a*), point el, the elastic limit, defines the onset of permanent set while point *a* represents 0.2 percent permanent set with respect to the original gauge length ($\varepsilon = 0.002$).

A measure of the "true" stress and strain can be obtained by taking simultaneous measurements of the load and cross-sectional area during the tensile test experiment. Text

Figure 3-4 shows a typical true stress-strain diagram for a ductile material. The curve between points *u* and *f* corresponds to a reduction in stress as the specimen necks down.
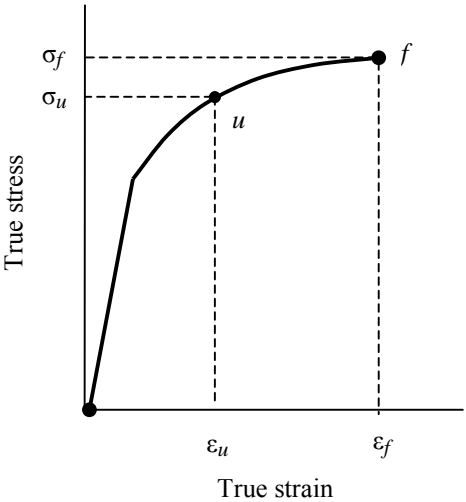
TEXT FIGURE 3-4: True stress-strain diagram plotted in Cartesian coordinates.

TEXT FIGURE 3-3: Tension specimen after necking.

CORRELATION OF STATE OF STRESS WITH TEST DATA

For design, we need to relate the expected state of stress in a part to the actual state of stress and thus, the material strength, as determined through the tensile test. We accomplish this by applying principal stresses since they characterize a state of stress independent of the original coordinate system.
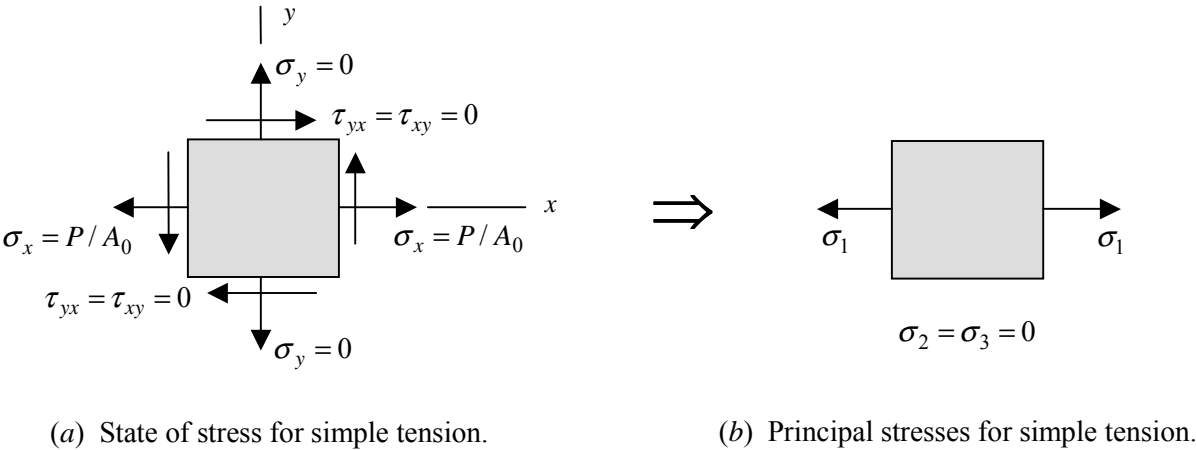

(*a*) State of stress for simple tension.

(*b*) Principal stresses for simple tension.

FIGURE 6A-1: Correlation of state of stress with principal stresses for simple tension.

Since the tensile test generates a uniaxial state of stress, the principal stresses can be defined as,

$$\sigma_1 = \sigma_{axial} = \frac{P}{A_0} \quad \text{and} \quad \sigma_2 = \sigma_3 = 0$$

When plotted on a Mohr's circle diagram, these stress values translate into what looks like a single circle passing through the origin where σ_2 is coincident with σ_3. Actually, there are still three circles on the Mohr's circle diagram. Two circles, defined by principal stresses (σ_1, σ_2) and (σ_1, σ_3), are drawn on top of each other. The third circle degenerates to a point defined by principal stresses (σ_2, σ_3).
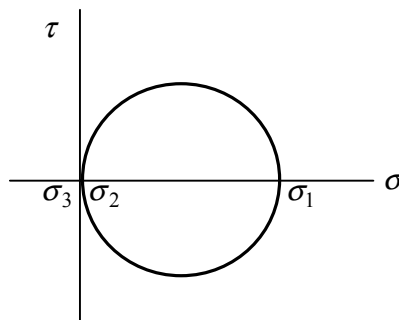

FIGURE 6A-2: Mohr's circle for simple tension.

DEVELOPMENT OF STATIC FAILURE THEORIES

Design for static loading dictates that all loading variables remain constant:

1. Magnitude of load is constant;
2. Direction of load is constant;
3. Point of application of the load is fixed.

These conditions, in conjunction with criteria specific to ductile and brittle materials, have been used in the development of the static failure theories outlined earlier.

Characteristics of Ductile Materials

1. The strain at failure is, $\varepsilon_f \geq 0.05$, or percent elongation greater than five percent.
2. Ductile materials typically have a well defined yield point. The value of the stress at the yield point defines the yield strength, S_y.
3. For typical ductile materials, the yield strength has approximately the same value for tensile and compressive loading $(S_{yt} \approx S_{yc} = S_y)$.

4. A single tensile test is sufficient to characterize the material behavior of a ductile material, S_y and S_{ut}.

Characteristics of Brittle Materials

1. The strain at failure is, $\varepsilon_f \leq 0.05$ or percent elongation less than five percent.
2. Brittle materials *do not* exhibit an identifiable yield point; rather, they fail by brittle fracture. The value of the largest stress in tension and compression defines the ultimate strength, S_{ut} and S_{uc} respectively.
3. The compressive strength of a typical brittle material is significantly higher than its tensile strength, $(S_{uc} \gg S_{ut})$.
4. Two material tests, a tensile test and a compressive test, are required to characterize the material behavior of a brittle material, S_{ut} and S_{uc}.

SUMMARY

This tutorial has attempted to provide a focused introduction to the development of static failure theory by summarizing the theories associated with specific material classifications. In addition, the experimental and analytical models, which have been employed historically to relate the experimental data to strength quantities used for static design, are presented. Subsequent tutorials, *Static Failure of Ductile Materials* and *Static Failure of Brittle Materials*, will respectively provide detailed reviews and examples, respectively, of the failure theories associated with ductile and brittle materials.