Complementary Events

Another important concept in probability theory is that of *complementary events.* When a die is rolled, for instance, the sample space consists of the outcomes 1, 2, 3, 4, 5, and 6. The event E of getting odd numbers consists of the outcomes 1, 3, and 5. The event of not getting an odd number is called the *complement* of event E, and it consists of the outcomes 2, 4, and 6.

The **complement of an event** E is the set of outcomes in the sample space that are not included in the outcomes of event E. The complement of E is denoted by $\bar{E}$ (read "E bar").

Example 4–10 further illustrates the concept of complementary events.

Example 4–10

Find the complement of each event.

a. Rolling a die and getting a 4
b. Selecting a letter of the alphabet and getting a vowel
c. Selecting a month and getting a month that begins with a J
d. Selecting a day of the week and getting a weekday

Solution

a. Getting a 1, 2, 3, 5, or 6
b. Getting a consonant (assume *y* is a consonant)
c. Getting February, March, April, May, August, September, October, November, or December
d. Getting Saturday or Sunday

The outcomes of an event and the outcomes of the complement make up the entire sample space. For example, if two coins are tossed, the sample space is HH, HT, TH, and TT. The complement of "getting all heads" is not "getting all tails," since the event "all heads" is HH, and the complement of HH is HT, TH, and TT. Hence, the complement of the event "all heads" is the event "getting at least one tail."

Since the event and its complement make up the entire sample space, it follows that the sum of the probability of the event and the probability of its complement will equal 1. That is, $P(E) + P(\bar{E}) = 1$. In Example 4–10, let E = all heads, or HH, and let $\bar{E}$ = at least one tail, or HT, TH, TT. Then $P(E) = \frac{1}{4}$ and $P(\bar{E}) = \frac{3}{4}$; hence, $P(E) + P(\bar{E}) = \frac{1}{4} + \frac{3}{4} = 1$.

The rule for complementary events can be stated algebraically in three ways.

Rule for Complementary Events

$$P(\bar{E}) = 1 - P(E) \quad \text{or} \quad P(E) = 1 - P(\bar{E}) \quad \text{or} \quad P(E) + P(\bar{E}) = 1$$

Stated in words, the rule is: *If the probability of an event or the probability of its complement is known, then the other can be found by subtracting the probability from 1.* This rule is important in probability theory because at times the best solution to a problem is to find the probability of the complement of an event and then subtract from 1 to get the probability of the event itself.